Dynamic Games and Applications
https://doi.org/10.1007/s13235-020-00359-8

Bargaining, Reference Points, and Limited Influence

Tarık Kara[1] **· Emin Karagözoğlu**[2,3] **· Elif Özcan-Tok**[4]

Abstract
We study the emergence of reference points in a bilateral, infinite horizon, alternating offers bargaining game. Players' preferences exhibit reference dependence, and their current offers have the potential to influence each other's future reference points. However, this influence is *limited* in that it expires in a finite number of periods. We first construct a subgame perfect equilibrium that involves an immediate agreement and study its properties. Later, we also show the existence of an equilibrium where agreement is reached with delay. We show that expiration lengths and initial reference points play a crucial role for the existence of this equilibrium. For instance, we show that equilibrium with a delayed agreement does not exist when the initial reference point is (0, 0). Finally, we provide comparative static analyses on model parameters, compare two variations of our model, and compare our findings with those of the closest paper to ours, Driesen et al. (Math Soc Sci 64:103–118, 2012).

Keywords Alternating offers · Bargaining games · Delay · Reference-dependent preferences · Recency effect · Reference points · Retrievability

JEL Classification C72 · C78 · D63 · D74

1 Introduction

Motivation and Related Literature A plethora of experimental studies in the last two decades almost unequivocally documented influence of reference points on bargaining behavior and outcomes.[1] One well-known critique of the theories that utilize exogenously given

[1] Ashenfelter and Bloom [2], Blount et al. [7], Kristensen and Gärling [37], Bohnet and Zeckhauser [8], Gächter and Riedl [18,19], Gimpel [20], Bartling and Schmidt [3], Herweg and Schmidt [24], Fehr et al. [16], Karagözoğlu and Riedl [33], Bolton and Karagözoğlu [9] and Karagözoğlu and Kocher [32] are only some of these studies, all of which reported that reference points —in the form of existing contracts, reservation prices, expired contracts, historical contractual conditions, informal agreements, (fairness) norms—significantly influence the whole bargaining process and the negotiated agreement.

✉ Emin Karagözoğlu
karagozoglu@bilkent.edu.tr

1 Bilkent University, Ankara, Turkey

2 Department of Economics, Bilkent University, 06800 Bilkent, Ankara, Turkey

3 CESifo-Munich, Munich, Germany

4 Central Bank of the Republic of Turkey, Ankara, Turkey

Published online: 11 July 2020

reference points is that almost any type of behavior can be explained with an appropriate choice of a reference point. This led many researchers to develop theoretical models where the reference point is endogenously derived (preferably from the observables) and/or elicit agents' reference points with experimental methods to check whether the elicited reference points, which were otherwise unobservable, can explain the observed behavior.[2]

This paper models the emergence of reference points and investigates their influence on bargaining behavior and outcomes in a bilateral, infinite horizon, alternating offers bargaining game [42,46], where players' preferences exhibit reference dependence. We allow past offers to influence players' reference points in later periods (see [12,45] for earlier examples). Players in our model are both gain-seeking and loss-averse. Accordingly, player i ($i = 1, 2$) weights payoffs above his reference point with γ_i and payoffs below his reference point with λ_i, where we assume $\lambda_i \geq \gamma_i$ (i.e., losses loom larger than gains).

A novel element we introduce is *limited influence*: an offer made to player i in the current period has the potential to influence his reference points for the next m_i periods in which he responds to player j's offers. In particular, player i's reference point at period $t > 1$ according to which he evaluates player j's current offer is assumed to be the highest (or the most generous) offer he received in the last m_i periods (in which he received offers). Therefore, in contrast to Driesen et al. [12], which assumed that player i's reference point at any given period $t > 1$ is the highest offer he received until t, we model those bargaining situations where past offers can have only a *limited influence* on the current reference points. In other words, the influence of past offers *expires* in finitely many periods (in our baseline model).

Our model is inspired by the *availability* heuristic or *retrievability* bias in decision making (see [27]) and the *order effect* (or the *recency effect*) in belief updating and intertemporal decision making (see [25]). Availability heuristic refers to a mental shortcut, which reveals itself as a heavy reliance on immediately or easily available information, experience or examples while making a decision. In his best-selling book, *Thinking Fast and Slow*, Daniel Kahneman argues "... recent occurrences are likely to be relatively more available than earlier occurences." Recency effect and order effect are arguably special cases of availability heuristic in intertemporal decision-making contexts since recency and order are good measures of availability or retrievability in such environments. In psychology, proponents of *decay theory* argue that a mere passage of time is sufficient for memory decay, and as such, a piece of information will be less available for retrieval as time passes. There is strong empirical evidence for these heuristics and biases. For instance, Bartos [4] formulated a theoretical model, which implied that the later an offer is made in bargaining, the more it influences players' decisions, and conducted an experiment, results of which provided empirical support for this prediction. DeBondt et al.'s [11] results suggested that even stock market professionals exhibit recency bias. In an influential study, Hogarth and Einhorn (1992) formulated a model of belief adjustment, which incorporates primacy, recency, and order effects, and experimentally found that recency bias is observed more frequently in complicated tasks. In an experiment designed to test the empirical validity of Bayesian updating, Grether [22] observed that subjects gave greater weights to most recently observed evidence. In advertising and marketing, various scholars reported decay in the influence of advertisements as the time between the exposure and the consumption decision increases (see [13,14]). Finally, Malmendier and Nagel [39] empirically identified a clear recency effect in inflation expectations.

[2] The reader is referred to Benartzi and Thaler [6], Shalev [44,45], Compte and Jehiel [10], Falk and Knell [15], Köszegi and Rabin [34–36], Gimpel [20], Li [38], Vartiainen [47], Abeler et al. [1], Baucells et al. [5], Giorgi and Post [21], Hyndman [26], Driesen et al. [12], Sarver [43], Roels and Su [41], Karagözoğlu and Keskin [31] and Karagözoğlu et al. [30] among others.

Results We, first, show that for any exogenously given initial reference point, there exists a subgame perfect equilibrium of the game, which induces an immediate agreement. A closer look at this equilibrium reveals that despite the immediate agreement result, *expiration lengths* systematically influence equilibrium behavior. More precisely, the influence of expiration lengths (i.e., m_1 and m_2) is concealed in reference points since m_1 and m_2 determine the corresponding sets of past offers from which the current reference points emerge, and rational players incorporate the information from the continuation game to their actions in the first period. Second, we also provide an equilibrium analysis of the game with unlimited influence (no expiration) which we later compare with that of the game with limited influence.

Third, we show the existence of an equilibrium with a delayed agreement, by constructing an example. In our example, players reach an agreement at $t = 3$. The set of parameter values we use in constructing this equilibrium offers insights into the causes of delay: (i) players start with high reference points, (ii) expiration lengths are short, and (iii) the breakdown probability is low. Therefore, the potential benefits (enlarging the utility possibilities frontier by making offers that reduce the opponent's reference point) from delaying the agreement outweigh potential costs (breakdown).

Fourth, we show that whenever the reference point is at the origin, there cannot be a (further) delay in reaching an agreement.

We also conduct comparative static analyses on players' equilibrium payoffs, for any possible pair of initial reference points. Our results reveal that (i) an increase in λ_i favors player j and (ii) an increase in γ_i favors player i. We compare the equilibria of the game with limited influence and the game with unlimited influence (i.e., $m_1 = m_2 = \infty$). Focusing on the immediate-agreement equilibrium, our comparison shows that equilibrium outcomes are identical, whereas equilibrium strategies are different. Finally, we compare our findings with those of Driesen et al. [12].

Comparison with Driesen et al. [12] It would be useful for the reader to know the differences between our model and the one closest to it, Driesen et al. [12], at this stage. As we mentioned above, the first and the most important difference is about the extent of the influence of past offers on current and future reference points. We allow that influence to be limited, whereas Driesen et al. [12] focus only on unlimited influence. Second, arguably less important, difference is: there is room for gain-seekingness in the utility function we use, whereas Driesen et al. [12] focus on only loss aversion. Third, we allow the initial reference point to be any element of $[0, 1] \times [0, 1]$, whereas Driesen et al. [12] assumed it to be $(0, 0)$. Note that this assumption may not be restrictive in their case, whereas the existence of our delayed-agreement equilibrium depends on the initial reference point in our model, and as such, the flexibility we bring is valuable. We leave comparing our results with those of Driesen et al. [12] to Section 4.

Organization The organization of the paper is as follows: Sect. 2 introduces the model. Section 3 and its subsections present our analysis of equilibria. Section 4 presents (i) results from comparative static analyses, (ii) a comparison of two variations of the model, and (iii) a comparison of our findings with those of Driesen et al. [12]. Finally, Sect. 5 ends by presenting concluding remarks. Throughout the paper, we relegate lengthy proofs to "Appendix."

2 The Model

We consider an infinite-horizon bargaining model in which two players, player 1 and 2, bargain over the division of a perfectly divisible pie of a unit size, following an alternating-offers bargaining protocol. More precisely, at odd periods $t = 1, 3, 5, \ldots$, player 1 makes an offer $z = (z_1, z_2)$, where $z_1 + z_2 = 1$ and player 2 decides whether to accept (a) or reject (r) the offer. Similarly, at even periods $t = 2, 4, 6, \ldots$, player 2 makes an offer and player 1 decides whether to accept (a) or reject (r) it. If an offer $z = (z_1, z_2)$ is accepted, the game ends with players receiving their corresponding agreed shares. At any period t, if an offer is rejected, then with probability $\delta \in (0, 1)$ the game continues to period $t + 1$ and with probability $1 - \delta$ (i.e., the breakdown probability) the game ends. If and when the game ends as a result of a breakdown, players do not receive any share from the pie (i.e., the shares of both players are equal to 0). The set of all possible (efficient) offers is denoted by

$$Z = \{(z_1, z_2) \in R_+^2 | z_1 + z_2 = 1\}.$$

For each player $i \in \{1, 2\}$, a strategy $\sigma_i = (\sigma_i^t)_{t=1}^{\infty}$ is a sequence of functions where σ_i^t maps any history up to period t to an offer or a response (i.e., a or r) depending on whose turn it is to make an offer at period t.

We use a framework similar to the ones developed in Shalev [45] and Driesen et al. [12] to study the influence of past actions on current decisions through their influence on players' reference points. In particular, player i's current reference point according to which he evaluates player j's current offer is determined by player j's past offers (and the exogenously given initial reference point). However, in contrast to Driesen et al. [12], the reference point of player i in our model is *not necessarily* the highest offer he received up to the period he has to take an action; instead, it is the highest offer he received in the last m_i periods in which he received offers, where m_i is finite. In other words, the influence of past offers on players' reference points expires in finite periods (after a certain number of periods, a past offer gets simply *too old* to constitute a reference point). For any $i \in \{1, 2\}$ and $t \in \mathbb{N}$, we will use r_i^t to denote the reference point of player i at period t. Assume that initial reference points are exogenous, $(r_1^1, r_2^1) \in [0, 1]^2$. Thus, after a sequence of offers $(z^s)_{s=1}^{t-1}$, the reference point of the agents at period t are (with the convention $z^0 = (z_1^0, z_2^0) = (r_1^1, r_2^1)$):

- If t is odd

$$r_1^t = \max \left\{ z_1^s | s \in \{t-1, t-3, \cdots, t-(2m_1-3), t-(2m_1-1)\} \cap \mathbb{Z}_+ \right\}$$
$$r_2^t = r_2^{t-1}$$

- If t is even

$$r_1^t = r_1^{t-1}$$
$$r_2^t = \max \left\{ z_2^s | s \in \{t-1, t-3, \cdots, t-(2m_2-3), t-(2m_2-1)\} \cap \mathbb{Z}_+ \right\}$$

Suppose that period $t + 1$ is player i's turn to make an offer and he offers $z = (z_i, z_j)$. If player j rejects the offer, player j's reference point in period $t + 2$ will be the maximum of the offers that were made in periods $t - (2m_j - 3), t - (2m_j - 5), \ldots, t - 3, t - 1$ and the last offer z_j, since the influence of offer at the period $t - (2m_j - 1)$ *expires*.

The following definitions will provide us some convenience in the following discussions. For odd periods t, let

$$\bar{r}_1^t = \max\left\{z_1^s | s \in \{t-1, t-3, \cdots, t-(2m_1-3)\} \cap \mathbb{Z}_+\right\},$$

and for even periods t,

$$\bar{r}_2^t = \max\left\{z_2^s | s \in \{t-1, t-3, \cdots, t-(2m_2-3)\} \cap \mathbb{Z}_+\right\}.$$

Thus, $\bar{r}_i^t$ is the maximum of the last $m_i - 1$ offers (or t offers if $m_i - 1 > t$) that the agent has received).

Hence, in the case of a rejection, the reference point of player i in the next period, after he received an offer z, will be $\max\{\bar{r}_i^t, z_i\}$. We assume that player i evaluates the offer he receives at period t, z_i^t, according to his current reference point, r_i^t.

We employ the functional form in Köszegi and Rabin [34] to incorporate reference-dependent preferences. More precisely, offers below the reference point are regarded as *losses*, whereas the offers above the reference point are regarded as *gains*. Accordingly, at period t, the utility of player i from the realization of z^t is given as follows (assuming that the current reference point is r_i^t):

$$u_i^t(z^t, r^t) = \begin{cases} z_i^t + \gamma_i(z_i^t - r_i^t) & \text{if } z_i^t \geq r_i^t \\ z_i^t + \lambda_i(z_i^t - r_i^t) & \text{if } z_i^t < r_i^t, \end{cases}$$

where $\lambda_i \geq \gamma_i \geq 0$. The first term, z_i^t, is the intrinsic *consumption utility*, which can be considered as the benefit player i obtains from consuming his share of the pie. The second term (i.e., $\lambda_i(z_i^t - r_i^t)$ or $\gamma_i(z_i^t - r_i^t)$) represents *gain-loss utility.* λ_i is the loss-aversion coefficient, whereas γ_i is the gain-seekingness coefficient. By taking $\lambda_i \geq \gamma_i$, we are assuming that players are more sensitive to losses than to gains, capturing the main idea in Kahneman and Tversky [28,29]. Note that the functional forms used in Shalev [45] and Driesen et al. [12] are special cases (i.e., $\gamma_i = 0$) of the functional form that we employ. We denote the game described above by Γ.

3 Equilibrium

In this section, we first focus on the subgame perfect equilibrium inducing immediate agreement with two variations: limited influence and unlimited influence. Second, we show the existence of a subgame perfect equilibrium with a delayed agreement. Throughout the paper, equilibrium means subgame perfect Nash equilibrium.

3.1 Equilibrium with Immediate Agreement

We first analyze the model with a limited influence (i.e., finite expiration lengths). Later, we examine a variation of our baseline model, with an unlimited influence (i.e., infinite expiration lengths).

