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Intraday efficiency-frequency nexus in the cryptocurrency markets[☆]

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ABSTRACT

This study investigates the nexus between weak-form efficiency and intraday sampling frequency for the highest capitalized cryptocurrencies. Applying a battery of long memory tests, we provide evidence of major discrepancies on the predictability of cryptocurrency returns for alternative high frequency intervals. Accordingly, efficiency demonstrates a U-shaped pattern with respect to alternative sampling frequencies, hence there exists an optimal intraday sampling frequency that maximizes the market efficiency. These findings have important implications for portfolio analysis, risk management, regulations and administrative rulings in the cryptocurrency markets.

1. Introduction

In today's global financial markets, cryptocurrencies, of which Bitcoin is the most prominent example, have been subject to noteworthy interest by theorists, analysts, traders and regulators. The exponential growth of their values coincides with the increased attention in media and finance literature, and moreover, they are acted as a new type of financial assets lately. Many central banks are now exploring new payment technologies, including cryptocurrencies and planning to launch their own digital currencies in order to better understand the principles and potential benefits. Chicago Mercantile Exchange (CME) and Chicago Board Options Exchange (CBOE) have begun to trade futures contracts on Bitcoin in December 2017. In a first for a major U.S. bank, J.P. Morgan launched its own cryptocurrency in February 2019.¹ The latest move by Facebook with its coin Libra has led companies around the world to consider cryptocurrency offering as a potential strategy.

Although cryptocurrencies have attracted special attention from academics and practitioners, there lacks a comprehensive study towards the intraday dynamics of cryptocurrencies, and worse, these research have almost exclusively focused on Bitcoin. Among these, [Urquhart \(2016\)](#), [Nadarajah and Chu \(2017\)](#), [Bariviera \(2017\)](#), [Yonghong et al. \(2018\)](#), [Vidal-Tomas and Ibanez \(2018\)](#), [Turatto et al. \(2019\)](#) and [Gerritsen et al. \(2019\)](#) investigate whether time varying behaviour of Bitcoin is predictable, which would be inconsistent with the Efficient Market Hypothesis (EMH) proposing that prices should follow a random walk ([Fama, 1970](#)). The same question has been analyzed at the intraday level by [Sensoy \(2019\)](#), [Corbet et al. \(2019\)](#) and [Akyildirim et al. \(2019\)](#). Such an analysis is uttermost important since any predictable patterns open the doors for trading strategies

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¹ <https://www.nytimes.com/2019/02/14/business/dealbook/jpmorgan-cryptocurrency-bitcoin.html>

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that can beat the market and generate abnormal returns.

Despite extensive research on the efficiency of Bitcoin, only a few other studies have investigated the return predictability of alternative cryptocurrencies. The majority of these studies provide evidence for inefficiency in cryptocurrency markets (Bouri et al., 2018; Brauneis and Mestel, 2018; Caporale et al., 2018; Phillip et al., 2018; Wei, 2018; Zhang et al., 2018; Chaim and Laurini, 2019; Charfeddine and Maouchi, 2019; Kochling et al., 2019; Hu et al., 2019; Mensi et al., 2019a; 2019b). To the best of our knowledge, only the work by Mensi et al. (2019b) uses high-frequency data of an alternative coin (Ethereum) in order to examine its long memory and weak-form efficiency property.

In this paper, we examine the weak-form efficiency property of four major cryptocurrencies, namely, Bitcoin, Ethereum, Ripple and Litecoin. We employ three strong methods (R/S, GHE and GPH) to estimate the Hurst exponent and test whether (i) efficiency depends on sampling frequency, (ii) there is an optimal sampling frequency that maximizes weak-form market efficiency, and (iii) optimal frequency differs from one cryptocurrency to another.

This study fills four gaps in the literature. First, the efficiency of Bitcoin has been examined in some depth yet other cryptocurrencies have so far been ignored. We investigate the efficiency of four most capitalized cryptocurrencies, including the Bitcoin.

Second, instead of using daily data like most of the previous studies do, we go down to high frequency level since algorithmic (especially high frequency) trading dominates the trading scene in today's modern financial markets. Moreover, high frequency data is more informative compared to the daily data and may yield more clear picture regarding portfolio analysis and risk management. From the point of the EMH, high frequency data has the edge over the traditional daily data in the empirical study of informational efficiency.

