

# ESSAYS ON STATUS SEEKING, BEQUESTS AND INEQUALITY

A Ph.D. Dissertation

by  
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September 2019



To my family and Büşra

**ESSAYS ON STATUS SEEKING, BEQUESTS  
AND INEQUALITY**

The Graduate School of Economics and Social Sciences  
of  
İhsan Doğramacı Bilkent University

by

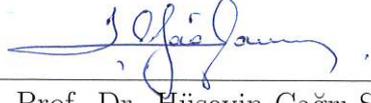
**MEHMET FATİH HARMANKAYA**

In Partial Fulfillment of the Requirements for the Degree of  
**DOCTOR OF PHILOSOPHY IN ECONOMICS**

**THE DEPARTMENT OF  
ECONOMICS  
İHSAN DOĞRAMACI BİLKENT UNIVERSITY  
ANKARA**

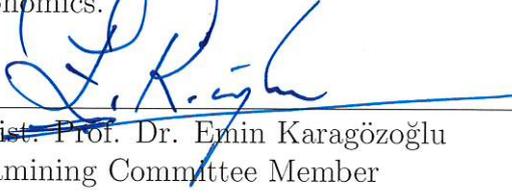
September 2019

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Supervisor

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ABSTRACT

ESSAYS ON STATUS SEEKING, BEQUESTS AND  
INEQUALITY

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Social status is the motivating force that governs the behavior of individuals. The tendency to desire higher social status affects household decision making activities. The quest for social status is mostly associated with reference dependent preferences related to economic decisions. This dissertation is made up of three essays on reference dependent preferences related to bequests and inequality. In this scope, this study presents a theoretic framework to analyze the effects of reference dependent preferences on the economy.

The first essay analyzes the effects of status quest on bequest distribution and household inequality. Focusing on the relative wealth dimension of social status, we develop a two-period overlapping generations model with heterogeneous agents. It is found that, the quest for social status modifies lifetime decisions and as a consequence, the trajectory of the overall economy. We show that, the

bequest motive and the concern for social status not only increase the stationary level of capital, but also enhance the household equality.

In the second essay, the implications of assuming different production function for the final good is studied in an overlapping generations economy model. In this analysis, social status is identified with relative transmissible wealth or bequest. In the long run, the social status concern increase the stationary level of capital. Moreover, inequality in a segregated economy made up of two groups which notably differ in their social status referent, is analyzed. It is shown numerically that, even when the only transmissible factor is wealth, group inequality persists in time. It is found that inequality can decrease in the long-term as long as the poorer group refers the richer group strongly enough.

In the third essay, we analyze the role of consumption envy on the resource distribution and household inequality. To do this, a non-overlapping generations renewable resource model is developed. Long run dynamics of the total renewable resource in the economy are analyzed, considering both linear and concave production functions. For the case of linear production function, the fraction of resources devoted to consumption is shown to increase with consumption envy. Thus, steady state level of the available resource in the economy decreases with the effect of consumption envy. Moreover, consumption envy is proven to increase the inequality between households in terms of wealth, consumption and renewable resources.

*Keywords:* Bequest, Household Inequality, Reference Dependent Preferences, Renewable Resource, Social Status

## ÖZET

# STATÜ ARAYIŞI, MİRAS VE EŞİTSİZLİK ÜZERİNE MAKALELER

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Sosyal statü, bireylerin davranışlarını belirleyen motive edici bir güçtür. Daha yüksek sosyal statü edinme arzusu yönündeki eğilim, hane halklarının karar verme faaliyetlerini etkilemektedir. Sosyal statü arayışı, çoğunlukla ekonomik kararlarla ilgili referans bağımlı tercihlerle ilişkilendirilmektedir. Bu tez, miras referansına bağımlı tercihler ve eşitsizlik üzerine üç makaleden oluşmaktadır. Bu kapsamda, referans bağımlı tercihlerin ekonomi üzerindeki etkilerini analiz etmek için teorik bir çerçeve sunulmuştur.

İlk makalede, sosyal statü arayışının, miras dağılımı ve hane halkları eşitsizliği üzerindeki etkileri analiz edilmiştir. Sosyal statünün göreceli varlık boyutuna odaklanarak, heterojen hane halklarından oluşan iki periyotlu ardışık nesiller modeli geliştirilmiştir. Sosyal statü arayışının, yaşam boyu kararları ve bunun sonucunda da genel ekonominin yörüngesini değiştirdiği bulunmuştur. Bu

çalışmada, miras bırakma güdüsünün ve sosyal statü kaygısının, yalnızca sabit sermaye seviyesini arttırmakla kalmayıp, aynı zamanda hane halkı eşitliğini de arttırdığı gösterilmiştir.

İkinci makalede, nihai ürün üretimi için farklı bir üretim fonksiyonu varsayımı kullanılarak, sermaye dinamikleri, ardışık nesiller ekonomi modelinde incelenmiştir. Bu analizde sosyal statü, göreceli aktarılabilir varlık veya miras ile tanımlanmıştır. Uzun vadede, sosyal statü kaygısının durağan denge sermaye seviyesini artırdığı gözlenmiştir. Ayrıca, sosyal statü referanslarında farklılık gösteren, iki grubun oluşturduğu ayrılmış bir ekonomideki eşitsizlik de analiz edilmiştir. Yapılan çalışmada, sonraki nesillere aktarılabilir tek faktör varlık olsa bile, grup eşitsizliğinin zaman içinde devam ettiği, numerik analizle gösterilmiştir. Zengin olan grubun, görece daha fakir olan grup tarafından, referans alınma seviyesi yeterince arttırdığında, eşitsizliğin uzun vadede azalabildiği gösterilmiştir.

Üçüncü makalede, tüketim kıskançlığının, kaynak dağılımı ve hane halkı eşitsizliği üzerindeki rolü analiz edilmiştir. Bunu yapmak için, yenilenebilir kaynak içeren ardışık olmayan nesiller modeli geliştirilmiştir. Ekonomideki toplam yenilenebilir kaynağın uzun dönem dinamikleri, hem doğrusal hem de içbükey üretim fonksiyonları dikkate alınarak incelenmiştir. Doğrusal üretim fonksiyonu için, tüketime ayrılan kaynakların oranının tüketim kıskançlığı ile arttığı gösterilmiştir. Böylece, ekonomideki mevcut kaynağın durağan denge seviyesinin, tüketim kıskançlığının etkisiyle azaldığı sonucuna ulaşılmıştır. Buna ek olarak, tüketim kıskançlığının, hane halkları arasında varlık, tüketim ve yenilenebilir kaynaklar açısından eşitsizliği arttırdığı kanıtlanmıştır.

*Anahtar Kelimeler:* Hane Halkı Eşitsizliği, Miras, Referans Bağımlı Tercihler,  
Sosyal Statü, Yenilenebilir Kaynak

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# CHAPTER 1

## INTRODUCTION

The pursuit for social status explains much of human behavior. It is impossible to prevent agents from interacting with one another. Agent's decisions are influenced by others in her society or neighborhood. Satisfaction levels depend on not only agents' own decisions, but also how they compare them with other members of the society. Weber (1922) presents 'social status' as a significant source of power and defines it as "an effective claim to social esteem in terms of positive or negative privileges." Pigou (1920) states that "a larger proportion of the satisfaction yielded by the incomes of rich people comes from their relative, rather than from their absolute amount of wealth." Duesenberry (1949) proposes relative income hypothesis which suggests that individual utility depends both absolute and relative income. It states that "ours is a society in which one of the principal social goals is a high standard of living" leading to an increase in expenditures. Easterlin (1974, 1995) claims that if the income of all others increase, the happiness of an agent will not increase. They all highlight that the desire of agents to increase expenditures depends on the relative expenditures of

the given society.

The ranking for the status includes education, age, wealth or occupation. Some empirical studies show that household decision making activities are directed by the quest for social status. Solnick and Hemenway (1998) analyze the data obtained from twelve different questions including education, attractiveness and income in a survey (257 faculty, students and staff at the Harvard School of Public Health) to provide some empirical results about relative standing. Income related survey questions are answered by half of the respondents as preferring to have 50% less real income but higher relative income. With their experimental study, Johansson-Stenman et al. (2002) support that agents want to allocate some of their resources to increase their hypothetical grand-children's relative standing in the society.

Status seeking and income inequality are interrelated concepts. Since the poorer people have more incentives to increase their social status, income inequality affects consumption and saving decisions of them. These agents increase savings, and as a result decrease consumption, to accumulate wealth for the future. Paskov et al. (2016) use repeated cross-sectional data from the European Social Survey between 2002-2014 and find that there exists negative relationship between income inequality and status seeking. Using Chinese Urban Household Survey data between 1997 and 2006, Jin et al. (2011) show that rise in income inequality can increase the level of status seeking savings. This inequality affects the consumption of the poorer households negatively. Bossmann et al. (2007) examine the role of bequests on the distribution of wealth. Using coefficient of variation as

an inequality measure, they show that bequests have diminishing effects on the wealth inequality.

How does reference dependent preferences affect the long-run dynamics of the economy? What are their effects on wealth distribution and inequality? With the aim of analyzing influences created by others, reference dependent preferences have been examined for long. (see, among others, Liu and Turnovsky, 2005; Alonso-Carrera et al., 2008; Garcia-Peñalosa and Turnovsky, 2008; Alvarez-Cuadrado and Long, 2012; Borissov, 2016). These papers address social status as an individual's level of consumption relative to the average level of consumption of others. In a capital-based economy, Liu and Turnovsky (2005) analyze the effects of consumption externalities on capital dynamics. Alonso-Carrera et al. (2008) study how the consumption externalities affects the optimality of the dynamic equilibrium in an economy displaying dynastic altruism. Focusing on distributional effects, Garcia-Peñalosa and Turnovsky (2008) investigate the link between consumption externalities and wealth inequality by considering two forms of heterogeneity, different initial wealth endowments and different reference consumption levels. They find that existence of externalities decreases wealth inequality. Alvarez-Cuadrado and Van Long (2012) study the effects of the consumption envy on inequality and shows that consumption envy increases wealth, bequest and consumption inequality. Borissov (2016) analyzes the change in inequality over time considering a family altruism type model. It considers positional concerns on agent's consumption and her heir's disposable income.

However, the aforementioned papers almost ignore that the large portion of the

wealth comes from bequests. As such, this thesis fulfils this gap in the literature by identifying social status with transmissible wealth and bequests. Household bequest is an important factor that determines the dynamics of the distribution of wealth, hence social status. Intergenerational transfers and bequests account for non-negligible percentage of wealth (see Kotlikoff et al., 1982, Modigliani, 1988 for US; Hayashi, 1986, Horioka, 2009, for Japan; Piketty, 2011, for France). Parents can leave bequest intentionally defined as altruism to increase their utility from the resources of their children (see Barro, 1974 and Becker, 1974). In line with this, we assume that parents bequeath to improve their heirs' social status. In this thesis, we consider reference dependent altruism to analyze the effects of social status pursuit on economic decisions and inequality. In a society made of altruistic households, agents care about their bequests relative to the average level in the economy.

This thesis consists of three essays centering on reference dependent altruism and inequality. In the first essay, we analyze how quest for social status affect the economy in terms of capital dynamics and wealth inequality. We use a two period overlapping generations model with heterogeneous agents. Agents differ in terms of productive ability and transmissible wealth. We consider 'joy of giving' bequests to explore the implications of status seeking on inequality. Preferences are defined on the comparison between individual and average bequests. This increases the overall fraction of resources devoted to bequests, with this effect being stronger for agents with lower income. Since the analysis of distribution of bequests requires consideration of wage and interest rates, we consider linear production technology to obtain analytical results. We find that bequests not

only increase steady state capital, as a bequest motive would always tend to do, but also reduce wealth inequality. The effects of bequest motive on inequality are also analyzed. We obtain that, up to some threshold value of bequest motive, inequality decreases. Indeed, beyond this threshold value, the society will be segregated.

Empirical evidence suggests that greater income inequality implies housing and neighborhood inequality and segregation. Reardon and Bischoff (2011) try to understand how variation in inequality has shaped patterns of income segregation between 1970-2000. They conclude that income segregation is affected from income inequality in terms of spatial segregation of poverty and geographic scale of income segregation. Sethi and Somanathan (2004) claim that, despite the decrease in group income inequality between median white and black households in US between 1967 and 1999, residential segregation remains in many large metropolitan areas with significant black populations. Levels of segregation are linked to the social and economic differences as they affect the quality of the living standards between different segregated groups. Thus, segregation encourages households mostly pay more attention to their neighborhood or to the groups consisting of the same ethnic or racial members instead of the whole society.

In the second essay, we extend the model of first essay to analyze a segregated economy in which households' bequest decisions depend on both their and other group's average bequest level together. We explore numerically the case of difference in status seeking across groups and study how inequality changes in a segregated economy. Despite equal access to the same labour and capital mar-

kets, and non-transmissible earning capability, we find that inequality persists in time. The initial inequality can diminish if the poorer group consider the richer group's reference strong enough. We also explore the effects of reference dependent altruism on the capital accumulation. We again consider reference dependent altruism in agents' preferences. However, this time our concern is on the capital dynamics. Therefore, we consider a concave, more specifically in Cobb-Douglas form, production function. It is shown that introducing reference dependence for bequests increases young households' savings. As a result, we find that steady state level of capital increases with the reference dependent altruism. This steady state level also increases with the weight of the bequests in preferences.

The link between reference dependent preferences and the environmental issues have also been investigated in the literature. It is shown by Ng and Wang (1993) that consumption and environmental degradation increases when relative income is considered. Howarth (1996) studies the implications of consumption externalities in a static model to offer optimal environmental policies. Alvarez-Cuadrado and Van Long (2011) examine the effect of consumption envy on resource dynamics under different property rights regimes. They obtain that envious agents over-exploit the natural resource, which leads to a lower level of steady state resource stock than the efficient level. Brechet and Lambrecht (2011) consider joy of giving type altruism model in which bequests act as a mechanism for the transmission of resources across generations. They explore whether a natural renewable resource can be managed efficiently or not. However, these papers do not analyze the investigation of the effects of consumption externalities on the wealth and long-run resource distribution. Our third essay analyzes the im-

plications of the consumption envy on households' wealth distribution and the resource dynamics.

In the third essay, we analyze the role of consumption envy on the dynamics of the distribution of household's wealth and resource. To do this, we consider a non-overlapping generations model in which each agent lives for one period and gives birth to one off-spring. Agents are heterogeneous in terms of productive ability and initial resource endowment. Agents have reference dependent preferences in terms of consumption and the resource. The references for consumption and resource are taken as the average of the agents in the economy. We analyze resource dynamics using both Cobb-Douglas and linear production functions. When we consider the model with Cobb-Douglas function, we obtain that the steady state level of resource depends on the threshold point for regeneration rate. Above this threshold point, economy exhibits a balanced growth path, below the threshold point resource stock exhausts immediately. When the regeneration rate is equal to this threshold value, the economy gets stuck at the initial average level of resource. When we consider linear production function, we obtain a unique steady state for the average level of the resource stock. The fraction of resources devoted to consumption increases with the consumption envy leading to lower level of resource stock. Moreover, consumption envy is proven to increase inequality of wealth, consumption and resource distribution. We also find that the degree of altruism increases wealth inequality after some threshold point.

## CHAPTER 2

# SOCIAL STATUS PURSUIT, DISTRIBUTION OF BEQUESTS AND INEQUALITY

Household bequests alter a country's income distribution permanently. Among the reasons of old agents to leave bequests, we find in the literature altruism (as in Barro, 1974, or Becker, 1974) and saving miscalculations, that is, accidental bequests (as in Hurd, 1987 and 1989). In line with Wei and Zhang (2011), we believe that households also bequeath to improve their heirs' social status. We focus on a society structured in families which behave as dynasties and which compete for social status and indirectly, for economic preeminence. This essay aims at studying the evolution of household wealth and the inequality among households in a setting where preferences are interdependent, hinging on social status. Here, social norms are endogenous and they evolve with families' decisions and the overall economy.

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Social status is understood as the “ranking of individuals based on their characteristics, assets and actions”, (p. 802, Weiss and Fershtman, 1998). The quest for social status explains much of human behavior since it provides overall favorable treatment which ranges from transfers of goods, natural leadership, and a myriad of symbolic gestures. It seems reasonable then to take into account the quest for social status in a model aiming at describing household decision making. And although social status may intervene in many dimensions, we restrict its benefits to the individual’s preferences. This has been the direction taken most frequently in the literature starting with Easterlin (1974, 1995) who asserted that individuals would not be happier if the income of all increased. In subsequent studies, it was proven that from the post-war US until the 90’s, there is no time trend in happiness although there is a clear trend for median national family income (Duncan, 1975, Maddison, 1991). Instead, it seems that the standards for a good life increase with income (Easterlin,1995).