3.1.1 Limited Influence

Due to the structure of the game, we first investigate players' expected continuation payoffs at any period of the game.[3] Now, consider an odd period t in which player 1 makes the offer $x \in Z$. Suppose that if player 2 rejects the offer x in period t, then he will propose $y \in Z$ in period $t+1$. Let $r_2 = r_2^t$ and $\bar{r}_2 = \bar{r}_2^t$. Note that in the case of a rejection, the reference point of player 2 in period $t+1$ will be $r_2^{t+1} = \max\{\bar{r}_2, x_2\}$. For player 2 to be indifferent between accepting the offer x made in period t and rejecting this offer and making the offer y in the next period, which is heuristically assumed to be accepted by player 1, we need

$$u_2^t(x, r^t) = \delta u_2^{t+1}(y, r^{t+1}) + (1-\delta)u_2^{t+1}(0, r^{t+1}). \tag{1}$$

The left-hand side of the equality is the utility that player 2 gets if he accepts x, whereas the right-hand side is his (expected) continuation utility (i.e., with probability δ, the game continues to the next period and player 2 offers y which is assumed to be accepted by player 1 or with probability $1-\delta$, the game ends and player 2 gets zero). Similarly, consider an even period t in which player 2 makes the offer $y \in Z$. Suppose that if player 1 rejects this offer, then he will propose $x \in Z$ in period $t+1$. Let $r_1 = r_1^t$ and $\bar{r}_1 = \bar{r}_1^t$. Note that in the case of a rejection, the reference point of player 1 in period $t+1$ will be $r_1^{t+1} = \max\{\bar{r}_1, y_1\}$. For player 1 to be indifferent between accepting the offer y made in period t and rejecting this offer and making the offer x in the next period, which is assumed to be accepted by player 2, we need

$$u_1^t(y, r^t) = \delta u_1^{t+1}(x, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}). \tag{2}$$

For the analysis of equilibrium under limited influence in the rest of the paper, we make the following additional assumption:

Assumption X A player at period t makes an offer that gives him a share more than the offer that he rejected at $t-1$, i.e., $x_1 > y_1$.
A detailed example on the role this assumption serves and why we need it is presented in "Appendix B."[4] Under this assumption, depending on the reference points, Equation (2) yields one of the following cases:

1. $r_1 \geq \bar{r}_1 > y_1$

   $a.\ x_1 \geq \bar{r}_1 : (1+\gamma_1)\delta x_1 = (1+\lambda_1)y_1 - \lambda_1 r_1 + (1-\delta)\lambda_1 \bar{r}_1 + \delta\gamma_1\bar{r}_1$
   $b.\ x_1 < \bar{r}_1 : (1+\lambda_1)\delta x_1 = (1+\lambda_1)y_1 - \lambda_1 r_1 + \lambda_1 \bar{r}_1$

2. $r_1 \geq y_1 \geq \bar{r}_1 : (1+\gamma_1)\delta x_1 = (1+\lambda_1+\delta\gamma_1+(1-\delta)\lambda_1)y_1 - \lambda_1 r_1.$
3. $y_1 > r_1 \geq \bar{r}_1 : (1+\gamma_1)\delta x_1 = (1+\gamma_1+\delta\gamma_1+(1-\delta)\lambda_1)y_1 - \gamma_1 r_1.$

Similarly, for Equation (1), we have:

I. $r_2 \geq \bar{r}_2 > x_2$

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[3] In the bargaining game Γ, (i) the pie is desirable, (ii) breakdown is the worst outcome, (iii) $u_i^t(x, r) > u_i^{t+1}(x, r)$ for each t, x and r, and (iv) u_i is continuous. The game is stationary in the sense that player i's preference between his share from the division x at period t and his share from division y at period $t+1$ is independent of t when period t is player i's turn to make an offer. Note that the reference point of player i at period $t+1$ is equal to his reference point at period t, i.e., $r_i^{t+1} = r_i^t$. These properties of Γ allow us to use the expected payoff at period $t+1$ as the continuation payoff of the game (see [40, pp. 33–34 and 74–75]).

[4] See [17] for a similar modeling assumption. Note that our assumption is weaker than their *endogenous commitment* assumption.

$a.\ y_2 \geq \bar{r}_2 : (1+\gamma_2)\delta y_2 = (1+\lambda_2)x_2 - \lambda_2 r_2 + (1-\delta)\lambda_2\bar{r}_2 + \delta\gamma_2\bar{r}_2$
$b.\ y_2 < \bar{r}_2 : (1+\lambda_2)\delta y_2 = (1+\lambda_2)x_2 - \lambda_2 r_2 + \lambda_2\bar{r}_2$

II. $r_2 \geq x_2 \geq \bar{r}_2 : (1+\gamma_2)\delta y_2 = (1+\lambda_2+\delta\gamma_2+(1-\delta)\lambda_2)x_2 - \lambda_2 r_2.$
III. $x_2 > r_2 \geq \bar{r}_2 : (1+\gamma_2)\delta y_2 = (1+\gamma_2+\delta\gamma_2+(1-\delta)\lambda_2)x_2 - \gamma_2 r_2.$

Considering the cases above together, we obtain 16 possible regions for the reference point $(r_1, r_2) \in [0,1]^2$. We will denote these mutually exclusive regions by $R_{1.a-I.a}, R_{1-I.b}, \ldots, R_{3-II}, R_{3-III}$. Let x^ω and y^ω be the offers associated with the corresponding region R_ω, where $\omega \in \{1.a-I.a, 1.a-I.b, 1.a-II, \ldots, 3-III\}$ for player 1 and player 2, respectively.

The following theorem describes a subgame perfect equilibrium of the game. Note that in deriving this equilibrium, we make Assumption X presented above. We are not able to prove Theorem 1 without this assumption, and we do not know whether the theorem is true without it.

Theorem 1 *Suppose that Assumption X holds. Take any period $t \geq 1$. Let the reference point be $(r_1^t, r_2^t) \in R_\omega$ and x^ω, y^ω be the offers associated with the corresponding region R_ω, where $\omega \in \{1.a-I.a, 1.a-I.b, 1.a-II, \ldots, 3-III\}$. For player 1, let σ_1^* be such that if t is odd, player 1 makes the offer x^ω and if t is even, player 1 accepts the offer z if and only if $z_1 \geq y_1^\omega$. For player 2, define the strategy σ_2^* in a similar way. The strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*)$ is a subgame perfect equilibrium of the bargaining game Γ.*

Proof of Theorem 1 The proof is relegated to "Appendix B." □

This subgame perfect equilibrium induces an immediate agreement. The equilibrium offers of the players depend on the region that contains their reference points. For instance, at any period t,

- if $(r_1, r_2) \in R_\omega$ where $\omega \in \{1.a-I.a, 1.a-I.b, 1.b-I.b, 1.b-I.b\}$, the equilibrium offers of the players depend on the current reference points r_i and $\bar{r}_i$ (maximum of the last $m_i - 1$ offers). In these cases, the current reference points are high, and in the case of rejection, the reference point of the responder in the next period will be equal to $\bar{r}_i$.
- On the other hand, if $(r_1, r_2) \in R_\omega$ where $\omega \in \{2-II, 2-III, 3-II, 3-III\}$, the equilibrium offers depend only on the current reference points r_i. In these regions, the reference point of the responder in the next period will be equal to the most recent offer he received. Hence, $\bar{r}_i$ will not influence the equilibrium offer.

Consequently, at any period t, the equilibrium strategy of player i associated with region R_ω directly depends on the reference points of both players for any $\omega \in \{1.a-I.a, 1.a-I.b, 1.a-II, \ldots, 3-III\}$, implying dependence on expiration lengths, m_i and m_j.

Equilibrium Outcome Theorem 1 states that players follow the strategy profile (σ_1^*, σ_2^*) in the equilibrium described: a player makes the offer x^ω or y^ω based on the relevant region R_ω for the reference point, and the agreement is reached immediately. This implies that at $t = 1$, player 1 makes the offer x^ω, where $(r_1^1, r_2^1) \in R_\omega$, and player 2 accepts the offer. For instance, if the initial reference points belong to region R_{3-III}, i.e., $(r_1^1, r_2^1) \in R_{3,III}$, then the equilibrium outcome of the bargaining game $x = (x_1, x_2)$ is given by

$$x_1 = \frac{\eta_1(\eta_2 - \gamma_2 r_2^1) - \delta(1+\gamma_2)(\eta_1 - \gamma_1 r_1^1)}{\eta_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$

$$x_2 = \frac{\eta_1\gamma_2 r_2^1 + \delta(1+\gamma_2)(\eta_1 - \gamma_1 r_1^1) - \delta^2(1+\gamma_1)(1+\gamma_2)}{\eta_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$

where $\eta_i = (1 + \gamma_i + \delta\gamma_i + (1-\delta)\lambda_i)$. The equilibrium division of the pie depends directly on initial reference points in this case. It implicitly depends on expiration lengths, since they are decisive in the evolution of the reference points, which are taken into account through expected continuation payoffs. Equilibrium outcomes are given in "Appendix A" for all regions R_ω, where $\omega \in \{1.a - I.a, 1.a - I.b, 1.a - II, \ldots, 3 - III\}$.

3.1.2 No Expiration (Unlimited Influence)

In this section, we remove the bounds on the number of periods an offer can influence future reference points. Hence, in any period $t > 1$, the reference points at period t are defined on the basis of the most generous offers players received up to t,

$$r_1^t = \max\{z_1^s | s = 0, 2, 4, 6, \ldots \le t\}$$
$$r_2^t = \max\{z_2^s | s = 0, 1, 3, 5, \ldots \le t\}$$

where z^s is the offer made at period s and $(z_1^0, z_2^0) = (r_1^1, r_2^1)$.

Suppose that period t is player i's turn to make an offer and he offers $z = (z_i, z_j)$. If player j rejects the offer z, his reference point period $t+1$ will be $r_j^{t+1} = \max\{r_j, z_j\}$.

Considering Equation (2) for the bargaining game with no expiration, we have the following three cases:

1. $r_1 > x_1 > y_1 : \delta x_1 = y_1$.
2. $x_1 \ge r_1 > y_1 : (1+\gamma_1)\delta x_1 = (1+\lambda_1)y_1 + \delta\gamma_1 r_1 - \delta\lambda_1 r_1$.
3. $x_1 > y_1 \ge r_1 : (1+\gamma_1)\delta x_1 = (1+\gamma_1+\delta\gamma_1+(1-\delta)\lambda_1)y_1 - \gamma_1 r_1$.

Similarly, for Equation (1), we have:

I. $r_2 > y_2 > x_2 : \delta y_2 = x_2$.
II. $y_2 \ge r_2 > x_2 : (1+\gamma_2)\delta y_2 = (1+\lambda_2)x_2 + \delta\gamma_2 r_2 - \delta\lambda_2 r_2$.
III. $y_2 > x_2 \ge r_2 : (1+\gamma_2)\delta y_2 = (1+\gamma_2+\delta\gamma_2+(1-\delta)\lambda_2)x_2 - \gamma_2 r_2$.

Let x^ω and y^ω be the associated offers with the regions R_ω where $\omega \in \{1 - I, 1 - II, \ldots, 3 - III\}$ for player 1 and player 2, respectively. The following theorem describes a subgame perfect equilibrium of the corresponding game.

Theorem 2 *Take any period $t \ge 1$. Let $(r_1^t, r_2^t) \in R_\omega$ where $\omega \in \{1-I, 1-II, \ldots, 3-III\}$. For player 1, let σ_1^* be such that if t is odd, player 1 makes the offer x^ω and if t is even, player 1 accepts the offer z if and only if $z_1 \ge y_1^\omega$. For player 2, define the strategy σ_2^* in a similar way. The strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*)$ is a subgame perfect equilibrium of the bargaining game Γ.*

Proof of Theorem 2 The proof of Theorem 2 is similar to that of Theorem 1, and it is relegated to "Appendix B." □

This game, too, has a subgame perfect equilibrium that induces an immediate acceptance. At any period $t \ge 1$,

- if $(r_1, r_2) \in R_{1-I}$, the equilibrium offers depend only on the continuation probability of the game, δ, and they are identical to the equilibrium offers in the standard Rubinstein game. This result is different from that of the game with limited influence. In the unlimited influence variation, the reference points are the maximum of the offers received up to period t. Hence, when the offer received is less than the reference point of the responder, his reference point in the next period remains the same. Thus, the current reference points do not influence the equilibrium strategies.

- In regions R_{1-II}, R_{1-III}, R_{2-I}, and R_{3-I}, the equilibrium offers depend on at least one player's current reference point, since the reference point of the responder in the next period may be equal to the offer he received most recently.
- In regions R_{2-II}, R_{2-III}, R_{3-II}, and R_{3-III}, the equilibrium offer of player i directly depends on the reference points of both players, which is the highest offer he received and the highest offer he made to player j.

Equilibrium Outcome Suppose that the initial reference points are in the region R_{3-III}, i.e., $(r_1^1, r_2^1) \in R_{3-III}$. In this case, the subgame perfect equilibrium outcome is as follows:

$$x_1 = \frac{\eta_1(\eta_2 - \gamma_2 r_2^1) - \delta(1+\gamma_2)(\eta_1 - \gamma_1 r_1^1)}{\eta_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$

$$x_2 = \frac{\eta_1\gamma_2 r_2^1 + \delta(1+\gamma_2)(\eta_1 - \gamma_1 r_1^1) - \delta^2(1+\gamma_1)(1+\gamma_2)}{\eta_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$

where $\eta_i = (1 + \gamma_i + \delta\gamma_i + (1-\delta)\lambda_i)$.

Since the game induces immediate agreement and the evolution of reference points is backward-looking, the subgame perfect equilibrium outcome of the game with unlimited influence presented here has the same formulation as the one in the game with limited influence. The equilibrium outcome depends explicitly on the initial reference points and implicitly on the expiration lengths. The loss aversion and gain-seekingness coefficients also influence the outcome. An increase in a player's loss aversion hurts him while it provides benefit to his opponent. Conversely, an increase in a player's gain-seekingness makes him better while hurting his opponent. Detailed comparative statics on these parameters are presented in the Discussion section.

3.2 Equilibrium with Delayed Agreement

In this section, we first show the existence of an equilibrium where the agreement is reached with delay in the model with limited influence. We do this by constructing an example. The equilibrium we construct and the parameter values we use provide useful insights in to the role of limited influence on bargaining.

Let $\delta = 0.9$ (hence the breakdown probability is 0.1), $\lambda_1 = \lambda_2 = 1$ and $\gamma_1 = \gamma_2 = 0.5$. Suppose that the initial reference points are both equal to 0.9, i.e., $r_1^1 = r_2^1 = 0.9$. Furthermore, the influence of a received offer (on a player's reference point) expires in one period for both players, $m_1 = m_2 = 1$. Now, consider a strategy $\tilde{\sigma} = (\tilde{\sigma_1}, \tilde{\sigma_2})$ such that

(i) In period $t = 1$, player 1 proposes $(1, 0)$. Player 2 accepts any offer $x = (x_1, x_2)$ if $x_2 \geq 0.4737$ and he rejects the offer otherwise,
(ii) In period $t = 2$, following a rejected offer in $t = 1$, player 2 proposes $(0, 1)$. Player 1 accepts any offer $y = (y_1, y_2)$ if $y_1 \geq 0.7545$, and he rejects the offer otherwise,
(iii) For any period $t \geq 3$, $\tilde{\sigma}^t = \sigma^{*t}$.

The following lemma will be useful in proving that the strategy profile presented above constitutes a subgame perfect equilibrium. It shows that if both players' reference points are equal to zero, then they reach an agreement without (further) delay.

Lemma 1 *Take any period* $t \geq 1$*. Suppose that* $(r_1^t, r_2^t) = (0, 0)$*. For any player, the proposer obtains the maximum utility by making the equilibrium offer associated with region* R_{3-III} *given that the responder follows* σ^**.*

Proof of Lemma 1 Take any period $t \geq 1$ and suppose that $(r_1^t, r_2^t) = (0, 0)$. Without loss of generality, assume that t is odd. According to the strategy σ^*, player 1 proposes x^{3-III}. If player 1 proposes x' such that $x'_2 < x_2^{3-III}$, then player 2 (following σ^*) rejects the offer. Now, player 2 (following σ^*) proposes $y^{3-III} = y = (y_1, y_2)$.

$$\begin{aligned}\delta u_1^{t+1}(y, r^{t+1}) + (1-d)u_1^{t+1}(0, r^{t+1}) &= \delta(y_1 + \gamma_1 y_1) - (1-\delta)\lambda_1 0 \\ &= \delta(1+\gamma_1)y_1 \\ &< \delta(1+\gamma_1)x_1^{3-III} = u_1^t(x^{3-III}).\end{aligned}$$

Equipped with this result, now we can formulate our proposition, which describes a subgame perfect equilibrium with a delayed agreement.

