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Race meets bargaining in product development

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We study product development in a firm, utilizing a multistage contest model (i.e., *race*) with an endogenous length (with one stage or two stages) between two workers. We model the payments to workers using the normatively appealing Nash bargaining solution. We analytically characterize the equilibrium effort levels of workers and describe the conditions under which a full-fledged final product (as opposed to, say, a prototype) is developed. We also provide an answer to the firm's problem of optimal incentive provision.

JEL CLASSIFICATION

C72; C78; D86; O31; O32

1 | INTRODUCTION

Most new product development or product innovation activities in companies or research centers are undertaken by teams rather than single agents (see Akgün & Lynn, 2002; Levi & Slem, 1995; Sethi, 2000; Sethi, Smith, & Park, 2001). It could be argued that a team, as a whole, has a collective interest yet its members may have competing interests regarding who among them achieves a breakthrough or contributes the most (see Beersma et al., 2003; Kistruck, Lount, Smith, Bergman, & Moss, 2016; Natter, Mild, Feurstein, Dorffner, & Taudes, 2001). Hence, balancing individual and collective incentives in these team environments is a challenge for firms (see Chang, Yeh, & Yeh, 2007; Garbers & Konradt, 2014; Hutchison-Krupat & Chao, 2014; Lazaric & Raybaut, 2014; Nyberg, Maltarich, Abdulsalam, Essman, & Cragun, 2018; Sarin & Mahajan, 2001). Naturally, two important questions arise: (i) to what extent do these incentives influence workers' behavior? and (ii) what are the optimal individual and collective incentives that take the *fairness* aspect into account? To answer such questions, it is of interest to theoretically study these environments with rich models that encompass both competitive and cooperative aspects of the interaction.

In this paper, we model product development in a firm. We describe this process as a potentially multistage contest (i.e., *race*) between two workers assigned to the corresponding product development project.[1] Our model incorporates the payments to workers as an outcome of a cooperative bargaining problem. The *fairness* aspect is introduced by utilizing the normatively appealing Nash bargaining solution (see Nash, 1950).[2] Moreover, the *disagreement point* in the bargaining problem is not exogenously given but rather depends on workers' performances in the race. More precisely, the better-performing worker receives an advantageous bargaining position, a la *Lockean desert*. Finally, in contrast to the *industry standard* in the literature on multistage contest games, our product development race does not have an exogenously fixed length. Instead, two workers need to reach a consensus on the length of the race. A direct consequence is that they may end up building only a prototype (i.e., one-stage race) or a full-fledged final product (i.e., two-stage race) depending on their productivity levels, the values of the prototype and the final product, the incentives provided by the firm, and the team-decision rule. We also study optimal incentive provision by the firm, which in turn influences workers' decisions regarding the race length.

A full description of our results would require going into the fine details of the model, which are relegated to Section 2. Hence, we only briefly mention some of the results here without providing any technical details. First, whether the final product *should* be developed depends on the relative values of the prototype and the final product as well as the workers' productivities in a natural way. Second, whether the final product *will* be developed in equilibrium depends not only on the same parameters but also on the incentive scheme and the team-decision rule determined by the firm management.

Our analyses of comparative statics and optimal incentive provision give potentially useful insights about the management of product innovation teams in real life. First, the values of the prototype and the final product must be determined as precisely as possible and clearly shared with the innovation team to avoid suboptimal choices in the

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innovation stage. In other words, the asymmetric information between the firm management and the innovation team regarding the precisely calculated values must be minimized. Second, the firm management must pay special attention to the decision-making processes within teams and the productivity characteristics of team members (e.g., in the team formation phase) to implement the first-best scenario, because there are situations where some team compositions and decision rules lead to a suboptimal outcome for the firm if the difference between the values of the prototype and the final product is not too large. Finally, the bonus schemes (i.e., incentives to encourage making breakthroughs and sharing information) must be properly balanced, because some incentive schemes may encourage one agent while discouraging the other one, which may lead to a suboptimal outcome for the firm.

Our paper makes a contribution to the theoretical investigation of dynamic contests in applied settings. The theoretical literature on contests produced various models of dynamic contests, such as race, tug-of-war, elimination contests, war of attrition, and repeated incumbency fights (see Konrad, 2012 for a review). The innovation contest model we formulate can be classified as a race. Hence, here, we focus on earlier studies that provided a theoretical investigation of a race model. Race, as a form of dynamic contest, was first formally studied in Harris and Vickers (1987), who modeled a research and development competition between two firms. Klumpp and Polborn (2006) modeled political campaign competition in the U.S. presidential elections as a race and derived predictions for the election outcomes that fit several stylized facts. Konrad and Kovenock (2009) studied a two-player, multistage race with intermediate prizes (for winning stage battles), characterized the unique subgame perfect Nash equilibrium, and conducted a rent-dissipation analysis. Sela (2011) analyzed a best-of-three contest, which is a special case of race, and compared its productivity level with that of one-stage all-pay auction. Gelder (2014) introduced a losing penalty to race, showed that it can prevent momentum from building up in favor of the front runner, leading to a last-stand behavior. Doğan, Karagözoğlu, Keskin, and Sağlam (2018) studied a multiplayer version, characterized the unique subgame perfect Nash equilibrium, and investigated how equilibrium changes compared with a two-player race. The main differences of our model from earlier works are as follows: (i) there is an endogenously and strategically determined winning threshold, as agents can choose to leave the game at the end of the first stage; (ii) winning prizes are state-dependent, as they are determined in a cooperative bargaining model that takes the contest outcome as an input; and (iii) jumps from some decision nodes to the last decision node are possible, as the firm management enforces a knowledge transfer from the winner in the first stage to the losing side, which indicates a strong cooperative behavior between the agents. The last assumption is relevant in the sense that it would be in the firm management's interest to make all information regarding the successfully built prototype common knowledge among agents because spending further resources for an already existing prototype would be rather inefficient (see Foss, Husted, & Michailova, 2010; Hu & Randel, 2014).

