Chapter 3
Colorimetry for LED Lighting

Abstract In this Chapter, we explain the basics of colorimetry and introduce the colorimetric tools useful for designing light sources.

Keywords Colorimetry · Color matching · Chromaticity diagram

For general lighting, a good white light source should help us perceive the real colors of objects as accurately as possible. Especially from the architectural point of view, we also need to be able to compare the white light emitted by different light sources. From the point of displays, the light sources should be able to reproduce the colors of objects as correctly as possible. To evaluate all these qualities of light sources, colorimetry plays an essential role. It provides us with a quantitative description of colors and gives us the tools kit to test the quality of a light source for various applications and compare different light sources.

The human color perception forms the basis of the colorimetry field. As we have discussed in the previous section, the cones are the photoreceptors that form the essence of our color perception. When we look at their sensitivity spectra (Fig. 2.2), we observe that three types of cones predominantly absorb across different parts of the visual spectrum. This is the reason why we perceive three primary colors. However, another important feature of these sensitivity spectra is that they also overlap very strongly. This means—from a mathematical point of view—they do not form an orthogonal basis. As a result, we may perceive different combinations of the light stimuli having different spectra as identically the same color. This enables to achieve perfect or at least satisfactory perceived color accuracy without mimicking the sun's spectrum.

The attempts to quantitatively describe the colors date back to the early 20th century with the color wheel and color triangle of J. C. Maxwell. There have been several additional efforts on this topic and in 1931, the International Commission on Illumination (CIE) introduced a standard quantitative description of color by mapping the perceived colors to a color space called CIE 1931. This color space makes use of three color matching functions: $\bar{x}$, $\bar{y}$, and $\bar{z}$, whose spectral distributions are given in Fig. 3.1 [1].

T. Erdem and H. V. Demir, *Color Science and Photometry for Lighting with LEDs and Semiconductor Nanocrystals*, Nanoscience and Nanotechnology,
https://doi.org/10.1007/978-981-13-5886-9_3

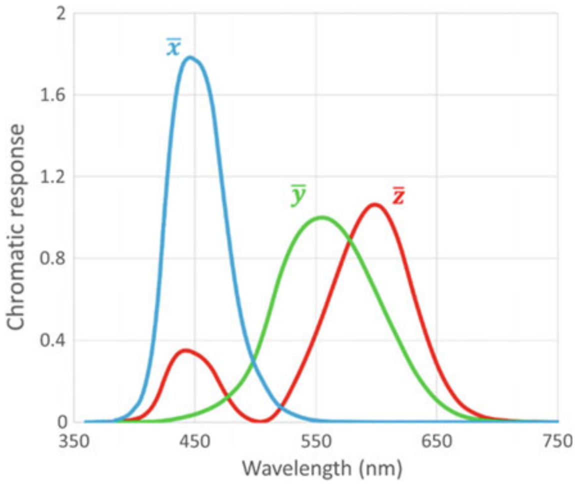

Fig. 3.1 Spectral distribution of the color matching functions used in CIE 1931 color space

In order to compute the color coordinates, we first calculate the so-called tristimulus values, X, Y, and Z by using Eqs. (3.1)–(3.3) for an arbitrary radiation spectrum of s(λ).

$$X = \int s(\lambda)\bar{x}(\lambda)d\lambda \tag{3.1}$$

$$Y = \int s(\lambda)\bar{y}(\lambda)d\lambda \tag{3.2}$$

$$Z = \int s(\lambda)\bar{z}(\lambda)d\lambda \tag{3.3}$$

The (x, y) chromaticity coordinates, which are also referred to as CIE 1931 chromaticity coordinates, are calculated using Eqs. (3.4)–(3.6). Instead of using three independent variables, the normalization reduces the coordinates to (x, y) as z = 1 − x − y. Since one of the three coordinates is dependent on the other remaining two, this methodology generates a two-dimensional color mapping as presented in Fig. 3.2.

$$x = \frac{X}{X + Y + Z} \tag{3.4}$$

$$y = \frac{Y}{X + Y + Z} \tag{3.5}$$

$$z = \frac{Z}{X + Y + Z} = 1 - x - y \tag{3.6}$$

Despite the fact that this color mapping is the most widely preferred chromaticity diagram, it has an inherent problem that the geometrical difference between the posi-

