Chapter 4
Metrics for Light Source Design

Abstract In this part of this brief, we summarize the metrics that need to be considered for designing light sources. We start with metrics on the shade of color and then continue with color rendering and photometry.

Keywords Color temperature · Color rendering index · Color quality scale · Luminous efficiency

4.1 Cool Versus Warm White Light: Correlated Color Temperature (CCT)

The chromaticity diagrams offering color uniformity are especially targeted for comparing the colors of different sources. For a white light source, one of the obvious illuminants whose color is compared with is the sun. Since the sun is a blackbody radiator, the shade of the white light radiated by the designed light source can be safely compared with the shade of a blackbody radiator whose spectral distribution P(λ) is given below.

$$P(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/(\lambda kT)} - 1} \tag{4.1}$$

where c is the speed of light, h is the Planck's constant, k is the Boltzmann constant, and T is the temperature.

The emission spectrum of a blackbody radiator is a function of its temperature. With the same analogy, the shade of the white light of an arbitrary white light source can be characterized by finding the temperature of the blackbody radiator whose color is closest to the color of the light source. This temperature is called the correlated color temperature (CCT). As opposed to the common usage in thermodynamics, high CCTs indicate a cool white-shade since a blackbody radiator at higher temperatures have a stronger bluish color tint. Similarly, a blackbody radiator at lower temperatures have a stronger red content giving its emission a warmer white shade

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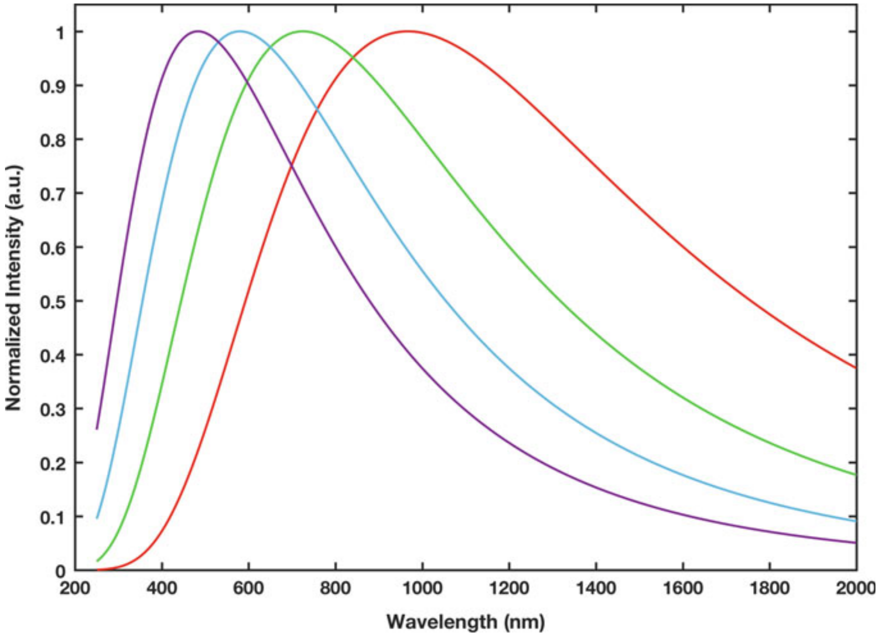

Fig. 4.1 Spectral power distribution of blackbody radiators at 3000 K (red), 4000 K (green), 5000 K (blue), and 6000 K (violet)

(Fig. 4.1). Traditionally, the CCT of an arbitrary light source is calculated using (u', v') chromaticity diagram (see Fig. 3.3). Incandescent light bulbs have CCTs around 3000 K and fluorescent tubes have varying CCTs from 3000 to 6500 K, whereas the CCT of the sun is close to 6000 K [1]. Having a warmer white shade (between 3000 and 4500 K) is more desirable for indoor lighting applications mainly for avoiding the disturbing effects of cool white light on the human biological clock. In the Appendix B of this brief we provide codes for calculating the correlated color temperature of a given spectral power distribution.