Social status is made of innate characteristics like being aristocrat, but also of household wealth and others like occupation or education. Worldwide, bequests determine to a great extent wealth. Kotlikoff et al. (1982) find that intergenerational transfers account for 80% of the US wealth, whereas it is a 20% for Modigliani (1988). In Japan, according to Hayashi (1986) they account for at least 9.6% and 20 years later, Horioka (2009) estimates that they account for 15%. Of all the social status features above, wealth or bequests are of particular interest. First, because they are economic decisions and second, they are not an immutable advantage. According to Kopczuk and Lupton (2007), three fourths of the elderly has a bequest motive and four fifths of the elderly wealth will be

bequeathed. Along these lines, we simplify the notion of social status and identify it with transmissible wealth or bequests. The extension to a wider definition of social status is far from trivial since it requires an adequate indicator for social status encompassing and weighting wealth, education, economic sector and the household history.

The literature on bequests proposes several motives. Altruism is the classical motive. As defined in Barro (1974) or Becker (1974), parents leave bequests because they earn utility from the resources of their children. Altruism has been challenged both applied and theoretically, being widely tested empirically (see for instance Wilhelm, 1996, Laitner and Juster, 1996). Results show that at best, altruism cannot explain all bequests. As Masson and Pestieu (1997) put it, if altruism was the reason to bequeath then the average age of a heir in developed countries would not be 45. Bequest should arrive earlier in life. Among other complementary motives, let us mention unintended and strategic bequests and egoism. Accidental bequests happen when the old agent does not manage her wealth adequately and leaves bequests unintentionally (see Hurd, 1987 and 1989). In Kopczuk and Lupton (2007), at least part of the bequest is proven an accident and this after controlling for various family characteristics, all found insignificant. Among others, Blinder (1974) and Hurd (1989) put forward a powerful motive: egoism. Parents get utility from the quantity bequeathed to their children, not from the amount the children actually consume. Laitner and Ohlsson (2001) compare the empirical accuracy of the egoistic versus the altruistic model. While the accidental and egoistic motives are supported for the US and Sweden, the altruistic model seems to fit only Sweden. Parents may also bequeath

strategically, exchanging bequests against received services. In Bernheim et al. (1985), it is found that children pay more attention to parents with bequeathable wealth. Another of the commonly invoked motives is risk aversion in the presence of incomplete annuity and health insurance markets. Joining risk aversion and the strategic motive, Perozek (1998) finds that the strategic behavior is not robust. For Wei and Zhang (2011), the main reason to bequeath in China is the quest for social status. In the context of a severely unbalanced sex ratio, only men with a high social status (wealth), will get married.

In this essay, we present an overlapping generations model where individual's preferences depend on consumption in the young and old age, but also on the relative bequest left to the following generation. Note that we distinguish here savings for later consumption and savings for intentional bequests. As aforementioned, our definition of social status only includes bequests, as a measure of transmissible wealth. This limitation enables us to underline the effects of competition on household wealth inequality. Furthermore, although we assume that individuals' skills are heterogeneous, skills are not transmissible so the only channel to exacerbate inequality is the unequal accumulation of wealth. When all households share the same view on positional bequests, we find that both savings and bequests increase with household wealth and current income. Additionally, the society-wide average bequest level reinforces the bequest motive, inducing all households to increase their bequest, reducing inequality.

Our essay relates to the literature analyzing the effect of social status pursuit on economic decisions and inequality. Some authors associate the quest for so-

cial status with reference dependent preferences, where the household welfare increases only when a given variable surpasses the referent value. If the referent only concerns consumption, then we enter the field of ‘keeping up with the Joneses’. Depending on the strength of the referent, on preferences, technology and inequality, looking up to the others may have different effects on growth in the long-run.<sup>1</sup> Focusing on distributional effects, Garcia-Peñalosa and Turnovsky (2008) find that the will to ‘keep up with the Joneses’ enhances household equality. However, when households bequeath, Caballé and Moro-Egido (2014) find that habits increase average wealth although they reduce stationary wealth mobility. In other papers, the referent is average household wealth and individuals enter into a wealth race which fosters overall economic growth.<sup>2</sup> There are fewer papers studying the link between bequest references and household inequality. Alvarez-Cuadrado and Long (2012) show that consumption envy can increase inequality among households. In a society made of altruistic households, caring about bequests in a prospect theoretic sense, Bogliacino and Ortoleva (2015) find that the society becomes more polarised and that the middle class disappears in finite time. Additionally, they prove that reference dependence does not harm growth. On the contrary, envy pushes agents to improve their relative situation. The paper closest to ours is Wei and Zhang (2011), where the Chinese gender imbalance induces parents to under-consume and accumulate wealth to bequeath as much as possible, to ensure their son’s future marriage. Oversaving of just a part of the households can drive down interest rates, which pushes in turn all

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<sup>1</sup>Brekke and Howarth (2002), Carroll et al. (1997), Corneo and Jeanne (1997), or Liu and Turnovsky (2005) find that ‘keeping up with the Joneses’ fosters growth in the long-run. For Fisher and Heijdra (2009) and Wendner (2010) there is a negative effect, while Brekke and Howarth (2002) or Rauscher (1997) find no long-run effect on growth.

<sup>2</sup>Konrad (1992), Cole et al. (1992), Fershtman et al. (1996), or Stark, (2006).

other households to oversave. In the end, like in the present essay, all households oversave which leads to household equality in the long-run.

The remainder of the essay is organized as follows. In Section 2.1, we present and discuss our model. In section 2.2, we analyze the dynamics of the overall economy while the stationary distribution of the household variables are analyzed in section 2.3. Finally, section 2.4 concludes.

## **2.1 The Economy**

This section presents the households, the firm and finishes with an analysis of the overall economy. As argued, in this section all households share the same preferences.

### **2.1.1 Households**

We consider an economy made by  $N$  households, of constant and identical size, indexed by  $i$ . A household is made of a young adult and an old adult. When young, individuals income is made of bequests from their parents and their labor income. They decide how to allocate their young-age resources between consumption and savings for the next period. When old, the individual retires, consumes a part of the first period savings and bequeaths the rest.

There are two potential sources of heterogeneity in the economy. First, households have different initial wealth. Second, individuals are endowed with varying

ability. As a result, young individuals at time  $t$  differ with respect to their productive ability,  $l_t^i$ , and the bequest inherited from their parents,  $b_t^i$ . That is, even if all families bequeathed equally, heterogeneity would persist since young individuals differ in their abilities. Each young agent draws  $l_t^i$  from an independent and identical distribution at the beginning of period  $t$ , with mean  $\bar{l}_t = 1$  and variance,  $\mathbb{V}(l_t^i) = \sigma_l^2$ . It results in a wage distribution with mean  $\bar{w}_t = w_t$  and standard deviation  $\sigma_{w_t} = w_t \sigma_l$ . Notice that the ability distribution is constant in time and identical across families. In this regard, we are not modelling here the transmission of human capital from one generation to another nor varying access to technology or education across families.

There is evidence that welfare depends on the relative situation of the family in the society and not only on absolute income (see Foster, 1998, Atkinson and Bourguignon, 2000, Ravallion, 2003, and Gupta et al., 2018). In this essay, preferences reflect these two perspectives. While individuals care about their absolute levels of consumption, they also obtain satisfaction by the relative position of their family in the society, measured by the relative wealth of the family. As a result family preferences depend on the choices of all other families. The young adult acts as the family leader, taking all decisions, deciding on current and future consumption, and about the amount to bequeath to the following generation. We assume that preferences are transmitted from parent to child, without any parental effort required. The lifecycle utility function for family  $i$ , born in period  $t$  is given by,

$$u(c_t^i, d_{t+1}^i, b_{t+1}^i) = \ln c_t^i + \beta \ln d_{t+1}^i + \theta \ln(b_{t+1}^i - \gamma \bar{b}_{t+1}), \quad (2.1)$$

where  $\beta < 1$  is the time discount factor.  $\theta$  governs the bequest motive and it also includes a discount factor. Our key behavioral assumption is that satisfaction from bequests does not depend only on the amount bequeathed, but rather depends on how it compares to the average bequest per capita of reference group.  $\bar{b}_t$  is the average bequest of generation born at time  $t$ , that is  $\bar{b}_t = \frac{1}{N} \sum_{i=1}^N b_t^i$ ; and  $0 < \gamma < 1$  is the measure of positional bequest concern.<sup>3</sup> A larger  $\theta$  implies that the individual cares more about her offspring or the prospective power of the family, whereas a larger  $\gamma$  indicates a larger influence of the society on the individual's welfare. For simplicity reasons, we assume that both  $\theta$  and  $\gamma$  are common to all families and constant in time. Both  $\theta$  and  $\gamma$  are crucial for our analysis, and their role is analyzed throughout the essay.<sup>4</sup>

In the current period  $t$ , the household revenue is made of bequests from the previous generation and their labor revenue,  $b_t^i + w_t l_t^i = b_t^i + w_t^i$ . Then, this revenue is split between consumption and savings,  $c_t^i + s_t^i$ . Note that savings will provide the agent both with old age consumption and the possibility to bequeath to the following generation, that is

$$c_t^i + s_t^i = b_t^i + w_t^i, \quad (2.2)$$

$$R_{t+1} s_t^i = d_{t+1}^i + b_{t+1}^i, \quad (2.3)$$

for every  $t$ , where  $R_{t+1}$  is the return rate on investment. The adult maximizes (2.1) subject to (2.2) and (2.3). Using the first order conditions, one can derive

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<sup>3</sup>The family utility is well defined if and only if  $b_{t+1}^i > \gamma \bar{b}_{t+1}$ , which depends on  $\gamma$ , average bequests  $\bar{b}_{t+1}$  as well as on the choice of  $b_{t+1}^i$ .

<sup>4</sup>Alternatively, we could have followed Duesenberry (1949) and modeled preferences as  $u(c_t^i, d_{t+1}^i, \frac{b_{t+1}^i}{\bar{b}_{t+1}})$ . We opt here for the simplest preference representation in the overlapping generations literature.

optimal savings and bequests of agent  $i$  in each period:

$$s_t^i = \frac{\beta + \theta}{1 + \beta + \theta}(b_t^i + w_t^i) + \frac{\gamma}{(1 + \beta + \theta)R_{t+1}}\bar{b}_{t+1}, \quad (2.4)$$

$$b_{t+1}^i = \frac{\theta R_{t+1}}{1 + \beta + \theta}(b_t^i + w_t^i) + \frac{\gamma(1 + \beta)}{1 + \beta + \theta}\bar{b}_{t+1}. \quad (2.5)$$

Equations (2.4) and (2.5) show that an increase in received bequests,  $b_t^i$ , or income raises savings and bequests to the next generation. If average future bequest increases, then all households increase savings that will allow for an increase in the level of future bequest.

The existence of a common social status definition and the fact that all households care equally about social status results in increasing rates of savings and bequests. From the point of view of a policy maker caring about inequality this is not necessarily good news if the rich accumulate wealth faster. We devote the next sections to study the underpinnings of inequality.

### 2.1.2 The Firm

There exists a unique final good. Output  $Y_t$  is produced by combining overall physical capital  $K_t$  and total labor  $L_t$  through a production function  $F(K_t, L_t)$ , homogenous of degree one. Taking the final good as the numeraire, let  $w_t$  stand for the unit salary and  $R_t$  the rate of return. At each period  $t$ , the firm maximizes net profits defined as

$$\max_{L_t, K_t} F(K_t, L_t) - w_t L_t - R_t K_t.$$

To investigate the effect of the status quest on household inequality, we need to examine the dynamic behavior of the household distribution of bequests. However, a complete analysis requires the dynamic analysis of the wage rate and the interest rate. To provide analytical results, following Alvarez-Cuadrado and Van Long (2012) and Caballe and Moro-Egido (2014), we consider a production technology linear in capital and labor,

$$F(K_t, L_t) = RK_t + wL_t,$$

under which the factor prices are independent of the degree of positional concerns and constant over time, i.e.  $w_t = w$ ,  $R_t = R$ , for all  $t$ .<sup>5</sup>

### 2.1.3 The Overall Economy

Individual optimal choices depend on the economy average bequest. We can rewrite (2.4) and (2.5) as a function of the household income,  $y_t^i = b_t^i + w_t^i$ , and the economy average income,  $\bar{y}_t$  given by:

$$\bar{y}_t = \frac{1}{N} \sum_{i=1}^N b_t^i + \frac{1}{N} \sum_{i=1}^N w_t^i = \bar{b}_t + w. \quad (2.6)$$

Then, we obtain the following average optimal savings, bequests, and consump-

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<sup>5</sup>Alvarez-Cuadrado and Van Long (2012) mentions that the numerical analysis of the dynamic behavior of bequests under a Cobb-Douglas production function is consistent with the analytical results obtained under the linear technology.

tion choices:

$$\bar{c}_t = \frac{1 - \gamma}{\theta + (1 + \beta)(1 - \gamma)} \bar{y}_t, \quad (2.7)$$

$$\bar{s}_t = \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} \bar{y}_t, \quad (2.8)$$

$$\bar{b}_{t+1} = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \bar{y}_t, \quad (2.9)$$

$$\bar{d}_{t+1} = \frac{\beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} R \bar{y}_t. \quad (2.10)$$

Naturally, an increase in average output per capita,  $\bar{y}_t$ , increases current and future consumption, savings and bequests, all else equal. Depending on the relative value of  $\theta$ , current consumption or bequests will be privileged as we show in the following lemma:

**Lemma 1.** *When the bequest motive is strong, that is, if  $\theta > \max\{\beta(1 - \gamma), (1 - \beta)(1 - \gamma)\}$ , an increase in  $\bar{y}_t$  privileges savings to current consumption, and bequests to consumption at old age (and vice versa).*

*Proof.* Results follow from direct derivation of the aggregate variables in (2.7)-(2.10). □

When the bequest motive is large enough, any increase in average income tends to reinforce the household position in the economy, privileging savings against consumption to increase the available resources next period. Along the same lines, when old, the household prefers to increase bequests rather than consumption to acquire a higher social status.

We can now use the results for the average household to characterize the behavior of individual households. Using equations (2.4) and (2.9), we obtain the optimal saving choice:

$$s_t^i = \frac{1}{1 + \beta + \theta} [(\beta + \theta)y_t^i + \phi_s \bar{y}_t], \quad (2.11)$$

where  $\phi_s = \frac{\gamma\theta}{\theta + (1+\beta)(1-\gamma)}$ . Similarly, using equation (2.11) with (2.4) and (2.5) we reach the remaining choices for the  $i$ 'th household:

$$c_t^i = \frac{1}{1 + \beta + \theta} (y_t^i - \phi_c \bar{y}_t), \quad (2.12)$$

$$d_{t+1}^i = \frac{\beta}{1 + \beta + \theta} R (y_t^i - \phi_d \bar{y}_t), \quad (2.13)$$

$$b_{t+1}^i = \frac{\theta}{1 + \beta + \theta} R (y_t^i + \phi_b \bar{y}_t), \quad (2.14)$$

with  $\phi_c = \phi_d = \phi_s$ ,  $\phi_b = \frac{\gamma(1+\beta)}{\theta + (1+\beta)(1-\gamma)}$ , and  $\phi_c = \phi_s = \phi_d < \phi_b$ . From (2.12) and (2.13), it obtains that  $d_{t+1}^i = \beta R c_t^i$ , showing that the model structural parameters affect  $c_t^i$  and  $d_{t+1}^i$  in the same direction. Future consumption will be larger than current consumption whenever future returns compensate for the sacrifice of current consumption, that is, whenever  $\beta R > 1$ . Consumption and bequests of the  $i$ th household are composed of two elements: her own lifetime income and the society average product. Note that inequalities in bequests can completely disappear when the society effect dominates, or they can grow when  $\gamma$  tends to zero, as in Alvarez-Cuadrado and van Long (2012).

Although all variables depend positively on household income, aggregated income increases savings and bequests and it decreases current and future consumption.