The organization of the paper is as follows. In Section 2, we introduce the model of product development in a firm as a race with endogenous length. In Section 3, we present our equilibrium analysis, comparative static analyses, and results on optimal incentive provision. In Section 4, we conclude by discussing our modeling assumptions and possible future research questions.

2 | THE MODEL

Consider two agents, denoted by $i \in \{1, 2\}$, working in the same firm and competing in a potentially multistage innovation contest. The first stage can be thought of as developing a prototype or an unfinished product, which still has some market value. The second stage can be thought of as developing a full-fledged final product, which naturally has a higher market value than the unfinished one. Below, we describe the sequence of events in these two stages in detail.

In the first stage, agents compete in a one-shot contest game such that the winner gains an advantageous position. One possible interpretation is that the winner of the first stage contest achieves a breakthrough that leads to a prototype for a new product. In this contest, each agent chooses an effort level, denoted by x for Agent 1 and y for Agent 2, and the winner is determined by a standard Tullock contest success function (see Tullock, 1980) where

$$p_1 = \frac{x}{x+y} \qquad \text{and} \qquad p_2 = \frac{y}{x+y}$$

are the corresponding winning probabilities for Agents 1 and 2, respectively.[3] As usual, contest efforts are assumed to be costly and irreversible: $C_i(e) = c_i e$ for each agent $i \in \{1, 2\}$ and effort level $e \in [0, \infty)$, where $c_i > 0$ denotes the constant marginal cost of effort for the respective player. The total compensation for workers' efforts in the development of the prototype is $V_p \geq 0$, and it will be allocated by the firm using the Nash bargaining solution. Noting that a higher disagreement payoff is a source of bargaining power in the Nash bargaining solution, in our model, the winner of the first stage contest is given a more advantageous disagreement point, $d_r \geq 0$, whereas the loser's disagreement point is normalized to 0.[4]

The game does not necessarily end after the first stage: agents have an option to continue working on the project to develop the final version of the product. In other words, our innovation contest model is one with an *endogenous length*. Each agent individually decides whether to stop or continue, and *the unanimity rule* determines whether the team will or not. In Section 3, we consider both cases for this rule separately: "both agents should agree to stop, otherwise they will continue" or "both agents should agree to continue, otherwise they will stop."

If agents jointly decide to continue, they forfeit their rights of earning V_p and proceed to the second stage in which there is another one-shot contest game. If that happens, it is assumed that all information regarding the successfully built prototype is shared with the losing agent in the first stage. This is the point at which cooperation between the two agents is emphasized.[5] In the second stage, agents collect a total earning of $V_f \geq V_p$ (again, to be allocated by the firm), capturing the fact that the full-fledged final product has a value greater than the prototype's. The contest structure and the allocation rule are assumed to be the same. Similarly, the winner in the second stage obtains an advantageous disagreement point, $d_r \geq 0$, whereas the loser's disagreement point is normalized to 0. However, now the winner of the first stage contest is additionally given an advantage in return for the prototype information shared, $d_s \in [0, d_r]$.

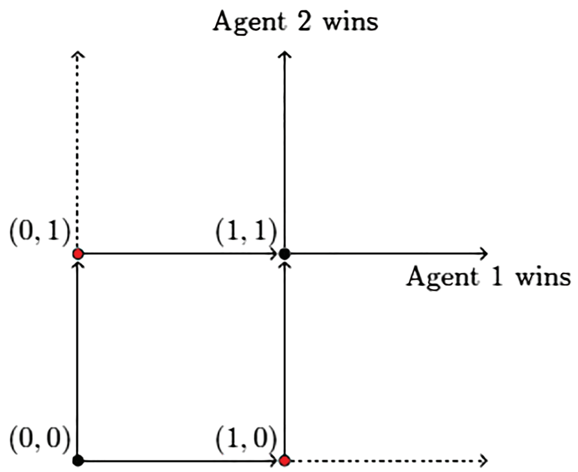

FIGURE 1 Product development race

In line with the Nash's axiomatic model of bargaining, we assume that $d_r \le V_p$ and $d_r + d_s \le V_f$. These imply that the possible disagreement points are elements of the bargaining set. Note that the formal definition of the Nash bargaining solution will be provided in Section 3.1 below.

Figure 1 depicts a *modified* version of the standard illustration of a race model as used in the contest theory literature. The modifications are made to capture our modeling assumptions. Here, a node represents the number of battle victories each agent has, for instance, at node (m, n), Agent 1 has m victories and Agent 2 has n victories. Two battle victories collected by an agent would move the game to a terminal node in which the agent wins the race.