4.2 Color Rendition: Color Rendering Index (CRI), Color Quality Scale (CQS), and Other Metrics

A critical parameter regarding the performance of a light source is its capability to render the real colors of the objects. When objects are illuminated with a high-quality light source, we expect to perceive the colors correctly. This requirement has to be addressed especially for the indoor lighting applications. Moreover, for outdoor lighting applications such as road lighting, a light source with good color rendering

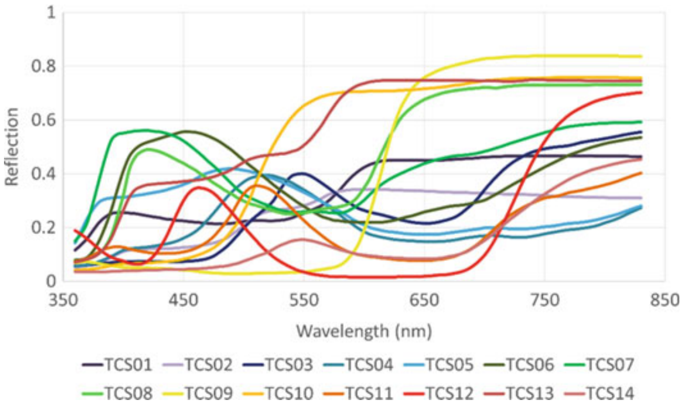

Fig. 4.2 Reflection spectra of the test color samples (TCS) used for calculating the color rendering index

capability was shown to increase the safety of roads and streets for pedestrians and drivers as good color rendition helps increase the color contrast [2].

This property of light sources has been proposed to be evaluated by various measures including the color discrimination index [3], color rendering capacity [4], feeling of contrast index [5], and flattery index [6]. However, these metrics have not attracted considerable attention in the lighting community to date. Therefore, we will not cover them here in detail and continue with two of the most commonly used color rendition metrics, which are the color rendering index (CRI) and the color quality scale (CQS) [7].

CRI was first introduced by CIE in 1971 [8] and later in 1995 its calculation method was revised [9]. It makes use of fourteen test samples whose reflection spectra are given in Fig. 4.2 and the table summarizing these spectra are given in Appendix A. The calculation assumes that the reference white light source, which is in general a blackbody radiator, renders the colors of objects perfectly. The calculation involves evaluating the performance of the test light sources by comparing reflection spectra of the reference and test light sources from the test color samples and calculating the associated color difference between these two light sources. This color difference data was then employed to calculate the CRI whose maximum value is 100 indicating a perfect color rendition capability. Its minimum value is -100 which indicates the worst color rendition performance. During the CRI calculation, a color rendering index value specific to each test sample is obtained. The general color rendering index is calculated by using the first eight test samples while the remaining six samples define the specific CRI. In general, a light source possessing CRI > 90 is considered to successfully render the real colors of objects [10].

Calculation of CRI starts with the determination of (u, v) coordinates of the reflection from the test sample i using the reference (dubbed with ref) and test light sources. Using Eqs. (4.2) and (4.3), (u, v) coordinates are transformed to (c, d) coordinates.

$$c = \frac{4 - u - 10v}{v} \tag{4.2}$$

$$d = \frac{1.708v + 0.404 - 1.481u}{v} \tag{4.3}$$

Subsequently, $(u^{**}_{text,i}, v^{**}_{text,i})$ coordinates are found using Eqs. (4.4) and (4.5).

$$u^{**}_{test,i} = \frac{10.872 + 0.404\frac{c_{ref}}{c_{test}}c_{test,i} - \frac{4d_{ref}}{d_{test}}d_{test,i}}{16.518 + 1.481\frac{c_{ref}}{c_{test}}c_{test,i} - \frac{d_{ref}}{d_{test}}d_{test,i}} \tag{4.4}$$

$$v^{**}_{test,i} = \frac{5.520}{16.518 + 1.481\frac{c_{ref}}{c_{test}}c_{test,i} - \frac{d_{ref}}{d_{test}}d_{test,i}} \tag{4.5}$$

Then, $(u^{**}_{text}, v^{**}_{text})$ are obtained using Eqs. (4.6) and (4.7).

