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ON THE EXISTENCE OF A PSEUDO-REGULAR BASIS IN SOME KÖTHE SPACES

Z. Ertuğrul, M. Kocatepe

Abstract

In this note, we study the relationship between the vanishing of the functor $\text{Ext}(E \times F, E \times F)$ for two Köthe spaces E and F, one of which having property (DN), and the existence of a pseudo-regular basis in the space $E \times F$ and $E\hat{\otimes}_\pi F$.

Introduction

In [2], Kocatepe has shown that if $\text{Ext}(\lambda(B), \lambda(A)) = 0$, for two Schwartz regular Köthe spaces with $\lambda(A)$ having propetry (DN) and $\lambda(B)$ having property $(\bar{\Omega})$,then $\lambda(A) \times \lambda(B)$ and $\lambda(A)\hat{\otimes}_\pi \lambda(B)$ have regular bases. In this note, we drop one of the conditions: $(\bar{\Omega})$. We take E regular, F with (DN) and by using some characteristics of the vanishing of Ext functor given in [4], show that if $\text{Ext}(E \times F, E \times F) = 0$, then $E \times F$ and $E\hat{\otimes}_\pi F$ have pseudo-regular bases, which is weaker than regularity (still unknown whether strictly weaker), but is strong enough to obtain almost all of the results can be obtained using regularity, especially the quasi-equivalence property, [1]. We note that in its most generality, the vanishing of $\text{Ext}^1(\lambda(A), \lambda(A)) = 0$, does not imply the existence of a pseudo-regular basis in $\lambda(A)$, since an example of such a space with no pseudo-regular basis has been given in [2].

Preliminaries

Let $E = \lambda(A)$ be a Fréchet Köthe space where $A = (a_i^k)$ is an infinite Köthe matrix such that for all $i, k \in \mathbb{N}$, we have $0 < a_i^k \leq a_i^{k+1}$.

Let $E = \lambda(A), F = \lambda(B)$ be two Köthe spaces. For the definition of the functor $\text{Ext}^1(E, F)$ and proof of the following fact we refer the reader to Vogt [5].

$\text{Ext}^1(E, F) = 0$ if and only if every short exact sequence $0 \to F \to G \to E \to 0$ of Frechet spaces and continuous linear maps splits.

Also we use in this paper $\text{Ext}(E, F)$ instead of $\text{Ext}^1(E, F)$.

The pair (E, F) is said to satisfy the condition (S^*), in [4], briefly $(E, F) \in (S^*)$, if the following holds :

$$\forall\,\mu\,\exists n_o,k\,\forall\,K,m\,\exists n,S\,\forall i,j:\ \frac{a_i^m}{b_j^k}\le S\max\left\{\frac{a_i^n}{b_j^K},\frac{a_i^{no}}{b_j^{\mu}}\right\}.$$

This condition has been rewritten in an apparently strengthened form in [4], Lemma 1.2. Namely, if $(E,F)\in(S^*)$ then either $E=l^1$ or (E,F) satisfy the following condition:

$$(S^*)_o\quad \forall\mu\,\exists\,n_o,k\forall K,m,R,>0\exists\,n,S\,\forall i,j:$$
$$\frac{a_i^m}{b_j^k}\le\max\left\{S\frac{a_i^n}{b_j^K},\frac{1}{R}\frac{a_i^{no}}{b_j^{\mu}}\right\}.$$

So, $(E,F)\in(S^*)_o$ is equivalent to

$$(S^*)_1\quad \forall\mu\,\exists n_o,k\,\forall K,m,R>0\,\exists\tilde{n},\tilde{S}\,\forall i,j:$$
$$\frac{{a_i}^m}{b_j^k}\le\max\left\{\tilde{S}\frac{a_i^{\tilde{n}}}{b_j^K},\frac{1}{R}\frac{a_i^{n_o}}{b_j^{\mu}}\right\}.$$

In the text, we shall also be using $(F,E)\in(S^*)_o$, which we write as

$$(S^*)_2\quad \forall\bar{\mu}\,\exists\bar{n_o},\bar{k}\,\forall K,m,R>0\,\exists\bar{n},\bar{S}\,\forall i,j:$$
$$\frac{b_j^m}{a_i^{\bar{k}}}\le\max\left\{\bar{S}\frac{b_j^{\bar{n}}}{a_i^K},\frac{1}{R}\frac{b_j^{\bar{n}o}}{a^{\bar{\mu}}}\right\}.$$

It was shown in [4] that $(E,F)\in(S^*)$ if and only if Ext $(E,F)=0$. If Ext $(E\times F,E\times F)=0$,then we have

Ext$(E,E)=$Ext$(F,F)=$Ext$(E,F)=$Ext$(F,E)=0$.