3 | THE RESULTS

This section is divided into three subsections. In the first part, we present the equilibrium analysis of the extensive form game formulated in Section 2. In the second part, we conduct comparative static analyses on $V_f - V_p$, d_r, d_s, c_1, and c_2. Finally, in the third part, we investigate optimal incentives that must be provided to workers by the firm management.

3.1 | Equilibrium analysis

Our product development race is a finite-horizon game with complete information. Hence, we can use backward induction to find the subgame perfect Nash equilibrium of the game. Along those lines, we first analyze a generic Nash bargaining problem. Suppose that two agents with linear utility functions share a pie of size $V > 0$ with disagreement points $d_1, d_2 \ge 0$ satisfying $d_1 + d_2 \le V$. The standard Nash bargaining solution (see Nash, 1950), which is given by

$$\arg\max_{a_1, a_2 \in [0, V]} (a_1 - d_1)(a_2 - d_2)$$

$$\text{subject to } a_1 + a_2 = V,$$

yields the following payoffs:

$$a_1^* = \frac{V + d_1 - d_2}{2} \qquad \text{and} \qquad a_2^* = \frac{V + d_2 - d_1}{2}.$$

Now, utilizing this generic solution, we apply backward induction below. Consider node (1, 1), which can only be reached if the team decided to continue developing the product. Without loss of generality, assume that it was Agent 1 who managed to develop the prototype in the first stage. This means that the race proceeded from node (1, 0) to node (1, 1). Recall that this brings an additional term to the disagreement point of Agent 1, who then aims to maximize

$$U_{1,0}^{c,1} = \frac{x_{1,0}}{x_{1,0} + y_{1,0}} \frac{V_f + d_r + d_s}{2} + \frac{y_{1,0}}{x_{1,0} + y_{1,0}} \frac{V_f - d_r + d_s}{2} - c_1 x_{1,0},$$

where $x_{1,0}$ and $y_{1,0}$ denote the respective efforts exerted by the agents and the superscript c indicates that agents *continued* to Stage 2. Similarly, Agent 2 aims to maximize

$$U_{1,0}^{c,2} = \frac{x_{1,0}}{x_{1,0} + y_{1,0}} \frac{V_f - d_r - d_s}{2} + \frac{y_{1,0}}{x_{1,0} + y_{1,0}} \frac{V_f + d_r - d_s}{2} - c_2 y_{1,0}.$$

After taking the first-order conditions, $\frac{\partial U_{1,0}^{c,1}}{\partial x_{1,0}} = 0$ and $\frac{\partial U_{1,0}^{c,2}}{\partial y_{1,0}} = 0$, the equilibrium analysis yields

$$x_{1,0}^* = \frac{c_2 d_r}{(c_1 + c_2)^2} \qquad \text{and} \qquad y_{1,0}^* = \frac{c_1 d_r}{(c_1 + c_2)^2},$$

with the equilibrium expected payoffs

$$EU_{1,0}^{c,1} = \frac{V_f - d_r + d_s}{2} + \frac{{c_2}^2 d_r}{(c_1 + c_2)^2} \text{ and } EU_{1,0}^{c,2} = \frac{V_f - d_r - d_s}{2} + \frac{{c_1}^2 d_r}{(c_1 + c_2)^2}. \tag{1}$$

On the other hand, if the team decided to stop at (1, 0), so that the game does not proceed to node (1, 1), they immediately collect their payoffs

$$EU_{1,0}^{s,1} = \frac{V_p + d_r}{2} \text{ and } EU_{1,0}^{s,2} = \frac{V_p - d_r}{2}, \tag{2}$$

where the superscript s indicates that agents *stopped* after Stage 1.

Accordingly, we can write that Agent 1 prefers to continue if

$$V_f - V_p > 2d_r - d_s - \frac{2{c_2}^2 d_r}{(c_1 + c_2)^2}, \tag{3}$$

whereas Agent 2 prefers to continue if

$$V_f - V_p > d_s - \frac{2{c_1}^2 d_r}{(c_1 + c_2)^2}. \tag{4}$$

It is then easy to see that for sufficiently high values of $V_f - V_p$, both agents prefer to continue; for sufficiently low values of $V_f - V_p$, both agents prefer to stop; and for intermediate values of $V_f - V_p$, either Agent 1 or Agent 2 prefers to continue, and in such a case, how the unanimity rule is applied would be critical in determining the outcome.

Notice that if it was Agent 2 who managed to develop the prototype in the first stage, so that the race follows the path of nodes $(0, 0) \to (0, 1) \to (1, 1)$, the equilibrium strategies and expected payoffs for both agents can symmetrically be written. Accordingly, we can write that Agent 1 prefers to continue if

$$V_f - V_p > d_s - \frac{2{c_2}^2 d_r}{(c_1 + c_2)^2}, \tag{5}$$

whereas Agent 2 prefers to continue if

$$V_f - V_p > 2d_r - d_s - \frac{2{c_1}^2 d_r}{(c_1 + c_2)^2}. \tag{6}$$

This completes the analysis of the second stage contest. For the one-shot contest game in the first stage, backward induction dictates that we should consider four possibilities: (i) agents would continue no matter who wins (*cc*), (ii) agents would continue if Agent 1 wins but stop if Agent 2 wins (*cs*), (iii) agents would stop if Agent 1 wins but continue if Agent 2 wins (*sc*), and (iv) agents would stop no matter who wins (*ss*).