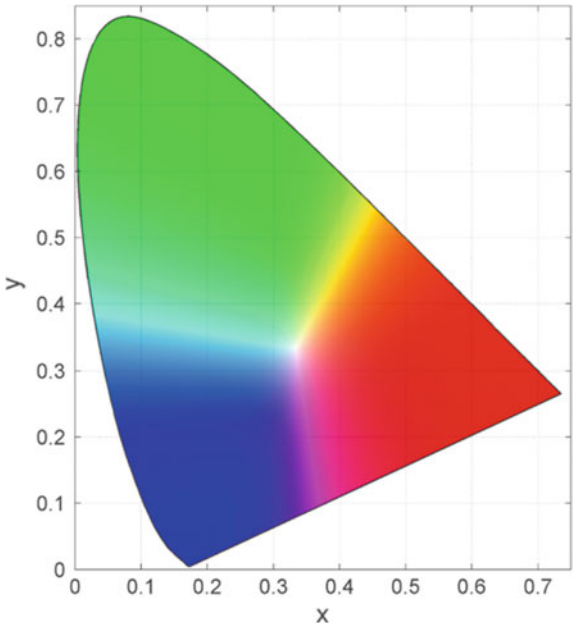

Fig. 3.2 (x, y) chromaticity diagram. This color gamut is also known as CIE 1931 chromaticity diagram

tions of pairs of colors does not consistently correspond to the perceived difference between the colors leading to nonuniform color distributions. As a solution to this problem, additional color mapping methodologies were proposed by CIE. Among them are the (u, v), (u', v'), and $L^*a^*b^*$ chromaticity diagrams.

The (u, v) and (u', v') coordinates are related to X, Y, and Z color coordinates using Eqs. (3.7)–(3.9). We present the (u', v') chromaticity diagram in Fig. 3.3. As we can clearly see, especially green and red colors are more equally distributed on this diagram.

$$u = u' = \frac{4X}{X + 15Y + 3Z} \tag{3.7}$$

$$v = \frac{6Y}{X + 15Y + 3Z} \tag{3.8}$$

$$v' = \frac{3}{2}v \tag{3.9}$$

Despite the improvements on (u', v') chromaticity diagrams in terms of color uniformity, this system still needed to be improved. In addition to this, the existing systems, which do not include the effect of the luminance on the color perception, needed to be modified to possess this information. These issues were addressed by CIE in 1976 and $(L^*a^*b^*)$ chromaticity diagram was introduced (Fig. 3.4). Contrary to the previous systems, $(L^*a^*b^*)$ is a three-dimensional color space that maps the perceived colors considering the effects of luminance. The corresponding color

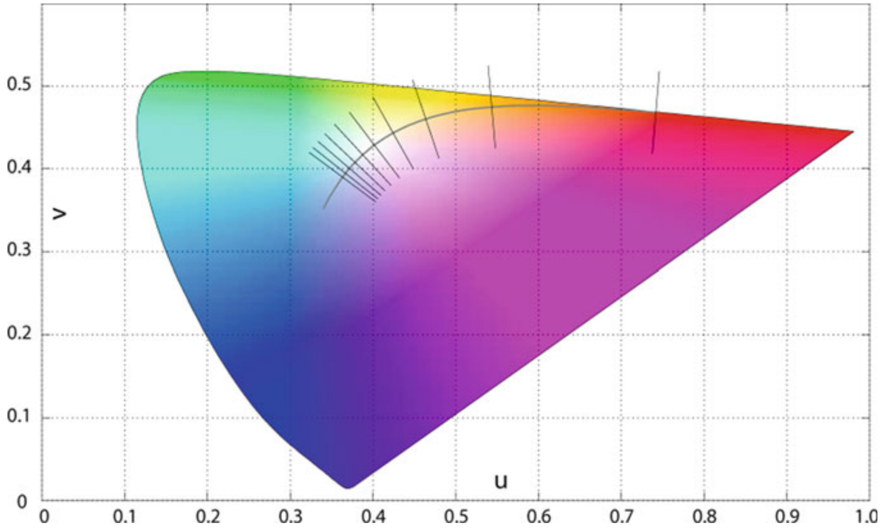

Fig. 3.3 (u′, v′) chromaticity diagram. Reproduced from Ref. [2]