A Köthe space E is said to have the property

(DN) if $\exists n_o\forall m\,\exists n,C>0\,\forall i:(a_i^m)^2\le Ca_i^{n_o}a_i^n$. If E has the property (DN) and Ext$(E,E)=0$, then E has a regular basis, [3]. A Köthe space E is said to be regular if

$$\forall i\,\text{and}\,k,\qquad \frac{a_i^{k+1}}{a_i^k}\le\frac{a_{i+1}^{k+1}}{a_{i+1}^k}$$

and it is called pseudo-regular, [1],if

$$\forall p\,\exists\,q\,\forall r>q\,\exists\,s>p\,\exists\,M>0:i\le j\Rightarrow\frac{a_i^r}{a_i^q}\le M\frac{a_j^s}{a_j^p}$$

.

First we prove the following Lemma :

Lemma *Let E, F be two Köthe space. If $Ext(E \times F, E \times F) = 0$, then there exist increasing sequences $(m(k))_k, (n(k))_k, (S_k)_k, (C_k)_k$ such that $\forall\, k, i, j$ we have*

$$\frac{S_{k+2}a_i^{m(k+2)}}{C_{k+1}b_j^{n(k+1)}} \leq \max\left\{\frac{S_{k+3}a_i^{m(k+3)}}{C_{k+2}b_j^{n(k+2)}}, \frac{S_{k+1}a_i^{m(k+1)}}{C_k b_j^{n(k)}}\right\} \tag{1}$$

and

$$\frac{C_{k+2}b_j^{n(k+2)}}{S_{k+1}a_i^{m(k+1)}} \leq \max\left\{\frac{C_{k+3}b_j^{n(k+3)}}{S_{k+2}a_i^{m(k+2)}}, \frac{C_{k+1}b_j^{n(k+1)}}{S_k a_i^{m(k)}}\right\} \tag{2}$$

Moreover, if F has the property (DN), then we can choose $(n(k))_k$ and $(C_k)_k$ in such a way that they satisfy (1) and (2) and also

$$(C_k b_j^{n(k)})^2 \leq (C_1 b_j^{n(1)})(C_{k+1}b_j^{n(k+1)}). \tag{3}$$

Proof. First we observe that, if for a given μ, n_o and k satisfy $(S^*)_o$, then for the same $\mu, (S^*)_o$ is satisfied for larger values of n_o and k. We determine $(m(k))_k, (n(k))_k, (S_k)_k, (C_k)_k$ inductively. Put $m(1) = n(1) = 1$ and $C_i = S_i = 1$ where $i = 1, 2, 3$. For $\mu = n(1)$ in $(S^*)_1$ and $\bar{\mu} = m(1)$ in $(S^*)_2$ there exists $n_o, k, \bar{n}_o, \bar{k}$. We put $n(2) = \max\{k, \bar{n}_o, n(1)+1\}$ and $m(2) = \max\{n_o, \bar{k}, m(1)+1\}$. For $\mu = n(2)$, and $\bar{\mu} = m(2)$, there exist again $n_o, k, \bar{n}_o, \bar{k}$. We put $n(3) = \max\{k, \bar{n_o}, n(2)+1\}$ and $m(3) = \max\{n_o, \bar{k}, m(2)+1\}$.

Now in $(S^*)_1$, put $\mu = n(1)$. By the above observation $n_o = m(2), k = n(2)$ satisfy $(S^*)_1$. Taking $K = n(3), m = m(3)$ and $R = 1$ there exist $\tilde{n} = \tilde{n}_4$ and $\tilde{S} = \tilde{S}_4$. In $(S^*)_2$, put $\bar{\mu} = m(1)$ then we have $\bar{n}_o = n(2), \bar{k} = m(2)$ taking $K = m(3), m = n(3)$ and $R = 1$ there exist $\bar{n} = \bar{n}_4$ and $\bar{S} = \bar{S}_4$. For $\mu = n(3)$ and $\bar{\mu} = m(3)$ there exist $n_o, k, \bar{n}_o, \bar{k}$. We put $n(4) = \max\{k, \bar{n}_o, n(3)+1, \bar{n}_4\}, m(4) = \max\{n_o, \bar{k}, m(3)+1, \tilde{n}_4\}$ and $S_4 = \tilde{S}_4 S_3 C_3, C_4 = \bar{S}_4 S_3 C_3$.