We analyze the equilibrium strategies case by case. In case (i), agents anticipate that they will jointly decide to proceed to the second stage independent of who wins in the first stage. Then, at node $(0,0)$, Agent 1 aims to maximize

$$U^{cc,1}_{0,0} = \frac{x^{cc}_{0,0}}{x^{cc}_{0,0} + y^{cc}_{0,0}} EU^{c,1}_{1,0} + \frac{y^{cc}_{0,0}}{x^{cc}_{0,0} + y^{cc}_{0,0}} EU^{c,1}_{0,1} - c_1 x^{cc}_{0,0},$$

where $x^{cc}_{0,0}$ and $y^{cc}_{0,0}$ denote the respective efforts exerted by the agents in this particular case. Similarly, Agent 2 aims to maximize

$$U^{cc,2}_{0,0} = \frac{x^{cc}_{0,0}}{x^{cc}_{0,0} + y^{cc}_{0,0}} EU^{c,2}_{1,0} + \frac{y^{cc}_{0,0}}{x^{cc}_{0,0} + y^{cc}_{0,0}} EU^{c,2}_{0,1} - c_2 y^{cc}_{0,0}.$$

The equilibrium analysis yields

$$x^{cc*}_{0,0} = \frac{c_2 d_s}{(c_1+c_2)^2} \text{ and } y^{cc*}_{0,0} = \frac{c_1 d_s}{(c_1+c_2)^2},$$

with the equilibrium expected payoffs

$$\begin{aligned} EU^{cc,1}_{0,0} &= \frac{V_f - d_r - d_s}{2} + \frac{{c_2}^2(d_r + d_s)}{(c_1+c_2)^2} \text{ and} \\ EU^{cc,2}_{0,0} &= \frac{V_f - d_r - d_s}{2} + \frac{{c_1}^2(d_r + d_s)}{(c_1+c_2)^2}. \end{aligned} \tag{7}$$

In case (ii), agents anticipate that they will proceed to the second stage if Agent 1 wins in the first stage and they will stop if Agent 2 wins in the first stage. At node $(0,0)$, Agent 1 aims to maximize

$$U^{cs,1}_{0,0} = \frac{x^{cs}_{0,0}}{x^{cs}_{0,0} + y^{cs}_{0,0}} EU^{c,1}_{1,0} + \frac{y^{cs}_{0,0}}{x^{cs}_{0,0} + y^{cs}_{0,0}} EU^{s,1}_{0,1} - c_1 x^{cs}_{0,0},$$

where $x^{cs}_{0,0}$ and $y^{cs}_{0,0}$ denote the respective efforts exerted by the agents in this particular case. Similarly, Agent 2 aims to maximize

$$U^{cs,2}_{0,0} = \frac{x^{cs}_{0,0}}{x^{cs}_{0,0} + y^{cs}_{0,0}} EU^{c,2}_{1,0} + \frac{y^{cs}_{0,0}}{x^{cs}_{0,0} + y^{cs}_{0,0}} EU^{s,2}_{0,1} - c_2 y^{cs}_{0,0}.$$

The equilibrium analysis yields

$$x^{cs*}_{0,0} = \frac{c_2\left((c_1+c_2)^2(V_f - V_p - d_s) - (2{c_2}^2 + 4c_1c_2)d_r\right)\left((c_1+c_2)^2(V_f - V_p + d_s) - 2{c_2}^2 d_r\right)^2}{2(c_1+c_2)^2\left((c_1+c_2)({c_2}^2(V_f - V_p + 2d_r + d_s) - {c_1}^2(V_f - V_p - d_s)) + 2c_1c_2(c_2 d_s + c_1(2d_r + d_s))\right)^2},$$

$$y^{cs*}_{0,0} = \frac{c_1\left((c_1+c_2)^2(V_f - V_p - d_s) - (2{c_2}^2 + 4c_1c_2)d_r\right)^2\left((c_1+c_2)^2(V_f - V_p + d_s) - 2{c_2}^2 d_r\right)}{2(c_1+c_2)^2\left((c_1+c_2)({c_2}^2(V_f - V_p + 2d_r + d_s) - {c_1}^2(V_f - V_p - d_s)) + 2c_1c_2(c_2 d_s + c_1(2d_r + d_s))\right)^2},$$

The equilibrium expected payoffs can be calculated by plugging these equilibrium strategies into the expected payoff functions $U^{cs,1}_{0,0}$ and $U^{cs,2}_{0,0}$ above.

Noting that case (iii) is symmetric to case (ii), here, we omit the respective equilibrium analysis. Finally, in case (iv), agents anticipate that they will jointly decide to stop after developing the prototype independent of who wins in the first stage. Then, at node $(0,0)$, Agent 1 aims to maximize

$$U^{ss,1}_{0,0} = \frac{x^{ss}_{0,0}}{x^{ss}_{0,0} + y^{ss}_{0,0}} EU^{s,1}_{1,0} + \frac{y^{ss}_{0,0}}{x^{ss}_{0,0} + y^{ss}_{0,0}} EU^{s,1}_{0,1} - c_1 x^{ss}_{0,0},$$

where $x^{ss}_{0,0}$ and $y^{ss}_{0,0}$ denote the respective efforts exerted by the agents in this particular case. Similarly, Agent 2 aims to maximize

$$U^{ss,2}_{0,0} = \frac{x^{ss}_{0,0}}{x^{s}_{0,0} + y^{ss}_{0,0}} EU^{s,2}_{1,0} + \frac{y^{ss}_{0,0}}{x^{s}_{0,0} + y^{ss}_{0,0}} EU^{s,2}_{0,1} - c_2 y^{ss}_{0,0}.$$