Third, there is a notable lack of studies examining the efficiency in cryptocurrency market for various time intervals. Ongoing literature on the efficiency mostly focused on price observations drawn at fixed time intervals. For example, Eross et al. (2019) decide to select 5 min frequency since they claim that due to noise trading, any higher frequency data is not as liquid compared to 5-min data which may lead to unreliable and spurious results. We, on the other hand, analyze the weak form efficiency in cryptocurrency markets at intervals of 1 min, 5, 10, 15, 30 and 60 min. Goodhart and O'Hara (1997) argue that it is important to use high enough frequency to capture intraday dynamics of the data, but at the same time low enough to avoid any undue noise. In that sense, we characterize the optimal trading frequency for efficiency in this dynamic market using various time intervals. To the best of our knowledge, such kind of an analysis has not been done by previous studies within this context.

Fourth, we employ three various powerful and contrasting tests (Rescaled Range (R/S) statistic, semi parametric GPH statistic and Generalized Hurst Exponent method) for randomness in order to prevent spurious results and to capture all the dynamics of those cryptocurrencies.

Our findings can be summarized as follows: (i) the efficiency in the cryptocurrency markets varies across frequencies, (ii) sample cryptocurrencies achieve maximum market efficiency around 5-min and 10-min sampling frequencies, thus there is a U-shaped pattern, and (iii) investors employing trading strategies are more likely to generate abnormal profits in the sample cryptocurrencies making trades on either very short (1 min) or the longest (60 min) sampling basis.

2. Methodology

We use the Hurst exponent H to measure long-range dependence.² Accordingly, the Hurst exponent lies in the interval $[0,1]$. On the basis of the H values, three categories can be identified: (i) the series are mean-reverting, returns are negatively correlated ($0 \leq H < 0.5$); (ii) the series are random, returns are uncorrelated, thus there is no long-memory in the series ($H = 0.5$); and (iii) the series are persistent, returns are positively correlated ($0.5 < H \leq 1$). Accordingly, while items (i) and (iii) imply inefficiency, item (ii) implies weak form efficient property of the analyzed time series.

We utilize three differing methods to estimate the Hurst exponents for four cryptocurrencies at six intraday sampling frequencies. The selected methods include rescaled range analysis (Hurst, 1951), the Geweke and Porter-Hudak (1983) estimator, and the generalized Hurst exponent (Barabasi and Vicsek, 1991). The following section briefly describes each method.

2.1. Rescaled range analysis (r/s)

Rescaled range analysis (R/S) was introduced by Hurst (1951) and further augmented by Mandelbrot (1972, 1975), Mandelbrot and Taqqu (1979) and Mandelbrot and Wallis (1968, 1969a, 1969b) in order to test for long memory property in time series.

To determine the difference between minimum and maximum values of running sums of deviations from the sample mean, renormalized by the sample standard deviation is the purpose behind the classical R/S test. The deviations are larger in case there is long-term memory.

As discussed by Mandelbrot and Taqqu (1979) and Taqqu et al. (1995), for a time series $\{X_t\}_{t \in T}$, $T = \{1, \dots, N\}$, with partial sum $Y(n) = \sum_{i=1}^n X_i$, $n = 1, \dots, N$, and sample variance $S^2(n) = (1/n) \sum_{i=1}^n X_i^2 - (1/n)^2 Y(n)^2$, the R/S statistic, or the rescaled adjusted

² Hurst exponent is a more robust tool to measure market efficiency compared to simple measures of autocorrelation or variance ratio tests. The autocorrelation function is inadequate in explaining non-linear temporal dependence in a series. Besides, it is sensitive to abrupt shifts or jumps in mean or volatility. On the other hand, variance ratio tests assumes normality of the data. In addition, these tests are asymptotic and hence for a finite sample, the sampling distributions of the test statistics are approximated based on its limiting distributions.

range, is given by: $\frac{R}{S}(n) = \frac{1}{S(n)} [\max_{0 \leq t \leq n} (Y(t) - \frac{t}{n}Y(n)) - \min_{0 \leq t \leq n} (Y(t) - \frac{t}{n}Y(n))]$.