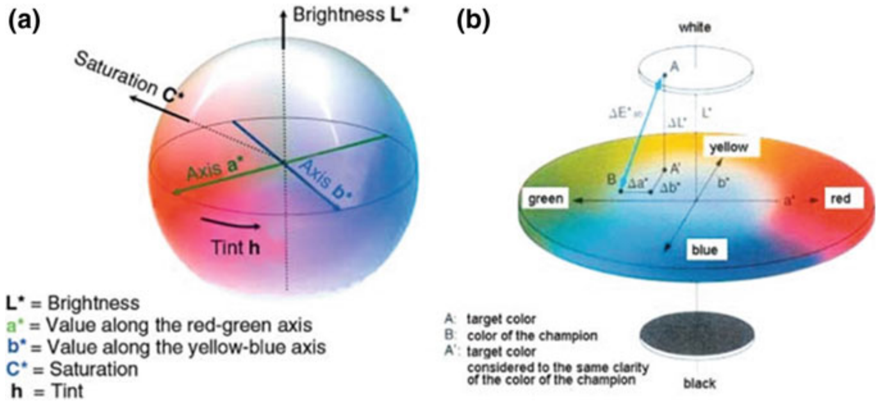

Fig. 3.4 Illustration of (**a**) the full CIE L*a*b* chromaticity diagram and (**b**) a cross-section. Reproduced with permission from Ref. [3]

coordinates are calculated using Eqs. (3.10)–(3.12), where X_n, Y_n, and Z_n are the nominally white object color stimulus, and calculated mostly using CIE standard illuminant A.

$$L^* = 116\left(\frac{Y}{Y_n}\right)^{1/3} \tag{3.10}$$

$$a^* = 500\left[\left(\frac{X}{X_n}\right)^{1/3} - \left(\frac{Y}{Y_n}\right)^{1/3}\right] \tag{3.11}$$

$$b^* = 200\left[\left(\frac{X}{X_n}\right)^{1/3} - \left(\frac{Z}{Z_n}\right)^{1/3}\right] \tag{3.12}$$

To express color differences under different conditions, color adaptation transformations are developed. This approach allows to a quantitative description of human color perception adaptation to different white point/white light changes under different illumination conditions. Here, we will summarize CMCCAT2000 method, which is essentially a developed version of the previous adaptation transformation CIECAT97. This transformation makes use of X, Y and Z values of a spectral power distribution usually from the reflected stimulus of the test source (called X_s, Y_s and Z_s), the chromaticity coordinates of the spectral power distribution of the test light source (dubbed with X_t, Y_t and Z_t), the chromaticity coordinates of the spectral power distribution of a reference light source such as standard D65 illuminant (dubbed as X_r, Y_r and Z_r), and finally the luminance values of test and reference adapting fields, named as L_{a1} and L_{a2}. The calculation starts with transforming X, Y, and Z tristimulus values of all input tristimulus values to R, G, and B values using the relation given below:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.7982 & 0.3389 & -0.1371 \\ -0.5918 & 1.5512 & 0.0406 \\ 0.0008 & 0.0239 & 0.9753 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{3.13}$$

Next, the degree of adaptation D is 1 if $D' > 1$, and it is 0 if $D' < 0$, and otherwise it is equal to D' where D' is found using Eq. (3.14).

$$D' = 0.08\log_{10}(0.5(L_{a1} + L_{a2})) + 0.76 - \frac{0.45(L_{a1} - L_{a2})}{L_{a1} + L_{a2}} \tag{3.14}$$

The adapted RGB values (R_c, G_c and B_c) are calculated using the relation given below in Eq. (3.15):

$$\begin{bmatrix} R_c \\ G_c \\ B_c \end{bmatrix} = \begin{bmatrix} D \times \frac{R_r}{R_t} + 1 - D & 0 & 0 \\ 0 & D \times \frac{G_r}{G_t} + 1 - D & 0 \\ 0 & 0 & D \times \frac{B_r}{B_t} + 1 - D \end{bmatrix} \begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix} \tag{3.15}$$

The adapted X, Y, Z tristimulus values (X_c, Y_c and Z_c) are found using Eq. (3.16).

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} 0.7982 & 0.3389 & -0.1371 \\ -0.5918 & 1.5512 & 0.0406 \\ 0.0008 & 0.0239 & 0.9753 \end{bmatrix}^{-1} \begin{bmatrix} R_c \\ G_c \\ Z_c \end{bmatrix} \tag{3.16}$$

Finally, the adapted x, y and z chromaticity coordinates are computed using Eqs. (3.4)–(3.6).

All of these calculations are necessary for evaluating the color rendition performance of the sources and the shade of the light. In Appendix A of this brief, we provide MATLAB codes for calculating the presented chromaticity coordinates along with color matching function tables.

References

1. "CIE Commission Proceedings," 1931
2. "By Adoniscik—Own work, CC BY 3.0, https://commons.wikimedia.org/w/index.php?curid=3838965
3. Lorusso S, Natali A, Matteucci C (2007) Colorimetry applied to the field of cultural heritage: examples of study cases. Conservation Sci Cultural Heritage 7:187–220