The equilibrium analysis yields

$$x^{ss*}_{0,0} = \frac{c_2 d_r}{(c_1+c_2)^2} \text{ and } y^{ss*}_{0,0} = \frac{c_1 d_r}{(c_1+c_2)^2},$$

with the equilibrium expected payoffs

$$EU^{ss,1}_{0,0} = \frac{V_p - d_r}{2} + \frac{{c_2}^2 d_r}{(c_1+c_2)^2} \text{ and } EU^{ss,2}_{0,0} = \frac{V_p - d_r}{2} + \frac{{c_1}^2 d_r}{(c_1+c_2)^2}. \tag{8}$$

This completes the equilibrium analysis.

3.2 | Comparative statics

Here, we conduct comparative static analyses on various model parameters. We mainly concentrate on inequalities (3) and (4) reported for the stop or continue decision after Agent 1 wins in the first stage. Those inequalities indicate that Agent 1 prefers to continue if

$$V_f - V_p > 2d_r - d_s - \frac{2{c_2}^2 d_r}{(c_1+c_2)^2}$$

and that Agent 2 prefers to continue if

$$V_f - V_p > d_s - \frac{2{c_1}^2 d_r}{(c_1+c_2)^2}.$$

The results are summarized in Figures 2 and 3. Also, note that the comparative statics for the inequalities in case Agent 2 wins in the first stage would similarly follow.

(a) **Changes in $V_f - V_p$:** It is clear that an increase in $V_f - V_p$ enlarges the parameter space where the second stage is reached and the final product is developed in equilibrium. For intermediate values of $V_f - V_p$, one agent prefers to continue and the other agent prefers to stop, and it would be up to the team-decision rule to determine whether the second stage is reached. On the other hand, if $V_f - V_p$ decreases to a sufficiently low level, then both agents would want to stop at the end of Stage 1. This is economically intuitive, because a higher difference between the total earnings would make it more worthwhile for both agents to continue.

(b) **Changes in d_r or d_s:** In inequality (3), the sign of the coefficient of d_r is positive and the sign of d_s is negative. The situation is the converse in inequality (4). Therefore, an increase in d_r or a decrease in d_s enlarges the set of parameter values under which Agent 1 prefers to stop after the first stage and Agent 2 prefers to continue to the second stage. This is economically intuitive, because such changes in these parameters would increase the importance of winning the first stage contest in case of stopping and decrease the importance of the same in case of continuing to Stage 2. As a result, it would be better for the contest winner (which is Agent 1, for the considered inequalities) to stop and for the contest loser (which is Agent 2) to continue.

(c) **Changes in c_1 or c_2:** Taking the derivatives of the right-hand sides of inequalities (3) and (4) with respect to the cost parameters, we see that in the corresponding inequality for agent $i \in \{1,2\}$,

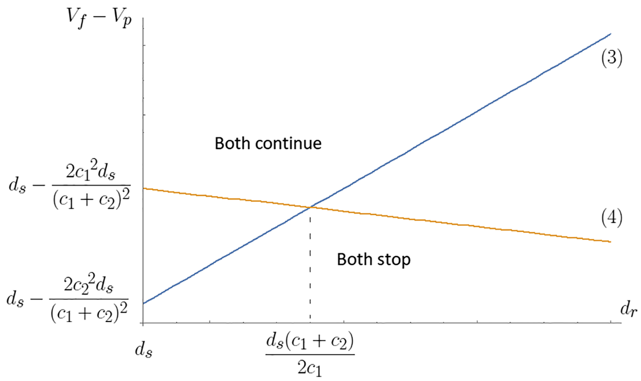

FIGURE 2 The regions for unanimous decisions with respect to changes in d_r (assuming $c_1 < c_2$)

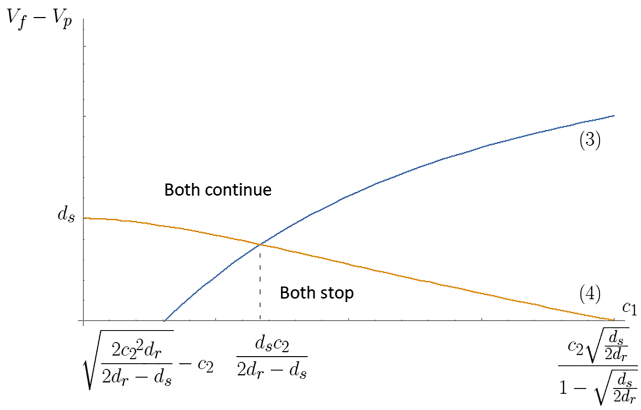

FIGURE 3 The regions for unanimous decisions with respect to changes in c_1 (assuming $c_1 < c_2$)

an increase in c_i leads to an increase while an increase in c_j leads to a decrease in the expression.[6] Accordingly, as c_i increases, the set of parameters under which agent i prefers to stop after Stage 1 would be enlarged. The same is true when c_j decreases. This is economically intuitive, because a higher marginal cost parameter for an agent results in a lower expected return form the contest, so that the agent would opt out of the contest if possible.