Proposition 1 *Let the initial reference points be* $(r_1^1, r_2^1) = (0.9, 0.9)$*. Furthermore, assume that* $(\delta, \lambda_1, \lambda_2, \gamma_1, \gamma_2) = (0.9, 1, 1, 0.5, 0.5)$ *and* $m_1 = m_2 = 1$*. The strategy profile* $\tilde{\sigma} = (\tilde{\sigma_1}, \tilde{\sigma_2})$ *is a subgame perfect equilibrium of the game* Γ*, where the agreement is reached with a delay (to be precise, at* $t = 3$*).*

Proof of Proposition 1 In the initial period, $(r_1^1, r_2^1) = (0.9, 0.9) \in R_{1.b-I.b}$. Suppose that player 1 proposes $z = (z_1, z_2)$, where $z_2 < 0.4737$, then player 2 rejects the offer. The game continues to the next period with probability δ, or it ends with probability $1-\delta$. If the game continues to the second period, the reference points will be $(r_1^2, r_2^2) = (0.9, z_2)$. Now, (r_1^2, r_2^2) can only belong to regions $R_{1.b-I.a}$, $R_{1.b-II}$ or $R_{1.b-III}$. Since in period $t = 3$ players follow the strategy σ^*, player 2 gets the maximum expected payoff by proposing $(0, 1)$. Player 1 rejects the offer $(0, 1)$. Once we return to $t = 1$, the offer $z = (1, 0)$ gives player 1 the maximum expected payoff. Hence, at $t = 3$, $(r_1^3, r_2^3) = (0, 0) \in R_{3-III}$. It follows from Lemma 1 that both players follow σ^*: player 1 proposes the equilibrium offer associated with this region x^{3-III} and player 2 accepts this offer. Thus, player 1 and player 2 obtain

$$\delta(\delta u_1^2((0,1), r^2) + (1-\delta)u_1^2(0, r^2)) + (1-\delta)u_1^1(0, r^1) = 0.6426$$
$$\delta(\delta u_2^2((0,1), r^2) + (1-\delta)u_2^2(0, r^2)) + (1-\delta)u_2^1(0, r^1) = 0.4824.$$

For the remaining periods $t \geq 3$, the proof follows from Lemma 1 and the proof of Theorem 1. We have that for any subgame and for any strategy σ_2, $u_2(\tilde{\sigma_1}, \tilde{\sigma_2}) \geq u_2(\tilde{\sigma_1}, \sigma_2)$. Hence, $\tilde{\sigma_2}$ is the best response of player 2 given that player 1 follows strategy $\tilde{\sigma_1}$. Similarly, for any subgame and for any strategy σ_1, $u_1(\tilde{\sigma_1}, \tilde{\sigma_2}) \geq u_1(\sigma_1, \tilde{\sigma_2})$. Therefore, $\tilde{\sigma}$ is a subgame perfect equilibrium of the game Γ with a delayed agreement.[5] □

Expiration length (or limited influence) plays an intuitive role in the construction of this equilibrium. First of all, the initial reference points are high, and as such, the potential benefits from making low (in particular, 0) offers and delaying the agreement are high. Second, expiration lengths are short, and as such, the utility possibilities can reach its maximal size (given loss-aversion and gain-seekingness parameters) in a short period of time (in particular, two

[5] Note that the deviation (from the strategies of the corresponding immediate-agreement equilibrium) that brings about the delayed-agreement equilibrium is a bilateral one, and as such, the immediate-agreement equilibrium and the delayed-agreement equilibrium coexist.

periods). Finally, the breakdown probability is low, and as such, the risk associated with making offers that will be rejected is low. Therefore, expected benefits of delaying the agreement outweigh expected costs. Along these lines, we conjecture that delayed-agreement equilibrium can be constructed for similar parameter values. For instance, the strategy profile given in Proposition 1 will constitute an equilibrium also when initial reference points are larger than 0.9 and/or breakdown probability is lower than 0.1. However, a full characterization of delayed-agreement equilibria is far from trivial and left for future research.

4 Discussion

In this section, we present (i) the results of comparative static analyses we conducted to study the effects of loss-aversion and gain-seekingness parameters on equilibrium outcomes, (ii) a closer look at the comparison of two variations of our model, and (iii) a comparison of our model/results with those of Driesen et al. [12].

Gain-Loss Parameters ($\boldsymbol{\lambda}$ and $\boldsymbol{\gamma}$) Here, we analyze how the changes in the model parameters influence the division of the pie in the equilibrium with immediate acceptance.[6] We examine the response of the share of player 1 to the changes in loss-aversion coefficients. The analysis for player 2 is similar. Differentiating x_1 with respect to λ_1 and λ_2, we obtain the following results for all regions $\omega \in \{1.a - I.a, 1.a - I.b, \ldots, 3 - III\}$:

$$\frac{dx_1^{\omega}}{d\lambda_1} \leq 0$$

$$\frac{dx_1^{\omega}}{d\lambda_2} \geq 0.$$

Hence, an increase in player 1's loss-aversion coefficient leads to a decrease in his equilibrium share, whereas an increase in player 2's loss-aversion coefficient leads to an increase in his equilibrium share.

Now, we investigate the impact of a change in parameters γ_1 and γ_2 on player 1's share in the equilibrium we studied. Differentiating x_1 with respect to γ_1 and γ_2, we obtain the following results for all regions $\omega \in \{1.a - I.a, 1.a - I.b, \ldots, 3 - III\}$:

$$\frac{dx_1^{\omega}}{d\gamma_1} \geq 0$$

$$\frac{dx_1^{\omega}}{d\gamma_2} \leq 0.$$

Hence, an increase in player 1's gain-seekingness coefficient leads to an increase in his equilibrium share, whereas an increase in his opponent's gain-seekingness coefficient leads to a decrease in his equilibrium share. Obtaining more than the reference point gives a player higher utility since the difference between the received share from the pie and the reference point is regarded as a gain. Hence, for low reference points (the equilibrium offer is greater than the reference point), gain-seekingness parameter becomes more important.

It is interesting to see that two urges (loss aversion and gain-seekingness) both of which tell basically the same thing to an agent (i.e.,*do not get less than* vs *get more than* your reference point) have different influences over his bargaining power.

6 The reader is referred to "Appendix C" for the mathematical derivations of the results presented here.

Comparison of the Variations of the Model Restricting our attention to the equilibria with immediate agreement, we can say that the equilibrium outcomes (in different regions) of the bargaining game with unlimited influence are the same with the equilibrium outcomes of the bargaining game with limited influence. This result possibly stems from the equilibrium inducing immediate agreement in combination with the backward focus of the reference point. In the initial period, we have $r_i = \bar{r}_i$ for each player i. Hence, the equilibrium offer in the game with limited influence is equal to the equilibrium offer in the game with unlimited influence. However, there are still some differences: the strategies are different for two variations of the model since these models differ in the evolution of reference points. Suppose the game is at period $t > m_i$. At this period, in the bargaining game with unlimited influence, the first offer player i received has still an influence on his reference point (and so on his actions), while in the bargaining game with limited influence the impact of the first offer on his reference point expires. Hence, in the future periods of the game with limited influence, r_i and $\bar{r}_i$ may differ, while for all periods of the game with unlimited influence we have $r_i = \bar{r}_i$. Further, the equilibrium strategies in the game with limited influence implicitly depend on expiration lengths since they are effective in the evolution of reference points.

Comparison with Driesen et al. [12] In our game with limited influence, the reference point of player i is equal to the maximum of the last m_i offers he received. In this framework, for any region, the equilibrium offers depend on the reference points of the players. Thus, they implicitly depend on the expiration lengths. However, in Driesen et al. [12], the equilibrium offers, for some regions, do not depend on the reference points. For instance, consider region $R_{1.b-I.b}$ in our model which corresponds to region R_{1-I} in Driesen et al. [12]. The equilibrium offers for this region in our game are

$$x_1^{1.b-I.b} = \frac{1}{1+\delta} + \frac{\delta(1+\lambda_2)\lambda_1(r_1-\bar{r}_1)-(1+\lambda_1)\lambda_2(r_2-\bar{r}_2)}{(1-\delta_2)(1+\lambda_1)(1+\lambda_2)}$$
$$y_1^{1.b-I.b} = \frac{\delta}{1+\delta} + \frac{(1+\lambda_2)\lambda_1(r_1-\bar{r}_1)-\delta(1+\lambda_1)\lambda_2(r_2-\bar{r}_2)}{(1-\delta_2)(1+\lambda_1)(1+\lambda_2)},$$

whereas the corresponding equilibrium offers for this region in Driesen et al. [12] are

$$x_1^{1-I} = \frac{1}{1+\delta} \quad y_1^{1-I} = \frac{\delta}{1+\delta},$$

Unless $r_i = \bar{r}_i$ for each player i, the equilibrium offers are different for some values of λ_1 and λ_2. Therefore, introducing the limited influence affects the equilibrium of the game. Note that for this region, the equilibrium offer in the game with limited influence can be decomposed into an expression of which the first part is equal to the equilibrium of Rubinstein bargaining game. Suppose that the initial reference points belong to region $R_{1.b-I.b}$, i.e., $(r_1^1, r_2^1) \in R_{1.b-I.b}$. In this region, the equilibrium outcome coincides with the equilibrium of Rubinstein model. However, for other regions, making this sort of comparison is difficult, since we have very complex expressions (see “Appendix A”).

In the bargaining game with unlimited influence, reference points evolve as in Driesen et al. [12], i.e., they are the maxima of the rejected offers. Another point that we differ from Driesen et al. [12] is the players’ evaluation of the offers above the reference point. As we mentioned before, the utility function they employ is a special case of ours; for each player i, $\gamma_i = 0$. For low values of reference points, the equilibrium offers in Driesen et al. [12] are not affected by them since the gain relative to the reference point does not have any impact on the utility. However, in our model the offers in the equilibrium depend on the reference points, even for low values of those. Furthermore, Driesen et al. [12] restrict their attention to the case where the initial reference point is $(0, 0)$, whereas we do not impose such a restriction. Arguably, as a consequence of these differences, they can prove that the constructed equilibrium is the unique subgame perfect equilibrium satisfying some certain properties, whereas we cannot.

Last but not least, we show the existence of an equilibrium where agreement is reached with delay, whereas Driesen et al. [12] completely focus on an equilibrium with immediate agreement. Arguably, the delayed-agreement equilibrium we construct is where the limited influence introduced in our paper has the most transparent impact, and as such, it significantly differentiates our results from those of Driesen et al. [12].

5 Concluding Remarks

We studied an infinite horizon, alternating offers bargaining game with endogenous reference points. In our model, (i) players have reference-dependent preferences, (ii) the initial reference point is exogenously given, (iii) but once the bargaining starts, the current reference point of player i depends on the most recent m_i offers player j made. To the best of our knowledge, this is the first paper that incorporates behavioral phenomena such as the recency effect/retrievability bias into bargaining model with endogenous reference points. We first showed that there exists a subgame perfect equilibrium with an immediate agreement and expiration lengths influence players' strategies but not their payoffs in this equilibrium. Second, we showed that for some parameter values, there also exists an equilibrium with a delayed agreement. Expiration length, the main novelty of our model, plays an important and intuitive role for the existence of this type of equilibrium. We show that nonzero initial reference points are crucial for the existence of such an equilibrium. Given the construction of our delayed-agreement equilibrium and the role expiration lengths play there, one is tempted to say that an equilibrium with a delayed agreement does not exist in the model with unlimited influence. However, formally proving that turns out to be far from trivial and as such is left for future research.

Acknowledgements We would like to thank an anonymous associate editor and three reviewers for constructive and helpful comments, which significantly improved the paper. Usual disclaimers apply.

A Definitions of the Regions and the Corresponding Equilibrium Strategies

We define the regions R_ω and find the corresponding equilibrium strategies/proposals , $x^\omega = (x_1^\omega, x_2^\omega)$, where $\omega \in \{1.a-I.a, 1.a-I.b, \ldots, 3-III\}$, in the model with limited influence. Note that $x_2^\omega = 1-x_1^\omega$ for each ω. Suppose that the game is at period t. Let $(r_1^t, r_2^t) = (r_1, r_2)$ and $(\bar{r}_1^t, \bar{r}_2^t) = (\bar{r}_1, \bar{r}_2)$.

A.1 Region 1.a-I.a

$$R_{1.a-I.a} = \Big\{(r_1, r_2)\Big| \frac{(1+\lambda_2)\lambda_1 r_1 + \delta(1+\gamma_1)(\zeta_2\bar{r}_2 - \lambda_2 r_2 + 1 + \lambda_2 - \delta(1+\gamma_2))}{(1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2) + (1+\lambda_2)\zeta_1}$$
$$< \bar{r}_1 \geq \frac{(1+\lambda_1)(\zeta_2\bar{r}_2 - \lambda_2 r_2 + 1 + \lambda_2) + \delta(1+\gamma_2)\lambda_1 r_1 - \delta(1+\gamma_2)(1+\lambda_1)}{(1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2) + \delta(1+\gamma_2)\zeta_1}$$
$$\text{and } \frac{\delta(1+\gamma_1)(\zeta_1\bar{r}_1 - \lambda_1 r_1 + 1 + \lambda_1 - \delta(1+\gamma_2)) - (1+\lambda_1)\lambda_2 r_2}{(1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2) - (1+\lambda_1)\zeta_2}$$
$$< \bar{r}_2 \leq \frac{(1+\lambda_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1 + 1 + \lambda_1 - \delta(1+\gamma_1)) + \delta(1+\gamma_1)\lambda_2 r_2}{(1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2) + \delta(1+\gamma_1)\zeta_2}\Big\},$$

where $\zeta_i = \delta\gamma_i + (1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{1.a-I.a} = \frac{(1+\lambda_1)(\zeta_2\bar{r}_2 - \lambda_2 r_2 + 1 + \lambda_2) - \delta(1+\gamma_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1 + 1 + \lambda_1)}{(1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2)}$$
$$y_1^{1.a-I.a} = \frac{\delta(1+\gamma_1)(\zeta_2\bar{r}_2 - \lambda_2 r_2 + 1 + \lambda_2) - (1+\lambda_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1) - \delta^2(1+\gamma_1)(1+\gamma_2)}{(1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2)}.$$

A.2 Region 1.a-I.b

$$R_{1.a-I.b} = \Big\{(r_1, r_2)\Big| \frac{(1+\lambda_2)\lambda_1 r_1 + \delta(1+\gamma_1)(\lambda_2\bar{r}_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2))}{(1+\lambda_1)(1+\lambda_2) - (1+\lambda_2)(\delta^2(1+\gamma_1) - \zeta_1)}$$
$$< \bar{r}_1 \leq \frac{(1+\lambda_1)(\lambda_2\bar{r}_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2)) + \delta(1+\lambda_2)\lambda_1 r_1}{(1+\lambda_1)(1+\lambda_2) - \delta(1+\lambda_2)(\delta(1+\gamma_1) - \zeta_1)}$$
$$\text{and } \bar{r}_2 > \frac{(1+\lambda_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1 + 1 + \lambda_1) + \delta(1+\gamma_1)(\lambda_2 r_2 - (1+\lambda_2))}{(1+\lambda_1)(1+\lambda_2) - \delta(1+\gamma_1)(\delta(1+\lambda_2) - \lambda_2)}\Big\},$$

where $\zeta_i = \delta\gamma_i + (1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{1.a-I.b} = \frac{(1+\lambda_1)(\lambda_2\bar{r}_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2)) - \delta(1+\lambda_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1)}{(1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\lambda_2)}$$
$$y_1^{1.a-I.b} = \frac{\delta(1+\gamma_1)(\lambda_2\bar{r}_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2)) - (1+\lambda_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1)}{(1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\lambda_2)}.$$