It is straightforward to compute the variables' elasticity to income:

$$\epsilon_{c_t}^i = \left(1 - \phi_c \frac{\bar{y}_t}{y_t^i}\right)^{-1}, \quad (2.15)$$

$$\epsilon_{d_{t+1}}^i = \left(1 - \phi_d \frac{\bar{y}_t}{y_t^i}\right)^{-1}, \quad (2.16)$$

$$\epsilon_{s_t}^i = \left(\theta + \beta - \phi_s \frac{\bar{y}_t}{y_t^i}\right)^{-1}, \quad (2.17)$$

$$\epsilon_{b_{t+1}}^i = \left(1 + \phi_b \frac{\bar{y}_t}{y_t^i}\right)^{-1}. \quad (2.18)$$

Hence, given the household preferences, elasticities verify that

$$0 < \epsilon_{s_t}, \epsilon_{b_{t+1}} < 1 < \epsilon_{c_t} = \epsilon_{d_{t+1}}.$$

Consumption is a luxury good, while bequests and savings are necessity goods. Indeed, consumption only becomes a necessity when  $\gamma$  or  $\theta$  tend to zero. A threshold arises for relative income. For households whose relative income,  $\frac{y_t^i}{\bar{y}_t}$  is below  $\frac{\gamma(1+\beta-\theta)}{\beta[\theta+(1+\beta)(1-\gamma)]}$ , the most necessary variable is bequests, followed by savings.

**Definition 1.** A sequence of household decisions  $\{(c_t^i, s_t^i, d_{t+1}^i, b_{t+1}^i)\}_{i=1, \dots, N; t=1, \dots, \infty}$  together with the unit salary and the interest rate  $\{w, R\}$  is an equilibrium if

- i) Individual's skills  $l_t^i$  are thrown from a skill distribution with  $\bar{l}_t = 1$  and  $var(l_t^i) = \sigma_l^2$ .
- ii) Households' decisions are optimal, satisfying equations (2.11), (2.12), (2.13) and (2.14) where household income is  $y_t^i = b_t^i + w^i$ , and average income is

defined by (2.6).

- iii) Labor and capital markets clear, so that in particular,  $L_t = N$ , for all  $t$ .
- iv) The firm maximizes profits at every period, and pays labor and capital at their marginal productivities.

## 2.2 The Dynamics of the Capital Stock

Assuming that physical capital depreciates completely from one period to next, total capital available in the economy next period,  $K_{t+1}$  results from households' savings, that is  $K_{t+1} = N\bar{s}_t$  or in per capita terms  $k_{t+1} = \bar{s}_t$ . Note that the average wage in the economy at time  $t$  was defined as  $\bar{w} = w\bar{l}_t$  where the average ability  $\bar{l}_t$  is by assumption equal to 1. Then, using equations (2.6), (2.8) and (2.9) along with the law of accumulation of physical capital, we obtain

$$k_{t+1} = \bar{s}_t = \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} \bar{y}_t,$$

where, by recursion, the average income can be written as

$$\bar{y}_t = \left[ \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} \right]^t R^t \bar{y}_0 + w \sum_{j=0}^{t-1} \left[ \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} \right]^j R^j.$$

The sum of the geometric series on the right hand side of this equation can be recast as

$$\sum_{j=0}^{t-1} \left[ \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} \right]^j R^j = \begin{cases} \frac{1 - \left[ \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} \right]^t R^t}{1 - \left[ \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} \right] R}, & \text{if } \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \neq 1 \\ t, & \text{otherwise.} \end{cases}$$

Note that if  $R < \frac{\theta + (1 + \beta)(1 - \gamma)}{\theta}$ , then the stock of physical capital converges to a unique steady state:<sup>6</sup>

$$k^* = \frac{\theta + \beta(1 - \gamma)}{(1 + \beta)(1 - \gamma) + \theta(1 - R)} w,$$

at which the average income takes the value

$$\bar{y}^* = \frac{\theta + (1 + \beta)(1 - \gamma)}{(1 + \beta)(1 - \gamma) + \theta(1 - R)} w.$$

In line with Wei and Zhang (2011), an increase in the bequest motive  $\theta$  induces all households to increase savings, which increases the stock of capital. Further, Proposition 2 in the upcoming Section 2.3 underlines the role of competition in the overall economy. When inter household comparisons increase by augmenting  $\gamma$ , household bequests must increase to remain in the lead. Young agents in our economy save for two reasons. The first reason is to finance old age consumption and the second, to leave bequests to their offsprings. The latter one includes positional concerns and it results affected by changes in the value of  $\gamma$ . The afore mentioned Proposition 1 shows that an increase in  $\gamma$  reinforces household

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<sup>6</sup>See Appendix A for all computational details.

competition, and it shifts savings from old-age consumption to bequests. In order to increase bequests, families need to save more, increasing this way the level of the steady state capital stock.

Using (2.6) together with equation (2.9), the following equation for the evolution of the average bequest is obtained:

$$\bar{b}_{t+1} = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R(\bar{b}_t + w). \quad (2.19)$$

The following proposition shows that a stationary value for the average bequest is attained:

**Proposition 1.** *If physical capital is at its steady state then average bequest  $\bar{b}_t$  also reaches a stable stationary state,*

$$\bar{b}^* = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma) - \theta R} R w, \quad (2.20)$$

*which is increasing in  $\gamma$  and  $\theta$ .*

*Proof.* Average bequests reach a steady state value if and only if the steady state value of the interest rate is small enough,

$$R < \frac{\theta + (1 + \beta)(1 - \gamma)}{\theta},$$

which is exactly the condition we had imposed to obtain  $k^*$ .

Taking the derivative of the  $\bar{b}^*$  with respect to  $\theta$  and  $\gamma$  gives:

$$\frac{\partial \bar{b}^*}{\partial \theta} = \frac{R(1+\beta)(1-\gamma)}{(\theta + (1+\beta)(1-\gamma) - \theta R)^2}$$

and

$$\frac{\partial \bar{b}^*}{\partial \gamma} = \frac{\theta R(1+\beta)}{(\theta + (1+\beta)(1-\gamma) - \theta R)^2}$$

which are positive. □

## 2.3 Stationary Distributions and Household Inequality

To analyze the influences of status quest on household inequality, we have to characterize the dynamic behavior of the distribution of bequest. To do so, consider that the stock of physical capital, the average income, and the factor prices take their steady state values so that we can rewrite (2.5) as:

$$b_{t+1}^i = c_1 b_t^i + c_2 l_t^i + c_3,$$

where  $c_1 = \frac{\theta}{1+\beta+\theta}R$ ,  $c_2 = \frac{\theta}{1+\beta+\theta}Rw$ ,  $c_3 = \frac{\gamma\theta(1+\beta)}{(1+\beta+\theta)(\theta+(1+\beta)(1-\gamma))}R\bar{y}^*$ . If we solve for  $b_t^i$  backwards, we obtain that

$$b_t^i = c_1^t b_0^i + c_2 \sum_{j=0}^{t-1} l_j^i c_1^{t-1-j} + c_3 \sum_{j=1}^{t-1} c_1^j.$$

Since at the steady state  $0 < c_1 < 1$ ,  $b_t^i$  exists for all  $t > 0$ . Then, we can compute

the expected value of household bequests at time  $t$ :

$$\mathbb{E}(b_t^i) = b_0^e c_1^t + c_2 \sum_{j=0}^{t-1} c_1^{t-1-j} + c_3 \sum_{j=1}^{t-1} c_1^j = b_0^e c_1^t + (c_2 + c_3) \frac{1 - c_1^t}{1 - c_1} = \frac{c_2 + c_3}{1 - c_1} + (b_0^e - c_2 - c_3) c_1^t,$$

where  $b_0^e$  is the expected value of initial bequests, and it is known.  $\mathbb{E}(b_t^i)$  is always positive and it converges to a constant when time increases. If the expected initial bequest is above  $c_2 + c_3$ , then the expected bequest decreases with time.

We compute next the variance of household  $i$  bequest:

$$\mathbb{V}(b_t^i) = c_2^2 \sigma^2 \frac{1 - c_1^{2t}}{1 - c_1^2},$$

which increases with time. In order to provide robust results, let us use the coefficient of variation, the quotient between the variable's standard deviation and its mean value, as the measure of inequality (see Bossmann et al., 2007 and Alvarez-Cuadrado and Long, 2012). We denote the coefficient of variation of a random variable  $X$  by  $CV(X)$ .

The coefficient of variation of  $b_t^i$  is

$$CV(b_t^i) = \frac{c_2 \sigma \left( \frac{1 - c_1^{2t}}{1 - c_1^2} \right)^{1/2}}{\frac{c_2 + c_3}{1 - c_1} + (b_0^e - c_2 - c_3) c_1^t}.$$

We can prove that if  $b_0^e > c_2 + c_3$ , then the inequality in bequests increases with time.<sup>7</sup> Let us summarize our results on the evolution of the bequest distribution.

If an economy is initially poorly endowed on average, then the expected bequest will decrease with time and household inequality in bequests will consequently

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<sup>7</sup>See Appendix B.

decrease. If on the contrary, the economy average bequest is high enough, then expected bequest will continuously increase. Nevertheless, although bequests increase on average, so does inequality. Therefore, bequests of the wealthier increase faster than bequests of the poor. Since the expected bequest increases in time, the distribution of bequests stretches. Hence our results show that the difference in bequests among any two households increases with time in rich economies.

Now we study the stationary distributions of the household's variables. That is, we assume that physical capital has attained its steady state, which induces average variables to achieve their steady state values as well. The analysis of the stationary distributions is of particular interest because it enables us to identify the drivers of inequality. Beforehand and for tractability reasons, we need the following assumption:

**Assumption 1.** *Individual's abilities and inherited bequests are uncorrelated, that is  $cov(l_t^i, b_t^i) = 0$ , for all  $i \in \{1, 2, \dots, N\}$ .*

Assumption 1 underlines that there is no skill transmission in our economy, and that abilities are independent of bequests.

Applying the variance operator on both sides of equation (2.14), the stationary value of bequest variance results:

$$\mathbb{V}[b^i] = \frac{\theta^2 R^2}{(1 + \beta + \theta)^2 - \theta^2 R^2} \sigma_l^2. \quad (2.21)$$

Indeed, the larger the spread of abilities, the larger the disparities in income,

which induces a larger variance in bequests. Using these results together with equations (2.11), (2.12) and (2.13), the first and second moments of  $c^i$ ,  $d^i$  and  $s^i$  obtain:

$$c^i \sim D(\mathbb{E}[c^i], \mathbb{V}[c^i]) \equiv D\left(\frac{1-\gamma}{\theta+(1+\beta)(1-\gamma)-\theta R}w, \frac{1}{(1+\beta+\theta)^2-\theta^2 R^2}\sigma_i^2\right) \quad (2.22)$$

$$d^i \sim D(\mathbb{E}[d^i], \mathbb{V}[d^i]) \equiv D\left(\frac{(1-\gamma)\beta R}{\theta+(1+\beta)(1-\gamma)-\theta R}w, \frac{\beta^2 R^2}{(1+\beta+\theta)^2-\theta^2 R^2}\sigma_i^2\right) \quad (2.23)$$

$$s^i \sim D(\mathbb{E}[s^i], \mathbb{V}[s^i]) \equiv D\left(\frac{\theta+\beta(1-\gamma)}{\theta+(1+\beta)(1-\gamma)-\theta R}w, \frac{(\beta+\theta)^2}{(1+\beta+\theta)^2-\theta^2 R^2}\sigma_i^2\right) \quad (2.24)$$

Using (2.20) and (2.21), the stationary distribution of bequests follows:

$$b^i \sim D(\mathbb{E}[b^i], \mathbb{V}[b^i]) \equiv D\left(\frac{\theta}{\theta+(1+\beta)(1-\gamma)-\theta R}Rw, \frac{\theta^2 R^2}{(1+\beta+\theta)^2-\theta^2 R^2}\sigma_i^2\right). \quad (2.25)$$

As a first measure of distributional disparities, we can make a direct comparison of the variables variances. If  $1 < \beta + \theta$ , then savings have a larger variance than first period consumption. Furthermore, if  $\beta < \theta$ , then bequests are less equally distributed than consumption in old age, that is  $\mathbb{V}[d^i] < \mathbb{V}[b^i]$ . In other words, when households preferences show a high concern about social status, then all households increase their bequests, which amplifies differences among families in the long-run in terms of wealth. The variable with the greater weight in preferences between old age consumption and bequests becomes the less dispersed in the long-run.

The following proposition shows how the household variables' distributions react to changes in different parameters:

**Proposition 2.** *An increase in the reference dependence parameter,  $\gamma$ , increases the mean values of household bequest, consumption and savings at the steady*

state. Focusing on the variables' variance, a decrease in  $\sigma_l^2$  decreases all variables variance. An increase in  $R$  increases the expected value and the variance of all variables.

*Proof.* See Appendix C for the results on  $\gamma$  and  $\sigma_l$ . Results regarding  $R$  can be proven taking the partial derivative of the expected value and variance of all variables with respect to  $R$ . □

Proposition 2 shows that  $\sigma_l$  is the unique exogenous parameter that can decrease the variance of all the endogenous variables at the same time. Hence, a first effective direction to reduce inequality would be to reduce  $\sigma_l^2$ . For instance, improving schooling attendance and quality and publicly subsidized professional training belong to this set of policies. The proposition also highlights the major role played by the interest rate in all long-term distributions. An increase in the rate of return magnifies the distance between rich and poor since the wealthier the more a household can benefit from the more advantageous market for capital.

Arising from individual household's optimal decisions, the expected value of  $c^i$  equals the present value of the expected value of  $d^i$ , that is

$$\mathbb{E}[c^i] = \frac{1}{\beta R} \mathbb{E}[d^i].$$

Old age consumption variance is  $(\beta R)^2$  times the variance of  $c^i$ . Hence if  $\beta R > 1$ , then  $d^i$  spreads more than  $c^i$ . Indeed, an increase in the rate of return, makes more profitable to postpone consumption.

Finally, let us underline that the weight of bequests in the utility function,  $\theta$ ,

as well as the time discount,  $\beta$ , increase the variables' variance, exacerbating inequality. Again, the wealthier can benefit more from the capital market and augment further the future looking variables.

Inequality can be measured regarding different economic and social variables. Most surely, none of them will provide us with the same ordering or indicate the same magnitude. Next we use the coefficient of variation to measure relative inequality. It turns out that consumption in the young and in the old age are equally unequal. Bequests is the less unequal variable in contrast to savings, which is the most unequal variable. Straightforward computations lead to the following ordering  $CV(b^i) < CV(c^i) = CV(d^i) < CV(s^i)$ . Furthermore, household wealth is more unequal than consumption,  $CV(c^i) < CV(y^i)$ . If we had to provide a picture of the economy at a given time  $t$ , bequests would be the more egalitarian variable of the economy. Note that this result is independent of the values of the bequest motive  $\theta$  and of the inter-household comparisons  $\gamma$ . Independently of the structure of the economy and the distribution of abilities, households devote their efforts to legate as much as possible. Indeed, households sacrifice young age consumption to increase savings in order to bequeath at the maximum of their capacities. Worse endowed households save relatively more than wealthier families in order to leave a strong bequest and to improve their household social status. This explains that the coefficient of variation for savings is the most unequal variable: the wealthier will save relatively less than the poorer. The last inequality shows that, despite the poorer families efforts, household wealth will remain more unequally distributed than consumption and bequests.

We explore next how inequality varies with the weight of average bequests in the utility function and with the importance of positional bequests:

**Proposition 3.** *Inequality of wealth, savings, consumption and bequest decrease with  $\gamma$ . An increase in  $\theta$  also decreases inequality in all variables but only up to a threshold level  $\bar{\theta}$ .*

*Proof.* See Appendix D. □

Together with Proposition 2, Proposition 3 proves that an increase of social status in preferences not only increases expected values of all variables but it also decreases inequality. Besides, augmenting the importance of bequests, fosters equality in wealth, consumption, and naturally, in bequests.

We find here the same underlying mechanism as underneath the covariance ranking. When the weight of social status increases (measured either as the weight of bequests in preferences or the weight of the group), all households will tend to increase bequests. In relative terms, less wealthy households will increase bequests further than wealthier households. Hence, in the short-term, inequality in consumption increases while bequests become more equal. This mechanism will be strong for a number of generations, which depends on preferences and productivity. Then, as the poorer accumulate wealth, improving their social status, they bequeath less strongly and increase their consumption. That is, in the transition period it is necessary to exacerbate inequality in consumption, while closing the gap in bequests and hence in social status.

Finally, if the weight of bequests in preferences increases beyond  $\bar{\theta}$ , then the

society will be segregated. Indeed, beyond  $\bar{\theta}$  a share of households will not be able to afford bequests, so that their optimal lifecycle decisions will not be directed by (2.1). In case of segregation, inequality will steadily grow since households leaving bequests will accumulate more wealth at all generations as in Bossmann et al. (2007). Households which cannot bequeath at one generation, will not be able to catch up since the gap among the bequeathing and the non-bequeathing households will broaden.