3.3 | Optimal incentive provision

In this section, we study an optimal design problem from the perspective of the firm management who employs the two agents in product development. Assume now that contest efforts are *productive* such that the first stage efforts either influence the revenue generated by the prototype (if the game ends after Stage 1) or the technology to be used in the final version production (if the game proceeds to Stage 2), whereas the second stage efforts only influence the revenue generated by the final version. All revenue is completely collected by the firm management. By specifying three items in the agents' contracts, namely, the (V_f, V_p) pair, the (d_r, d_s) pair, and the unanimity rule, the firm aims to maximize its net earnings, calculated as the total revenue collected minus the total winning prize offered to its workers.

In the following analysis, we assume that the revenue generated by the prototype is $A(e_w^{\mathrm{I}})^\theta$ where $A > 0$ is the technology level, $\theta \in (0,1)$ is the productivity parameter, and e_w^{I} denotes the effort exerted by the winner in the first stage.[7] Also, we assume that the revenue generated by the final version is $A_k(e_w^{\mathrm{II}})^\phi$ for any $k \in \{L, H\}$ where $A_k \geq 0$ is the new technology level, $\phi \in (0,1)$ is the new productivity parameter, and e_w^{II} is the effort exerted by the winner in the second stage. In this context, $A_L = 0$ is normalized to a zero technology level,[8] whereas $A_H > 0$ represents a high technology level that will be achieved if a given effort threshold is reached in the first stage.

Before starting the optimal design analysis, notice that the firm simply cares about the stop or continue decision; hence, cases (ii) and (iii) above, where the agents would continue if one of them wins in the first stage but would stop if the other agent wins, can only be suboptimal. Accordingly, we concentrate on cases (i) and (iv) above. The firm maximizes its revenue in two cases separately: (a) maximization after Stage 1 (corresponding to case (iv)) and (b) maximization after Stage 2 (corresponding to case (i)).

In case (a), given our result in the previous subsection, we know that the equilibrium winning probability for agent $i \in \{1,2\}$ turns out to be $c_j/(c_1+c_2)$. The firm solves the following problem:

$$\max_{V_p \geq d_r} A\left(\frac{c_2}{c_1+c_2}\left(x_{0,0}^{ss*}\right)^\theta + \frac{c_1}{c_1+c_2}\left(y_{0,0}^{ss*}\right)^\theta\right) - V_p$$

$$\max_{V_p \geq d_r} A\left(\frac{{c_1}^{1+\theta}{d_r}^\theta + {c_2}^{1+\theta}{d_r}^\theta}{(c_1+c_2)^{1+2\theta}}\right) - V_p.$$

The former equation utilizes the facts that (i) Agent 1 wins the first stage contest with probability $c_2/(c_1+c_2)$, and because she would exert $x_{0,0}^{ss*}$ amount of effort, the revenue generated in this stage would be $A\left(x_{0,0}^{ss*}\right)^\theta$; (ii) Agent 2 wins the first stage contest with probability $c_1/(c_1+c_2)$, and because she would exert $y_{0,0}^{ss*}$ amount of effort, the revenue generated in this stage would be $A\left(y_{0,0}^{ss*}\right)^\theta$; and (iii) all revenue would be collected by the firm, but the firm is supposed to pay a total of V_p to Agents 1 and 2. Furthermore, the latter is the same problem with $x_{0,0}^{ss*}$ and $y_{0,0}^{ss*}$ being replaced by their values found in Section 3.1 above.

The solution to this problem is trivial: $V_p^* = d_r$. This indicates that the firm chooses an extreme value for the winner's disagreement point, so that it would be as if the Nash solution is not used in the equilibrium. But then, one can further maximize over d_r:

$$\max_{d_r \geq 0} \frac{A\left({c_1}^{1+\theta} + {c_2}^{1+\theta}\right)}{(c_1+c_2)^{1+2\theta}}{d_r}^\theta - d_r. \tag{9}$$

Taking the derivative with respect to d_r and setting it equal to zero yields

$$\frac{A\theta\left({c_1}^{1+\theta} + {c_2}^{1+\theta}\right)}{(c_1+c_2)^{1+2\theta}}{d_r}^{\theta-1} = 1$$

$$d_r^{s*} = \left[\frac{A\theta\left({c_1}^{1+\theta} + {c_2}^{1+\theta}\right)}{(c_1+c_2)^{1+2\theta}}\right]^{\frac{1}{1-\theta}}. \tag{10}$$

To sum up, if the firm prefers the agents to stop after Stage 1, $V_p^* = d_r^{s*}$ is the optimal winning prize that should be offered to the winner in the first stage, while the loser receives nothing.

In case (b), as mentioned earlier, there exists a threshold level for total exerted effort, such that if it is reached, the firm achieves a high technology of production, $A_H > 0$; but if not, it ends up with a zero technology level, $A_L = 0$. Given that the equilibrium effort for each agent $i \in \{1,2\}$ is given by

$$\frac{c_j d_s}{(c_1+c_2)^2},$$

so that the total equilibrium effort is $d_s/(c_1+c_2)$, let $d_{crit}>0$ be the respective critical level for d_s, which returns the total effort threshold in the equilibrium.