A.3 Region 1.a-II

$$R_{1.a-II} = \Big\{(r_1, r_2)\Big| \frac{\kappa_2\lambda_1 r_1 + \delta(1+\gamma_1)(\lambda_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2))}{\kappa_1\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$
$$< \bar{r}_1 \leq \frac{\delta(1+\gamma_2)\lambda_1 r_1 + (1+\lambda_1)(\lambda_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2))}{(1+\lambda_1)\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2) + \delta(1+\gamma_2)\zeta_1}$$
$$\text{and } r_2 \geq \frac{\delta(1+\gamma_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1) + \delta(1+\lambda_1)(1+\gamma_2) - \delta^2(1+\gamma_1)(1+\gamma_2)}{(1+\lambda_1)(\kappa_2 - \lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2)} > \bar{r}_2\Big\},$$

where $\zeta_i = \delta\gamma_i + (1-\delta)\lambda_i$ and $\kappa_i = 1+\lambda_i+\delta\gamma_i+(1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{1.a-II} = \frac{(1+\lambda_1)(\lambda_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2)) - \delta(1+\gamma_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1)}{(1+\lambda_1)\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$
$$y_1^{1.a-II} = \frac{\delta(1+\gamma_1)(\lambda_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2)) - \kappa_2(\zeta_1\bar{r}_1 - \lambda_1 r_1)}{(1+\lambda_1)\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}.$$

A.4 Region 1.a-III

$$R_{1.a-III} = \Bigg\{(r_1, r_2)\Bigg| \frac{\eta_2\lambda_1 r_1 + \delta(1+\gamma_1)(\gamma_2 - \gamma_2 r_2 + (1-\delta)(1+\lambda_2))}{\kappa_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$
$$< \bar{r}_1 \leq \frac{\delta(1+\gamma_2)\lambda_1 r_1 + (1+\lambda_1)(\gamma_2 - \gamma_2 r_2 + (1-\delta)(1+\lambda_2))}{(1+\lambda_1)\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2) + \delta(1+\gamma_2)\zeta_1}$$
$$\text{and } r_2 < \frac{\delta(1+\gamma_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1) + \delta(1+\lambda_1)(1+\gamma_2) - \delta^2(1+\gamma_1)(1+\gamma_2)}{(1+\lambda_1)(\kappa_2 - \gamma_2) - \delta^2(1+\gamma_1)(1+\gamma_2)}\Bigg\},$$

where $\zeta_i = \delta\gamma_i + (1-\delta)\lambda_i$, $\kappa_i = 1+\lambda_i+\delta\gamma_i+(1-\delta)\lambda_i$ and $\eta_i = 1+\gamma_i+\delta\gamma_i+(1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{1.a-III} = \frac{(1+\lambda_1)(\gamma_2 - \gamma_2 r_2 + (1-\delta)(1+\lambda_2)) - \delta(1+\gamma_2)(\zeta_1\bar{r}_1 - \lambda_1 r_1)}{(1+\lambda_1)\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$
$$y_1^{1.a-III} = \frac{\delta(1+\gamma_1)(\gamma_2 - \gamma_2 r_2 + (1-\delta)(1+\lambda_2)) - \eta_2(\zeta_1\bar{r}_1 - \lambda_1 r_1)}{(1+\lambda_1)\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}.$$

A.5 Region 1.b-I.b

$$R_{1.b-I.b} = \Bigg\{(r_1, r_2)\Bigg|\bar{r}_1 > \frac{\delta(1+\lambda_2)(\lambda_1 r_1 - (1+\lambda_1)) - (1+\lambda_1)(\lambda_2 r_2 - \lambda_2\bar{r}_2 - (1+\lambda_2))}{(1-\delta^2)(1+\lambda_1)(1+\lambda_2) + \delta(1+\lambda_2)\lambda_1}$$
$$\text{and } \bar{r}_2 > \frac{\delta(1+\lambda_1)(\lambda_2 r_2 - (1+\lambda_2)) - (1+\lambda_2)(\lambda_1 r_1 - \lambda_1\bar{r}_1 - (1+\lambda_1))}{(1-\delta^2)(1+\lambda_1)(1+\lambda_2) + \delta(1+\lambda_1)\lambda_2}\Bigg\},$$

The equilibrium strategies are

$$x_1^{1.b-I.b} = \frac{\delta(1+\lambda_2)(\lambda_1 r_1 - \lambda_1\bar{r}_1 - (1+\lambda_1)) - (1+\lambda_1)(\lambda_2 r_2 - \lambda_2\bar{r}_2 - (1+\lambda_2))}{(1-\delta^2)(1+\lambda_1)(1+\lambda_2)}$$
$$y_1^{1.b-I.b} = \frac{(1+\lambda_2)(\lambda_1 r_1 - \lambda_1\bar{r}_1) - \delta(1+\lambda_1)(\lambda_2 r_2 - \lambda_2\bar{r}_2 - (1-\delta)(1+\lambda_2))}{(1-\delta^2)(1+\lambda_1)(1+\lambda_2)}.$$

A.6 Region 1.b-II

$$R_{1.b-II} = \Bigg\{(r_1, r_2)\Bigg|\bar{r}_1 > \frac{\delta(1+\gamma_2)\lambda_1 r_1 + (1+\lambda_1)(\lambda_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2))}{(1+\lambda_1)\kappa_2 - \delta^2(1+\lambda_1)(1+\gamma_2) + \delta(1+\gamma_2)\lambda_1},$$
$$r_2 \geq \frac{\delta(1-\delta)(1+\lambda_1)(1+\delta\gamma_2) - \delta^2(1+\lambda_1)(1+\gamma_2) - \delta(1+\gamma_2)(\lambda_1 r_1 - \lambda_1\bar{r}_1)}{(1+\lambda_1)\kappa_2 - \delta^2(1+\lambda_1)(1+\gamma_2) + \delta(1+\lambda_1)\lambda_2} > \bar{r}_2\Bigg\},$$

where $\kappa_i = 1+\lambda_i+\delta\gamma_i+(1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{1.b-II} = \frac{\delta(1+\gamma_2)(\lambda_1 r_1 - \lambda_1\bar{r}_1) + (1+\lambda_1)(\lambda_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2))}{(1+\lambda_1)\kappa_2 - \delta^2(1+\lambda_1)(1+\gamma_2)}$$

$$y_1^{1.b-II} = \frac{\kappa_2(\lambda_1 r_1 - \lambda_1 \bar{r}_1) + \delta(1+\lambda_1)(\lambda_2 - \lambda_2 r_2 - (1-\delta)(1+\lambda_2))}{(1+\lambda_1)\kappa_2 - \delta^2(1+\lambda_1)(1+\gamma_2)}.$$

A.7 Region 1.b-III

$$R_{1.b-III} = \Big\{(r_1, r_2)\Big| \bar{r}_1 > \frac{\delta(1+\gamma_2)\lambda_1 r_1 + (1+\lambda_1)(\gamma_2 - \gamma_2 r_2 + (1-\delta)(1+\lambda_2))}{(1+\lambda_1)\eta_2 - \delta^2(1+\lambda_1)(1+\gamma_2) + \delta(1+\gamma_2)\lambda_1}$$
$$\text{and } r_2 < \frac{\delta(1+\lambda_1)(1+\gamma_2) - \delta^2(1+\lambda_1)(1+\gamma_2) - \delta(1+\gamma_2)(\lambda_1 r_1 - \lambda_1 \bar{r}_1)}{(1+\lambda_1)\eta_2 - \delta^2(1+\lambda_1)(1+\gamma_2) - (1+\lambda_1)\gamma_2}\Big\},$$

where $\eta_i = 1 + \gamma_i + \delta\gamma_i + (1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{1.b-III} = \frac{\delta(1+\gamma_2)(\lambda_1 r_1 - \lambda_1 \bar{r}_1) + (1+\lambda_1)(\gamma_2 - \gamma_2 r_2 + (1-\delta)(1+\lambda_2))}{(1+\lambda_1)\eta_2 - \delta^2(1+\lambda_1)(1+\gamma_2)}$$
$$y_1^{1.b-III} = \frac{\eta_2(\lambda_1 r_1 - \lambda_1 \bar{r}_1) + \delta(1+\lambda_1)(\gamma_2 - \gamma_2 r_2 + (1-\delta)(1+\lambda_2))}{(1+\lambda_1)\eta_2 - \delta^2(1+\lambda_1)(1+\gamma_2)}.$$

A.8 Region 2-II

$$R_{2-II} = \Big\{(r_1, r_2)\Big| r_1 \geq \frac{\delta(1+\gamma_1)\kappa_2 - \delta(1+\gamma_1)(\delta(1+\gamma_2) + \lambda_2 r_2)}{(\kappa_1 - \lambda_1)\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)} \geq \bar{r}_1$$
$$\text{and } r_2 \geq \frac{\delta(1+\gamma_2)(\kappa_1 - \lambda_1 r_1) - \delta^2(1+\gamma_1)(1+\gamma_2)}{\kappa_1(\kappa_2 - \lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2)} \geq \bar{r}_2\Big\},$$

where $\kappa_i = 1 + \lambda_i + \delta\gamma_i + (1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{2-II} = \frac{\kappa_1(\kappa_2 - \lambda_2 r_2) - \delta(1+\gamma_2)(\kappa_1 - \lambda_1 r_1)}{\kappa_1\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$
$$y_1^{2-II} = \frac{\kappa_2(\delta(1+\gamma_1) + \lambda_1 r_1) - \delta(1+\gamma_1)(\delta(1+\gamma_2) + \lambda_2 r_2)}{\kappa_1\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}.$$

A.9 Region 2-III

$$R_{2-III} = \Big\{(r_1, r_2)\Big| r_1 \geq \frac{\delta(1+\gamma_1)\eta_2 - \delta(1+\gamma_1)(\delta(1+\gamma_2) + \gamma_2 r_2)}{(\kappa_1 - \lambda_1)\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)} \geq \bar{r}_1$$
$$\text{and } r_2 < \frac{\delta(1+\gamma_2)(\kappa_1 - \lambda_1 r_1) - \delta^2(1+\gamma_1)(1+\gamma_2)}{\kappa_1(\eta_2 - \gamma_2) - \delta^2(1+\gamma_1)(1+\gamma_2)}\Big\},$$

where $\kappa_i = 1 + \lambda_i + \delta\gamma_i + (1-\delta)\lambda_i$ and $\eta_i = 1 + \gamma_i + \delta\gamma_i + (1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{2-III} = \frac{\kappa_1(\eta_2 - \gamma_2 r_2) - \delta(1+\gamma_2)(\kappa_1 - \lambda_1 r_1)}{\kappa_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$
$$y_1^{2-III} = \frac{\eta_2(\delta(1+\gamma_1) + \lambda_1 r_1) - \delta(1+\gamma_1)(\delta(1+\gamma_2) + \gamma_2 r_2)}{\kappa_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}.$$

A.10 Region 3-III

$$R_{3-III} = \left\{(r_1, r_2) \middle| r_1 < \frac{\delta(1+\gamma_1)\eta_2 - \delta(1+\gamma_1)(\delta(1+\gamma_2)+\gamma_2 r_2)}{(\eta_1-\gamma_1)\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)} \right.$$
$$\left. \text{and } r_2 < \frac{\delta(1+\gamma_2)(\eta_1-\gamma_1 r_1) - \delta^2(1+\gamma_1)(1+\gamma_2)}{\eta_1(\eta_2-\gamma_2) - \delta^2(1+\gamma_1)(1+\gamma_2)} \right\},$$

where $\eta_i = 1+\gamma_i+\delta\gamma_i+(1-\delta)\lambda_i$ for all $i = 1, 2$. The equilibrium strategies are

$$x_1^{3-III} = \frac{\eta_1(\eta_2-\gamma_2 r_2) - \delta(1+\gamma_2)(\eta_1-\gamma_1 r_1)}{\eta_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}$$
$$y_1^{3-III} = \frac{\eta_2(\delta(1+\gamma_1)+\gamma_1 r_1) - \delta(1+\gamma_1)(\delta(1+\gamma_2)+\gamma_2 r_2)}{\eta_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)}.$$

We do not describe the regions $\{R_{1.b-I.a}, R_{2,I.a}, R_{2,I.b}, R_{3,I.a}, R_{3,I.b}, R_{3,II}\}$ since they are the symmetric versions of the regions $\{R_{1.a-I.b}, R_{1.a,II}, R_{1.b,II}, R_{1.a,III}, R_{1.b,III}, R_{2,III}\}$, respectively. Hence, the corresponding equilibrium strategies can be found similarly. For the model with unlimited influence, the regions can be defined and the corresponding equilibrium strategies can be easily evaluated by assuming $(r_1, r_2) = (\bar{r}_1, \bar{r}_2)$.

B Proofs

Here, we give an explicit example of why we need Assumption X in the limited influence case.

Example Suppose that the game is at an odd period t and $r_1^t \geq \bar{r}_1^t > y_1^\omega$. We try to show that x^ω is the equilibrium offer. Hence, we need to show that proposing x^ω is the best response of player 1. Now, assume that player 1 makes an offer z such that $z_1 > x_1^\omega$. Since $z_2 < x_2^\omega$, player 2 rejects the offer. Player 1's expected continuation utility is

$$\begin{aligned} EU_1 &= \delta u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\ &= \delta[y_1^\omega + \lambda_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\ &= \delta(1+\lambda_1)y_1^\omega - \lambda_1 r_1. \end{aligned}$$

On the other hand, the equilibrium offer x^ω and its acceptance yield the following utility

$$AU_1 = (1+\lambda_1)x_1^\omega - \lambda_1 r_1.$$

Here, unless we make the assumption $x_1 > y_1$, we cannot decide whether $AU_1 \geq EU_1$ or $EU_1 > AU_1$. Therefore, in order to proceed, we need to make this assumption. We do not know whether Theorem 1 holds without this assumption though.

For the proof of Theorem 1, we first recall the Corollary of Theorem in Hendon et al. [23], which we will employ in our proof: *one-deviation principle* holds in infinite horizon extensive-form games, which are continuous at infinity.

Definition 1 (*Continuity at infinity*) A game is continuous at infinity if for any player i and for any $\varepsilon > 0$, there exists a period $\bar{t}$ such that if two strategy profiles σ and σ' satisfy for all $s \leq \bar{t}$, $\sigma^s = \sigma'^s$, then $|U_i(\sigma) - U_i(\sigma')| < \varepsilon$, where $U_i(\sigma)$ is the sum of the discounted utilities accrued at each period in strategy profile σ.

Lemma 2 *The bargaining game Γ is continuous at infinity.*

Proof of Lemma 2 Let $\varepsilon > 0$ and let σ, σ' be strategy profiles satisfying $\sigma^s = \sigma'^s$ for all $s \leq \bar{t}$ where $\bar{t} > \max_{i=1,2} \log_\delta \left(\frac{\varepsilon}{1+\gamma_i+\lambda_i}\right)$. Let $\bar{\sigma}$ be the strategy profile that maximizes U_i satisfying $(\bar{\sigma}^s)_{s=1}^{\bar{t}} = (\sigma^s)_{s=1}^{\bar{t}}$. Hence, in $\bar{\sigma}$ player i gets the entire pie in period $\bar{t}+1$.

$$\bar{U}_i = \max_{\bar{\sigma} \text{ s.t. } (\bar{\sigma}^s)_{s=1}^{\bar{t}}=(\sigma^s)_{s=1}^{\bar{t}}} U_i(\bar{\sigma}) = \delta^{\bar{t}}(1+\gamma_i(1-r_i^{\bar{t}+1})) + (1-\delta)\sum_{s=1}^{\bar{t}} \delta^{s-1} u_i^s(0)$$

Assume $\underline{\sigma}$ be the strategy profile that minimizes U_i satisfying $(\underline{\sigma}^s)_{s=1}^{\bar{t}} = (\sigma^s)_{s=1}^{\bar{t}}$, that is the strategy profile leading to perpetual disagreement.

$$\underline{U}_i = \min_{\underline{\sigma} \text{ s.t. } (\underline{\sigma}^s)_{s=1}^{\bar{t}}=(\sigma^s)_{s=1}^{\bar{t}}} U_i(\underline{\sigma}) = (1-\delta)\sum_{s=1}^{\infty} \delta^{s-1} u_i^s(0)$$

Note that the largest payoff difference between any two strategy profiles implying same actions in the first $\bar{t}$ periods is $\bar{U}_i - \underline{U}_i$. Utilizing this observation,

$$\begin{aligned}
|U_i(\sigma) - U_i(\sigma')| &\leq \bar{U}_i - \underline{U}_i \\
&= \delta^{\bar{t}}(1+\gamma_i(1-r_i^{\bar{t}+1})) + (1-\delta)\sum_{s=1}^{\bar{t}} \delta^{s-1} u_i^s(0) \\
&\quad - (1-\delta)\sum_{s=1}^{\bar{t}} \delta^{s-1} u_i^s(0) - (1-\delta)\sum_{s=\bar{t}+1}^{\infty} \delta^{s-1} u_i^s(0) \\
&= \delta^{\bar{t}}(1+\gamma_i(1-r_i^{\bar{t}+1})) - (1-\delta)\sum_{s=\bar{t}+1}^{\infty} \delta^{s-1}(-\lambda_i r_i^s) \\
&\leq \delta^{\bar{t}}(1+\gamma_i) + (1-\delta)\sum_{s=\bar{t}+1}^{\infty} \delta^{s-1}\lambda_i \\
&= \delta^{\bar{t}}(1+\gamma_i) + (1-\delta)\delta^{\bar{t}}\lambda_i \frac{1}{1-\delta} \\
&= \delta^{\bar{t}}(1+\gamma_i+\lambda_i) < \varepsilon.
\end{aligned}$$

Hence, the game is continuous at infinity. □

Proof of Theorem 1 We prove that σ^* is a subgame perfect equilibrium of the game. Utilizing *Lemma 1*, it is enough to check that no player can make a profitable deviation from his strategy σ_i^* in one single period, given that his opponent plays the strategy σ_j^*.