To calculate H for a return series of length T , we divide the series into N contiguous sub-periods with length ν , where $N\nu = T$. For each sub-period, we calculate the mean value and construct the mean adjusted series, and cumulative deviate series. We then calculate the range, which is defined as the maximum value of the profile minus the minimum of the profile, and the standard deviation of the original time series for each sub-period. Each range is normalized by dividing by the corresponding standard deviation and forms a rescaled range so that the average rescaled range for a given sub-period of length $(R/S)_\nu$ is obtained.

The rescaled range scales as $(R/S)_\nu \approx c\nu^H$, where c is a finite constant independent of ν (Di Matteo, 2007; Taqqu et al., 1995). To uncover the scaling law, simple ordinary least squares regression is used to estimate the equation $\log(R/S)_\nu = \log(c) + H\log(\nu)$, where the slope coefficient is the estimate of the Hurst exponent.

2.2. Geweke and Porter-Hudak (GPH) model

The second method used to calculate H is the log-periodogram estimator (also known as GPH estimator) of Geweke and Porter-Hudak (1983) which is one of the widespread semiparametric estimation methods. The GPH estimator of the long memory parameter, d , is obtained from the application of the simple linear regression based on periodogram. The periodogram is defined as:

$$I(\lambda_k) = \frac{1}{2\pi T} \left| \sum_{j=1}^N X_j e^{ij\lambda_k} \right|^2.$$

In this setup, $I(\lambda_k)$ stands for the k^{th} periodogram point and it is defined as the squared absolute values of the Fourier transform of the time series X_j . The estimate d is determined running OLS on $\log(I(\lambda_k)) = a - d \log\{4\sin^2(\frac{\lambda_k}{2})\} + \varepsilon_k$ where $\lambda_k = 2\pi k/N$, $k = 1, 2, \dots, n$; ε_k is the residual term and λ_k represents the $n = \sqrt{N}$ Fourier frequencies. The Hurst exponent equals to $d + 0.5$. Namely, the null hypothesis in GPH test is that 'there is no long memory ($d = 0$)'.

Using synthetic data, Geweke and Porter-Hudak (1983) developed the asymptotic theory which proved to be reliable in samples of 50 observations or more. Namely, the conventional interpretation of the least squares statistics is reasonable in large samples.

2.3. Generalized hurst exponent approach

The third method used to generate the parameter H is the generalized Hurst exponent (GHE) which was introduced by Barabasi and Vicsek (1991). Di Matteo et al. (2003) re-explored this method for financial time series which is based on scaling of q^{th} order moments of the distribution of the increments. The GHE is a generalization of the approach proposed by Hurst (1951). The q^{th} order moment of the distribution of the increments of the stochastic variable $S(t)$, with $t = (1, 2, \dots, \Delta t)$ is given by $K_q(\tau) = \frac{\langle |S(t+\tau) - S(t)|^q \rangle}{\langle |S(t)|^q \rangle}$ where τ can vary between 1 and τ_{\max} and $\langle \dots \rangle$ denotes the sample average over the time window. The generalized Hurst exponent $H(q)$ is defined for each time scale τ and each parameter q as $K_q(\tau) \propto \tau^{qH(q)}$.

For $q = 1$, $H(1)$ characterizes the scaling behaviour of the absolute increments and is very similar to the original Hurst exponent. The scaling exponent for $q = 2$ is connected to the scaling of the autocorrelation function of the increments. Hence, the estimated generalized Hurst exponent $H(2)$ is comparable with the estimated H of R/S .

GHE combines sensitivity to any type of dependence in the data and simple algorithm. Besides, it is less sensitive to outliers than the R/S analysis, which relies on maxima and minima (Di Matteo et al., 2005). Moreover, Barunik and Kristoufek (2010) questioned the behaviour of the Hurst exponent estimate for non-normal process with heavy tails since the returns of the financial markets are not normally distributed and are heavy-tailed. Comparing various methodologies; they show that R/S and GHE are robust to heavy tails in the underlying data whilst GHE provides the lowest variance and bias as well.

3. Data and results

Data comes from the digital asset store Kaiko which offers tick-by-tick trade data for 6,000+ currency pairs across 32+ exchanges including Bitfinex, the world's leading cryptocurrency trading platform. We examine dollar-denominated data from Bitfinex exchange for 4 cryptocurrencies, choosing those with the highest market capitalization, namely BitCoin, Ethereum, Ripple and Litecoin. More information on those cryptocurrency markets is provided in Table 1 below.