Then, the firm solves the following problem:

$$\max_{V_f \geq d_r+d_s} A_k \left[\frac{c_2}{c_1+c_2}\left(\frac{c_2}{c_1+c_2}\left(x^*_{1,0}\right)^{\phi}+\frac{c_1}{c_1+c_2}\left(y^*_{1,0}\right)^{\phi}\right) + \frac{c_1}{c_1+c_2}\left(\frac{c_2}{c_1+c_2}\left(x^*_{0,1}\right)^{\phi}+\frac{c_1}{c_1+c_2}\left(y^*_{0,1}\right)^{\phi}\right)\right] - V_f,$$

for any $k \in \{L, H\}$. This problem is similarly formulated. However, now, the winning probabilities in both stages are taken into account. In particular, Agent 1 wins the first stage contest with probability $c_2/(c_1+c_2)$, which moves the race to node (1, 0). Then, in the second stage, Agent 1 wins the contest with probability $c_2/(c_1+c_2)$, and because she would exert $x^*_{1,0}$ amount of effort, the revenue generated in this stage would be $A_k\left(x^*_{1,0}\right)^{\phi}$. If Agent 2 wins the second stage contest instead, which occurs with probability $c_1/(c_1+c_2)$, the generated revenue would be $A_k\left(y^*_{1,0}\right)^{\phi}$. Moreover, in case it is Agent 2 who wins the first stage contest, $x^*_{0,1}$ and $y^*_{0,1}$ should be used in the calculation of the generated revenues, respectively. Similar to above, all revenue would be collected by the firm, but the firm is supposed to pay a total of V_f to Agents 1 and 2.

Because, by symmetry, the terms in the parentheses are equal to each other, the maximization problem reduces to

$$\max_{V_f \geq d_r+d_s} A_k \left(\frac{c_2}{c_1+c_2}\left(x^*_{1,0}\right)^{\phi}+\frac{c_1}{c_1+c_2}\left(y^*_{1,0}\right)^{\phi}\right) - V_f$$

$$\max_{V_f \geq d_r+d_s} A_k \left(\frac{{c_1}^{1+\theta}{d_r}^{\phi}+{c_2}^{1+\theta}{d_r}^{\phi}}{(c_1+c_2)^{1+2\phi}}\right) - V_f.$$

This is very similar to the problem in case (a). We know that the solution is trivial: $V_f^*=d_r+d_s$. But then one can further maximize over d_r and d_s:

$$\max_{d_r \geq 0,\ d_s \geq 0} \frac{A_k\left({c_1}^{1+\phi}+{c_2}^{1+\phi}\right)}{(c_1+c_2)^{1+2\phi}}{d_r}^{\phi} - d_r - d_s. \tag{11}$$

This immediately implies that d_s should take its minimum value. Given that zero technology level can never be optimal, because it would always return a zero revenue in the second stage, we can conclude that the firm prefers $d_s^*=d_{crit}$, aiming to achieve a technology level of A_H. Then, taking the derivative with respect to d_r and setting it equal to zero yields

$$\frac{A_H\phi\left({c_1}^{1+\phi}+{c_2}^{1+\phi}\right)}{(c_1+c_2)^{1+2\phi}}{d_r}^{\phi-1}=1$$

$$d_r^{c*}=\left[\frac{A_H\phi\left({c_1}^{1+\phi}+{c_2}^{1+\phi}\right)}{(c_1+c_2)^{1+2\phi}}\right]^{\frac{1}{1-\phi}}. \tag{12}$$

To sum up, if the firm prefers the agents to continue to Stage 2, $V_f^*=d_r^{c*}+d_{crit}$ should be offered as the total prize in the second stage, while the winner in the first stage guarantees d_{crit} of it.

The optimization analysis above singles out the optimal selections of d_r and d_s values for each possible case: (a) and (b). The final step is to compare the respective net earnings in order to identify which case would be preferred by the firm. Now, let $\mathcal{E}_s^*(A)$ be the maximum level of net earnings the firm collects after Stage 1, which can be calculated by setting $V_p=d_r^{s*}$ and implementing d_r^{s*} in (10) into Equation (9).

[Correction added on 21 December 2020 after first online publication: The citation of equations has been updated in this version.]

Similarly, let $\mathcal{E}_c^*(A_H)$ be the maximum level of net earnings the firm collects after Stage 2 in case of high technology level, which can be calculated by setting $V_f=d_r^{c*}+d_{crit}$ and implementing d_r^{c*} in (12), $d_s^*=d_{crit}$, and $A_k=A_H$ into Equation (11). These values are

$$\mathcal{E}_s^*(A)=\frac{(1-\theta)\left(A\theta(c_1+c_2)^{-1-2\theta}\left(c_1^{1+\theta}+c_2^{1+\theta}\right)\right)^{\frac{1}{1-\theta}}}{\theta} \tag{13}$$

and

$$\mathcal{E}_c^*(A_H)=\frac{(1-\phi)\left(A_H\phi(c_1+c_2)^{-1-2\phi}\left(c_1^{1+\phi}+c_2^{1+\phi}\right)\right)^{\frac{1}{1-\phi}}-\phi d_{crit}}{\phi}. \tag{14}$$