Suppose that the game is at period t. Let $(r_1^t, r_2^t) = (r_1, r_2) \in R_\omega$, where $\omega \in \{1.a - I.a, 1.a - I.b, \ldots, 3 - III\}$, and let $(\bar{r}_1^t, \bar{r}_2^t) = (\bar{r}_1, \bar{r}_2)$. Denote the utility of player i by u_i^* when he follows the strategy σ_i^*.

The current period t may be either odd or even. First, we investigate the case where t is odd. Hence, it is player 1's turn to make an offer. Suppose that player 1 offers $z \in Z$.

Case 1.a $\omega \in \{1.a - I.a, 1.a - I.b, 1.a - II, 1.a - III\} : r_1 \geq \bar{r}_1 > y_1^\omega$ and $x_1^\omega \geq \bar{r}_1$.

1.a.1 We first analyze the case where $x_1^\omega \geq r_1$. We have three possible subcases: (i) $z_1 = x_1^\omega$, (ii) $z_1 < x_1^\omega$, and (iii) $z_1 > x_1^\omega$.

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1) = (1+\gamma_1)x_1^\omega - \gamma_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following σ_2^* since $z_2 > x_2^\omega$. So, player 1 gets

$$u_1^t(z, r^t) \leq z_1 + \gamma_1(z_1 - r_1) < (1+\gamma_1)x_1^\omega - \gamma_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer since $z_2 < x_2^\omega$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, player 1's expected continuation utility equals to

$$\begin{aligned}
\delta u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) &= \delta[y_1^\omega + \lambda_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\
&= \delta y_1^\omega + \delta\lambda_1 y_1^\omega - \lambda_1 r_1 \\
&< y_1^\omega + \lambda_1 y_1^\omega - \lambda_1 r_1 \\
&= y_1^\omega + \lambda_1 \underbrace{(y_1^\omega - r_1)}_{<0} \\
&\leq y_1^\omega + \gamma_1(y_1^\omega - r_1) \\
&< x_1^\omega + \gamma_1(x_1^\omega - r_1) = u_1^*.
\end{aligned}$$

To summarize, making an offer z satisfying $z_1 = x_1^\omega$ maximizes player 1's utility, given that player 2 follows σ_2^*. Thus, following σ_1^* is optimal.

1.a.2 Now, we analyze the case where $x_1^\omega < r_1$. Similarly, we have three possible subcases: (i) $z_1 = x_1^\omega$, (ii) $z_1 < x_1^\omega$, and (iii) $z_1 > x_1^\omega$.

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1) = (1+\lambda_1)x_1^\omega - \lambda_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following σ_2^* since $z_2 > x_2^\omega$. So, player 1 gets

$$u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1) < (1+\lambda_1)x_1^\omega - \lambda_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer since $z_2 < x_2^\omega$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, player 1's expected continuation utility is

$$\begin{aligned}
\delta u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) &= \delta[y_1^\omega + \lambda_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\
&= \delta(1+\lambda_1)y_1^\omega - \lambda_1 r_1 \\
&< (1+\lambda_1)y_1^\omega - \lambda_1 r_1 \\
&< (1+\lambda_1)x_1^\omega - \lambda_1 r_1 = u_1^*.
\end{aligned}$$

To summarize, making an offer z satisfying $z_1 = x_1^\omega$ maximizes player 1's utility, given that player 2 follows σ_2^*. Thus, following σ_1^* is optimal.

Case 1.b $\omega \in \{1.b - I.a, 1.b - I.b, 1.b - II, 1.b - III\} : r_1 \geq \bar{r}_1 > y_1^\omega$ and $x_1^\omega < \bar{r}_1$.

In this case, we also have three possible subcases: (i) $z_1 = x_1^\omega$, (ii) $z_1 < x_1^\omega$, and (iii) $z_1 > x_1^\omega$.

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1) = (1 + \lambda_1)x_1^\omega - \lambda_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following σ_2^* since $z_2 > x_2^\omega$. So, player 1 gets

$$u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1) < (1 + \lambda_1)x_1^\omega - \lambda_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer since $z_2 < x_2^\omega$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, player 1's expected continuation utility is

$$\begin{aligned}
&\delta u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&\quad = \delta[y_1^\omega + \lambda_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\
&\quad = \delta(1+\lambda_1)y_1^\omega - \lambda_1 r_1 \\
&\quad < (1+\lambda_1)y_1^\omega - \lambda_1 r_1 \\
&\quad < (1+\lambda_1)x_1^\omega - \lambda_1 r_1 = u_1^*.
\end{aligned}$$

The last inequality follows from Assumption X. To summarize, making an offer z satisfying $z_1 = x_1^\omega$ maximizes player 1's utility, given that player 2 follows σ_2^*. Thus, following σ_1^* is optimal.

Case 2 $\omega \in \{2 - I.a, 2 - I.b, 2 - II, 2 - III\} : r_1 \geq y_1^\omega \geq \bar{r}_1$

In this region, we either have $x_1^\omega \geq r_1$ or $x_1^\omega < r_1$. We analyze these cases separately.

2.1. We first investigate the case where $x_1^\omega \geq r_1$. We have three subcases:

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1) = (1 + \gamma_1)x_1^\omega - \gamma_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following σ_2^* since $z_2 > x_2^\omega$. So, player 1 gets

$$u_1^t(z, r^t) \leq z_1 + \gamma_1(z_1 - r_1) < (1 + \gamma_1)x_1^\omega - \gamma_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1 - \delta$, the game ends. So, player 1's expected continuation utility is

$$\begin{aligned}
\delta u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) &= \delta[y_1^\omega + \lambda_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\
&= \delta y_1^\omega + \delta\lambda_1 y_1^\omega - \lambda_1 r_1 \\
&< y_1^\omega + \lambda_1 y_1^\omega - \lambda_1 r_1 \\
&= y_1^\omega + \lambda_1 \underbrace{(y_1^\omega - r_1)}_{\leq 0} \\
&\leq y_1^\omega + \gamma_1(y_1^\omega - r_1) \\
&< x_1^\omega - \gamma_1(x_1^\omega - r_1) = u_1^*.
\end{aligned}$$

To summarize, making an offer z satisfying $z_1 = x_1^\omega$ maximizes player 1's utility, given that player 2 follows σ_2^*. Thus, following σ_1^* is optimal.

2.2. Now, we analyze the second case, $x_1^\omega < r_1$.

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1) = (1 + \lambda_1)x_1^\omega - \lambda_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following σ_2^* since $z_2 > x_2^\omega$. So, player 1 gets

$$u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1) < (1 + \lambda_1)x_1^\omega - \lambda_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer since $z_2 < x_2^\omega$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, player 1's expected continuation utility is

$$\begin{aligned}
\delta u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) &= \delta[y_1^\omega + \lambda_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\
&= \delta y_1^\omega - \delta\lambda_1 y_1^\omega - \lambda_1 r_1 \\
&< (1 + \lambda_1)y_1^\omega - \lambda_1 r_1 \\
&< (1 + \lambda_1)x_1^\omega - \lambda_1 r_1 = u_1^*.
\end{aligned}$$

The last inequality follows from Assumption X. To summarize, making an offer z satisfying $z_1 = x_1^\omega$ maximizes player 1's utility, given that player 2 follows σ_2^*. Thus, following σ_1^* is optimal.

Case 3 $\omega \in \{3 - I.a, 3 - I.b, 3 - II, 3 - III\} : y_1^\omega > r_1 \geq \bar{r}_1$
We have three possible subcases:

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1) = (1 + \gamma_1)x_1^\omega - \gamma_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following σ_2^* since $z_2 > x_2^\omega$. So, player 1 gets

$$u_1^t(z, r^t) \leq z_1 + \gamma_1(z_1 - r_1) < (1 + \gamma_1)x_1^\omega - \gamma_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer since $z_2 < x_2^\omega$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, player 1's expected continuation utility is

$$\begin{aligned}
\delta u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) &= \delta[y_1^\omega + \gamma_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\
&< \delta[y_1^\omega + \gamma_1(y_1^\omega - r_1)] - (1-\delta)\gamma_1 r_1 \\
&= (1 + \gamma_1)\delta y_1^\omega - \gamma_1 r_1 \\
&< (1 + \gamma_1)y_1^\omega - \gamma_1 r_1 \\
&< (1 + \gamma_1)x_1^\omega - \gamma_1 r_1 = u_1^*.
\end{aligned}$$

The last inequality follows from Assumption X. To summarize, making an offer z satisfying $z_1 = x_1^\omega$ maximizes player 1's utility, given that player 2 follows σ_2^*. Thus, following σ_1^* is optimal.

Second, we analyze the case where t is even (i.e., player 2's turn to make an offer). Suppose that player 2 offers $z \in Z$. If player 1 accepts the offer, he gets

$$u_1^t(z, r^t) = \begin{cases} z_i + \gamma_i(z_i - r_i) & \text{if } z_i \geq r_i \\ z_i + \lambda_i(z_i - r_i) & \text{if } z_i < r_i \end{cases}$$

If player 1 rejects the offer, his reference point will be $\max\{\bar{r}_1, z_1\}$. With probability δ, the game continues to the next period and player 1 offers x^ω and with probability $1-\delta$, the game ends. Hence, player 1's expected continuation utility in the case of rejection equals to

$$\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}).$$

Case 1.a $\omega \in \{1.a - I.a, 1.a - I.b, 1.a - II, 1.a - III\}$: $r_1 \geq \bar{r}_1 > y_1^\omega$ and $x_1^\omega \geq \bar{r}_1$
In this region, note that the following equality holds in the equilibrium:

$$(1+\gamma_1)\delta x_1^\omega = (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + (1-\delta)\lambda_1\bar{r}_1 + \delta\gamma_1\bar{r}_1.$$

We analyze the optimal decisions of player 1 in two subcases: (1) $z_1 \geq y_1^\omega$ and (2) $z_1 < y_1^\omega$.

1.a.1 First, suppose that $z_1 \geq y_1^\omega$. Further, there are two more cases: (i) $z_1 \geq r_1$ or (ii) $z_1 < r_1$.

(i) Suppose that $z_1 \geq r_1$. If player 1 accepts the offer, he gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1).$$

If player 1 rejects the offer, his reference point will be $\max\{\bar{r}_1, z_1\}$. With probability δ, the game continues to the next period and player 1 offers x^ω and with probability $1-\delta$, the game ends. So, the rejection gives player 1

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1})\\
&\quad\leq \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1\max\{\bar{r}_1, z_1\}\\
&\quad= (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1\\
&\quad= (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + (1-\delta)\lambda_1\bar{r}_1 + \delta\gamma_1\bar{r}_1 - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1\\
&\quad\leq (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + (1-\delta)\lambda_1 z_1 + \delta\gamma_1 z_1 - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1\\
&\quad\leq (1+\lambda_1)z_1 - \lambda_1 r_1\\
&\quad= z_1 + \lambda_1(z_1 - r_1)\\
&\quad\leq z_1 + \gamma_1(z_1 - r_1) = u_1^*.
\end{aligned}$$

In this case, accepting the offer gives player 1 a (weakly) higher utility than the rejection does. If $z_1 \geq y_1^\omega$, accepting the offer z is optimal.

(ii) Now, suppose that $z_1 < r_1$. If player 1 accepts the offer, he gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1).$$

If player 1 rejects the offer, he gets

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1})\\
&\quad\leq \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1\max\{\bar{r}_1, z_1\}\\
&\quad= (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1\max\{\bar{r}_1, z_1\} - (1-\delta)\lambda_1\max\{\bar{r}_1, z_1\}\\
&\quad= (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + (1-\delta)\lambda_1\bar{r}_1 + \delta\gamma_1\bar{r}_1 - \delta\gamma_1\max\{\bar{r}_1, z_1\}\\
&\qquad -(1-\delta)\lambda_1\max\{\bar{r}_1, z_1\}\\
&\quad\leq (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + (1-\delta)\lambda_1\bar{r}_1 + \delta\gamma_1\bar{r}_1 - \delta\gamma_1\bar{r}_1 - (1-\delta)\lambda_1\bar{r}_1\\
&\quad= (1+\lambda_1)y_1^\omega - \lambda_1 r_1\\
&\quad\leq (1+\lambda_1)z_1 - \lambda_1 r_1 = u_1^*.
\end{aligned}$$

In this case, accepting the offer gives player 1 a higher utility than the rejection does. If $z_1 \geq y_1^\omega$, accepting the offer z is optimal.

1.a.2 Second, suppose that $z_1 < y_1^\omega$. If player 1 accepts the offer, he gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1).$$

If player 1 rejects the offer, he gets

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&= \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&= (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1\bar{r}_1 - (1-\delta)\lambda_1\bar{r}_1 \\
&= (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + (1-\delta)\lambda_1\bar{r}_1 + \delta\gamma_1\bar{r}_1 - \delta\gamma_1\bar{r}_1 - (1-\delta)\lambda_1\bar{r}_1 \\
&= (1+\lambda_1)y_1^\omega - \lambda_1 r_1 \\
&> (1+\lambda_1)z_1 - \lambda_1 r_1 = u_1^*.
\end{aligned}$$

In this case, rejecting the offer gives player 1 a higher utility than accepting does. Thus, if $z_1 < y_1^\omega$, then rejecting the offer z is optimal.

To summarize, accepting the offer z is optimal if and only if $z_1 \geq y_1^\omega$.

Case 1.b $\omega \in \{1.b - I.a, 1.b - I.b, 1.b - II, 1.b - III\}$: $r_1 \geq \bar{r}_1 > y_1^\omega$ and $x_1^\omega < \bar{r}_1$.

In this region, note that the following equality holds in the equilibrium:

$$(1+\lambda_1)\delta x_1^\omega = (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + \lambda_1\bar{r}_1.$$

We analyze the optimal decisions of player 1 in two subcases: (1) $z_1 \geq y_1^\omega$ and (2) $z_1 < y_1^\omega$.

1.b.1 First, suppose that $z_1 \geq y_1^\omega$. Further, there are two more cases: (i) $z_1 \geq r_1$ or (ii) $z_1 < r_1$.