Let $m(1), ..., m(k+2); n(1), ..., n(k+2); S_1, ..., S_{k+2}; C_1, ..., C_{k+2}$ be determined in this way.

Now, for $\mu = n(k)$ we have $n_o = m(k+1), k = n(k+1)$. We apply $(S^*)_1$ to $K = n(k+2), m = m(k+2), R = \frac{S_{k+2}C_k}{C_{k+1}S_{k+1}}$ and obtain $\tilde{n} = \tilde{n}_{k+3}, \tilde{S} = \tilde{S}_{k+3}$. For $\bar{\mu} = m(k)$, we have $\bar{n}_o = n(k+1), \bar{k} = m(k+1)$, we apply $(S^*)_2$ to $K = m(k+2), m = n(k+2), R = \frac{C_{k+2}S_k}{C_{k+1}S_{k+1}}$ and obtain $\bar{n} = \bar{n}_{k+3}$ .and $\bar{S}_{k+3}$. To find $m(k+3)$ and $n(k+3)$, we again consider $\mu = n(k+2)$, and $\bar{\mu} = m(k+2)$, then there exist $n_o, k, \bar{n}_o, \bar{k}$. We put $n(k+3) = \max\{k, \bar{n}_o, n(k+2)+1, \bar{n}_{k+3}\}$ and $m(k+3) = \max\{n_o, \bar{k}, m(k+2)+1, \tilde{n}_{k+3}\}$ such that

$$\frac{a_i^{m(k+2)}}{b_j^{n(k+1)}} \leq \max\left\{\frac{\tilde{S}_{k+3}a_i^{m(k+3)}}{b_j^{n(k+2)}}, \frac{C_{k+1}S_{k+1}a_i^{m(k+1)}}{C_k S_{k+2}b_j^{n(k)}}\right\}$$

and

$$\frac{b_j^{n(k+2)}}{a_i^{m(k+1)}} \leq \max\left\{\frac{\bar{S}_{k+3}b_j^{n(k+3)}}{a_i^{m(k+2)}}, \frac{C_{k+1}S_{k+1}b_j^{n(k+1)}}{S_kC_{k+2}a_i^{m(k)}}\right\}.$$

We put $S_{k+3} = \frac{\bar{S}_{k+3}S_{k+2}C_{k+2}}{C_{k+1}}$ and $C_{k+3} = \frac{\bar{S}_{k+3}C_{k+2}S_{k+2}}{S_{k+1}}$. Hence we have(1)and (2). Finally, when F has (DN), we may choose

$$n(k+3) \geq \max\{k, \bar{n}_o, n(k+2)+1, \bar{n}_{k+3}\} \quad \text{and}\, C_{k+3} \geq \frac{\bar{S}_{k+3}C_{k+2}S_{k+2}}{S_{k+1}}$$

such that both (2) and (3) hold.
Now we can prove the main result of this note : □

Theorem *Let E and F be Schwartz Köthe spaces where E is regular and F has property (DN). If $Ext(E\times F, E\times F) = 0$ then $E\times F$ and $E\hat{\otimes}_\pi F$ have pseudo-regular bases.*

Proof. Let $A = (a_i^k)$ and $B = (b_j^k)$ be the Köthe matrices for E, F, respectively. We choose sequences according to the above Lemma and use $(S_ka_i^{m(k)}), (C_kb_j^{n(k)})$ which are equivalent to A and B respectively. Then we have (1) and (2). By using Proposition 1.5. in [4],(1) gives

$$\frac{S_{k+2}a_i^{m(k+2)}}{C_{k+1}b_j^{n(k+1)}} \leq \frac{S_{k+3}a_i^{m(k+3)}}{C_{k+2}b_j^{n(k+2)}} \quad \text{for some k}$$

$$\Rightarrow \frac{S_{l+2}a_i^{m(l+2)}}{C_{l+1}b_j^{n(l+1)}} \leq \frac{S_{l+3}a_i^{m(l+3)}}{C_{l+2}b_j^{n(l+2)}} \quad \text{for all} \quad l \geq k. \tag{4}$$