$$u^{**}_{test,i} = \frac{10.872 + 0.404c_{ref} - 4d_{ref}}{16.518 + 1.481c_{ref} - d_{ref}} \tag{4.6}$$

$$u^{**}_{test,i} = \frac{5.520}{16.518 + 1.481c_{ref} - d_{ref}} \tag{4.7}$$

The color shifts for each test sample (ΔE^{**}_i) are calculated with Eqs. (4.8)–(4.11)

$$\Delta L^{**} = \left(25Y^{\frac{1}{3}}_{ref,i} - 17\right) - \left(25Y^{\frac{1}{3}}_{test,i} - 17\right) = L^{**}_{ref,i} - L^{**}_{test,i} \tag{4.8}$$

$$\Delta u^{**} = 13L^{**}_{ref,i}\left(u_{ref,i} - u_{ref}\right) - 13L^{**}_{test,i}\left(u_{test,i} - u_{test}\right) \tag{4.9}$$

$$\Delta v^{**} = 13L^{**}_{ref,i}\left(v_{ref,i} - v_{ref}\right) - 13L^{**}_{test,i}\left(v_{test,i} - v_{test}\right) \tag{4.10}$$

$$\Delta E^{**}_i = \sqrt{(\Delta L^{**})^2 + (\Delta u^{**})^2 + (\Delta v^{**})^2} \tag{4.11}$$

Following the computation of the color shift, CRI for each test sample is calculated using Eq. (4.12). Finally, the general CRI can be found using Eq. (4.13).

$$CRI_i = 100 - 4.6\Delta E^*_i \tag{4.12}$$

$$CRI = \frac{1}{8}\sum_{i=1}^{8} CRI_i \tag{4.13}$$

In Appendix B of this brief, we also provide MATLAB codes for calculating the CRI for a given spectral power distribution.

Although CRI still remains as the most frequently used measure of color rendition, it suffers from various issues [7, 11, 12]. One of them is the utilization of an improper uniform color space. Another issue is the assumption that the used reference sources render the colors perfectly is not always correct e.g., at very low and very high CCTs. These problems cause inaccurate results especially for the light sources having saturated color components. In addition to this, the arithmetic mean used during the calculation of CRI allows for the compensation of a low CRI value belonging to a certain test sample by the high CRIs of other test samples.

These problems of CRI are later addressed by Davis and Ohno who introduced the color quality scale (CQS) as an alternative to CRI [7]. CQS and CRI both employ the same reference sources. However, the CQS makes use of fifteen commercially available Munsell samples, all having highly saturated colors. This selection is based on the observation that a light source successfully rendering the saturated colors also successfully renders the unsaturated colors successfully [7]. This is especially important for the narrow-band emitters such as LED and nanocrystal-based light sources. Different than CRI, CQS employs the L*a*b* color space, which is a more uniform color space compared to (u, v) color space. Another improvement in CQS compared to CRI is the addition of a saturation factor that neutralizes the effect of increasing the object chroma under the test illuminant with respect to a reference source. Furthermore, CQS does not allow the compensation of a poorly rendered test source by other successfully rendered sources by calculating the root-mean-square of individual color differences. Another fine-tuning in CQS compared to CRI is the change of the scale from the range of -100 to 100 to the range of 0 to 100. Finally, in CQS a correction for the low CCTs is introduced, and the final value of the CQS is determined.

The calculation of CQS employs 15 Munsell test samples whose reflection spectra we provide in Fig. 4.3 and tabulate in Appendix A of this brief.

An important difference of CQS compared to CRI is the reference light source, which is assumed to render the real colors of the objects perfectly. If the correlated color temperature of the test source is less than 5000 K, the reference source is the usual blackbody radiator. In the case that the correlated color temperature is between 5000 and 7000 K, the reference light source is calculated using Eqs. (4.14)–(4.18) as follows:

$$x = -4.7070 \times 10^9/T^3 + 2.9678 \times 10^6/T^2 + 0.09911 \times 10^3/T + 0.244063 \tag{4.14}$$

$$y = 3x^2 + 2.87x - 0.275 \tag{4.15}$$

$$m_1 = \frac{-1.3515 - 1.7703x + 5.9114y}{0.0241 + 0.2562x - 0.7341y} \tag{4.16}$$

$$m_2 = \frac{0.03 - 31.4424x + 30.0717y}{0.0241 + 0.2562x - 0.7341y} \tag{4.17}$$