(i) Let $z_1 \geq r_1$. If player 1 accepts the offer, he gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1).$$

If player 1 rejects the offer, his reference point will be $\max\{\bar{r}_1, z_1\}$. With probability δ, the game continues to the next period and player 1 offers x^ω and with probability $1-\delta$, the game ends. So, the rejection gives player 1

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&= \delta[x_1^\omega + \lambda_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&= (1+\lambda_1)\delta x_1^\omega - \delta\lambda_1 z_1 - (1-\delta)\lambda_1 z_1 \\
&= (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + \lambda_1\bar{r}_1 - \lambda_1 z_1 \\
&\leq (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + \lambda_1\bar{r}_1 - \lambda_1\bar{r}_1 \\
&= y_1^\omega + \lambda_1(y_1^\omega - r_1) \\
&\leq y_1^\omega + \gamma_1(y_1^\omega - r_1) \\
&\leq z_1 + \gamma_1(z_1 - r_1) = u_1^*.
\end{aligned}$$

In this case, accepting the offer gives player 1 a higher utility than the rejection does. If $z_1 \geq y_1^\omega$, accepting the offer z is optimal.

(ii) Now, let $z_1 < r_1$. If player 1 accepts the offer, he gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1).$$

If player 1 rejects the offer, he gets

$$\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1})$$

$$
\begin{aligned}
&= \delta[x_1^\omega + \lambda_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&= (1+\lambda_1)\delta x_1^\omega - \delta\lambda_1 \max\{\bar{r}_1, z_1\} - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&= (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + \lambda_1 \bar{r}_1 - \lambda_1 \max\{\bar{r}_1, z_1\} \\
&\le (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + \lambda_1 \bar{r}_1 - \lambda_1 \bar{r}_1 \\
&\le z_1 + \lambda_1(z_1 - r_1) = u_1^*.
\end{aligned}
$$

In this case, accepting the offer gives player 1 a higher utility than the rejection does. If $z_1 \ge y_1^\omega$, accepting the offer z is optimal.

1.b.2 Second, suppose that $z_1 < y_1^\omega$. Accepting the offer yields

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1).$$

If player 1 rejects the offer, he gets

$$
\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&\quad = \delta[x_1^\omega + \lambda_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad = (1+\lambda_1)\delta x_1^\omega - \lambda_1 \bar{r}_1 \\
&\quad = (1+\lambda_1)y_1^\omega - \lambda_1 r_1 + \lambda_1 \bar{r}_1 - \lambda_1 \bar{r}_1 \\
&\quad > z_1 + \lambda_1(z_1 - r_1) = u_1^*.
\end{aligned}
$$

In this case, rejecting the offer gives player 1 a higher utility than accepting does. If $z_1 < y_1^\omega$, then rejecting the offer z is optimal.

To summarize, accepting the offer z is optimal if and only if $z_1 \ge y_1^\omega$.

Case 2 $\omega \in \{2 - I.a, 2 - I.b, 2 - II, 2 - III\}$: $r_1 \ge y_1^\omega \ge \bar{r}_1$

In this region, note that the following equality holds in the equilibrium:

$$(1+\gamma_1)\delta x_1^\omega = (1 + \lambda_1 + \delta\gamma_1 + (1-\delta)\lambda_1)y_1^\omega - \lambda_1 r_1.$$

We analyze the optimal decisions of player 1 in two subcases: (1) $z_1 \ge y_1^\omega$ and (2) $z_1 < y_1^\omega$.

2.1. Suppose that $z_1 \ge y_1^\omega$. Accepting the offer yields

$$u_1^* = u_1^t(z, r^t) = \begin{cases} z_i + \gamma_i(z_i - r_i) & \text{if } z_i \ge r_i \\ z_i + \lambda_i(z_i - r_i) & \text{if } z_i < r_i \end{cases}$$

If player 1 rejects the offer, his reference point will be $\max\{\bar{r}_1, z_1\}$. With probability δ, the game continues to the next period and player 1 offers x^ω and with probability $1-\delta$, the game ends. So, the rejection gives player 1

$$
\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&\quad \le \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad = (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1 \\
&\quad = (1 + \lambda_1 + \delta\gamma_1 + (1-\delta)\lambda_1)y_1^\omega - \lambda_1 r_1 - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1 \\
&\quad \le (1+\lambda_1)y_1^\omega + \delta\gamma_1 y_1^\omega + (1-\delta)\lambda_1 y_1^\omega - \lambda_1 r_1 - \delta\gamma_1 y_1^\omega - (1-\delta)\lambda_1 y_1^\omega \\
&\quad = y_1^\omega + \lambda_1(y_1^\omega - r_1) \\
&\quad \le z_1 + \lambda_1(z_1 - r_1) \le u_1^*.
\end{aligned}
$$

In this case, accepting the offer gives player 1 a higher utility than the rejection does. If $z_1 \ge y_1^\omega$, accepting the offer z is optimal.

2.2. Now, suppose that $z_1 < y_1^\omega$. Accepting the offer yields

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1).$$

If player 1 rejects the offer, he gets

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&\quad = \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad = (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1 \max\{\bar{r}_1, z_1\} - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad = (1 + \lambda_1 + \delta\gamma_1 + (1-\delta)\lambda_1)y_1^\omega - \lambda_1 r_1 - \delta\gamma_1 \max\{\bar{r}_1, z_1\} \\
&\qquad -(1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad \geq (1+\lambda_1)y_1^\omega + (\delta\gamma_1 + (1-\delta)\lambda_1) \max\{\bar{r}_1, z_1\} - \lambda_1 r_1 \\
&\qquad -(\delta\gamma_1 + (1-\delta)\lambda_1) \max\{\bar{r}_1, z_1\} \\
&\quad = (1+\lambda_1)y_1^\omega - \lambda_1 r_1 \\
&\quad > z_1 + \lambda_1(z_1 - r_1) = u_1^*.
\end{aligned}$$

In this case, rejecting the offer gives player 1 a higher utility than accepting does. If $z_1 < y_1^\omega$, then rejecting the offer z is optimal.

To summarize, accepting the offer z is optimal if and only if $z_1 \geq y_1^\omega$.

Case 3 $\omega \in \{3 - I.a, 3 - I.b, 3 - II, 3 - III\}$: $y_1^\omega > r_1 \geq \bar{r}_1$

In this region, note that the following equality holds in the equilibrium:

$$(1+\gamma_1)\delta x_1^\omega = (1 + \gamma_1 + \delta\gamma_1 + (1-\delta)\lambda_1)y_1^\omega - \gamma_1 r_1.$$

We analyze the optimal decisions of player 1 in two subcases: (1) $z_1 \geq y_1^\omega$ and (2) $z_1 < y_1^\omega$.

3.1. Suppose that $z_1 \geq y_1^\omega$. If player 1 accepts the offer, he gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1).$$

If player 1 rejects the offer, his reference point will be $\max\{\bar{r}_1, z_1\}$. With probability δ, the game continues to the next period and player 1 offers x^ω and with probability $1-\delta$, the game ends. So, rejection gives player 1

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&\quad \leq \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad = (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1 \\
&\quad = (1 + \gamma_1 + \delta\gamma_1 + (1-\delta)\lambda_1)y_1^\omega - \gamma_1 r_1 - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1 \\
&\quad \leq (1+\gamma_1)y_1^\omega + (\delta\gamma_1 + (1-\delta)\lambda_1)z_1 - \gamma_1 r_1 - (\delta\gamma_1 + (1-\delta)\lambda_1)z_1 \\
&\quad = (1+\gamma_1)y_1^\omega - \gamma_1 r_1 \\
&\quad \leq z_1 + \gamma_1(z_1 - r_1) = u_1^*.
\end{aligned}$$

In this case, accepting the offer gives player 1 a higher utility than rejection does. If $z_1 \geq y_1^\omega$, accepting the offer z is optimal.

3.2. Now, suppose that $z_1 < y_1^\omega$. Accepting the offer yields

$$u_1^t(z, r^t) = \begin{cases} z_i + \gamma_i(z_i - r_i) & \text{if } z_i \geq r_i \\ z_i + \lambda_i(z_i - r_i) & \text{if } z_i < r_i \end{cases}$$

If player 1 rejects the offer, he gets

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&\quad = \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{\bar{r}_1, z_1\})] - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad = (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1 \max\{\bar{r}_1, z_1\} - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad = (1+\gamma_1+\delta\gamma_1+(1-\delta)\lambda_1)y_1^\omega - \gamma_1 r_1 - \delta\gamma_1 \max\{\bar{r}_1, z_1\} \\
&\qquad - (1-\delta)\lambda_1 \max\{\bar{r}_1, z_1\} \\
&\quad > (1+\gamma_1)y_1^\omega + (\delta\gamma_1+(1-\delta)\lambda_1)\max\{\bar{r}_1, z_1\} - \gamma_1 r_1 \\
&\qquad - (\delta\gamma_1+(1-\delta)\lambda_1)\max\{\bar{r}_1, z_1\} \\
&\quad = (1+\gamma_1)y_1^\omega - \gamma_1 r_1 \\
&\quad > z_1 + \gamma_1(z_1 - r_1) \geq u_1^*.
\end{aligned}$$

In this case, rejecting the offer gives player 1 a higher utility than accepting does. If $z_1 < y_1^\omega$, then rejecting the offer z is optimal.

To summarize, accepting the offer z is optimal if and only if $z_1 \geq y_1^\omega$.

Considering all cases, we have the following result: it is optimal to accept the offer z if $z_1 \geq y_1^\omega$ and to reject it otherwise, which means following σ_1^* is optimal.

Proof of Theorem 2 We prove that σ^* is a subgame perfect equilibrium. Utilizing *Lemma 1*, it is enough to check that no player can make a profitable deviation from his strategy σ_i^* in one single period, given that his opponent plays σ_j^*. Suppose that the game is at period t. Let $(r_1^t, r_2^t) = (r_1, r_2) \in R_\omega$ where $\omega \in \{1-I, 1-II, \ldots, 3-III\}$.

The current period t may be either odd or even. First, we investigate the case where t is odd. Hence, player 1 makes an offer $z \in Z$.

Case 1 $\omega \in \{1-I, 1-II, 1-III\} : r_1 > x_1^\omega > y_1^\omega$

In this case, the reference point of player 1 is greater than his share in the associated offer with the region R_ω. We have three possible subcases: (i) $z_1 = x_1^\omega$, (ii) $z_1 < x_1^\omega$, and (iii) $z_1 > x_1^\omega$.

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1) = (1+\lambda_1)x_1^\omega - \lambda_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following strategy σ_2^* since $z_2 > x_2^\omega$. So, player 1 gets

$$u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1) = (1+\lambda_1)z_1\lambda_1 r_1 < (1+\lambda_1)x_1^\omega - \lambda_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer since $z_2 < x_2^\omega$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, player 1's expected continuation utility is

$$\begin{aligned}
u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) &= \delta[y_1^\omega + \lambda_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\
&= (1+\lambda_1)\delta y_1^\omega - \delta\lambda_1 r_1 - (1-\delta)\lambda_1 r_1 \\
&= (1+\lambda_1)\delta y_1^\omega - \lambda_1 r_1 \\
&< (1+\lambda_1)\delta x_1^\omega - \lambda_1 r_1 = u_1^*.
\end{aligned}$$

To summarize, making the offer z satisfying $z_1 = x_1^\omega$ gives player 1 the maximum utility given that player 2 follows σ_2^*. Thus, σ_1^* is optimal.

Case 2 $\omega \in \{2-I, 2-II, 2-III\} : x_1^\omega \geq r_1 > y_1^\omega$

In this case, the reference point of player 1 belongs to the region which is bounded by his and his opponent's share from the associated offer with the region R_ω. Again, we have three distinct subcases: (i) $z_1 = x_1^\omega$, (ii) $z_1 < x_1^\omega$, and (iii) $z_1 > x_1^\omega$.

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1) = (1+\gamma_1)x_1^\omega - \gamma_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following σ_2^* since $z_2 > x_2^\omega$.

If $r_1 > z_1$, then player 1 gets

$$\begin{aligned} u_1^t(z, r^t) &= z_1 + \lambda_1(z_1 - r_1) \leq z_1 + \gamma_1(z_1 - r_1) \\ &= (1+\gamma_1)z_1 - \gamma_1 r_1 < (1+\gamma_1)x_1^\omega - \gamma_1 r_1 = u_1^*. \end{aligned}$$

If $r_1 \leq z_1$, then player 1 gets

$$u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1) < (1+\gamma_1)x_1^\omega - \gamma_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer since $z_2 < x_2^\omega$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, player 1's expected continuation utility is

$$\begin{aligned} u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) &= \delta[y_1^\omega + \lambda_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\ &< \delta[y_1^\omega + \gamma_1(y_1^\omega - r_1)] - (1-\delta)\gamma_1 r_1 \\ &= (1+\gamma_1)\delta y_1^\omega - \gamma_1 r_1 \\ &< (1+\gamma_1)\delta x_1^\omega - \gamma_1 r_1 = u_1^*. \end{aligned}$$

To summarize, making the offer z satisfying $z_1 = x_1^\omega$ gives player 1 the maximum utility given that player 2 follows σ_2^*. Thus, σ_1^* is optimal.

Case 3 $\omega \in \{3-I, 3-II, 3-III\} : x_1^\omega > y_1^\omega \geq r_1$

In this case, the reference point of the first player is less than both his share offer and his opponent's share from the associated offer with the region R_ω. We have three possible values for his share obtained from the offer z.

(i) If $z_1 = x_1^\omega$, then player 2 accepts the offer by following σ_2^*. So, player 1 gets

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1) = (1+\gamma_1)x_1^\omega - \gamma_1 r_1.$$

(ii) If $z_1 < x_1^\omega$, then player 2 accepts the offer by following σ_2^* since $z_2 > x_2^\omega$.

If $r_1 > z_1$, then player 1 gets

$$\begin{aligned} u_1^t(z, r^t) &= z_1 + \lambda_1(z_1 - r_1) \leq z_1 + \gamma_1(z_1 - r_1) \\ &= (1+\gamma_1)z_1 - \gamma_1 r_1 < (1+\gamma_1)x_1^\omega - \gamma_1 r_1 = u_1^*. \end{aligned}$$

If $r_1 \leq z_1$, then player 1 gets

$$u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1) < (1+\gamma_1)x_1^\omega - \gamma_1 r_1 = u_1^*.$$

(iii) If $z_1 > x_1^\omega$, then player 2 rejects the offer since $z_2 < x_2^\omega$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, player 1's expected continuation utility is

$$u_1^{t+1}(y^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) = \delta[y_1^\omega + \gamma_1(y_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1$$

$$
\begin{aligned}
&< \delta[y_1^\omega + \gamma_1(y_1^\omega - r_1)] - (1-\delta)\gamma_1 r_1 \\
&= (1+\gamma_1)\delta y_1^\omega - \gamma_1 r_1 \\
&< (1+\gamma_1)\delta x_1^\omega - \gamma_1 r_1 = u_1^*.
\end{aligned}
$$

To summarize, making the offer z satisfying $z_1 = x_1^\omega$ gives player 1 the maximum utility given that player 2 follows σ_2^*. Thus, σ_1^* is optimal.

Now, we analyze the case where t is even (i.e., player 2's turn to make an offer). Hence, player 2 makes an offer $z \in Z$. If player 1 accepts the offer, then he gets

$$
u_1^t(z, r^t) = \begin{cases} z_i + \gamma_i(z_i - r_i) & \text{if } z_i \geq r_i \\ z_i + \lambda_i(z_i - r_i) & \text{if } z_i < r_i \end{cases}
$$

On the other hand, if player 1 rejects the offer, his reference point will be $\max\{r_1, z_1\}$. With probability δ, the game continues to the next period and player 2 offers y^ω and with probability $1-\delta$, the game ends. Hence, player 1's expected continuation utility in the case of rejection is

$$
\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}).
$$

Case 1 $\omega \in \{1-I, 1-II, 1-III\} : r_1 > x_1^\omega > y_1^\omega$

In this case, recall that the following equality holds in equilibrium:

$$
\delta x_1^\omega = y_1^\omega.
$$

We analyze the optimal decisions of player 1 in two subcases: (1) $z_1 \geq r_1$ and (2) $z_1 < r_1$.