Similarly from (2) we get

$$\frac{C_{k+2}b_j^{n(k+2)}}{S_{k+1}a_i^{m(k+1)}} \leq \frac{C_{k+3}b_j^{n(k+3)}}{S_{k+2}a_i^{m(k+2)}} \quad \text{for some k}$$

$$\Rightarrow \frac{C_{l+2}b_j^{n(l+2)}}{S_{l+1}a_i^{m(l+1)}} \leq \frac{C_{l+3}b_j^{n(l+3)}}{S_{l+2}a_i^{m(l+2)}} \quad \text{for all} \quad l \geq k. \tag{5}$$

Since E and F are Schwartz space, can find increasing sequences of indices (m_i) and (n_i) such that

$$\cdots \leq \frac{C_2b_{n_i}^{n(2)}}{C_1b_{n_i}^{n(1)}} \leq \frac{S_3a_{m_i+1}^{m(3)}}{S_2a_{m_i+1}^{m(2)}} \leq \cdots \leq \frac{S_3a_{m_{i+1}}^{m(3)}}{S_2a_{m_{i+1}}^{m(2)}} \leq \frac{C_2b_{n_i+1}^{n(2)}}{C_1b_{n_i+1}^{n(1)}} \leq \cdots \tag{6}$$

If (e_i) and (f_j) denote the canonical bases for E and F respectively, we have that

$$(x_n) = (\cdots, f_{n_i}, e_{m_i+1}, \cdots, e_{m_{i+1}}, f_{n_{i+1}}, \cdots)$$

is a pseudo-regular basis for $E \times F$. To see this, let $m < n, m, n \in \mathbb{N}$. The following four cases are possible :

(1) $x_m = e_r, x_n = f_s$ with $m_i+1 \leq r \leq m_{i+1}, n_j+1 \leq s \leq n_{j+1}, i \leq j$. Then $r < s$ and we have

$$\frac{S_3 a_r^{m(3)}}{S_2 a_r^{m(2)}} \overset{(6)}{\leq} \frac{C_2 b_s^{n(2)}}{C_1 b_s^{n(1)}} \overset{(DN)}{\leq} \frac{C_4 b_s^{n(4)}}{C_3 b_s^{n(3)}}$$

and so by (5), we get

$$\frac{S_{k+1} a_r^{m(k+1)}}{S_k a_r^{m(k)}} \leq \frac{C_{k+2} b_s^{n(k+2)}}{C_{k+1} b_s^{n(k+1)}} \; \forall k \geq 2.$$

(2) $x_m = e_r, x_n = e_s$ with $m_i + 1 \leq r \leq m_{i+1}, m_j + 1 \leq s \leq m_{j+1}, i \leq j$. This case follows from the regularity of the matrix $(S_k a_i^{m(k)})$.

(3) $x_m = f_r, x_n = e_s$ with $n_{j-1} + 1 \leq r \leq n_j, m_i + 1 \leq s \leq m_{i+1}, j \leq i$. In this case we have

$$\frac{C_2 b_r^{n(2)}}{C_1 b_r^{n(1)}} \leq \frac{S_3 a_s^{m(3)}}{S_2 a_s^{m(2)}}$$

Then using (4), we get

$$\frac{C_{k+1} b_r^{n(k+1)}}{C_k b_r^{n(k)}} \leq \frac{S_{k+2} a_s^{m(k+2)}}{S_{k+1} a_s^{m(k+1)}} \quad \forall k \geq 1.$$

(4) $x_m = f_r, x_n = f_s$ with $n_i + 1 \leq r \leq n_i, n_j + 1 \leq s \leq n_j, i \leq j$. This case also follows from the regularity of the matrix $(C_k b_i^{n(k)})$.

Hence these four cases show that $E \times F$ has a pseudo-regular basis.

Next we consider $E \hat{\otimes}_\pi F$. We set

$$A_i^k = \frac{S_{k+1} a_i^{m(k+1)}}{S_2 a_i^{m(2)}}, \quad B_j^k = \frac{C_k b_j^{n(k)}}{C_1 b_j^{n(1)}}, \; k \geq 1.$$

The matrices (A_i^k) and (B_j^k) are regular and

$$A_i^1 = 1, A_i^k \geq 1 \forall i, k, B_j^1 = 1, B_j^k \geq 1 \forall j, k \quad \text{and} \, (B_j^k)^2 \leq (B_j^1)(B_j^{k+1}) = B_j^{k+1}$$

.

