Preface

The cooperation group "Discrete and Continuous Methods in the Theory of Networks" was granted financial support by

ZiF

Zentrum für interdisziplinäre Forschung (Centre for Interdisciplinary Research) at Bielefeld University in the summer of 2012 and has begun its investigations in the spring of 2013.

When originally designing this programme, our goal was twofold: on the one hand, we wanted to investigate the possibility of formalising questions from applied sciences—including, but not limited to, theoretical physics and biology—in a graph- or network-based language, making them feasible by mathematical methods. On the other hand, we wanted to bridge the distance between the analysis of difference operators on graphs and differential operators on metric graphs, which 10 years ago used to be studied by different and mostly disconnected communities active in the areas of spectral graph theory, potential theory, spectral geometry, Dirichlet form theory, stochastic analysis, semigroup theory, or dynamical systems.

In the framework of this cooperation group, we have organised a series of short, regular meetings spread all over the 5 years our group was supported. This format has allowed us to work intensively, concentrating meeting-wise on a specific subject; to elaborate our thoughts over time, multiplying our output and adapting our topics to the newest circumstances of research in the fields related to network science; to rebuild our team of collaborators according to our previous experiences, especially promoting productive attitude, interest in/ability at interactions with other group members, intellectual curiosity, promising ideas. We—three mathematicians—have been on our mettle to adapt and respect different scientific cultures concerning diverse issues like speed of publication, hierarchy of research work by graduate students, readiness for idea exchanges before publications, understanding of what kind of advances are worth a publication, and what the main content of an article

should be (Simply a proof? A new experiment? A new phenomenon observed heuristically?).

Throughout the years, we have invited to attend our meetings at the **ZiF** physicists, mathematicians, biologists, neuroscientists, theoreticians of complex networks, a linguist, a logistician, a computer scientist. Reflecting the interdisciplinary mission and the agreeable lack of output pressure of **ZiF**, these open discourses have provided the group with new challenges. Many scientific discussions were led within our group, which highly profited from the stimulating atmosphere at **ZiF**: the activity of dozens of invited scientists resulted in over a hundred published articles. The current volume collects a selection of interesting, original contributions based on scientific questions aroused during the workshops or on topics presented at the conclusive conference held in Bielefeld in December 2017.

In spite of the interdisciplinary nature of our past activities, when planning this volume we have decided to rather focus on our programme's mathematical core: we believe that our knowledge and skills are the best we can contribute to other scientific fields as well as to other mathematical areas.

Selecting papers for this publication was also a way to reflect the current status of the theory of networks and its manifold applications as well as to indicate research directions which we consider most interesting and promising in the medium term. Especially, the last decade saw the birth and the development of investigations on spectral estimates for quantum graphs, much along the lines of classical spectral geometry for manifolds; it is not immodest to say that our programme at **ZiF** was partially responsible for the outburst of this topic. We are glad to present, among others, five papers from this ever-growing area. Also, the dichotomy between graphs and metric graphs has been surpassed and the distance between the theories of finite and infinite graphs is being progressively bridged.

The topics of the papers in this volume can be roughly clustered as follows:

- **Spectral quantum graph geometry**: papers by J.B. Kennedy; by J. Rohleder and C. Seifert; by N. Nicolussi;
- **Spectral theory of discrete graphs**: papers by H. Ge, B. Hua, and A. Lin; by S. Liu, N. Peyerimhoff, and A. Vdovina;
- **Quantum graphs as chaotic systems**: papers by M. Ławniczak, M. Białous, V. Yunko, S. Bauch, and L. Sirko; by H.A. Weidenmüller;
- **Approximations of quantum graphs**: paper by C. Cacciapuoti;
- **Zeta functions for graphs**: paper by A. Karlsson;
- **Few-body systems on metric graphs**: papers by J. Bolte and J. Kerner; by S. Egger;
- **Non-linear differential equations on metric graphs**: papers by S. Dovetta and L. Tentarelli; by J. von Below and J.A. Lubary;
- **Complex networks**: papers by M.-T. Hütt and A. Lesne; by K. Taglieber and U. Freiberg.
- **Applications of networks**: papers by S. Bonaccorsi and S. Turri (epidemics); by M.T. Fairhurst (social networks).