$$R(\lambda) = D_1(\lambda) + m_1 D_2(\lambda) + m_2 D_3(\lambda) \tag{4.18}$$

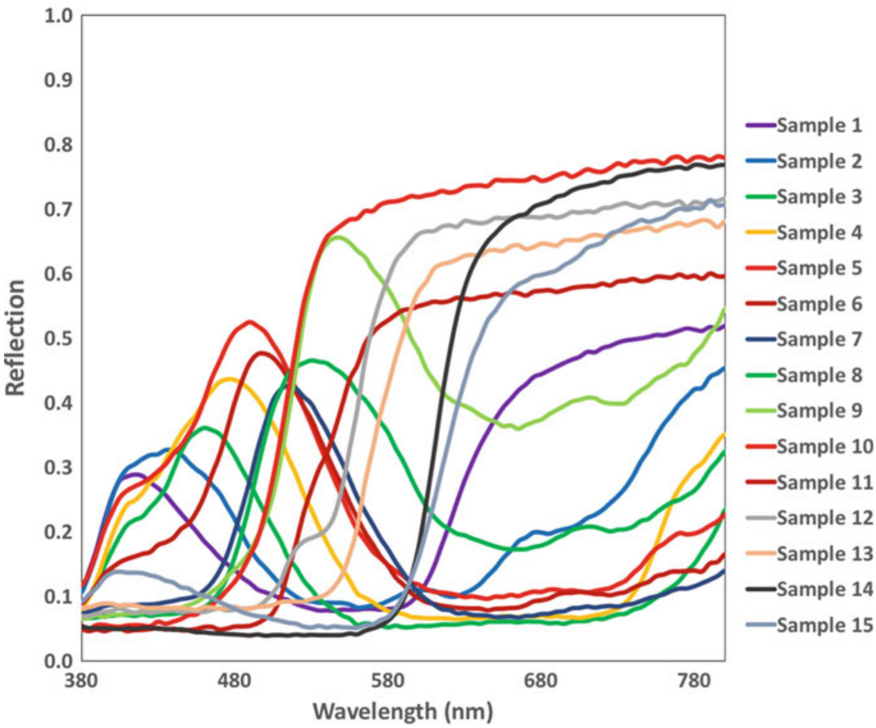

Fig. 4.3 Reflection spectra of 15 Munsell samples used in the calculation of CQS

where T stands for the correlated color temperature, D_i stands for the ith CIE standard daylight illuminants whose spectral power distributions are provided in Appendix A of the brief.

In the case that the correlated color temperature of the test light source is more than 7000 K, x is modified using Eq. (4.19):

$$x = -2.0064 \times 10^9/T^3 + 1.9018 \times 10^6/T^2 + 0.24748 \times 10^3/T + 0.23704 \tag{4.19}$$

Next, the intensities of the reference and test source are scaled such that their Y chromaticity coordinates become 100.

After calculating the reference source and scaling both reference and test sources, we are now ready to calculate the differences of the reflected colors when Munsell samples are illuminated with the reference and test sources. For this purpose, the reflected spectra $q_{ref,i}$ and $q_{test,i}$ from a Munsell sample *i* illuminated by the reference and test sources, respectively, are calculated as follows:

$$q_{ref,i}(\lambda) = r_i(\lambda)R(\lambda) \tag{4.20}$$

$$q_{test,i}(\lambda) = r_i(\lambda)s(\lambda) \tag{4.21}$$

where R(λ) and s(λ) are the reference and test sources, respectively, whose Y values were scaled to 100. These reflection spectra are then used to calculate the L*a*b* coordinates for both $q_{ref,i}$ and $q_{test,i}$ where the nominal white source is selected as the reference source R(λ). An important point here is that L*a*b* coordinates of the $q_{test,i}$ are calculated after carrying out chromatic adaptation transformation to the test illuminant using CMCCAT2000 method. The inputs of this transformation are the X, Y and Z tristimulus values of (1) $q_{test,i}(\lambda)$ (whose Y is set to 100), (2) test source s(λ), (3) adapting white source R(λ) (whose Y is set to 100), (3) adapting background luminance set to 1000, and (4) surround luminance set to 1000. Based on these calculated L*a*b* coordinates, the saturation difference of the reflected color $\Delta C_{ab,i}$ from sample *i* between the $q_{ref,i}(\lambda)$ and $q_{test,i}(\lambda)$ are found using Eq. (4.22):

$$\Delta C_{ab,i} = \sqrt{a_{ref,i}^2 + b_{ref,i}^2} - \sqrt{a_{test,i}^2 + b_{test,i}^2} \tag{4.22}$$