1.1. First, suppose that $z_1 \geq r_1$. If player 1 accepts the offer, he gets his share z_1 plus the relative gain from the reference point. Since the offer is greater than his reference point, he perceives the difference between two as a gain and this gain is scaled by γ_1. Accepting yields

$$
u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1).
$$

If player 1 rejects the offer, his reference point will be $\max\{r_1, z_1\}$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, rejection gives player 1

$$
\begin{aligned}
&u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&\quad = \delta[x_1^\omega + \lambda_1(x_1^\omega - \max\{r_1, z_1\})] - (1-\delta)\lambda_1 \max\{r_1, z_1\} \\
&\quad < \delta[x_1^\omega + \gamma_1(x_1^\omega - z_1)] - (1-\delta)\gamma_1 z_1 \\
&\quad = (1+\gamma_1)\delta x_1^\omega - \gamma_1 z_1 \\
&\quad < (1+\gamma_1)y_1^\omega - \gamma_1 r_1 \\
&\quad < (1+\gamma_1)z_1 - \gamma_1 r_1 = u_1^*
\end{aligned}
$$

In this case, acceptance gives player 1 a higher utility than rejection does. If $z_1 \geq y_1^\omega$, accepting the offer z_1 is optimal.

1.2 Second, suppose that $z_1 < r_1$. In the case of acceptance, player 1 gets his share from the offer z_1. However, since the offer is less than his reference point, player 1 perceives this offer as a loss relative to the reference point. The difference between z_1 and r_1 negatively affects his utility and this effect is scaled by λ_1. Accepting the offer yields

$$
u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1).
$$

If player 1 rejects the offer, he gets

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&= \delta[x_1^\omega + \lambda_1(x_1^\omega - \max\{r_1, z_1\})] - (1-\delta)\lambda_1 \max\{r_1, z_1\} \\
&= \delta[x_1^\omega + \lambda_1(x_1^\omega - r_1)] - (1-\delta)\lambda_1 r_1 \\
&= (1+\lambda_1)\delta x_1^\omega - \lambda_1 r_1 \\
&= (1+\lambda_1)y_1^\omega - \lambda_1 r_1.
\end{aligned}$$

Hence, $\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \geq u_1^*$ if and only if $z_1 \geq y_1^\omega$. In this case, accepting the offer satisfying $z_1 \geq y_1^\omega$ and rejecting it otherwise is optimal.

Case 2 $\omega \in \{2-I, 2-II, 2-III\} : x_1^\omega \geq r_1 > y_1^\omega$
In this case, recall that the following equality holds in equilibrium:

$$(1+\gamma_1)\delta x_1^\omega = (1+\lambda_1)y_1^\omega + \delta\gamma_1 r_1 - \delta\lambda_1 r_1.$$

We analyze the optimal decisions of player 1 in two subcases: (1) $z_1 \geq r_1$ and (2) $z_1 < r_1$.

2.1. Suppose that $z_1 \geq r_1$. If player 1 accepts the offer, he gets his share z_1 plus the relative gain from the reference point. Since the offer is greater than his reference point, he perceives the difference between the two as a gain and this gain is scaled by γ_1 . Accepting yields

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1).$$

If player 1 rejects the offer, his reference point will be $\max\{r_1, z_1\}$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, rejection gives player 1

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&= \delta[x_1^\omega + \lambda_1(x_1^\omega - \max\{r_1, z_1\})] - (1-\delta)\lambda_1 \max\{r_1, z_1\} \\
&< \delta[x_1^\omega + \lambda_1(x_1^\omega - z_1)] - (1-\delta)\lambda_1 z_1 \\
&< \delta[x_1^\omega + \gamma_1(x_1^\omega - z_1)] - (1-\delta)\gamma_1 z_1 \\
&= (1+\gamma_1)\delta x_1^\omega - \gamma_1 z_1 \\
&= (1+\lambda_1)y_1^\omega + \delta\gamma_1 r_1 - \delta\lambda_1 r_1 - \gamma_1 z_1 \\
&< (1+\gamma_1)z_1 - \gamma_1 r_1 = u_1^*.
\end{aligned}$$

In this case, acceptance gives player 1 a higher utility than rejection does. If $z_1 \geq y_1^\omega$, accepting the offer z_1 is optimal.

Now, suppose that $z_1 < r_1$. In the case of acceptance, player 1 gets his share from the offer z_1. However, since the offer is less than his reference point, player 1 perceives this offer as a loss relative to the reference point. The difference between z_1 and r_1 negatively affects his utility, and this effect is scaled by λ_1. Accepting the offer yields

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1).$$

If player 1 rejects the offer, he gets

$$\begin{aligned}
&\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \\
&= \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{r_1, z_1\})] - (1-\delta)\lambda_1 \max\{r_1, z_1\} \\
&= (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1 \max\{r_1, z_1\} - (1-\delta)\lambda_1 \max\{r_1, z_1\}
\end{aligned}$$

$$= (1+\lambda_1)y_1^\omega + \delta\gamma_1 r_1 - \delta\lambda_1 r_1 - \delta\gamma_1 \max\{r_1, z_1\}$$
$$- (1-\delta)\lambda_1 \max\{r_1, z_1\}$$
$$= (1+\lambda_1)y_1^\omega + \delta\gamma_1 r_1 - \delta\lambda_1 r_1 - \delta\gamma_1 r_1 - (1-\delta)\lambda_1 r_1$$
$$= (1+\lambda_1)\delta y_1^\omega - \lambda_1 r_1.$$

Thus, $\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \geq u_1^*$ if and only if $z_1 \geq y_1^\omega$.
In this case, accepting the offer satisfying $z_1 \geq y_1^\omega$ and rejecting it otherwise are optimal.

Case 3 $\omega \in \{3-I, 3-II, 3-III\} : x_1^\omega > y_1^\omega \geq r_1$
Recall that the following equality holds in equilibrium:

$$(1+\gamma_1)\delta x_1^\omega = (1+\gamma_1+\delta\gamma_1+(1-\delta)\lambda_1)y_1^\omega - \gamma_1 r_1.$$

Again, we analyze the optimal decisions of player 1 in two distinct subcases: (1) $z_1 \geq r_1$ and (2) $z_1 < r_1$.

3.1. Suppose that $z_1 \geq r_1$. If player 1 accepts the offer, he gets his share z_1 plus the relative gain from the reference point. Since the offer is greater than his reference point, he perceives the difference between two as a gain and this gain is scaled by γ_1. Accepting yields

$$u_1^* = u_1^t(z, r^t) = z_1 + \gamma_1(z_1 - r_1).$$

If player 1 rejects the offer, his reference point will be $\max\{r_1, z_1\}$. With probability δ, the game continues to the next period and player 2 offers y^ω, and with probability $1-\delta$, the game ends. So, rejection gives player 1

$$\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1})$$
$$= \delta u_1^{t+1}(x^\omega) - (1-\delta)\lambda_1 \max\{r_1, z_1\}$$
$$\leq \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{r_1, z_1\})] - (1-\delta)\lambda_1 \max\{r_1, z_1\}$$
$$= (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1$$
$$= (1+\gamma_1)y_1^\omega + \delta\gamma_1 y_1^\omega + (1-\delta)\lambda_1 y_1^\omega - \gamma_1 r_1 - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1$$
$$< (1+\gamma_1)y_1^\omega + \delta\gamma_1 z_1 + (1-\delta)\lambda_1 z_1 - \gamma_1 r_1 - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1$$
$$= (1+\gamma_1)y_1^\omega - \gamma_1 r_1$$

In this case, acceptance gives player 1 a higher utility than rejection does. If $z_1 \geq y_1^\omega$, accepting the offer z_1 is optimal.

3.2. Now, suppose that $z_1 < r_1$. In the case of acceptance, player 1 gets his share from the offer z_1. However, since the offer is less than his reference point, player 1 perceives this offer as a loss relative to the reference point. The difference between z_1 and r_1 negatively affects his utility, and this effect is scaled by λ_1. Accepting the offer yields

$$u_1^* = u_1^t(z, r^t) = z_1 + \lambda_1(z_1 - r_1).$$

If player 1 rejects the offer, he gets

$$\delta u_1^{t+1}(x^\omega, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1})$$
$$= \delta[x_1^\omega + \gamma_1(x_1^\omega - \max\{r_1, z_1\})] - (1-\delta)\lambda_1 \max\{r_1, z_1\}$$
$$= (1+\gamma_1)\delta x_1^\omega - \delta\gamma_1 \max\{r_1, z_1\} - (1-\delta)\lambda_1 \max\{r_1, z_1\}$$
$$= (1+\gamma_1)y_1^\omega + \delta\gamma_1 y_1^\omega + (1-\delta)\lambda_1 y_1^\omega - \gamma_1 r_1 - \delta\gamma_1 \max\{r_1, z_1\}$$

$$
\begin{aligned}
&- (1-\delta)\lambda_1 \max\{r_1, z_1\}\\
&> (1+\gamma_1)y_1^{\omega} + \delta\gamma_1 z_1 + (1-\delta)\lambda_1 z_1 - \gamma_1 r_1 - \delta\gamma_1 z_1 - (1-\delta)\lambda_1 z_1\\
&= (1+\gamma_1)\delta y_1^{\omega} - \gamma_1 r_1 > (1+\gamma_1)\delta z_1 - \gamma_1 r_1 = u_1^*.
\end{aligned}
$$

Thus, $\delta u_1^{t+1}(x^{\omega}, r^{t+1}) + (1-\delta)u_1^{t+1}(0, r^{t+1}) \geq u_1^*$ if and only if $z_1 \geq y_1^{\omega}$.

In this case, accepting the offer satisfying $z_1 \geq y_1^{\omega}$ and rejecting it otherwise are optimal.

Considering all cases, we have the following result: it is optimal to accept the offer z if $z_1 \geq y_1^{\omega}$ and to reject it otherwise, which implies that following σ_1^* is optimal. □

C Comparative Statics

Since we have $r_i = \bar{r}_i$ in the initial period and the subgame perfect equilibrium that we focus here induces immediate agreement, we obtain equilibrium outcomes by assuming $r_i = \bar{r}_i$ in the equilibrium strategies described in "Appendix B."

C.1 Region 1.a-I.a

$$
\begin{aligned}
\frac{dx_1^{1.a-I.a}}{d\lambda_1} &\leq \frac{\begin{array}{c}-\delta(1+\gamma_2)\theta(1-\delta r_1)-\delta(1+\gamma_2)(1+\lambda_2)(\delta\gamma_1 r_1-\delta\lambda_1 r_1)-\delta(1+\gamma_2)\theta\delta r_1\\ -\delta^3(1+\gamma_1)(1+\gamma_2)^2+\delta(1+\gamma_2)(1+\lambda_2)(\delta\gamma_1 r_1-\delta\lambda_1 r_1+1+\lambda_1)\end{array}}{\theta^2} = 0,\\
\frac{dx_1^{1.a-I.a}}{d\lambda_2} &\geq \frac{(1+\lambda_1)(-\delta(1+\lambda_2)r_2 - (\delta\gamma_2 r_2 - \delta\lambda_2 r_2) + \delta(1+\gamma_2)r_2)}{(1+\lambda_2)\theta} = 0,\\
\frac{dx_1^{1.a-I.a}}{d\gamma_1} &\geq \frac{\begin{array}{c}-\delta^2(1+\gamma_2)\theta r_1+\delta^3(1+\gamma_2)^2(\delta\gamma_1 r_1-\delta\lambda_1 r_1+1+\lambda_1)\\ +\delta^2(1+\gamma_2)\theta r_1-\delta^3(1+\gamma_2)^2(\delta\gamma_1 r_1-\delta\lambda_1 r_1+1+\lambda_1)\end{array}}{\theta^2} = 0,\\
\frac{dx_1^{1.a-I.a}}{d\gamma_2} &\geq \frac{\begin{array}{c}\delta(1+\lambda_1)\theta r_2-\delta^2(1+\lambda_1)(1+\gamma_1)(\delta\gamma_2 r_2-\delta\lambda_2 r_2+1+\lambda_2)\\ -\delta(1+\lambda_1)\theta r_2+\delta^2(1+\lambda_1)(1+\gamma_1)(\delta\gamma_2 r_2-\delta\lambda_2 r_2+1+\lambda_2\end{array}}{\theta^2} = 0,
\end{aligned}
$$

where $\theta = (1+\lambda_1)(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2)$.

C.2 Region 1.a-I.b

$$
\begin{aligned}
\frac{dx_1^{1.a-I.b}}{d\lambda_1} &= \frac{\delta^2(1+\gamma_1)(1-\delta)(-1+r_1+\delta r_1)}{\theta^2} \leq 0,\\
\frac{dx_1^{1.a-I.b}}{d\lambda_2} &= 0,\\
\frac{dx_1^{1.a-I.b}}{d\gamma_1} &\geq \frac{-\delta^2\theta r_1 + \delta^2(1-\delta)(1+\lambda_1) - \delta^2(1-\delta)(1+\lambda_1) + \delta^2\theta r_1}{\theta^2} = 0,\\
\frac{dx_1^{1.a-I.b}}{d\gamma_2} &= 0,
\end{aligned}
$$

where $\theta = (1+\lambda_1) - \delta^2(1+\gamma_1)$.

C.3 Region 1.a-II

$$\frac{dx_1^{1.a-II}}{d\lambda_1} \le \frac{\delta^2(1+\gamma_2)\kappa_2 r_1((1+\lambda_1)-(1+\gamma_1)+(\gamma_1-\lambda_1))}{\theta^2} = 0,$$
$$\frac{dx_1^{1.a-II}}{d\lambda_2} = \frac{(1+\lambda_1)(1-\delta)(1+(1-\delta)+\kappa_2 r_2)}{\kappa_2\theta} \ge 0,$$
$$\frac{dx_1^{1.a-II}}{d\gamma_1} \ge \frac{\substack{-\delta^2(1+\gamma_2)\theta r_1+\delta^3(1+\gamma_2)^2(\delta\gamma_1 r_1-\delta\lambda_1 r_1)\\ +\delta^2(1+\gamma_2)\theta r_1-\delta^3(1+\gamma_2)^2(\delta\gamma_1 r_1-\delta\lambda_1 r_1)}}{\theta^2} = 0,$$
$$\frac{dx_1^{1.a-II}}{d\gamma_2} = \frac{\delta(\delta\gamma_1 r_1-\delta\lambda_1 r_1)((1+\lambda_1)-\delta(1+\gamma_1)+\delta^2\lambda_1-\delta^2\gamma_1)}{\theta^2} \le 0,$$

where $\theta = (1+\lambda_1)\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)$.

C.4 Region 1.a-III

$$\frac{dx_1^{1.a-III}}{d\lambda_1} \le \frac{\delta^2(1+\gamma_2)\eta_2 r_1((1+\lambda_1)-(1+\gamma_1)+(\gamma_1-\lambda_1))}{\theta^2} = 0,$$
$$\frac{dx_1^{1.a-III}}{d\lambda_2} > \frac{\substack{(1-\delta)(1+\lambda_1)(\theta-(1+\lambda_1)(\gamma_2-\gamma_2 r_2+(1-\delta)(1+\lambda_2))\\ -\theta+(1+\lambda_1)(\gamma_2-\gamma_2 r_2+(1-\delta)(1+\lambda_2)))+\theta r_2}}{\theta^2} \ge 0,$$
$$\frac{dx_1^{1.a-III}}{d\gamma_1} \ge \frac{\substack{-\delta^2(1+\gamma_2)\theta r_1+\delta^3(1+\gamma_2)^2(\delta\gamma_1 r_1-\delta\lambda_1 r_1)\\ +\delta^2(1+\gamma_2)\theta r_1-\delta^3(1+\gamma_2)^2(\delta\gamma_1 r_1-\delta\lambda_1 r_1)}}{\theta^2} = 0,$$
$$\frac{dx_1^{1.a-III}}{d\gamma_2} \le -\frac{(1-\delta)(1+\lambda_1)(1+\lambda_2)r_2}{(1+\gamma_2)\theta} \le 0,$$

where $\theta = (1+\lambda_1)\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)$.