## 2.4 Conclusion

This essay has proposed a simple benchmark to analyze the effect of social status pursuit on the evolution of the distribution of bequests and household inequality. In our model, households can modify the social position of their heirs leaving extensive bequests. We have shown that the larger the bequest motive and the social status concern, the less the household inequality.

There are some important issues worth studying in future research. First issue is the transmissibility of abilities. If ability to earn a wage is inherited, then initial agent heterogeneity will be three dimensional. In the absence of public policies, we wonder whether the less able could escape a sort of trap. Then building on this, we could introduce education as in Moav and Neeman (2008). There, at equilibrium, the rich have a better education and do not need to show their status with their consumption. Introducing education in our set-up could diversify equilibria, and the role of a policy maker as education provider comes out as crucial for household decision making. In this regard Lu (2018) analyzes the

effects of status concern based on the agent's relative education level on economic growth, and Tournemaine and Tsoukis (2015) studies the effects of consumption envy on agents' choice between public or private education. Second, we could use our framework to analyze the role of the bequest motive in a segregated economy suffering from group inequality. A challenging project would be to apply our set-up to study the dynamics of rural-urban inequality in India as studied in Mallick (2014). Although there are powerful reasons to explain segregated behavior as access to high education or to fair mortgage markets, social referents may also play a role. In this regard, we would analyze the role of referents in group inequality and economic growth. Finally, we could also consider to introduce a financial sector into the model to study the roles of financial regulation, access to credit and of corruption. Financial development plays a key role in economic growth, although its consequences on inequality depend on a myriad other economic and social dimensions. In Agnello et al. (2012), it is found that inequality is reduced upon financial reforms, that the larger the government the more inequality is reduced and that trade could eventually hinder convergence. In the extended version of this model in a segregated economy, it could be utterly interesting to analyze the role of the financial markets (and its access and quality) in intra and inter group inequality and ultimately on overall economic growth.

## CHAPTER 3

# REFERENCE DEPENDENT ALTRUISM AND SEGREGATION

Positional concerns become an important aspect in economic models to analyze the interactions between agents in a society. Status concerns are one of the important features that affect the individual's well-being in the society or neighborhood. The role of positional concerns has been studied in many different contexts like consumption, leisure and production externalities.

There are plenty of papers analyzing these externalities. In terms of consumption externalities, Abel (1990) analyzes the effects of such externalities on asset prices under both “keeping up with Joneses” and “habit formation” framework. Ljungqvist and Uhlig (2000) explore its effects on optimal taxation; Liu and Turnovsky (2005) show how consumption and production externalities affect the capital accumulation. Alonso-Carrera et al. (2008) study the effects of consumption externalities on the optimality of dynamic equilibrium in an economy displaying dynastic altruism. Mino and Nakamoto (2012) examine the role of consumption externalities on equilibrium dynamics of a standard neoclassical growth

model in which agents are heterogeneous. They consider two groups of infinitely-lived heterogeneous agents. Alvarez-Cuadrado and Van Long (2012) show the effects of consumption envy on inequality in an altruistic household model.

The literature on the link between bequest related positional concerns and economic growth is limited. In this essay, we consider reference dependent altruism and analyze the effects of it on capital accumulation and segregated economy. Borissov (2016) considers a family altruism type model where there exist positional concerns on agent's consumption and her heir's disposable income, and asks the questions whether saving differences between rich and poor lead an increase in inequality over time and whether wealth distribution affects aggregate dynamics. It is shown that if the consumption related positional concerns are not sufficiently low or offspring related positional concerns are not sufficiently high, then the population splits into two classes at the steady-state. Bogliacino and Ortoleva (2015) use prospect theory to model reference dependent consumers in terms of endowments and study the effects of initial distribution of endowments on the growth. Breitmoser and Tan (2014) propose a theory of reference dependent altruism where agents' degree of altruism is affected from reference points. They estimate the model parameters on novel experimental data on majority bargaining.

In this essay, in line with previous chapter, we present an overlapping generations model where individual's preferences depend on consumption in the young and old age, but also on the relative bequest left to the following generation. We assume that agents are heterogeneous in terms of productive ability and non-transmissible

wealth. The steady-state dynamics of capital and bequests are analyzed using a concave, more specifically Cobb-Douglas production function. We find that stronger bequest related positional concerns increase individual savings in order to bequeath more to the following generations. This triggers an increase in the steady-state equilibrium level of capital and bequests.

This essay closes considering a segregated economy made up of two groups, which notably differ in their status referent. When the only transmissible factor is wealth, then group inequality disappears with time even in a growing economy, as long as the poorer group builds its social referent including the richer group. Although there are powerful reasons to explain segregated behavior as access to high education or to fair mortgage markets, social referents may also play a role. In this regard, we analyze the role of referents in inequality and economic growth. Despite equal access to the same labor and capital markets, and non-transmissible earning capability, inequality persists in time. Furthermore, only if the poorer group looks up to the richer group strong enough, the initial inequality can diminish. Charles et al. (2009) find that there exist striking differences in consumption patterns in the US, regarding visible expenditures: Blacks and Hispanics consume roughly 30% more than Whites, although all three groups spend the same percentage in other goods. They also show that the differences are actually driven by total income, rather than race. Sethi and Somanathan (2004) explore how race and income interact to determine residential location. They show that black households face lower neighborhood quality and segregation can be stable for sufficiently large or small racial income disparities. Reardon and Bischoff (2011) show that there exists robust relationship between income inequality and income

segregation by investigating the growth in income inequality in three dimensions: the spatial segregation of poverty, race specific patterns and geographic scale of segregation.

The remainder of the essay is organized as follows. In Section 3.1, the model is presented and discussed. In Section 3.2, the dynamics of the overall capital and bequests are analyzed. The benchmark model in Section 3.1 is extended to a segregated economy in Section 3.3. Finally, Section 3.4 concludes.

## **3.1 The Model**

This section presents the firm, the households and finishes with an analysis of the average household. In this section, all households share the same preferences. This assumption is relaxed in Section 3.3, where the economy is segregated in two groups.

### **3.1.1 The Firm**

The firm uses physical capital  $K_t$  and labor  $L_t$  as inputs to produce a single (numeraire) good  $Y_t$  that can be consumed or invested.  $F(K_t, L_t)$  is the production function that is homogeneous of degree one and satisfying Inada conditions. Let  $w_t$  stand for the unit salary and  $R_t$  the rate of return. Markets are assumed to

be competitive and wage rate and interest rate are determined as follows:

$$w_t = f(k_t) - k_t f'(k_t), \quad (3.1)$$

$$R_t = f'(k_t), \quad (3.2)$$

where  $k_t = \frac{K_t}{L_t}$  stands for physical capital per capita and function  $f(\cdot)$  is production per capita.

### 3.1.2 Households

Let us consider a two-period overlapping generations model where  $N$ , constant number of agents, indexed by  $i$ , are born in period  $t$ . A household consists of one parent and one child. In the first period, individuals earn labor income and get inheritance from their parents. The sum of income is divided between consumption and savings. In the second period, the individual retires and allocates first period savings to second period consumption and bequest. Households are altruistic toward their descendants, deriving warm glow utility from the bequests.

As in the second chapter, individuals differ in terms of productive ability and initial bequest. Labor productivity is the realization of random variable that is identically and independently distributed with mean  $\bar{l}_t = 1$  and variance,  $\mathbb{V}(l_t^i) = \sigma_l^2$ . This results in a wage distribution with mean  $\bar{w}_t = w_t$  and standard deviation  $\sigma_{w_t} = w_t \sigma_l$ .

Households bequeath to their descendants at the beginning of the second period. Utility derived from bequest depends not only on absolute level but also how it is

compared with the rest of the society. Accordingly, the life-cycle utility function for household  $i$ , born in period  $t$  is given by:

$$u(c_t^i, d_{t+1}^i, b_{t+1}^i) = \ln c_t^i + \beta \ln d_{t+1}^i + \theta \ln(b_{t+1}^i - \gamma \bar{b}_{t+1}), \quad (3.3)$$

by choosing consumption  $c_t^i$  when young,  $d_{t+1}^i$  when old and  $b_{t+1}^i$  the amount to be bequeathed to his heirs.  $\beta \in (0, 1)$  is the time discount factor and  $\theta$  is the weight of relative bequest in the life-time utility.  $\bar{b}_t$  is the average bequest of generation born at time  $t$ , that is  $\bar{b}_t = \frac{1}{N} \sum_{i=1}^N b_t^i$  and  $0 < \gamma < 1$  is the measure of positional bequest concern.<sup>1</sup>  $\theta$  and  $\gamma$  are assumed to be common for all households and constant over time.

In the first period of his life, the budget constraint for the agent is

$$c_t^i + s_t^i = b_t^i + w_t^i, \quad (3.4)$$

where  $b_t^i$  denotes inheritance received from parent and  $w_t^i$  denotes stochastic income. In the second period, the budget constraint is

$$R_{t+1}s_t^i = d_{t+1}^i + b_{t+1}^i, \quad (3.5)$$

where for every  $t$ , where  $R_{t+1}$  is the return rate on investment. The individual maximizes (3.3) subject to (3.4) and (3.5). Using the first order conditions, one

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<sup>1</sup>The household utility is well defined if and only if  $b_{t+1}^i > \gamma \bar{b}_{t+1}$ , which depends on  $\gamma$ , average bequests  $\bar{b}_{t+1}$  as well as on the choice of  $b_{t+1}^i$ .

can derive optimal savings and bequests of agent  $i$  in each period:

$$s_t^i = \frac{\beta + \theta}{1 + \beta + \theta}(b_t^i + w_t^i) + \frac{\gamma}{(1 + \beta + \theta)R_{t+1}}\bar{b}_{t+1}, \quad (3.6)$$

$$b_{t+1}^i = \frac{\theta R_{t+1}}{1 + \beta + \theta}(b_t^i + w_t^i) + \frac{\gamma(1 + \beta)}{1 + \beta + \theta}\bar{b}_{t+1}. \quad (3.7)$$

Income raise in period  $t$  increases savings and period  $t + 1$  bequests for the  $i$ th household. Average level of future bequest also behaves in the same direction with the income increasing savings and future bequest of  $i$ th household.

### 3.1.3 The Average Household

In order to analyze dynamics of overall capital and bequest, it is enough to have society average values of variables. We can rewrite (3.6) and (3.7) as a function of the household income,  $y_t^i = b_t^i + w_t^i$ , and the economy average income,  $\bar{y}_t$  given by:

$$\bar{y}_t = \frac{1}{N} \sum_{i=1}^N b_t^i + \frac{1}{N} \sum_{i=1}^N w_t^i = \bar{b}_t + \bar{w}_t. \quad (3.8)$$

Then, the following average optimal savings, bequests, and consumption choices are:

$$\bar{c}_t = \frac{1 - \gamma}{\theta + (1 + \beta)(1 - \gamma)}\bar{y}_t, \quad (3.9)$$

$$\bar{s}_t = \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)}\bar{y}_t, \quad (3.10)$$

$$\bar{b}_{t+1} = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)}R_{t+1}\bar{y}_t, \quad (3.11)$$

$$\bar{d}_{t+1} = \frac{\beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)}R_{t+1}\bar{y}_t. \quad (3.12)$$

Following equations (3.9)-(3.12), an increase in average output per capita,  $\bar{y}_t$ , increases current and future consumption, savings and bequests.

## 3.2 The Dynamics of the Capital and Bequests

The level of the capital available in the economy does not depend on the individual income levels. Assuming that physical capital depreciates completely from one period to next, total capital available in the economy next period,  $K_{t+1}$  results from households' savings, that is  $K_{t+1} = N\bar{s}_t$  or in per capita terms  $k_{t+1} = \bar{s}_t$ . Using equations (3.8) and (3.10) along with the law of accumulation of physical capital, we obtain that:

$$k_{t+1} = \bar{s}_t = \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} \left[ \frac{\theta}{\theta + \beta(1 - \gamma)} R_t \bar{s}_{t-1} + \bar{w}_t \right],$$

where  $R_t \bar{s}_{t-1} = R_t k_t$ . Thus, using that  $R_t = f'(k_t)$ , we reach the following equation which describes  $k_{t+1}$  as a function of past per capita capital and the model parameters:

$$k_{t+1} = \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} \left[ f(k_t) - \frac{\beta(1 - \gamma)}{\theta + \beta(1 - \gamma)} f'(k_t) k_t \right]. \quad (3.13)$$

Noteworthy, the households' variability of skills does not have any effect on the economy at the aggregated level. Since all individuals throw their abilities from the same random distribution, and that human capital is not inherited, aggregation smooths out individuals' differences eliminating any stochasticity.

**Definition 2.** A value  $k^*$  of physical capital is a steady state solution of equation (3.13) if and only if  $k^*$  satisfies

$$k^* = \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} \left[ f(k^*) - \frac{\beta(1 - \gamma)}{\theta + \beta(1 - \gamma)} f'(k^*) k^* \right].$$

A sufficient condition for the existence of a unique interior steady state is the concavity of the right hand side of (3.13) in  $k_t$ . To show the effects of reference parameter and the bequest motive on the steady state level of the physical capital and the average level of bequest, we consider a Cobb-Douglas production function in the following proposition:

**Proposition 4.** *Under a Cobb-Douglas production function,  $f(k_t) = Ak_t^\alpha$  with  $A > 0$ ,  $0 < \alpha < 1$ , the stock of physical capital reaches a stable steady state,*

$$k^* = \left( \frac{A(\theta + \beta(1 - \gamma)(1 - \alpha))}{\theta + (1 + \beta)(1 - \gamma)} \right)^{\frac{1}{1 - \alpha}}, \quad (3.14)$$

*which is increasing in  $\gamma$  and  $\theta$ .*

*Proof.* Note from (3.13) that

$$\frac{\partial k_{t+1}}{\partial k_t} = A\alpha \frac{\theta + \beta(1 - \gamma)(1 - \alpha)}{\theta + (1 + \beta)(1 - \gamma)} k_t^{\alpha-1}.$$

Substituting  $k_t$  by its steady state value, we obtain that  $\frac{\partial k_{t+1}}{\partial k_t} |_{k^*} = \alpha < 1$ . Then, in order to analyze the effect of the reference parameter  $\gamma$  and the bequest motive,

measured by  $\theta$ , we compute the partial derivatives of  $k^*$  with respect to  $\theta$  and  $\gamma$ :

$$\frac{\partial k^*}{\partial \gamma} = \frac{A}{1-\alpha} \left[ \frac{A(\theta + \beta(1-\gamma)(1-\alpha))}{\theta + (1+\beta)(1-\gamma)} \right]^{\frac{\alpha}{1-\alpha}} \frac{\theta(1+\alpha\beta)}{[\theta + (1+\beta)(1-\gamma)]^2},$$

and

$$\frac{\partial k^*}{\partial \theta} = \frac{A}{1-\alpha} \left[ \frac{A(\theta + \beta(1-\gamma)(1-\alpha))}{\theta + (1+\beta)(1-\gamma)} \right]^{\frac{\alpha}{1-\alpha}} \frac{(1-\gamma)(1+\alpha\beta)}{[\theta + (1+\beta)(1-\gamma)]^2},$$

which are always positive. □

Old age consumption and bequests are financed by the savings of the young agents in the first period. An increase in the degree of altruism induces all households to increase savings, which increases the steady-state level of capital. When the measure of reference dependence increases, family bequests must increase to remain in the lead. In order to increase bequests, families need to save more, increasing this way the level of the steady state capital stock.

The average wage in the economy at time  $t$  was defined as  $\bar{w} = w\bar{l}_t$  where the average ability  $\bar{l}_t$  is by assumption equal to 1. Using (3.8) together with equation (3.11), the following equation for the evolution of the average bequest is obtained:

$$\bar{b}_{t+1} = \frac{\theta}{\theta + (1+\beta)(1-\gamma)} R_{t+1}(\bar{b}_t + w_t b_t). \quad (3.15)$$

The following proposition shows that a stationary value for the average bequest is attained:

**Proposition 5.** *Under a Cobb-Douglas production function, if physical capital*

is at its steady state then average bequest  $\bar{b}_t$  also reaches a stable stationary state,

$$\bar{b}^* = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma) - \theta R^*} R^* w^*, \quad (3.16)$$

that is increasing in  $\gamma$  and  $\theta$  where  $R^* = f'(k^*) = A\alpha k^*\alpha - 1$  and  $w^* = f(k^*) - k^* f'(k^*) = A(1 - \alpha)k^*\alpha$ .