Subsequently, the L*a*b* Euclidian color difference ΔE_i between $q_{test,i}$ (λ) and $q_{ref,i}$ (λ) is found as shown below:

$$\Delta E_i = \sqrt{\left(L_{ref,i} - L_{test,i}\right)^2 + \left(a_{ref,i} - a_{test,i}\right)^2 + \left(b_{ref,i} - b_{test,i}\right)^2} \tag{4.23}$$

The corrected color difference $\Delta E_{c,i}$ becomes $\Delta E_{c,i} = \Delta E_i - \Delta C_{ab,i}$ if $\Delta C_{ab,i}$ is greater than zero, otherwise $\Delta E_{c,i}$ becomes equal to ΔE_i. The total color difference is found by finding the root mean square of the corrected color differences as expressed in Eq. (4.24):

$$\Delta E_{rms} = \sqrt{\frac{1}{15}\sum_{i=1}^{15} \Delta E_{c,i}^2} \tag{4.24}$$

An important improvement of CQS over CRI is the introduction of a correlated color temperature factor. Finding this factor requires the calculation of the gamut area F_{total} for each Munsell sample *i* (if *i* = 15, *i* + 1 is assumed to be 1). The calculation is carried out using Eqs. (4.25)–(4.30):

$$A_i = \sqrt{a_i^2 + b_i^2} \tag{4.25}$$

$$B_i = \sqrt{a_{i+1}^2 + b_{i+1}^2} \tag{4.27}$$

$$C_i = \sqrt{(a_{i+1} - a_i)^2 + (b_{i+1} - b_i)^2} \tag{4.27}$$

$$t_i = \frac{A_i + B_i + C_i}{2} \tag{4.28}$$

$$F_i = \sqrt{t_i(t_i - A_i)(t_i - B_i)(t_i - C_i)} \tag{4.29}$$

$$F_{total} = \sum_{i=1}^{15} F_i \tag{4.30}$$

If F_{total} is greater than 8210 K, the correlated color temperature factor f_{CCT} becomes 1, otherwise f_{CCT} is $F_{total}/8210$. Finally, the CQS is calculated using Eq. (4.31):

$$CQS = 10\log\left(e^{\frac{100-3.105\times\Delta E_{rms}}{10}+1}\right) \times f_{CCT} \tag{4.31}$$

4.3 Photometry: Stimulus Useful for the Human Eye

The first pair of radiometric-photometric quantities that we introduce here is the radiant and luminous flux. Radiant flux is basically the power radiated by a light source and has units of W_{opt}. The luminous flux (Φ), on the other hand, is defined as the useful optical radiation for the human eye, expressed in units of lumen (lm), and calculated by using Eq. (4.32) where $P_R(\lambda)$ and $V(\lambda)$ stand for the spectral radiant flux and the photopic eye sensitivity function, respectively.

$$\Phi = 683\frac{lm}{W_{opt}}\int P_R(\lambda)V(\lambda)d\lambda \tag{4.32}$$

Another important radiometric quantity is the irradiance, which is the optical power per unit area and expressed in units of W_{opt}/m^2. The illuminance is the irradiance subject to the photopic human eye sensitivity function, and it has units of lm/m^2 or equivalently lux. Given the spectral irradiance $P_I(\lambda)$, the illuminance (IL) is expressed as in Eq. (4.33). The illuminance is a quantity which is used to assess the effect of the lighting on the human circadian cycle.

$$IL = 683\frac{lm}{W_{opt}}\int P_I(\lambda)V(\lambda)d\lambda \tag{4.33}$$

Among the most important pairs of radiometric-photometric quantities we can include are the radiance and luminance. The radiance that is expressed in $W_{opt}/(m^2sr)$ is the optical power per solid angle per unit area. For a spectral radiance $P_L(\lambda)$, the luminance L that is the optical radiance useful to human eye is found in units of $lm/(m^2sr)$, or equivalently cd/m^2 using Eq. (4.34). The calculation makes use of photopic eye sensitivity function as given by

$$L = 683\frac{lm}{W_{opt}}\int P_L(\lambda)V(\lambda)d\lambda \tag{4.34}$$

where $V(\lambda)$ is the photopic eye sensitivity function.