Most of the contributions have direct implications in the study of fundamental problems in natural sciences (close to applications), while also more traditional topics often contain radical ideas opening new perspectives in research.

ZiF's generous support to our research may have ended, but this does certainly not imply the end of our scientific collaborations. We continue to work on the related subjects, keeping contacts established during our meetings at **ZiF**.

This is also the right place to thank all our invitees of the last years, without whom our project could have never be so successful: Riccardo Adami, Felix Ali Mehmeti, Lior Alon, Patricia Alonso Ruiz, Ramy Badr, Rami Band, Moritz Beber, Till Becker, Joachim von Below, Gregory Berkolaiko, Ginestra Bianconi, Türker Biyikoğlu, Jens Bolte, Stefano Bonaccorsi, Jonathan Breuer, Claudio Cacciapuoti, Radu C. Cascaval, Vsevolod Chernyshev, Andrea Corli, Taskin Deniz, Simone Dovetta, Sebastian Egger, Merle Fairhurst, Mareike Fischer, Uta Freiberg, Christoph Fretter, Júlia Gallinaro, Nebojša Gašparović, Federica Gregorio, Jiao Gu, Michael Hinz, Matthias Hofmann, Danijela Horak, Bobo Hua, Marc-Thorsten Hütt, Dina Irofti, Patrick Joly, Jürgen Jost, Stojan Jovanović, Maryna Kachanovska, Cristopher Kaiser-Bunbury, Michael Kaplin, Anders Karlsson, Moritz Kaßmann, Matthias Keller, James Kennedy, Joachim Kerner, Kosmas Kosmidis, Aleksey Kostenko, Maria Kozlova, Marjeta Kramar Fijavž, Hafida Laasri, Francisco Lacerda, Fereshteh Lagzi, Michał Ławniczak, Corentin Léna, Annick Lesne, Jiří Lipovský, Shiping Liu, Wenlian Lu, Alexander Lück, Yuri Maistrenko, Gabriela Malenová, Claudio Marchi, Benjamin Mauroy, Bojan Mohar, Fumito Mori, Jacob Muller, Anna Muranova, Serge Nicaise, Noema Nicolussi, Diego Noja, Philipp-Jens Ostermeier, Gábor Pete, Mats-Erik Pistol, Matteo Polettini, Mason A. Porter, Olaf Post, Jonathan Rohleder, Stefan Rotter, Mostafa Sabri, Sadrah Sadeh, Ruben Sanchez Garcia, Holger Schanz, Jonathan Schiefer, Marcel Schmidt, Michael Schwarz, Andrea Serio, Leszek Sirko, Uzy Smilansky, Adrian Spener, Rune Suhr, Klemens Taglieber, Lorenzo Tentarelli, Christiane Tretter, Françoise Truc, Konstantinous Tsougkas, Francesco Tudisco, Hande Tunçel Gölpek, Stephen J. Watson, Hans Arwed Weidenmüller, Melchior Wirth, Wolfgang Woess, and Verena Wolf.

Last but not least, we would like to express our warmest thanks to the whole staff of the **ZiF** and in particular to the executive secretary Dr Britta Padberg, who throughout these years provided us with an exceptional and highly stimulating environment for research and exchange of ideas and skills—and did so in a competent, interested, and hearty way; to the research secretary Ms Mo Tschache and to the academic coordinator Dr Marc Schalenberg, who constantly supported us with the planning and the everyday management of our workshops; and to Ms Trixi Valentin and Ms Marina Hoffmann of the conference office for their invaluable assistance as the conference was organised and then carried out.

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