Depending on parameter values, when $\mathcal{E}_s^*(A)>\mathcal{E}_c^*(A_H)$, the firm would choose $V_p=V_f=d_r^{s*}$, $d_r=d_r^{s*}$, and $d_s=0$. If the firm further sets the unanimity rule in such a way that "both agents should agree to continue, otherwise they will stop," one can see that agents would stop, because the winner in the first stage prefers stopping before moving onto the second stage. On the other hand, if $\mathcal{E}_c^*(A_H)>\mathcal{E}_s^*(A)$, then the firm would choose $V_p=d_r^{c*}$, $V_f=d_r^{c*}+d_{crit}$, $d_r=d_r^{c*}$, and $d_s=d_{crit}$. If the firm further sets the unanimity rule in such a way that "both agents should agree to stop, otherwise they will continue," one can see that agents would continue, because the loser in the first stage prefers moving onto the second stage. This results in a high technology level. With these arguments, we have shown that all optimal selections found above are *consistent* with the respective cases, because the team-decision rule can optimally be selected to induce the desired outcome. This completes the analysis of optimal incentive provision.

Returning back to the discussion of competition versus cooperation, recall that our model has a competition aspect in both stages, but cooperation occurs after Stage 1 (in the form of information sharing) only if agents decide to continue to Stage 2. Accordingly, because it is optimal for the firm management to incentivize its workers to continue when Equation (14) turns out to be greater than Equation (13), one can say that the same condition stands for the cases in which the firm chooses to enhance cooperation between these agents. Not surprisingly, this happens when the firm finds innovating the final product worthwhile, for instance, when it is easier to achieve a high technology level (i.e., d_{crit} is sufficiently low) and/or the revenue is generated more efficiently in the second stage is (i.e., A_H is sufficiently high).

4 | CONCLUSION

We formulate a model of product development where (i) two agents in a firm compete to receive a more advantageous share of earnings from a new product, (ii) the competition between agents is modeled as a race with an endogenous length (with one stage or two stages), and (iii) the agents' contest payoffs are determined using the Nash bargaining solution with endogenously determined disagreement points depending on the race outcome. Agents may choose not to proceed to the second stage, which would lead to a semideveloped product (i.e., a prototype) that has a lower market value than the full-fledged final product. We analytically solve for the subgame perfect Nash equilibrium of the corresponding sequential game, describe the conditions under which the final product is developed, conduct comparative static analyses on model parameters, and also analyze the firm management's optimal incentive design problem.

Our paper contributes to the literature on applied industrial organization (more precisely, product development and optimal contract design) as well as to the literature on contest theory. Our contribution to the applied literature is the introduction of a rich and tractable model of product innovation, which allows us to study (i) cooperation and competition in product development teams, (ii) fair and optimal incentives to be given to workers, (iii) knowledge sharing in teams, and (iv) influence of different team-decision rules on product development. Our theoretical innovation is threefold: ours is the first model to study a race with (i) an endogenous length, (ii) state-dependent prizes, and (iii) a cooperative bargaining game embedded in it (i.e., prize determination).

A few words about our modeling assumptions is in order. We model product development process and the corresponding race between agents with two stages (representing a prototype and a final product). We could have had more than two stages. However, that would have complicated the analysis yet not have brought significantly new insights on top of what the current model offers. A similar argument is valid regarding a possible extension to a multi-agent setup. Another possible extension would be about the form of the effort cost function. We expect our qualitative results to hold under a more general form of cost functions (e.g., increasing and convex). Finally, in line with Nash (1950), the source of bargaining power comes from disagreement points in our model. Another way of introducing bargaining power could be by using the asymmetric Nash bargaining solution, but we expect that this way of incorporating bargaining power would likely lead to a less tractable model without bringing significantly new insights.

Our model can be used in studying further questions related to product development teams. For instance, future research may model the demand side in greater detail by introducing (heterogenous) consumer preferences both for the prototype and the final product and then by explicitly solving for their utility maximization problems. Another potentially fruitful venue could be modeling duopolistic (or, more generally, oligopolistic) competition using the framework we introduced here. Finally, the optimal formation of product development teams can be studied to answer questions such as "Should teams be formed with similar or different agents?"

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NOTES

1 Throughout the paper, we use the words *agents* and *workers* interchangeably.

2 Fairness and the Nash bargaining solution are almost synonymously used in empirical and applied work due to the appealing normative properties of this solution. We follow that tradition here and argue that the Nash bargaining solution guarantees that the division of contest prizes is made in a *fair* fashion.

3 Note that once agents exert efforts in a product development contest, then the corresponding product is developed for sure. In reality, whether a product is developed or not may be a stochastic function of efforts. Because such a stochastic process is not the focus of this paper, here we assume a deterministic outcome for the sake of simplicity.

4 A disagreement point indicates what each agent receives in case of bargaining failure.

5 In practice, firms use various methods to encourage/implement collaboration in teams. The interested reader is referred to Jassawalla and Sashittal (2006), Gratton and Erickson (2007), and Dodgson (2014) for relevant discussions.

6 When we refer to agent $i \in \{1, 2\}$ as a generic player, the other agent is denoted by j.

7 This functional form is chosen for tractability reasons. We expect no qualitative change in our results for other strictly concave revenue functions.

8 This corresponds to the normalization of a case in which the technology for the final product is too low, such that the firm management would prefer to collect whatever revenue they can from the prototype. As it will be more clear after the analysis, ending up with such a technology level for the final product can never be optimal for the firm.

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