Then $E\hat{\otimes}_\pi F$ is isomorphic to the Köthe space $\lambda(C)$ where $(C^k_{m,n}) = (A^k_m B^k_n)$. We define $n_o = 0$ and

$$I = \bigcup_{i=1}^{\infty}\{(m,n) : m_i + 1 \leq m \leq m_{i+1}, n \leq n_i\}.$$

and

$$J = \bigcup_{i=0}^{\infty}\{(m,n) : m \leq m_{i+1}, n_i + 1 \leq n \leq n_{i+1}\}.$$

Then $I \cup J = \mathbb{N} \times \mathbb{N}, I \cap J = \emptyset$. Now we define a matrix $(D^k_{m,n})$ by

$$D^k_{m,n} = \begin{cases} A^k_m & \text{if} \quad (m,n) \in I \\ B^k_n & \text{if} \quad (m,n) \in J \end{cases}$$

First we show that the matrices $(C^k_{m,n})$ and $(D^k_{m,n})$ are equivalent. If $(m,n) \in I$, then there is a unique i such that $m_i + 1 \leq m \leq m_{i+1}, n \leq n_i$. Then

$$\frac{B^{k+1}_{n_i}}{B_{n_i}} = \frac{C_{k+1} b^{n(k+1)}_{n_i}}{C_k b^{n(k)}_{n_i}} \overset{(*)}{\leq} \frac{S_{k+2} a^{m(k+2)}_{m_i+1}}{S_{k+1} a^{m(k+1)}_{m_i+1}} = \frac{A^{k+1}_{m_i+1}}{A^k_{m_i+1}}$$

where $(*)$ follows from (6) and (4). So

$$B^k_n \leq B^k_{n_i} \leq \frac{B^{k+1}_{n_i}}{B^k_{n_i}} \leq \frac{A^{k+1}_{m_i+1}}{A^k_{m_i+1}} \leq \frac{A^{k+1}_m}{A^k_m}$$

from which it follows that $C^k_{m,n} = B^k_n A^k_m \leq A^{k+1}_m = D^{k+1}_{m,n}$. If $(m,n) \in J$, then there is a unique i such that $m \leq m_{i+1}, n_i + 1 \leq n \leq n_{i+1}$. We have

$$\frac{S_3 a^{m(3)}_{m_{i+1}}}{S_2 a^{m(2)}_{m_{i+1}}} \overset{(6)}{\leq} \frac{C_2 b^{n(2)}_{n_i+1}}{C_1 b^{n(1)}_{n_i+1}} \overset{(DN)}{\leq} \frac{C_4 b^{n(4)}_{n_i+1}}{C_3 b^{n(3)}_{n_i+1}} \Rightarrow \frac{C_3 b^{n(3)}_{n_i+1}}{S_2 a^{m(2)}_{m_{i+1}}} \leq \frac{C_4 b^{n(4)}_{n_i+1}}{S_3 a^{m(3)}_{m_{i+1}}}.$$

So by (5), we have

$$\frac{C_{k+1} b^{n(k+1)}_{n_i+1}}{S_k a^{m(k)}_{m_{i+1}}} \leq \frac{C_{k+2} b^{n(k+2)}_{n_i+1}}{S_{k+1} a^{m(k+1)}_{m_{i+1}}} \quad \text{for all } k \geq 2,$$

Then

$$\frac{A^k_{m_{i+1}}}{A^1_{m_{i+1}}} = \frac{S_{k+1} a^{m(k+1)}_{m_{i+1}}}{S_2 a^{m(2)}_{m_{i+1}}} = \frac{S_{k+1} a^{m(k+1)}_{m_{i+1}}}{S_k a^{m(k)}_{m_{i+1}}} \cdots \frac{S_3 a^{m(3)}_{m_{i+1}}}{S_2 a^{m(2)}_{m_{i+1}}}$$

$$\leq \frac{C_{k+2} b^{n(k+2)}_{n_i+1}}{C_{k+1} b^{n(k+1)}_{n_i+1}} \cdots \frac{C_4 b^{n(4)}_{n_i+1}}{C_3 b^{n(3)}_{n_i+1}} = \frac{C_{k+2} b^{n(k+2)}_{n_i+1}}{C_3 b^{n(3)}_{n_i+1}} = \frac{B^{k+2}_{n_i+1}}{B^3_{n_i+1}}.$$

Hence

$$A^k_m = \frac{A^k_m}{A^1_m} \leq \frac{A^k_{m_{i+1}}}{A^1_{m_{i+1}}} \leq \frac{B^{k+2}_{n_{i+1}}}{B^3_{n_{i+1}}} \leq \frac{B^{k+2}_n}{B^3_n} \leq \frac{B^{2k-1}_n}{B^k_n}$$

from which it follows that $C^k_{m,n} \leq B^{2k-1}_n = D^{2k-1}_{m,n}$.