C.5 Region 1.b-I.a

$$\frac{dx_1^{1.b-I.a}}{d\lambda_1} = 0,$$
$$\frac{dx_1^{1.b-I.a}}{d\lambda_2} \ge \frac{-\delta\theta r_2+\delta(1-\delta)(1+\gamma_2)+\delta\theta r_2-\delta(1-\delta)(1+\gamma_2)}{\theta^2} = 0,$$
$$\frac{dx_1^{1.b-I.a}}{d\gamma_1} = 0,$$
$$\frac{dx_1^{1.b-I.a}}{d\gamma_2} \le -\frac{\substack{-\delta\theta(1-r_2)+\delta^2(1+\lambda_2)-\delta^3(1+\gamma_2)\\ +\delta(1-\delta)(1+\lambda_2)-\delta\theta r_2}}{\theta^2} = 0,$$

where $\theta = (1+\lambda_2) - \delta^2(1+\gamma_2)$.

C.6 Region 1.b-I.b

$$\frac{dx_1^{1.b-I.b}}{d\lambda_1} = 0,$$

$$\frac{dx_1^{1.b-I.b}}{d\lambda_2} = 0,$$
$$\frac{dx_1^{1.b-I.b}}{d\gamma_1} = 0,$$
$$\frac{dx_1^{1.b-I.b}}{d\gamma_2} = 0.$$

C.7 Region 1.b-II

$$\frac{dx_1^{1.b-II}}{d\lambda_1} = 0,$$
$$\frac{dx_1^{1.b-II}}{d\lambda_2} = \frac{(1-\delta)r_2}{\kappa_2 - \delta^2(1+\gamma_2)} \geq 0,$$
$$\frac{dx_1^{1.b-II}}{d\gamma_1} = 0,$$
$$\frac{dx_1^{1.b-II}}{d\gamma_2} = -\frac{\delta(1-\delta)(\lambda_2 - \lambda_2 r_2 + (1-\delta)(1+\lambda_2))}{(\kappa_2 - \delta^2(1+\gamma_2))^2} \leq 0.$$

C.8 Region 1.b-III

$$\frac{dx_1^{1.b-III}}{d\lambda_1} = 0,$$
$$\frac{dx_1^{1.b-III}}{d\lambda_2} = \frac{(1-\delta)r_2}{\eta_2 - \delta^2(1+\gamma_2)} \geq 0,$$
$$\frac{dx_1^{1.b-III}}{d\gamma_1} = 0,$$
$$\frac{dx_1^{1.b-III}}{d\gamma_2} \leq \frac{(-\delta+\delta^2)(\eta_2 - \delta^2(1+\gamma_2)) - r_2(1-\delta)(1+\delta+\lambda_2)}{\theta^2} \leq 0,$$

where $\theta = \eta_2 - \delta^2(1+\gamma_2)$

C.9 Region 2-I.a

$$\frac{dx_1^{2-I.a}}{d\lambda_1} = -\frac{\delta(1-\delta)(1+\gamma_2)\kappa_1 r_1}{\theta} \leq 0,$$
$$\frac{dx_1^{2-I.a}}{d\lambda_2} = \frac{(1-\delta)\kappa_1 r_2}{\theta} \geq 0,$$
$$\frac{dx_1^{2-I.a}}{d\gamma_1} = \frac{\delta(1-\delta)(1+\lambda_1)(1+\gamma_2)r_1}{(1+\gamma_1)\theta} \geq 0,$$
$$\frac{dx_1^{2-I.a}}{d\gamma_2} = -\frac{(1-\delta)(1+\lambda_2)\kappa_1 r_2}{(1+\gamma_2)\theta} \leq 0,$$

where $\theta = \kappa_1(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2)$.

C.10 Region 2-I.b

$$\frac{dx_1^{2-I.b}}{d\lambda_1} = -\frac{(1-\delta)(1+\kappa_1)r_1}{(1+\gamma_1)\theta} \leq 0,$$
$$\frac{dx_1^{2-I.b}}{d\lambda_2} = 0,$$
$$\frac{dx_1^{2-I.b}}{d\gamma_1} = \frac{\delta^2(1-\delta)(\lambda_1 - \lambda_1 r_1 + (1-\delta)(1+\lambda_1))}{\theta^2} \geq 0,$$
$$\frac{dx_1^{2-I.b}}{d\gamma_2} = 0,$$

where $\theta = \kappa_1 - \delta^2(1+\gamma_1)$.

C.11 Region 2-II

$$\frac{dx_1^{2-II}}{d\lambda_1} = \frac{\delta(1+\gamma_2)(-1-(1-\delta)+r_1)}{\theta} \leq 0,$$
$$\frac{dx_1^{2-II}}{d\lambda_2} = \frac{(1+(1-\delta))\kappa_1 r_2}{\theta} \geq 0,$$
$$\frac{dx_1^{2-II}}{d\gamma_1} = \frac{\delta(1-\delta)(1+\lambda_1)(1+\gamma_2)r_1}{(1+\gamma_1)\theta} \geq 0,$$
$$\frac{dx_1^{2-II}}{d\gamma_2} = \frac{\kappa_1(\delta(1+\gamma_2) - 2\kappa_2 + \lambda_2 r_2 - \kappa_2 r_2)}{(1+\gamma_2)\theta},$$

where $\theta = \kappa_1\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)$.

C.12 Region 2-III

$$\frac{dx_1^{2-III}}{d\lambda_1} = \frac{\delta(1+\gamma_2)(-1-(1-\delta)+r_1)}{\theta} \leq 0,$$
$$\frac{dx_1^{2-III}}{d\lambda_2} = \frac{(1-\delta)\kappa_1 r_2}{\theta} \geq 0,$$
$$\frac{dx_1^{2-II}}{d\gamma_1} = \frac{\delta(1-\delta)(1+\lambda_1)(1+\gamma_2)r_1}{(1+\gamma_1)\theta} \geq 0,$$
$$\frac{dx_1^{2-III}}{d\gamma_2} = -\frac{(1-\delta)(1+\lambda_2)\kappa_1 r_2}{(1+\gamma_2)\theta} \leq 0,$$

where $\theta = \kappa_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)$.

C.13 Region 3-I.a

$$\frac{dx_1^{3-I.a}}{d\lambda_1} < \frac{\delta(1-\delta)(1+\gamma_2)(-\delta(1+\gamma_1) - \delta(1+\gamma_1)r_1 - \delta)}{\eta_1\theta} \leq 0,$$

$$\frac{dx_1^{3-I.a}}{d\lambda_2} > \frac{\delta\eta_1(-(1+\lambda_2)r_2 - \gamma_2 r_2 + \lambda_2 r_2 + (1+\gamma_2)r_2)}{(1+\lambda_2)\theta} = 0,$$
$$\frac{dx_1^{3-I.a}}{d\gamma_1} > \frac{\delta(1-\delta^2)(1+\lambda_1)(1+\gamma_2)r_1}{\eta_1\theta} \geq 0,$$
$$\frac{dx_1^{3-I.a}}{d\gamma_2} < -\frac{\begin{array}{c}\delta\eta_1\theta r_2+\delta^2\eta_1(1+\gamma_1)(\delta\gamma_2+\delta\lambda_2)r_2\\ -\delta\eta_1\theta r_2-\delta^2\eta_1(1+\gamma_1)(\delta\gamma_2-\delta\lambda_2)r_2\end{array}}{\theta^2} = 0,$$

where $\theta = \eta_1(1+\lambda_2) - \delta^2(1+\gamma_1)(1+\gamma_2)$.

C.14 Region 3-I.b

$$\frac{dx_1^{3-I.b}}{d\lambda_1} < \frac{-\delta(1-\delta)\theta + \delta(1-\delta)(\gamma_1 - \gamma_1 r_1 + (1-\delta)(1+\lambda_1))}{\theta^2} \leq 0,$$
$$\frac{dx_1^{3-I.b}}{d\lambda_2} = 0,$$
$$\frac{dx_1^{3-I.b}}{d\gamma_1} \geq \frac{\delta(1-\delta)(r_1(1+\lambda_1+\delta\gamma_1) - \delta\gamma_1)}{\theta^2} \geq 0,$$
$$\frac{dx_1^{3-I.b}}{d\gamma_2} = 0,$$

where $\theta = \eta_1 - \delta^2(1+\gamma_1)$.

C.15 Region 3-II

$$\frac{dx_1^{3-II}}{d\lambda_1} < \frac{\delta(1-\delta)(1+\gamma_2)(-\gamma_1 r_1 - \delta(1+\gamma_1)r_1)}{\eta_1\theta} \leq 0,$$
$$\frac{dx_1^{3-II}}{d\lambda_2} = \frac{(1-\delta)\eta_1 r_2}{\theta} \geq 0,$$
$$\frac{dx_1^{3-II}}{d\gamma_1} > \frac{\delta(1-\delta^2)(1+\gamma_2)(\lambda_1 - \gamma_1)r_1}{\eta_1\theta} \geq 0,$$
$$\frac{dx_1^{3-II}}{d\gamma_2} < \frac{\delta(\gamma_1 - \lambda_1 - (1-\delta)(1+\lambda_1))r_1}{\theta} \leq 0,$$

where $\theta = \eta_1\kappa_2 - \delta^2(1+\gamma_1)(1+\gamma_2)$.

C.16 Region 3-III

$$\frac{dx_1^{3-III}}{d\lambda_1} \leq -\frac{\delta(1-\delta)\gamma_1(1+\gamma_2)r_1}{\eta_1\theta} \leq 0,$$
$$\frac{dx_1^{3-III}}{d\lambda_2} > \frac{(1-\delta)\eta_1 r_2}{\theta} \geq 0,$$
$$\frac{dx_1^{3-III}}{d\gamma_1} > \frac{\delta(1-\delta^2)(1+\lambda_1)(1+\gamma_2)r_1}{\eta_1\theta} \geq 0,$$

$$\frac{dx_1^{3-III}}{d\gamma_2} < \frac{(1-\delta)\eta_1(-1-\gamma_2-\lambda_2))}{(1+\gamma_2)\theta} \leq 0,$$

where $\theta = \eta_1\eta_2 - \delta^2(1+\gamma_1)(1+\gamma_2)$.

References

1. Abeler J, Falk A, Goette L, Huffman D (2011) Reference points and effort provision. Am Econ Rev 101(2):470–492
2. Ashenfelter O, Bloom DE (1984) Models of arbitrator behavior: theory and evidence. Am Econ Rev 74:111–124
3. Bartling B, Schmidt KM (2015) Reference points, social norms, and fairness in contract renegotiations. J Eur Econ Assoc 13:98–129
4. Bartos OJ (1964) A model of negotiation and the recency effect. Sociometry 27(3):311–326
5. Baucells M, Weber M, Welfens F (2011) Reference-point formation and updating. Manag Sci 57(3):506–519
6. Benartzi S, Thaler RH (1995) Myopic loss aversion and the equity premium puzzle. Q J Econ 110(11):73–92
7. Blount S, Thomas-Hunt MC, Neale MA (1996) The price is right—or is it? A reference point model of two-party price negotiations. Organ Behav Hum Decis Process 68(1):1–12
8. Bohnet I, Zeckhauser R (2004) Social comparisons in ultimatum bargaining. Scand J Econ 106(3):495–510
9. Bolton GE, Karagözoğlu E (2016) On the influence of hard leverage in a soft leverage bargaining game: the importance of credible claims. Games Econ Behav 9:164–179
10. Compte O, Jehiel P (2007) Bargaining with reference dependent preferences. Unpublished work
11. DeBondt WFM, Thaler RH (1990) Do security analysts overreact? Am Econ Rev 80:52–57
12. Driesen B, Perea A, Peters H (2012) Alternating offers bargaining with loss aversion. Math Soc Sci 64:103–118
13. Ephron E (1997) Recency planning. J Advert Res 37(4):61–65
14. Ephron E, McDonald C (2002) Media scheduling and carry-over effects: is adstock a useful tv planning tool? J Advert Res 42(4):66–70
15. Falk A, Knell M (2004) Choosing the joneses: endogenous goals and reference standards. Scand J Econ 106(3):417–435
16. Fehr E, Hart O, Zehnder C (2015) How do informal agreements and revision shape contractual reference points? J Eur Econ Assoc 13(1):1–28
17. Fershtman C, Seidmann DJ (1993) Deadline effects and inefficient delay in bargaining with endogenous commitment. J Econ Theory 60:306–321
18. Gächter S, Riedl A (2005) Moral property rights in bargaining with infeasible claims. Manag Sci 51(2):249–263
19. Gächter S, Riedl A (2006) Dividing justly in bargaining problems with claims. Soc Choice Welf 27(3):571–594
20. Gimpel H (2007) Loss aversion and reference-dependent preferences in multi-attribute negotiations. Group Decis Negot 16(4):303–319
21. Giorgi EGD, Post T (2011) Loss aversion with a state-dependent reference point. Manag Sci 57(6):1094–1110
22. Grether DM (1992) Testing bayes rule and the representativeness heuristic: some experimental evidence. J Econ Behav Organ 17(1):31–57
23. Hendon E, Jacobsen HJ, Sloth B (1996) The one-shot deviation principle for sequential rationality. Games Econ Behav 12(2):274–282
24. Herweg F, Schmidt KM (2015) Loss aversion and inefficient renegotiation. Rev Econ Stud 82(1):297–332
25. Hogarth RM, Einhorn HJ (1992) Order effects in belief updating: the belief-adjustment model. Cogn Psychol 24(1):1–55
26. Hyndman K (2011) Repeated bargaining with reference-dependent preferences. Int J Game Theory 40:527–549
27. Kahneman D, Tversky A (1974) Judgment under uncertainty: heuristics and biases. Science 185:1124–1131

28. Kahneman D, Tversky A (1979) Prospect theory: an analysis of decision under risk. Econometrica 47(2):263–291
29. Kahneman D, Tversky A (1991) Loss aversion in riskless choice: a reference dependent model. Q J Econ 107(4):1039–1061
30. Karagözoğlu E, Keskin K, Özcan-Tok E (2019) Between anchors and aspirations: a new family of bargaining solutions. Rev Econ Des 23(1–2):53–73
31. Karagözoğlu E, Keskin K (2018) Endogenous reference points in bargaining. Math Methods Oper Res 88(2):283–295
32. Karagözoğlu E, Kocher MG (2019) Bargaining under time presssure from deadlines. Exp Econ 22(2):419–440
33. Karagözoğlu E, Riedl A (2015) Performance information, production uncertainty, and subjective entitlements in bargaining. Manag Sci 61(11):2611–2626
34. Köszegi B, Rabin M (2006) A model of reference-dependent preferences. Q J Econ 121(4):1133–1165
35. Köszegi B, Rabin M (2007) Reference-dependent risk attitudes. Am Econ Rev 97(4):1047–1073
36. Köszegi B, Rabin M (2009) Reference-dependent consumption plans. Am Econ Rev 99(3):909–36
37. Kristensen H, Gärling T (2000) Anchor points, reference points and counteroffers in negotiations. Group Decis Negot 9(6):493–505
38. Li D (2007) Bargaining with history-dependent preferences. J Econ Theory 136(1):695–708
39. Malmendier U, Nagel S (2016) Learning from inflation experiences. Q J Econ 131(1):53–87
40. Osborn MJ, Rubinstein A (1990) Bargaining and markets. Academic Press, San Diego
41. Roels G, Su X (2014) Optimal design of social comparison effects: setting reference groups and reference points. Manag Sci 60(3):606–627
42. Rubinstein A (1982) Perfect equilibrium in a bargaining model. Econometrica 50(1):97–109
43. Sarver T (2012) Optimal reference points and anticipation. Unpublished work
44. Shalev J (2000) Loss aversion equilibrium. Int J Game Theory 29(2):269–287
45. Shalev J (2002) Loss aversion and bargaining. Theor Decis 52(3):201–232
46. Stahl I (1971) Bargaining theory. Stockholm School of Economics, Stockholm
47. Vartiainen H (2007) Collective choice with endogenous reference outcome. Games Econ Behav 58(1):172–180