For each trade, the data includes a GMT timestamp, amount of cryptocurrency traded and the cryptocurrency price in terms of USD. For each interval, we use that interval's closing price (instead of the volume-weighted average price) to calculate log-returns, at six different interval lengths (i.e. 1, 5, 10, 15, 30, and 60 min). Returns are calculated by taking the difference in the logarithm of two consecutive prices. Sample data for the cryptocurrencies start from 01/04/13, 09/03/16, 10/08/17 and 19/05/13 for Bitcoin,

Table 1
Summary of the sample cryptocurrencies (as of 06.05.2019).

Name	Market Cap (USD)	Price (USD)	Circulating Supply	Sample starts from
BTC	101,019,023,292	5,712.11	17,685,062 BTC	01/04/13
ETH	17,060,582,832	161.02	105,951,478 ETH	09/03/16
XRP	12,601,538,021	0.299416	42,087,046,846 XRP	10/08/17
LTC	4,506,090,567	73.09	61,647,458 LTC	19/05/13

Source: <https://coinmarketcap.com/>

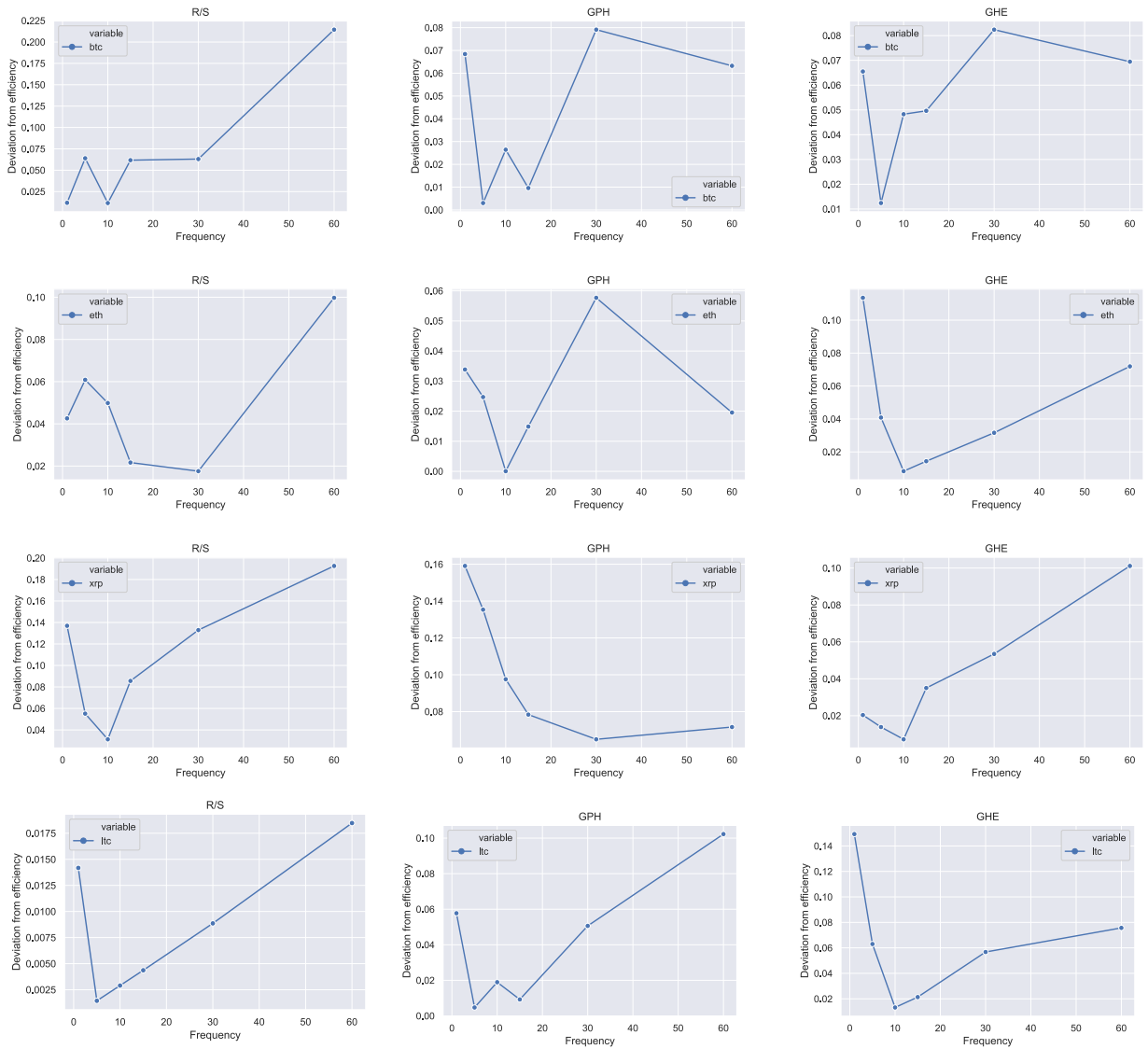


Fig. 1. The evolution of informational efficiency of each cryptocurrency across different frequencies with three techniques .

Ethereum, Ripple and Litecoin, respectively. For all coins, the sample ends on 23/06/2018. After estimating the Hurst exponents using R/S, GHE and GPH methods, we take their differences from 0.5 to determine the deviation from the efficiency. Accordingly, the less the deviations from 0.5, the more efficient the cryptocurrency is. Fig. 1 presents the evolution of informational efficiency of each cryptocurrency across different frequencies. The plots clearly show that the degree of efficiency changes from one frequency to another. The optimal sampling frequencies that maximize weak form market efficiency are reported in Table 2.

Accordingly, we can achieve more efficiency with 5-min and 10-min frequencies than 1-min, 15-min or 60-min frequencies. More specifically, 5-min is mostly optimal for BTC and XRP whereas 10m is optimal for ETH and LTC in our sample period. These results are consistent with these Andersen (2000) who states that the 5-min interval is the highest at which enough data is available to