*Proof.* Average bequests reach a steady state value if and only if the steady state value of the interest rate is small enough,

$$R^* < \frac{\theta + (1 + \beta)(1 - \gamma)}{\theta}.$$

Or otherwise stated, if the steady state for aggregated capital is large enough. Substituting  $R^*$  by  $A\alpha k^*\alpha - 1$  and then  $k^*$  using (3.14), we obtain that the proposition holds if and only if

$$\alpha\theta < \theta + \beta(1 - \alpha)(1 - \gamma),$$

which is always true.

Taking the derivative of the  $\bar{b}^*$  with respect to  $\theta$  and  $\gamma$  gives:

$$\frac{\partial \bar{b}^*}{\partial \theta} = \frac{A\alpha \left[ \frac{A(\theta + \beta(1 - \gamma)(1 - \alpha))}{\theta + (1 + \beta)(1 - \gamma)} \right]^{\frac{\alpha}{1 - \alpha}} \left[ \beta(1 - \gamma) + \frac{\alpha(1 + \alpha\beta)(1 - \gamma)(\beta(1 - \gamma) + \theta)\theta}{(1 - \alpha)((1 + \beta)(1 - \gamma) + \theta)((1 - \gamma)\beta(1 - \gamma) + \theta)} \right]}{[\beta(1 - \gamma) + \theta]^2}$$

and

$$\frac{\partial \bar{b}^*}{\partial \gamma} = \frac{A\alpha\theta \left[ \frac{A(\theta + \beta(1 - \gamma)(1 - \alpha))}{\theta + (1 + \beta)(1 - \gamma)} \right]^{\frac{\alpha}{1 - \alpha}} \left[ \beta + \frac{\alpha(1 + \alpha\beta)(\beta(1 - \gamma) + \theta)\theta}{(1 - \alpha)((1 + \beta)(1 - \gamma) + \theta)((1 - \gamma)\beta(1 - \gamma) + \theta)} \right]}{[\beta(1 - \gamma) + \theta]^2}$$

which are positive. □

### 3.3 Bequests and Growth in a Segregated Economy

This section examines the quest for social status as the source of persistent inequality among differentiated groups in a segregated economy. As it turns out, group inequality may never disappear in the long-run even in the extreme case when the poorer group imitates the richer group disregarding their own group bequest decisions.

Suppose the economy is made of two groups, A and B.<sup>2</sup> Each group has a different referent regarding bequests, so that for group A the bequest reference level is

$$\hat{b}_{A,t} = \eta_A \bar{b}_{A,t} + (1 - \eta_A) \bar{b}_{B,t}, \quad (3.17)$$

where  $\bar{b}_{A,t}$  and  $\bar{b}_{B,t}$  are the time  $t$  average bequest of groups A and B, respectively.  $\eta_A$  measures the weight of A's own group in the bequest reference and it lies between 0 and 1. The larger  $\eta_A$ , the stronger the link between generations within A. Similarly,  $1 - \eta_A$  measures the relevance of group B in A's referent. If  $\eta_A = 1$ , then the group A social referent is made exclusively of its own members. When on the contrary,  $\eta_A = 0$ , then individuals in group A only consider group B's

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<sup>2</sup>Our results easily extend to the case of  $N \in \mathbb{N}$  groups.

bequests. Similarly, the bequest reference for group B at time  $t$  is

$$\hat{b}_{B,t} = (1 - \eta_B)\bar{b}_{A,t} + \eta_B\bar{b}_{B,t},$$

with  $0 \leq \eta_B \leq 1$ . Then, the vector of bequests can be written as

$$\begin{pmatrix} \hat{b}_{A,t} \\ \hat{b}_{B,t} \end{pmatrix} = \begin{pmatrix} \eta_A & 1 - \eta_A \\ 1 - \eta_B & \eta_B \end{pmatrix} \begin{pmatrix} \bar{b}_{A,t} \\ \bar{b}_{B,t} \end{pmatrix}.$$

If  $\eta_A = \eta_B = 1$ , then we obtain naturally the benchmark model analyzed in previous sections. Groups do not interact socially, and each group reaches their own steady state. This is the case in which inequality among groups is maximal. It suffices that one of the groups opens its referent to launch the dynamics towards equality.

We assume that all households within a group share the same preferences over consumption and social status. Household  $i$  in group A aims at

$$\max_{\{c_t^i, d_{t+1}^i, b_{t+1}^i\}} \ln c_t^i + \beta \ln d_{t+1}^i + \theta \ln (b_{t+1}^i - \gamma \hat{b}_{t+1}^A),$$

subject to the young and old age budget constraints (3.4) and (3.5), plus (3.17).

The household optimal decisions are given by

$$s_t^i = \frac{\theta + \beta}{1 + \theta + \beta} (b_t^i + w_t^i) + \frac{\gamma}{(1 + \theta + \beta)R_{t+1}} \hat{b}_{t+1}^A, \quad (3.18)$$

$$b_{t+1}^i = \frac{\theta}{1 + \theta + \beta} R_{t+1} (b_t^i + w_t^i) + \frac{\gamma(1 + \beta)}{1 + \theta + \beta} \hat{b}_{t+1}^A. \quad (3.19)$$

Similar optimal decisions obtain for group  $B$ . After some algebra, aggregated group bequests can be expressed as

$$\begin{pmatrix} \bar{b}_{A,t+1} \\ \bar{b}_{B,t+1} \end{pmatrix} = \mathcal{B} \begin{pmatrix} \bar{b}_{A,t} + w_t \\ \bar{b}_{B,t} + w_t \end{pmatrix} = \begin{pmatrix} \mathcal{B}_{1,1} & \mathcal{B}_{1,2} \\ \mathcal{B}_{2,1} & \mathcal{B}_{2,2} \end{pmatrix} \begin{pmatrix} \bar{b}_{A,t} + w_t \\ \bar{b}_{B,t} + w_t \end{pmatrix}, \quad (3.20)$$

where

$$\begin{cases} \mathcal{B}_{1,1} = \frac{\theta R_{t+1}}{D} [\theta + (1 + \beta)(1 - \gamma\eta_B)], \\ \mathcal{B}_{1,2} = \frac{\theta R_{t+1}}{D} [\theta + (1 + \beta)(1 - \gamma\eta_A)], \\ \mathcal{B}_{2,1} = \frac{\theta R_{t+1}}{D} \frac{\gamma(1+\beta)(1-\gamma\eta_B)[\theta+(1+\beta)(1-\gamma\eta_B)]}{1+\theta+\beta}, \\ \mathcal{B}_{2,2} = \frac{\theta R_{t+1}}{D} \left( \frac{D}{\theta+(1+\beta)(1-\gamma\eta_B)} + \frac{\gamma(1+\beta)(1-\gamma\eta_A)}{1+\theta+\beta} \right), \end{cases}$$

and

$$D = \theta + (1 + \beta)[1 - \gamma(\eta_A + \eta_B)] - \gamma^2(1 + \beta)^2(1 - \eta_A - \eta_B).$$

Average group's savings can also be expressed as a function of  $\{\bar{b}_{A,t}, \bar{b}_{B,t}\}$  and  $w_t$ :

$$\begin{pmatrix} \bar{s}_{A,t} \\ \bar{s}_{B,t} \end{pmatrix} = \begin{pmatrix} \varsigma_{1,1} & \varsigma_{1,2} \\ \varsigma_{2,1} & \varsigma_{2,2} \end{pmatrix} \begin{pmatrix} \bar{b}_{A,t} + w_t \\ \bar{b}_{B,t} + w_t \end{pmatrix}, \quad (3.21)$$

with

$$\left\{ \begin{array}{l} \varsigma_{1,1} = \frac{\theta+\beta}{1+\theta+\beta} + \frac{\gamma}{1+\theta+\beta} [\eta_A \mathcal{B}_{1,1} + (1-\eta_A) \mathcal{B}_{2,1}], \\ \varsigma_{1,2} = \frac{\gamma}{1+\theta+\beta} [\eta_A \mathcal{B}_{1,2} + (1-\eta_A) \mathcal{B}_{2,2}], \\ \varsigma_{2,1} = \frac{\gamma}{1+\theta+\beta} [\eta_B \mathcal{B}_{1,1} + (1-\eta_B) \mathcal{B}_{2,1}], \\ \varsigma_{2,2} = \frac{\theta+\beta}{1+\theta+\beta} + \frac{\gamma}{1+\theta+\beta} [\eta_B \mathcal{B}_{1,2} + (1-\eta_B) \mathcal{B}_{2,2}]. \end{array} \right.$$

Let us assume that population is constant, and so are the groups sizes. Denoting total population by  $N$ , the size of group A and group B are  $N_A = \mu N$  and  $N_B = (1-\mu)N$ , respectively, for  $0 < \mu < 1$ . Under these assumptions, physical capital is also a function of  $\{\bar{b}_{A,t}, \bar{b}_{B,t}\}$  and  $w_t$ :

$$k_{t+1} = \mu \bar{s}_{t,A} + (1-\mu) \bar{s}_{t,B}. \quad (3.22)$$

The definition of equilibrium for the segregated economy follows:

**Definition 3.** A sequence of household decisions  $\{(c_{A,t}^i, s_{A,t}^i, d_{A,t}^i, b_{A,t}^i)\}_{i=1,\dots,N_A; t=1,\dots,\infty}$  and  $\{(c_{B,t}^i, s_{B,t}^i, d_{B,t}^i, b_{B,t}^i)\}_{i=1,\dots,N_B; t=1,\dots,\infty}$  together with the time sequence of unit salary and interest rates  $\{(w_t, R_t)\}_{t=1,\dots,\infty}$  is an equilibrium if:

- i) Skills of individuals from both regions,  $l_{A,t}^i$  and  $l_{B,t}^i$  are thrown from the same skill distribution with  $\bar{l}_t = 1$  and  $var(l_t^i) = \sigma_l^2$ , for all  $t$ .
- ii) Households' decisions are optimal and described by (3.18) and (3.19). Average group bequests and savings are described by (3.20) and (3.21), respectively.

- iii) Group A's population is a share  $\mu$  of total population and B's a fraction  $1 - \mu$ .
- iv) There exists a unique firm, which maximizes profits at every period, and which pays labor and capital at their marginal productivities as in (3.1) and (3.2).

Note then that the system made by (3.20) and (3.22) is backward looking, that is, predetermined. Hence, for a given distribution of initial bequests and overall capital, we can obtain the trajectory of physical capital in time, which crucially depend on  $\gamma$ ,  $\eta_A$ ,  $\eta_B$  and  $\mu$ . We prove in Appendix E that a steady state solution exists for the group bequests and capital.

Due to the difficulty of providing with intuitive analytic results, we turn next into a numerical illustration of the economy. We illustrate the short and long-term behavior of an economy made of two groups, A and B, which differ in their initial bequest without loss of generality. Group A is assumed to be better endowed than group B. Regarding the constructions of the group's referent, we follow Ferrer-i-Carbonell (2005), and assume that the rich build their referent more strongly on their group than on B. Taking this view to the extreme, throughout the exercises,  $\eta_A$  is fixed to 1.<sup>3</sup> Table 3.1 provides the benchmark parameters values for preferences and production. In what concerns groups size and initial bequests, it is assumed that group A represents 25% of the population and it starts with an initial capital of 25 units of final good. In contrast, the 75% of the population represented by Group B starts with a level of physical capital of

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<sup>3</sup>Qualitative results do not change for different values of  $\eta_A$  and  $A$ .

2 units.

Table 3.1: Parameters Values

$A$	Technological parameter	5
$\alpha$	Output elasticity	0.75
$\beta$	Weight of old age consumption	0.5
$\theta$	Weight of bequest	0.25
$\gamma$	Weight of the bequest reference	0.1
$\eta_A$	Weight of group A's in its own reference	1
$k(0)$	Initial capital	30
$b_A(0)$	Group A's initial bequest	25
$b_B(0)$	Group B's initial bequest	2
$\mu$	Relative size of group A	0.25

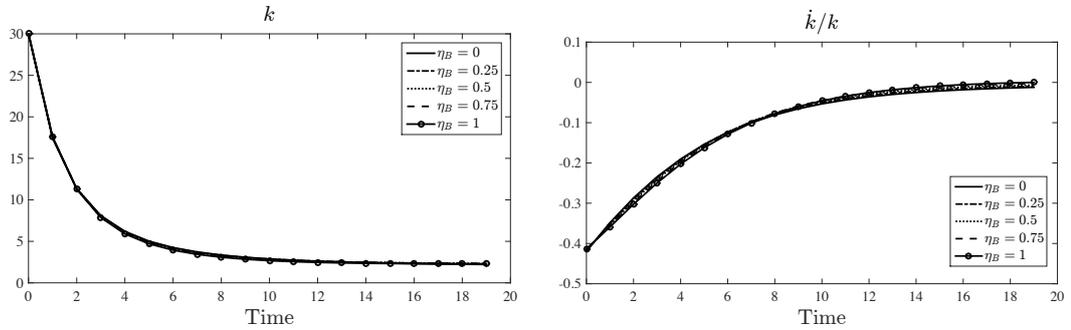


Figure 3.1: Evolution of  $k$  and Its Growth Rate

In our exercises, A's referent only depends on group A average. In contrast, all individuals in B compare their bequests with the average of A, as well as B's. Even if the effect of A on B's decisions is relatively small, the fact that B opens to the other group will reduce group inequality in the long-run. Nevertheless, our results show that initial disparities do not disappear even in the long-run.

Throughout the exercises, we consider a set of parameter values under which there exist a unique stable steady state and analyze the evolution of capital, groups' average bequests and consumption depending on group B's referent parameter  $\eta_B$ . We make  $\eta_B$  take values 0, 0.25, 0.5, 0.75 and 1. Simulation results are

displayed in Figures 3.1-3.3. Focusing first on the overall economy dynamics, Figure 3.1 shows that physical capital decreases from the first generation.<sup>4</sup> The adjustment to the long-run value is made essentially by the first five generations. Note how capital halves from the first to the second generation, then it halves again from the second to the third generation. From the third generation onwards, the growth rate of capital is less steep.<sup>5</sup> Despite variations in group B's bequest motive, physical capital evolves almost identically in all scenarios.

Let us underline some stylized facts. Group A's consumption and bequests are always higher than group B's. This means that the initial inequality in wealth persists in time and it determines the groups' future. As Figure 3.3 shows, bequest inequality decreases with  $1 - \eta_B$ , that is, the harder group B looks up to group A. Regarding group A's evolution, note that all variables increase with  $\eta_B$ . The larger  $\eta_B$ , the lower B's bequests and hence, the lower B's wealth. Hence, group A becomes relatively wealthier the larger  $\eta_B$ .

Regarding group B, notice first that the less B looks up to A, that is, the higher  $\eta_B$ , the lower their wealth. As a consequence, both B's consumption and bequests also decrease. Then, note that B's trajectories are not always monotone. Take the case  $\eta_B = 0$ . The first generation sacrifices current and future consumption to bequest imitating A. Actually, initial consumption is at its minimum for  $\eta_B = 0$ . Then, after one generation, B increases consumption and reaches the maximum. While bequests also reach a maximum after one generation, they decrease afterwards.

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<sup>4</sup>The horizontal axis on the figures represents time. Here, the unit of time is one generation.

<sup>5</sup>If group A were sufficiently large, then the level of the steady state of physical capital would have been larger than the initial condition and capital would have increased in the transition. Nevertheless, in that case, differences in B's trajectories are less noticeable and hence less interesting for the illustration purposes.

This behavior is not at odds with their vocation to imitate A. On the contrary, they do not need to sacrifice consumption further since they are following A, who is decreasing bequests.

Finally, note how group B variables tend to the same long-run value when  $\eta_B$  is 0, 0.25 and 0.5. Hence, there exists a threshold value for the reference parameter or an optimal mixture between groups A and B bequest behavior so that B reaches the highest steady state while preserving its identity.