Although the luminance levels are traditionally calculated using photopic eye sensitivity function, there is a need to quantitatively express accurate luminance levels in different visual regimes, especially for the mesopic vision regime, which corresponds to the road lighting conditions. In 2010, CIE addressed this problem by publishing a recommended system called CIE 191:2010. According to this recommendation, the mesopic vision regime falls into any photopic luminance levels between 0.005 and 5 cd/m^2. When the luminance level is below 0.005 cd/m^2, the vision regime is considered to be the scotopic regime while the luminance greater than 5 cd/m^2 corresponds to the photopic vision regime [13]. The mesopic luminance L_{mes} is found using Eq. (4.35) where $V_{mes}(\lambda)$ is the mesopic eye sensitivity function whose maximum value is 1, λ_0 is 555 nm, and $P(\lambda)$ is the spectral radiance.

$$L_{mes} = 683/V_{mes}(\lambda_0) \int P(\lambda) V_{mes}(\lambda) d\lambda \tag{4.35}$$

The mesopic eye sensitivity function is suggested to be a linear combination of the photopic and scotopic eye sensitivity functions, calculated using Eq. (4.36) where $V(\lambda)$ and $V'(\lambda)$ stand for the photopic and scotopic eye sensitivity functions, respectively. M(m) is a normalization constant equating the maximum value of $V_{mes}(\lambda)$ to 1, and m is the coefficient that sets the contribution of scotopic and photopic eye sensitivity functions according to visual adaptation conditions.

$$M(m)V_{mes}(\lambda) = mV(\lambda) + (1 - m)V'(\lambda) \tag{4.36}$$

Here m is 0 if L_{mes} is greater 5 cd/m^2, and m is 1 if L_{mes} is smaller than 0.005 cd/m^2. The intermediate values of m and L_{mes} are found using an iterative approach employing the relations in Eqs. (4.37) and (4.38) and setting m_0 to 0.5.

$$L_{mes,n} = \frac{m_{n-1}L_p + (1 - m_{n-1})L_s V'(\lambda_0)}{m_{n-1} + (1 - m_{n-1})L_s V(\lambda_0)} \tag{4.37}$$

$$m_n = a + b\log_{10}\left(L_{mes,n}\right) \tag{4.38}$$

where a and b are 0.7670 and 0.3334, respectively, n is the step of iteration, m_n is always between 0 and 1, L_s and L_p are the scotopic and photopic luminances, respectively, and $V(\lambda_0)$ and $V'(\lambda_0)$ are the photopic and scotopic eye sensitivity function values at 550 nm. The iteration is continued until the difference between m_n and m_{n-1} becomes negligibly low.

From the device point of view, achieving the desired luminance levels is important. However, this is just one part of the performance, also the efficiency of the light-emitting devices should be high. There are two metrics that need to be considered while designing an efficient light source. The first one is the optical efficiency of the device. It basically evaluates how efficiently the radiated light can be perceived by the human eye. This metric is called the luminous efficacy of the optical radiation (LER), which is calculated using Eq. (4.39). In this equation, $P(\lambda)$ stands for the spectral radiation and $V(\lambda)$ is the eye sensitivity function at the vision regime of interest.

LER has units of lm/W_{opt} and takes a maximum value of 683 lm/W_{opt}, which can only be achieved by a monochromatic light source emitting at 555 nm. An excellent white light source should have LER > 350 lm/W_{opt} [10].

$$LER = \frac{683\,\mathrm{lm}/\,W_{opt} \int P(\lambda)V(\lambda)d\lambda}{\int P(\lambda)V(\lambda)d\lambda} \tag{4.39}$$

The second efficiency metric evaluates how efficiently the sources radiate light per supplied electrical power. This metric that disregards the human perception specifications is called the wall plug efficiency or power conversion efficiency, which is essentially the total collected optical power divided by electrical power. When we consider the human perception, on the other hand, the efficiency metric should include the luminous flux. The resulting quantity is known as the luminous efficiency (LE), computed using Eq. (4.40) where $P(\lambda)$ is the spectra radiance and P_{elect} is the electrical power. The unit of LE is lm/W_{elect}. Today, the LEs of the efficient light sources are in the proximity of 150 lm/W_{elect} [14].

$$LE = \frac{683\,\mathrm{lm}/\,W_{opt} \int P(\lambda)V(\lambda)d\lambda}{P_{elect}} \tag{4.40}$$

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