Table 2

Optimal sampling frequencies maximizing weak form market efficiency with respect to different techniques for estimating Hurst exponent.

Cryptocurrency	R/S	GPH	GHE
BTC	10 m	5 m	5 m
ETH	30 m	10 m	10 m
XRP	5 m	5 m	10 m
LTC	10 m	30 m	10 m

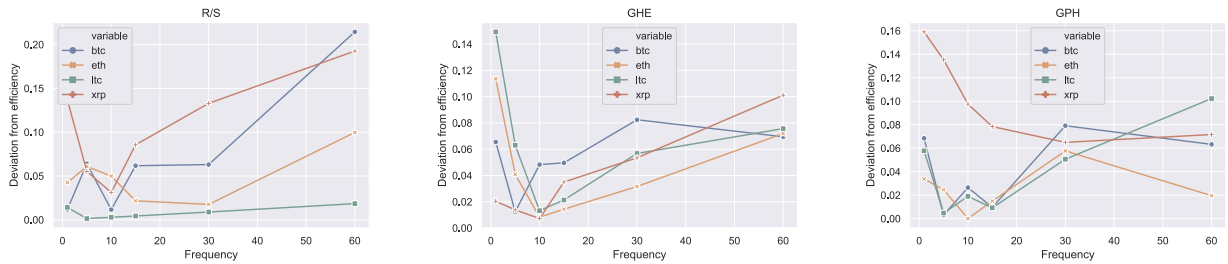


Fig. 2. The evolution of informational efficiency of four cryptocurrency across different frequencies with three techniques.

reflect the high-frequency behaviour of the series while also at the same time enough observations free of noise problems are provided.

For each currency, the results show that neither 1 min frequency nor 60 min frequency is optimal in terms of efficiency. The efficiency is lower when we use 60 min frequency rather than 5- or 10 min frequency since a higher-frequency market enables investors to respond to new information and to adjust their asset holdings more quickly. On the other hand, 1 min frequency data does not give more efficiency. Except the noise in the data, as a shortcoming, the high-frequency market gets 'thinner' and thus attracts less investors since they prefer not to trade expecting superior relative returns in future periods, which leads to low transaction volume and relative illiquidity. This trade-off, together with the timing of incoming information, results in the optimal trading frequency of 5- and 10 min intervals. The test results also indicate that the degree of informational efficiency differs from one cryptocurrency to another. Fig. 2 plot the evolution of efficiencies of four markets across different time intervals together.

The results of the R/S test show that LTC is the most efficient cryptocurrency for each frequency in our sample period. Moreover, the R/S test provide evidence that BTC is more efficient than XRP in 10-min data interval which is optimal frequency for both cryptocurrencies. Investors in the XRP market can generate more abnormal profits by implementing trading strategies, compared to BTC.