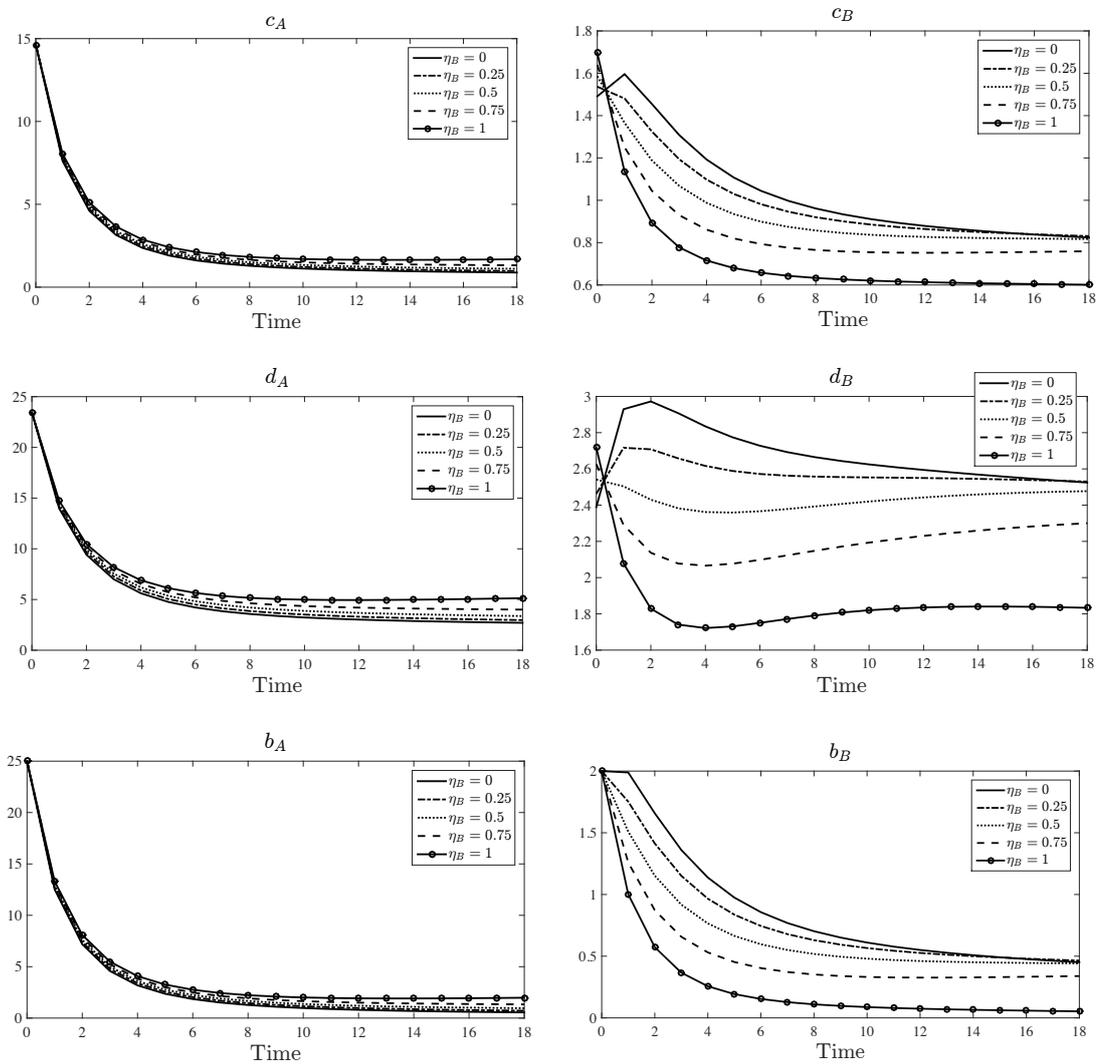


Figure 3.2: Groups' Consumption and Bequests

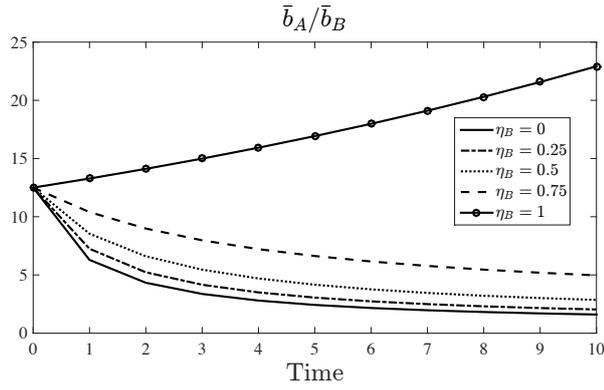


Figure 3.3: Bequest Ratio,  $\frac{\bar{b}_A}{\bar{b}_B}$

### 3.4 Conclusion

This essay analyzed the impact of positional concerns related to bequests on dynamics of the economy. Individuals bequeath more to enhance their heirs' social status. An increase in the positional concern leads to a high level of savings which is resulted higher steady-state total capital and bequest levels compared to absence of positional concerns. In the last section, we analyzed the inequality for segregated economies. In segregated economies, with different referents for bequests, inequality decreases but it persists in the long-run.

## CHAPTER 4

# CONSUMPTION ENVY AND INEQUALITY IN A RESOURCE ECONOMY

The aim of this essay is to analyze the effects of positional concerns on resource exploitation and household inequality. We present a non-overlapping generations resource economy model where households are heterogeneous in terms of their productive abilities and initial wealth. The households are assumed to have a warm glow, *joy of giving*, type bequest motive (see Andreoni, 1989). We consider that households have two types of positional concerns: consumption related and bequest related. As such they derive utility not only from their relative consumption but also from the relative amount of resources that they bequeath to their children. The reference levels of consumption and bequest are taken as the average consumption and bequest levels of all households living in the same period.

Large fraction of human behavior can be explained by social status seeking. Interdependent preferences have been acknowledged in the literature for long time. It is admitted that the consumer's satisfaction level also depends on how she com-

compares herself with the rest of the society. Duesenberry (1949) suggests that the decision of people includes not only their own income but also relative income (see, for models with positional concern, Abel, 1990, Ireland, 1994, Easterlin, 1995, Liu and Turnovsky, 2005). To examine the equity premium puzzle, Abel (1990) introduces a “catching up with the Joneses” utility function which depends on the individual’s consumption level relative to the average level of consumption. Ireland (1994) analyzes the behavior of consumers in a society that individuals care what others think. Individuals attempt to show a higher level of status than their real status with the way of consumption signals. Easterlin (1995) supports the role of consumption externalities empirically. In a capital-based economy, Liu and Turnovsky (2005) show that consumption externalities have no effect on the steady-state level of capital as long as labor is inelastically supplied.

Garcia-Peñalosa and Turnovsky (2008) investigate the link between consumption externalities and wealth inequality by considering two forms of heterogeneity, different initial wealth endowment and different reference consumption levels. They show the conditions where the equilibrium with heterogeneous agents coincides with representative agent case and also find that existence of externalities decrease inequality. In addition to this, Alvarez-Cuadrado and Van Long (2012) study the effects of consumption envy on inequality and find that envy increases wealth and consumption inequalities. In a prospect theoretic model, Bogliacino and Ortoleva (2015) advocate the reference dependence on capital endowments left to the offspring which pushes households to improve their relative situation.

The relationship between positional concerns and the environmental issues have

also been analyzed in Ng and Wang (1993) and Howarth (1996). The former paper suggests that positional concern for income may lead to an increase in private consumption and a decrease in environmental quality. The latter one studies the implications of consumption externality in a static model to offer optimal environmental taxes. The closest paper to our essay is Alvarez-Cuadrado and Van Long (2011) that examine the effects of consumption envy on resource dynamics under different property-rights regimes in a continuous time model. They analyze resource exploitation and reach that envious agents over-exploit the natural resources, that leads to a lower steady-state stock than the efficient level. However, these models do not include the analysis of the effects of consumption envy on the wealth and resource distribution. This essay fulfils this gap by considering a non-overlapping generations resource model to explore the implications of the consumption envy on the dynamics of the households' wealth and resource distribution.

In this essay, bequest acts as a mechanism for the transmission of the resource across generations in a resource based economy (see Brechet and Lambrecht, 2011)<sup>1</sup>. Each household owns some portion of the available renewable resource in the economy. The extraction of the resource by the household is costless. As we consider joy of giving motive, households are eager to leave more unexploited resource to their descendants in order to increase the following generations' revenue from the renewable resource.

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<sup>1</sup>As examples for renewable resource dynamics models, Brechet and Lambrecht (2011) explore whether a natural renewable resource can be managed efficiently or not. Koskela et al. (2002) characterize the dynamic, efficiency and stability of the equilibrium by incorporating a renewable resource into an overlapping generations model without capital where the resource serves both as a store of value and input in the production for the consumption.

We characterize the resource dynamics under both strictly concave and linear production functions. In particular, we consider a Cobb–Douglas production function and show that the steady state level of the average resource depends on the value of the regeneration rate. There exists a threshold level of regeneration rate, above which the economy exhibits a balanced growth path; below which the resource stock declines exponentially and exhausts ultimately. If the regeneration rate is equal to the threshold level of regeneration, the economy gets stuck at the initial level of average resource in the economy.

When we consider a linear production technology and concentrate only on the effects of the consumption envy by ignoring the bequest related positional concern for the sake of tractability, we obtain that there exists a unique steady state at which the average resource level decreases with the degree of consumption envy. We also show that the steady-state level of average resource increases with the resource regeneration rate and the degree of altruism. Proposing the coefficient of variation as a measure of inequality (see Bossmann et al., 2007, Alvarez-Cuadrado and Van Long, 2012), we reach the following results on inequality. Firstly, consumption envy increases wealth inequality as well as resource and consumption inequality. Secondly, if the degree of altruism is strong enough, inequality increases. This result contrasts with Bossmann et al. (2007) which propose equalizing effects of bequests in a capital economy. Lastly, in line with the effects of the degree of altruism, if the resource regeneration rate is strong enough, wealth and resource inequality increases.

The remainder of the essay is organized as follows. In Section 4.1, we describe a

non-overlapping generations model with renewable resource in which households are altruistic. Section 4.2 analyzes dynamics of the resource available in the economy with both Cobb-Douglas and linear production technologies. The stationary distribution of the household variables are characterized and implications are presented in Section 4.3. Section 4.4 summarizes our results and conclude the essay.

## 4.1 The Model

We consider a non-overlapping generations economy where each individual lives for one period. Population consists of  $N$  individuals and assumed to be constant. Individuals receive income from supplying one unit of labor to the firms. The agents are altruistic in terms of resource. They bequeath a portion of renewable resource to their descendants which can be sold to the firms or saved for the following generations. We exclude capital from our setup in order to concentrate on the effects of positional concerns on resource dynamics and inequality.

Let us describe the resource dynamics. At the beginning of each period,  $i$ th household is endowed with  $z_t^i$  level of resource. She decides the level,  $e_t^i$ , how much to extract and sells this extracted amount to the firms. The remaining stock is left to the following generations. There is no extraction cost of the resource. The  $i$ th dynasty's stock regenerates at a linear rate  $\pi$  (see Mourmouras, 1991), which has evolution dynamics as:

$$z_{t+1}^i = \pi(z_t^i - e_t^i) \tag{4.1}$$

Individuals within a given generation differ with respect to their productive ability  $a_t^i$  and resource level inherited from their parents  $z_t^i$ . Each agent draws  $a_t^i$  from an independent and identical distribution at the beginning of the period  $t$ , where  $\bar{a}_t = 1$  and  $var(a_t^i) = \sigma_a^2$ . Hence, resulting wage distribution is  $\bar{w}_t = w_t$  and standard deviation is  $\sigma_{w_t} = w_t \sigma_a$ .

We concentrate on the  $i$ th household, born in period  $t$ , who inelastically supplies her endowment of labor and earns income  $w_t^i = a_t^i w_t$ . She spends all her income to the consumption  $c_t^i$ . This gives the budget constraint for the individual  $i$  as:

$$c_t^i = q_t e_t^i + w_t^i \quad (4.2)$$

where  $w_t^i$  is stochastic income,  $c_t^i$  denotes consumption at period  $t$ ,  $e_t^i$  is the amount of the resource that is extracted and sold to the firm by the individual,  $q_t$  is the price of the resource.

As we have joy of giving type altruism, the life-cycle utility function for the individual born in period  $t$  is given by:

$$u(\hat{c}_t^i, \hat{z}_{t+1}^i) = \ln(c_t^i - \gamma \bar{c}_t) + \beta \ln(z_{t+1}^i - \eta \bar{z}_{t+1}) \quad (4.3)$$

where  $\beta$  governs the degree of altruism. An agent derives utility from consumption including how she compares her level of consumption and unextracted resource bequeathed to her heir,  $z_{t+1}^i$ , with that of others from the same generation. We adopt additive specification for consumption and bequest, relative to the others as  $\hat{c}_t^i = c_t^i - \gamma \bar{c}_t$  and  $\hat{z}_{t+1}^i = z_{t+1}^i - \eta \bar{z}_{t+1}$  where  $\bar{c}_t = \frac{1}{N_t} \sum_{i=1}^{N_t} c_t^i$  is the average

consumption of generation born at time  $t$  and  $0 < \gamma < 1$  is the measure of positional consumption concern. Similarly,  $\bar{z}_{t+1} = \frac{1}{N_t} \sum_{i=1}^{N_t} z_{t+1}^i$  is the average bequest and  $0 < \eta < 1$  is the measure of bequest related positional concern. Households maximize (4.3) subject to (4.1), (4.2) and the non-negativity constraints.

By means of the first order necessary conditions of optimality, we derive the extracted amount of resource, bequeathed amount of resource and consumption of an agent in each period:

$$e_t^i = \frac{q_t z_t^i - \beta w_t^i}{q_t(\beta + 1)} + \frac{\beta\gamma}{q_t(\beta + 1)} \bar{c}_t - \frac{\eta}{\pi(1 + \beta)} \bar{z}_{t+1} \quad (4.4)$$

$$z_{t+1}^i = \pi \left( \frac{\beta}{1 + \beta} \frac{q_t z_t^i + w_t^i}{q_t} - \frac{\beta\gamma}{(1 + \beta) q_t} \bar{c}_t + \frac{\eta}{\pi(1 + \beta)} \bar{z}_{t+1} \right), \quad (4.5)$$

$$c_t^i = \frac{q_t z_t^i + w_t^i}{(1 + \beta)} + \frac{\beta\gamma}{(1 + \beta)} \bar{c}_t - \frac{\eta}{\pi(1 + \beta)} q_t \bar{z}_{t+1} \quad (4.6)$$

Following (4.5) and (4.6), we characterize the optimal behavior of an average household. Defining household wealth as the sum of wage and revenue that may occur from selling the total amount of available resource  $y_t^i = q_t z_t^i + w_t^i$  (and  $\bar{y}_t = q_t \bar{z}_t + \bar{w}_t$ ) and associating it with (4.4), we reach the following consumption and resource levels for the average household:

$$\bar{c}_t = \frac{(1 - \eta)}{(1 + \beta(1 - \gamma) - \eta)} \bar{y}_t, \quad (4.7)$$

$$\bar{z}_{t+1} = \frac{\pi\beta(1-\gamma)}{(1+\beta(1-\gamma)-\eta)} \frac{\bar{y}_t}{q_t} \quad (4.8)$$

$$\bar{e}_t = \frac{(1-\eta)q_t\bar{z}_t - \beta(1-\gamma)\bar{w}_t}{(1+\beta(1-\gamma)-\eta)q_t} \quad (4.9)$$

An increase on the average level of household wealth increases consumption and bequeathed level of resource of the average household. We can now use the results for the average household to characterize the behavior of the households. Individuals' optimal choices depend on the average level of resource in the society. Using equations (4.4)-(4.8), we reach the optimal choices which are given by:

$$c_t^i = \frac{1}{(1+\beta)} (y_t^i + \beta\phi\bar{y}_t), \quad (4.10)$$

$$z_{t+1}^i = \frac{\pi\beta}{(1+\beta)q_t} (y_t^i - \phi\bar{y}_t), \quad (4.11)$$

$$e_t^i = \frac{q_t z_t^i - \beta w_t^i}{(1+\beta)q_t} + \frac{\beta}{(1+\beta)} \phi \frac{\bar{y}_t}{q_t}, \quad (4.12)$$

where  $\phi = \frac{(\gamma-\eta)}{(1+\beta(1-\gamma)-\eta)} \begin{matrix} \leq \\ \geq \end{matrix} 0$  if and only if  $\gamma \begin{matrix} \leq \\ \geq \end{matrix} \eta$ .

Consumption and bequeathed level of resource are composed of two components: household's lifetime wealth and the influence of the average level of wealth in the society. They both increase with the household's own wealth. Consumption and bequeathed level of resource move in opposite directions with respect to the average level of wealth in the society. Moreover, harvesting decision of agent  $i$ ,  $e_t^i$ , increases with the bequeathed level of resource of her ancestor and decreases with her wage.