$D^k_{m,n} \leq C^k_{m,n}$ for all k, m, n is obvious

Finally we show that the matrix $(D^k_{m,n})$ is pseudo-regular when the elements (m,n) are ordered as follows :

$$\begin{array}{ll}
(1,n_o+1),\cdots,(m_1,n_o+1), & (1,n_o+2),\cdots,(m_1,n_o+2),\cdots \\
& \cdots,(1,n_1),\cdots,(m_1,n_1), \\
(m_1+1,1),\cdots,(m_1+1,n_1), & (m_1+2,1),\cdots,(m_1+2,n_1),\cdots \\
& \cdots,(m_2,1),\cdots,(m_2,n_1),\cdots \\
\cdots, & \\
(1,n_{i-1}+1),\cdots,(m_i,n_{i-1}+1), & (1,n_{i-1}+2),\cdots,(m_i,n_{i-1}+2), \\
& \cdots,(1,n_i),\cdots,(m_i,n_i), \\
(m_i+1,1),\cdots,(m_i+1,n_i), & (m_i+2,1),\cdots,(m_i+2,n_i),\cdots \\
& \cdots,(m_{i+1},1),\cdots,(m_{i+1},n_i), \\
(1,n_i+1),\cdots,(m_{i+1},n_i+1), & (1,n_i+2),\cdots,(m_{i+1},n_i+2),\cdots \\
& \cdots,(1,n_{i+1}),\cdots,(m_{i+1},n_{i+1}),\cdots
\end{array}$$

Let the ordered pair (r,u) appear before the ordered pair (s,v) in the above ordering. We have four cases :

$$\begin{array}{ll}
(1)(r,u) \in I, & (s,v) \in I \\
(2)(r,u) \in I, & (s,v) \in J \\
(3)(r,u) \in J, & (s,v) \in I \\
(4)(r,u) \in J, & (s,v) \in J
\end{array}$$

Now we show the pseudo-regularity of $(D^k_{m,n})$ in each case:

(1) $m_i+1 \leq r \leq m_{i+1}, u \leq n_i, m_j+1 \leq s \leq m_{j+1}, v \leq n_j, i \leq j$. Then $r \leq s$. Then by regularity of (A^k_i),

$$\frac{A^{k+1}_r}{A^k_r} \leq \frac{A^{k+1}_s}{A^k_s} \quad \text{which is equivalent to} \quad \frac{D^{k+1}_{r,u}}{D^k_{r,u}} \leq \frac{D^{k+1}_{s,v}}{D^k_{s,v}}.$$

(2) $m_i+1 \leq r \leq m_{i+1}, u \leq n_i, s \leq m_{j+1}, n_j+1 \leq v \leq n_{j+1}, i \leq j$, so $r < v$. As we have shown before, we have

$$\frac{D_{r,u}^{k+1}}{D_{r,u}^{k}} = \frac{A_r^{k+1}}{A_r^k} \leq \frac{B_v^{k+2}}{B_v^{k+1}} = \frac{D_{s,v}^{k+2}}{D_{s,v}^{k+1}}.$$

(3) $r \leq m_i, n_{i-1}+1 \leq u \leq n_i, v \leq n_j, m_j+1 \leq s \leq m_{j+1}, i \leq j$. Then $u \leq s$. So we have

$$\frac{D_{r,u}^{k+1}}{D_{r,u}^{k}} = \frac{B_u^{k+1}}{B_u^k} \leq \frac{A_s^{k+2}}{A_s^{k+1}} = \frac{D_{s,v}^{k+2}}{D_{s,v}^{k+1}}.$$

(4) $m_i+1 \leq r \leq m_{i+1}, u \leq n_i, s \leq m_{j+1}, n_j+1 \leq v \leq n_{j+1}, i \leq j$. This case can be shown as case (1), it follows from the regularity of (B_i^k).

□

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BAZI KÖTHE UZAYLARINDA YAKLAŞIK-DÜZGÜN TABANLARIN VARLIĞI HAKKINDA

Özet

Bu makalede biri (DN) özelliğine sahip iki Köthe uzayının çarpım uzaylarının Ext funktorunun sıfır olması ile çarpım uzaylarının yaklaşık -düzgün (pseudo-regular) tabanlarının olması arasındaki ilişki çalışıldı.

Z. Ertuğrul & M. Kocatepe
University of Bilkent
Department of Mathematics
06533 Bilkent, Ankara, TURKEY

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