The results from GHE test reveal that 10 min is optimal frequency for all cryptocurrencies, except for BTC. BTC is the most inefficient cryptocurrency at the 10 min level of data whereas it is the most efficient at the 5 min sampling frequency (where the optimal frequency is attained for BTC). Not only GHE test, but the GPH test also validates the same fact that BTC is the most efficient cryptocurrency at 5-min sampling. However, the results of the GPH test provides evidence that ETH is most efficient one among these 4 cryptocurrencies at 10 min intervals. One possible explanation could be related to that ETH has the second-highest market capitalization and supports much more functionality than Bitcoin (Chen et al., 2017). Moreover, according to the GPH method, XRP is more inefficient than LTC and ETH at 30 min data sampling.

To characterize the impact of frequency on the efficiency better, we consider Spearman correlations between the efficiency rankings within the same methodology but across different frequencies. Spearman's rank-order correlation allows us to evaluate the association between variables that are measured at the ordinal level. The averages of Spearman correlation coefficients in Table 3 are 0.43, 0.29 and 0 for R/S, GPH and GHE methods, respectively. The low correlations suggest that there is no significant linear relationship between those 6 different frequencies. Put it differently, the efficiency in cryptocurrency market varies across frequencies so the frequency matters.

Table 3

Spearman correlation coefficients between sampling frequencies for different Hurst exponent estimation techniques.

R/S	30 m	5 m	10 m	15 m	30 m	60 m
1 m	1.0	-0.4	0.6	0.4	0.4	-0.2
5 m	-0.4	1.0	0.4	0.4	0.4	0.8
10 m	0.6	0.4	1.0	0.4	0.4	0.2
15 m	0.4	0.4	0.4	1.0	1.0	0.8
30 m	0.4	0.4	0.4	1.0	1.0	0.8
60 m	-0.2	0.8	0.2	0.8	0.8	1.0
GPH	1 m	5 m	10 m	15 m	30 m	60 m
1 m	1.0	0.2	1.0	0.4	0.6	0.4
5 m	0.2	1.0	0.2	0.8	-0.2	0.0
10 m	1.0	0.2	1.0	0.4	0.6	0.4
15 m	0.4	0.8	0.4	1.0	0.4	-0.4
30 m	0.6	-0.2	0.6	0.4	1.0	-0.4
60 m	0.4	0.0	0.4	-0.4	-0.4	1.0
GHE	1 m	5 m	10 m	15 m	30 m	60 m
1 m	1.0	0.8	0.4	-0.6	0.0	-0.2
5 m	0.8	1.0	-0.2	-0.8	-0.4	0.4
10 m	0.4	-0.2	1.0	0.4	0.8	-0.8
15 m	-0.6	-0.8	0.4	1.0	0.8	-0.2
30 m	0.0	-0.4	0.8	0.8	1.0	-0.4
60 m	-0.2	0.4	-0.8	-0.2	-0.4	1.0

4. Conclusion

This study contributes to the debate surrounding the market efficiency of cryptocurrencies using intraday data at six different frequencies (1, 5, 10, 15, 30 and 60 min). We examine the frequency tie with the efficiency in four main cryptocurrency markets (Bitcoin, Ethereum, Ripple and Litecoin) using Hurst exponents estimated by various techniques. The analysis shows major discrepancies on the predictability of cryptocurrency returns for alternative frequency intervals. More specifically, the outcomes provide strong evidence on weak-form informational efficiency for 5m and 10m series whilst informationally inefficiency for 115 and 60 minutely returns, hence market efficiency displaying a U-shaped pattern with respect to sampling frequency. These findings provide a guide for investors and policy makers. In particular, high-frequency traders who use algorithmic trading strategies can generate abnormal profits in the selected cryptocurrencies by trading on 1 min or 60 min basis. On the other hand, sophisticated traders placing orders within 5- or 10 min scales are less likely to beat the market. In addition to this, to protect unsophisticated traders and to prevent adverse consequences of long memory patterns on them, financial market regulators may consider analyzing these markets at different frequencies and introduce regulations to improve efficiency levels.

Further research can focus on (i) whether these findings are stylized facts for only cryptocurrencies or are they valid for other asset classes as well; and (ii) if the same efficiency-frequency nexus holds for Bitcoin futures just as the Bitcoin spot market.

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