**Proposition 6.** *If  $\gamma \neq \eta$ , at any given time, one good (either consumption or bequest) is necessity while the other good is a luxury. When bequest is a luxury good (i.e.,  $\gamma > \eta$ ), the proportion of the lifetime income bequeathed is greater for wealthier individuals since*

$$\frac{\partial \left( \frac{z_{t+1}^i}{y_t^i} \right)}{\partial y_t^i} = \frac{\pi\beta}{(1+\beta)q_t} \phi \frac{\bar{y}_t}{(y_t^i)^2} > 0 \iff \gamma > \eta;$$

*Proof.* Define income elasticity of consumption and bequest as  $\varepsilon_{c^i}^{y^i} = \left(1 + \beta\phi\frac{\bar{y}_t}{y_t^i}\right)^{-1}$  and  $\varepsilon_{z^i}^{y^i} = \left(1 - \phi\frac{\bar{y}_t}{y_t^i}\right)^{-1}$ , respectively. Then,  $\varepsilon_{c^i}^{y^i} \leq 1$  and  $\varepsilon_{z^i}^{y^i} \geq 1$  if and only if  $\gamma \geq \eta$ .  $\square$

According to the Alvarez-Cuadrado *et al.* (2012, p. 952), “the high concentration of bequests on the upper tail of the wealth distribution” is empirically well-documented and thus bequests are generally considered to be luxury goods. In a similar vein, we assume  $\gamma > \eta$ , in general, and  $\gamma = \eta$  only as a limit case.

**Proposition 7.** *When  $\gamma = \eta$ , i.e. the positional concerns for consumption and bequests weigh equal, the individual choices are not affected by the average levels of income. Moreover, the individual motions are only governed by individual levels of  $y_t^i$ ,  $z_t^i$  and  $w_t^i$ ,*

$$c_t^i = \frac{1}{(1+\beta)} y_t^i, \tag{4.13a}$$

$$z_{t+1}^i = \frac{\pi\beta}{(1+\beta)q_t} y_t^i, \tag{4.13b}$$

$$e_t^i = \frac{q_t z_t^i - \beta w_t^i}{(1+\beta)q_t}. \tag{4.13c}$$

*Proof.* When  $\gamma = \eta$  then,  $\phi = 0$ . The rest follows directly.  $\square$

## 4.2 Dynamics of Aggregate Resource

The level of the renewable resource available in the economy does not depend on the individual income levels:

$$Z_{t+1} = N_t \bar{z}_{t+1} = N_t (\pi(\bar{z}_t - \bar{e}_t)). \quad (4.14)$$

Assuming zero population growth, using (4.8) and (4.14) together, we analyze the evolution of the average resource using both Cobb-Douglas and linear production functions.

### 4.2.1 Cobb-Douglas Production Technology

Following (4.8), we have resource dynamics as:

$$\bar{z}_{t+1} = \frac{\pi\beta(1-\gamma)}{(1+\beta(1-\gamma)-\eta)} \frac{q_t \bar{z}_t + \bar{w}_t}{q_t}. \quad (4.15)$$

In order to pinpoint the steady state of renewable energy stock available in the economy, one has to take into account that factor prices  $q_t$  and  $\bar{w}_t$ , which in turn depends on the economic variables. We consider a Cobb-Douglas (homogeneous of degree one) production in the economy using resource stock and labor to see the existence as well as the comparative statics of the steady state.

$$f(\bar{e}_t) = \bar{e}_t^\alpha, \alpha \in (0, 1).$$

Under competitive markets, the factors will be paid their marginal products,

$$q_t = f'(\bar{e}_t) = \alpha \bar{e}_t^{\alpha-1}, \quad (4.16)$$

$$\bar{w}_t = f(\bar{e}_t) - \bar{e}_t f'(\bar{e}_t) = (1 - \alpha) \bar{e}_t^\alpha. \quad (4.17)$$

From equation (4.9), we have,

$$\begin{aligned} \bar{e}_t &= \frac{(1 - \eta)}{(1 + \beta(1 - \gamma) - \eta)} \bar{z}_t - \frac{\beta(1 - \gamma)}{(1 + \beta(1 - \gamma) - \eta)} \frac{\bar{w}_t}{q_t} \\ &= \frac{(1 - \eta)}{(1 + \beta(1 - \gamma) - \eta)} \bar{z}_t - \frac{\beta(1 - \gamma)}{(1 + \beta(1 - \gamma) - \eta)} \frac{1 - \alpha}{\alpha} \bar{e}_t. \end{aligned}$$

Thus,

$$\bar{e}_t = \frac{\alpha(1 - \eta)}{((1 - \eta)\alpha + \beta(1 - \gamma))} \bar{z}_t. \quad (4.18)$$

Plugging (4.16), (4.17) and (4.18) into (4.15), (4.15) will reduce to

$$\bar{z}_{t+1} = \frac{\pi\beta(1 - \gamma)}{\alpha(1 - \eta) + \beta(1 - \gamma)} \bar{z}_t. \quad (4.19)$$

At the steady state we have  $\bar{z}_{t+1} = \bar{z}_t = z^*$  where from (4.19) we have,

$$z^* = \frac{\pi\beta(1 - \gamma)}{\alpha(1 - \eta) + \beta(1 - \gamma)} z^*.$$

Thus, the steady state value of the resource stock is determined by the coefficient

$$\frac{\pi\beta(1 - \gamma)}{((1 - \eta)\alpha + \beta(1 - \gamma))} = \pi \left( 1 + \frac{\alpha(1 - \eta)}{\beta(1 - \gamma)} \right)^{-1}.$$

**Proposition 8.** *Define the threshold level of regeneration as  $\bar{\pi} = \left( 1 + \frac{\alpha(1 - \eta)}{\beta(1 - \gamma)} \right)$ .*

*Then,*

1. If  $\pi = \bar{\pi}$ ,  $z^* = z_0$ , i.e. the only steady state is the initial level of average resource level;
2. If  $\pi > \bar{\pi}$ , then the resource stock grows unboundedly with growth rate  $\left(\frac{\pi\beta(1-\gamma)}{\alpha(1-\eta)+\beta(1-\gamma)} - 1\right) > 0$  and the economy exhibits balanced growth path;
3. If  $\pi < \bar{\pi}$ , then the resource stock declines exponentially at a rate  $\left(\frac{\pi\beta(1-\gamma)}{\alpha(1-\eta)+\beta(1-\gamma)} - 1\right) < 0$  and is exhausted ultimately at the steady state<sup>2</sup>, i.e.  $z^* = 0$ .

Thus, the threshold level of regeneration determines the behavior of the resource stock. However, this threshold is sensitive to the key parameters of the economy.

**Proposition 9.** *The threshold level of regeneration,  $\bar{\pi}$ , increases with  $\alpha$  and  $\gamma$ ; and decreases with  $\beta$  and  $\eta$ .*

*Proof.* Taking derivative of  $\bar{\pi}$  with respect to the given parameters will give  $\frac{\partial \bar{\pi}}{\partial \alpha} = \frac{(1-\eta)}{\beta(1-\gamma)} > 0$ ;  $\frac{\partial \bar{\pi}}{\partial \gamma} = \frac{\alpha(1-\eta)}{\beta(1-\gamma)^2} > 0$ ;  $\frac{\partial \bar{\pi}}{\partial \beta} = -\frac{\alpha(1-\eta)}{\beta^2(1-\gamma)} < 0$ ;  $\frac{\partial \bar{\pi}}{\partial \eta} = \frac{-\alpha}{\beta(1-\gamma)} < 0$ .  $\square$

**Proposition 10.** *Suppose  $\pi \neq \bar{\pi}$ .*

1. If  $\pi > \bar{\pi}$ , then the growth rate of resources increases with  $\pi, \beta$  and  $\eta$ ; and decreases with  $\alpha$  and  $\gamma$ .
2. If  $\pi < \bar{\pi}$ , then the growth rate of resources increases with  $\alpha$  and  $\gamma$ ; and decreases with  $\pi, \beta$  and  $\eta$ .

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<sup>2</sup>When the resource is not renewable, which is  $\pi = 1$ , at the steady state the resource become extinct.

*Proof.* Taking derivative of  $\left(\frac{\pi\beta(1-\gamma)}{\alpha(1-\eta)+\beta(1-\gamma)} - 1\right)$  with respect to the given parameters will give the results.  $\square$

**Proposition 11.** *For more patient agents, the threshold level of regeneration,  $\bar{\pi}$ , increases more rapidly with respect to the positional concern on bequest,  $\eta$ , and decreases more rapidly with respect to the envy on consumption,  $\gamma$ ; since  $\frac{\partial^2 \bar{\pi}}{\partial \gamma \partial \beta} < 0$  and  $\frac{\partial^2 \bar{\pi}}{\partial \eta \partial \beta} > 0$ .*

In other words, envy for consumption in more patient societies can induce sustained growth; while positional concerns for bequests can cause exhaustion for the resources since patience pushes the threshold level of regeneration below and above, respectively.

## 4.2.2 Linear Production Technology

Output is produced by combining resource  $e_t$  and labor  $l_t$  where  $F(e_t, l_t)$  is linear function. Labor is supplied inelastically, wage and resource price are constant over time:

$$F(e_t, l_t) = wl_t + qe_t. \quad (4.20)$$

Considering constant factor prices  $q$  and  $w$ , and assuming no reference dependence in bequest,  $\eta = 0$ , we identify the steady state level of average renewable resource available in the economy. At the steady state, the average level of the resource,  $\bar{z}_{t+1} = \bar{z}_t = \bar{z}^*$  is obtained as:

$$\bar{z}^* = \frac{\pi\beta(1-\gamma)w}{q(\beta(1-\gamma)(1-\pi) + 1)}. \quad (4.21)$$

The steady state value,  $\bar{z}^*$  is positive if  $(\pi - 1)\beta(1 - \gamma) < 1$ , which is  $\pi < 1 + \frac{1}{\beta(1-\gamma)}$ . Under this condition, the steady state value satisfies uniqueness and interiority.

**Proposition 12.** *The steady state level of the average resource decreases with  $\gamma$  and increases with  $\beta$  and  $\pi$ .*

*Proof.* Taking derivative of  $\bar{z}^*$  with respect to the given parameters will give:

$$\begin{aligned}\frac{\partial \bar{z}^*}{\partial \gamma} &= -\frac{\pi \beta w}{q(\beta(1-\gamma)(1-\pi)+1)^2} < 0, \\ \frac{\partial \bar{z}^*}{\partial \beta} &= \frac{\beta w(1-\gamma)(\beta(1-\gamma)+1)}{q(\beta(1-\gamma)(1-\pi)+1)^2} > 0, \\ \frac{\partial \bar{z}^*}{\partial \pi} &= \frac{\pi w(1-\gamma)}{q(\beta(1-\gamma)(1-\pi)+1)^2} > 0.\end{aligned}$$

□

An increase in the positional concern,  $\gamma$ , would increase the usage of the resource to compensate for the reduction in the utility coming from relative consumption. This leads to a decrease in the steady state level of the resource of agents. Thus, average level of the resource decreases with consumption envy. Fisher and Heijdra (2009) consider perpetual-youth model which shows that consumption externalities decrease steady-state of capital. Similar to their results, Alvarez-Cuadrado and Van Long (2012) also confirm that the steady state capital decreases with the degree of consumption envy. An increase in envy shifts the resources to reference dependent consumption leading to a decrease in the fraction of income saved. Second chapter of this dissertation considers an economy where agents' altruism

level depends on society average. In that chapter, it is shown that this reference dependence increases steady-state level of aggregate capital. In this chapter, for a resource-based economy, we show that consumption envy decreases the average level of the resource in the steady-state.

Increasing the degree of altruism,  $\beta$ , leads to an increase in the average steady state stock level. Agents want to bequeath more resource to their descendants as they get more utility from it. Lastly, increase in the regeneration rate of the resource,  $\pi$ , increases the steady state level of the average resource. In our framework with linear production technology, consumption externalities affect the steady state level.

### 4.3 Stationary Distributions and Inequality

In this section, our aim is to explore the effects of consumption envy on resource stock and wealth inequality. Since the scope of the study is to analyze the effect of consumption envy on inequality, we need to examine the dynamic behavior of the distribution of the resource. To do this, we concentrate on the influences of the consumption externalities on the resource distribution among the society. Since this requires that factor prices should be taken into account as well, in line with Alvarez-Cuadrado and Van Long (2012) as in chapter 2, we use a linear production technology in wage and resource.

At the steady state, assuming that there is no envy on bequeathed level of resource,  $\eta = 0$ , and taking  $q$  and  $w$  constant together with steady state values, we

calculate the explicit equation for the evolution of average level of bequeathed resource using equation (4.15):

$$\bar{z}_{t+1} = \pi \left[ \frac{q\beta(1-\gamma)\bar{z}_t + \beta(1-\gamma)\bar{w}_t}{q[\beta(1-\gamma) + 1]} \right]. \quad (4.22)$$

To ensure average level of the resource to reach positive steady state:

$$\pi < 1 + \frac{1}{\beta(1-\gamma)}, \quad (4.23)$$

which is the same condition in order to have a positive interior steady state for the average level of resource in the society. Under the linear production manifested in equation (4.20), by taking expectations of both sides of (4.11), we obtain the expected value of  $i$ th household resource as,

$$E[z^i] = \bar{z} = \frac{\pi\beta(1-\gamma)}{(1 + (1-\pi)\beta(1-\gamma))} \frac{w}{q}, \quad (4.24)$$

which is unique and positive.

It is assumed that the wage earned in the labor market and the level of the resource inherited from the previous generation are uncorrelated, namely  $cov(w_t^i, z_t^i) = 0$ . This means that the abilities are independent of the bequeathed level of resource. Now we can express the variance of the resource by applying variance operator to both sides of the equation (4.11):

$$var[z^i] = \frac{\beta^2\pi^2}{q^2[(1+\beta)^2 - \beta^2\pi^2]} \sigma_w^2. \quad (4.25)$$

Note from (4.25) that, to get positive variance we should further have  $\pi < 1 + \frac{1}{\beta}$ . By using these results together with (4.12), the mean and the variance for consumption are:

$$E[c^i] = \bar{c} = \frac{1}{1 + \beta(1 - \gamma)(1 - \pi)} \bar{w}, \quad (4.26)$$

$$\text{var}[c^i] = \frac{1}{(1 + \beta)^2 - \beta^2 \pi^2} \sigma_w^2. \quad (4.27)$$

**Proposition 13.** *An increase in the consumption envy,  $\gamma$ , decreases the mean values of resource, increases the mean value of consumption at the steady state.*

*Proof.* Taking the derivatives of mean values of variables with respect to  $\gamma$  gives the following results:

$$\frac{\partial \bar{z}}{\partial \gamma} < 0, \quad \frac{\partial \bar{c}}{\partial \gamma} > 0.$$

□

Increase in consumption envy decreases the average resource level and increases the average consumption at the steady state. Households increase their consumption in order to compensate the increase in consumption envy. While doing this, they are selling more of the resource, which decreases the society average of the remaining renewable resource.

We follow Bossmann et al.(2007) and use coefficient of variation which is a convenient measure of the relative inequality in the distribution of random variable. From (4.24)-(4.27) we obtain the following measures of inequality for resource

and consumption:

$$CV[w^i] = \frac{\sigma_w}{\bar{w}}, \quad (4.28)$$

$$CV[z^i] = \frac{\sigma_z}{\bar{z}} = \frac{\sqrt{\frac{\beta^2 \pi^2}{(1+\beta)^2 - \beta^2 \pi^2}} (1 + (\pi - 1)\beta(\gamma - 1)) \sigma_w}{\pi \beta (1 - \gamma)} \frac{\sigma_w}{\bar{w}}, \quad (4.29)$$

$$CV[c^i] = \frac{\sigma_c}{\bar{c}} = \frac{\sqrt{\frac{1}{(1+\beta)^2 - \beta^2 \pi^2}} (1 + (\pi - 1)\beta(\gamma - 1)) \sigma_w}{\pi \beta (1 - \gamma)} \frac{\sigma_w}{\bar{w}}. \quad (4.30)$$

We compute the coefficient of variation also for wealth as:

$$CV[y^i] = \frac{\sigma_y}{\bar{y}} = \frac{\sqrt{\frac{(1+\beta)^2}{(1+\beta)^2 - \beta^2 \pi^2}} (1 + (\pi - 1)\beta(\gamma - 1)) \sigma_w}{1 + \beta(1 - \gamma)} \frac{\sigma_w}{\bar{w}}. \quad (4.31)$$

Distribution of renewable resource is less unequal than consumption  $CV[z^i] < CV[c^i]$  and wealth is more unequal than consumption,  $CV[c^i] < CV[y^i]$ . This shows that, in order to bequeath more resource to their descendants, households sacrifice their consumption. We explore next that how inequality varies with the consumption envy, regeneration rate and degree of altruism.

**Proposition 14.** *Coefficient of variation of wealth, namely inequality of wealth increases with the consumption envy,  $\gamma$ . Consumption inequality and resource inequality also increase with  $\gamma$ .*

*Proof.* Taking the derivatives of coefficient of variations with respect to  $\gamma$  gives

positive results.

$$\begin{aligned}\frac{\partial CV[y^i]}{\partial \gamma} &= \frac{\pi \beta \sqrt{\frac{(1+\beta)^2}{(1+\beta)^2 - \beta^2 \pi^2}} \sigma_w}{(1 + \beta - \beta \gamma)^2 \bar{w}} > 0, \\ \frac{\partial CV[c^i]}{\partial \gamma} &= (\pi - 1) \beta \sqrt{\frac{1}{(1 + \beta)^2 - \beta^2 \pi^2}} \frac{\sigma_w}{\bar{w}} > 0, \\ \frac{\partial CV[z^i]}{\partial \gamma} &= \frac{\sqrt{\frac{1}{(1+\beta)^2 - \beta^2 \pi^2}} \sigma_w}{(1 - \gamma)^2 \bar{w}} > 0.\end{aligned}$$

□

Proposition 14 shows that an increase in the value of consumption envy increases inequality of all variables including wealth. As the poorer households devote more resources to increase consumption, they bequeath lesser resource to their descendants which exacerbates inequality.

**Proposition 15.** *Inequality of wealth, consumption and level of the resource decreases with the resource regeneration rate,  $\pi$ , up to a threshold level of  $\tilde{\pi}$  which is  $\tilde{\pi} = \frac{(1+\beta)^2(1-\gamma)}{\beta[1+\beta(1-\gamma)]}$ . This threshold value decreases with the consumption envy  $\gamma$ .*

*Proof.* Taking the derivatives of coefficient of variations with respect to  $\pi$  gives following results:

$$\begin{aligned}\frac{\partial CV[y^i]}{\partial \pi} &= \frac{\beta (1 + \beta)^2 [1 + (\pi - 1)\beta^2(\gamma - 1) - \gamma - \beta(-2 + \pi + 2\gamma)]}{((\pi - 1)\beta - 1)^2 \sqrt{\frac{(1+\beta)^2}{(1+\beta)^2 - \beta^2 \pi^2}} (1 + \beta + \pi\beta)^2 (-1 + \beta(\gamma - 1))} \frac{\sigma_w}{\bar{w}}, \\ \frac{\partial CV[c^i]}{\partial \pi} &= \frac{[1 + (\pi - 1)\beta^2(\gamma - 1) - \gamma - \beta(-2 + \pi + 2\gamma)] \sigma_w}{\sqrt{\frac{1}{(1+\beta)^2 - \beta^2 \pi^2}} (1 + \beta(2 + \beta - \pi^2\beta))^2 (\gamma - 1)} \frac{\sigma_w}{\bar{w}}, \\ \frac{\partial CV[z^i]}{\partial \pi} &= \beta [-1 + \gamma + \beta ((2 + \beta) + \pi (1 + \beta - \beta\gamma))] \left( \frac{1}{(1 + \beta)^2 - \beta^2 \pi^2} \right)^{3/2} \frac{\sigma_w}{\bar{w}}.\end{aligned}$$

are all negative until  $\pi < \tilde{\pi}$ .

Taking derivative of  $\tilde{\pi}$  with respect to  $\gamma$  gives

$$\frac{\partial \tilde{\pi}}{\partial \gamma} = -\frac{(1 + \beta)^2}{\beta(1 + \beta - \beta\gamma)^2}$$

which is clearly negative. □

This proposition explores the impact of the regeneration rate of resource on the inequality. If the regeneration rate is less than a threshold value  $\bar{\pi}$ , inequality decreases with this rate. However, if it is larger than this threshold, inequality acts reversely and increases up to the most possible maximum level of  $\pi$  which is given in equation (4.23).

**Proposition 16.** *Inequality of wealth, consumption and level of the resource decreases with the degree of altruism,  $\beta$ , up to a threshold level of  $\bar{\beta}$  which is  $\bar{\beta} = \frac{\pi(1-\gamma)+\gamma}{\pi^2-\pi(1-\gamma)+\gamma}$ . This threshold value decreases with the consumption envy  $\gamma$ .*

*Proof.* The impact of  $\beta$  on resource and wealth inequality is proportional to its impact on consumption inequality. Hence, we prove here that an increase in  $\beta$  decreases inequality in consumption up to a threshold level. The results for other variables follow then. Taking the derivatives of  $CV[c^i]$  with respect to  $\beta$ :

$$\frac{\partial CV[c^i]}{\partial \beta} = \frac{\beta\pi^2 + \pi(1 + \beta)(\gamma - 1) - (1 + \beta)\gamma\frac{\sigma_w}{\bar{w}}}{((1 + \beta)^2 - \beta^2\pi^2)^{3/2}}$$

which is negative up to  $\bar{\beta} = \frac{\pi(1-\gamma)+\gamma}{\pi^2-\pi(1-\gamma)+\gamma}$ .

Taking derivative of  $\bar{\beta}$  with respect to  $\gamma$  gives negative result. □

Using warm glow altruism and in the absence of envy, Bossmann et al. (2007) state that bequests reduce wealth inequality. If the degree of altruism,  $\beta$ , is not strong enough, our results are consistent with their findings. However, if it is strong enough, contrary to their results, strengthening this degree increases wealth and consumption inequality.

## 4.4 Conclusion

This essay has proposed a non-overlapping generations economy in which households privately own a renewable resource stock. Agents are altruistic in the sense that they get utility from the transfer of the unextracted part of their resources to their heirs. They derive utility both from consumption and the bequest relative to the average values of the society. We have analyzed the implications of consumption and bequest envy on the dynamics of the resource and inequality.

When the production technology is in concave form and both type of positional concerns exist, the steady-state level of resource stock exhibits three different consequences such as unique steady-state, grows with balanced growth rate or declines exponentially. When it is linear and only consumption related positional concern exists, we obtain a unique steady state level for the average resource stock available in the economy that is decreasing with the consumption envy. Considering consumption envy in the model leads resource-poor households to sell higher fraction of their stock than resource-rich households. Lastly, we obtain that resource and wealth inequality increases with consumption related positional concern and decreases with the resource regeneration rate up to some threshold

level. Our findings show the relationship between natural resource dynamics, inequality and preference interdependence.

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## APPENDICES

### A Dynamics of Physical Capital

We have  $\bar{y}_t = \bar{b}_t + w$ . Note that  $\bar{b}_t = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \bar{y}_{t-1}$ . Then,  $\bar{y}_t = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \bar{y}_{t-1} + w$ . As  $\bar{y}_{t-1} = \bar{b}_{t-1} + w$ , we get

$$\bar{y}_t = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \bar{b}_{t-1} + \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R w + w.$$

Again, replacing for  $\bar{b}_{t-1}$ , we obtain that

$$\bar{y}_t = \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \bar{y}_{t-2} + \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R w + w.$$

Hence

$$\bar{y}_t = \left( \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \right)^2 \bar{y}_{t-2} + \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R w + w.$$

Iterating in the same way, we obtain

$$\begin{aligned} \bar{y}_t &= \left( \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \right)^2 (\bar{b}_{t-2} + w) + \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R w + w \\ \bar{y}_t &= \left( \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \right)^2 \bar{b}_{t-2} + \left( \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \right)^2 w + \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R w + w, \\ \bar{y}_t &= \left( \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \right)^3 \bar{y}_{t-3} + \left( \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \right)^2 w + \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R w + w, \end{aligned}$$

so that

$$\bar{y}_t = \left[ \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} \right]^t R^t \bar{y}_0 + w \sum_{j=0}^{t-1} \left[ \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} \right]^j R^j.$$

The sum of the geometric series on the right-hand side of this equation can be recast as

$$\sum_{j=0}^{t-1} \left[ \frac{\theta}{(1 + \beta)(1 - \gamma) + \theta} \right]^j R^j = \begin{cases} \frac{1 - \left[ \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} \right]^t R^t}{1 - \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R}, & \text{if } \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \neq 1 \\ t, & \text{otherwise.} \end{cases}$$

Substituting this in the equation:  $k_{t+1} = \bar{s}_t = \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} \bar{y}_t$ , leads to the evolution of the capital stock.

Note that if  $\frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R < 1$ , then the stock of physical capital converges to a steady state. Indeed, we have

$$k^* = \lim_{t \rightarrow \infty} \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} \left[ \left( \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \right)^t \bar{y}_0 + w \sum_{j=0}^{t-1} \left( \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R \right)^j \right],$$

so that

$$k^* = \frac{\theta + \beta(1 - \gamma)}{\theta + (1 + \beta)(1 - \gamma)} \frac{1}{1 - \frac{\theta}{\theta + (1 + \beta)(1 - \gamma)} R} w = \frac{\theta + \beta(1 - \gamma)}{(1 + \beta)(1 - \gamma) + \theta(1 - R)} w.$$

## B Evolution of Inequality in Time

Taking time derivative of  $CV(b_t^i)$ , we obtain that

$$\begin{aligned} \frac{\partial CV(b_t^i)}{\partial t} &= \frac{c_2 \sigma}{(1 - c_1^2)^{1/2}} \frac{1}{2} (1 - c_1^{2t})^{-1/2} (-2c_1^{2t} \ln c_1) \frac{1}{\frac{c_2 + c_3}{1 - c_1} + (b_0^e - c_2 - c_3)c_1^t} \\ &+ \frac{c_2 \sigma}{(1 - c_1^2)^{1/2}} (1 - c_1^{2t})^{1/2} \frac{-(b_0^e - c_2 - c_3)c_1^t \ln c_1}{\frac{c_2 + c_3}{1 - c_1} + (b_0^e - c_2 - c_3)c_1^t}. \end{aligned}$$

## C Proof of Proposition 2

Taking derivatives of  $\mathbb{E}(b_t^i)$ ,  $\mathbb{E}(c_t)$ ,  $\mathbb{E}(d_t^i)$  and  $\mathbb{E}(s_t)$  given in equations (2.22)-(2.25) with respect to  $\gamma$  will give following results:

$$\begin{aligned}\frac{\partial \bar{b}}{\partial \gamma} &= \frac{(1 + \beta)\theta R}{[\theta(R - 1) + (1 + \beta)(\gamma - 1)]^2} w > 0, \\ \frac{\partial \bar{c}}{\partial \gamma} &= \frac{\theta(R - 1)}{[\theta(R - 1) + (1 + \beta)(\gamma - 1)]^2} w > 0, \\ \frac{\partial \bar{d}}{\partial \gamma} &= \frac{\theta(R - 1)\beta R}{[\theta(R - 1) + (1 + \beta)(\gamma - 1)]^2} w > 0, \\ \frac{\partial \bar{s}}{\partial \gamma} &= \frac{\theta(R\beta + 1)}{[\theta(R - 1) + (1 + \beta)(\gamma - 1)]^2} w > 0.\end{aligned}$$

which are all positive.

Let us compute the derivative of the variables' variance with respect to  $\sigma_t^2$ :

$$\begin{aligned}\frac{\partial \mathbb{V}(c^i)}{\partial \sigma_t^2} &= \frac{1}{(1 + \beta + \theta)^2 - \theta^2 R^2} > 0, \\ \frac{\partial \mathbb{V}(b^i)}{\partial \sigma_t^2} &= \frac{\theta^2 R^2}{[1 + \beta + \theta]^2 - \theta^2 R^2} > 0, \\ \frac{\partial \mathbb{V}(s^i)}{\partial \sigma_t^2} &= \frac{(\beta + \theta)^2}{(1 + \beta + \theta)^2 - \theta^2 R^2} > 0, \\ \frac{\partial \mathbb{V}(d^i)}{\partial \sigma_t^2} &= \frac{\beta^2 R^2}{(1 + \beta + \theta)^2 - \theta^2 R^2} > 0.\end{aligned}$$

## D Proof of Proposition 3

Taking derivatives of CV's that we found in the previous proposition we prove that inequality decrease with  $\gamma$ :

$$\begin{aligned}
\frac{\partial CV(b^i)}{\partial \gamma} &= -\frac{(1+\beta)}{[(1+\beta+\theta)^2 - (\theta R)^2]^{1/2}} \frac{\sigma_w}{w} < 0, \\
\frac{\partial CV(c^i)}{\partial \gamma} &= -\frac{\theta(R-1)}{(1-\gamma)^2 [(1+\beta+\theta)^2 - \theta^2 R^2]^{1/2}} \frac{\sigma_w}{w} < 0, \\
\frac{\partial CV(s^i)}{\partial \gamma} &= -\frac{(R\beta+1)\theta(\beta+\theta)}{[(1+\beta+\theta)^2 - \theta^2 R^2]^{1/2} (\beta+\theta-\beta\gamma)^2} \frac{\sigma_w}{w} < 0, \\
\frac{\partial CV(y^i)}{\partial \gamma} &= -\frac{R\theta(\beta+1)(1+\beta+\theta)}{[(1+\beta+\theta)^2 - \theta^2 R^2]^{1/2} [(1+\beta)(1-\gamma)+\theta]^2} \frac{\sigma_w}{w} < 0.
\end{aligned}$$

The impact of  $\theta$  on bequests and wealth inequality is proportional to its impact on consumption inequality. Hence, we prove here that an increase in  $\theta$  decreases inequality in consumption up to a threshold level. The results for the other variables follow then trivially. By definition, the coefficient of variation of consumption is:

$$CV(c^i) = \frac{(1+\beta)(1-\gamma) - \theta(R-1)}{[(1+\beta+\theta)^2 - \theta^2 R^2]^{1/2} (1-\gamma)} \frac{\sigma_w}{w}.$$

We have then

$$\frac{\partial CV(c^i)}{\partial \theta} = \frac{(1+\beta)[(1-\gamma)\theta R^2 + (1+\beta+\theta)(\gamma-R)]}{[(1+\beta+\theta)^2 - \theta^2 R^2]^{3/2} (1-\gamma)} \frac{\sigma_w}{w}.$$

This derivative is negative if and only if  $(1-\gamma)\theta R^2 + (1+\beta+\theta)(\gamma-R) < 0$  which implies

$$\theta < \frac{(R-\gamma)(1+\beta)}{(R-1)(R-\gamma(R+1))} = \bar{\theta}.$$

Furthermore,

$$\frac{\partial \bar{\theta}}{\partial \gamma} = \frac{R^2(1 + \beta)}{(R - 1)(R(-1 + \gamma) + \gamma)^2} > 0.$$

## E The Steady State of the Segregated Economy

Associated to equations (3.20) and (3.22), there exists a steady state solution, along which  $\bar{b}_{A,t+1} = \bar{b}_{A,t} = \bar{b}_A$ ,  $\bar{b}_{B,t+1} = \bar{b}_{B,t} = \bar{b}_B$  and  $k_{t+1} = k_t = k^*$ .

$$(I_{2 \times 2} - \mathcal{B}) \begin{pmatrix} \bar{b}_A \\ \bar{b}_B \end{pmatrix} = \mathcal{B} \begin{pmatrix} w(\bar{k}) \\ w(\bar{k}) \end{pmatrix}.$$

Since  $I_{2 \times 2} - \mathcal{B}$  is invertible, bequests can be written as a function of the steady state of capital:

$$\begin{pmatrix} \bar{b}_A \\ \bar{b}_B \end{pmatrix} = (I_{2 \times 2} - \mathcal{B})^{-1} \mathcal{B} \begin{pmatrix} w(\bar{k}) \\ w(\bar{k}) \end{pmatrix}$$

Denoting by  $C(\bar{k})$  the  $2 \times 1$  matrix

$$C(\bar{k}) = (I_{2 \times 2} - \mathcal{B})^{-1} \mathcal{B} \begin{pmatrix} w(\bar{k}) \\ w(\bar{k}) \end{pmatrix} = \begin{pmatrix} C_1(\bar{k}) \\ C_2(\bar{k}) \end{pmatrix},$$

we can write the steady state value of capital as the solution to:

$$\bar{k} = [\mu s_{1,1} + (1 - \mu) s_{2,1}] C_1(\bar{k}) + [\mu s_{1,2} + (1 - \mu) s_{2,2}] C_2(\bar{k}) + w(\bar{k}) [\mu (s_{1,1} + s_{1,2}) + (1 - \mu) (s_{2,1} + s_{2,2})].$$