

On the Importance of Sequencing of Markets in Monetary Economies

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Abstract. This paper studies money as working capital in a general equilibrium model. We argue that the way transactions are settled is the main determinant of the presence or lack of working capital in a cash-in-advance economy. In a production cycle, if the wage payments are made before sales proceeds are collected, firms have a financing need. This need alone brings, in a long run equilibrium, a deviation of real wages from marginal product of labor due to a 'working capital premium' in output prices. In contrast, if sales revenues can be collected before production costs are paid, then the working capital premium vanishes. These results are obtained in an economy with borrowing constraints, full equity financing, and optimal dividend policy.

Keywords. Working capital premium, fiat money, cash-in-advance, limited participation, equity financing, dividend policy.

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1 Introduction

This paper draws attention to the striking difference in competitive equilibrium that arises from simply changing the order of good and factor market payments in a

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cash-in-advance model. In particular we observe that a premium on prices over production costs is obtained if wage payments come first in a production cycle. We call this extra amount the working capital premium and show that it prevails in an all-equity financed economy. We also show that the firms tend to hold currency as working capital if and only if the factor payments are required to be made before the sales revenue is collected.

The observation that cash constraints in the labor market may drive the equilibrium real wage below the marginal product of labor has been made in an all debt financed economy. Fuerst (1992), considering a representative family model with cash-in-advance constraints in all markets, points out that the equilibrium real wage is inversely related to the nominal interest rate. A similar observation in a slightly different context is made by Carlstrom and Fuerst (1995). The business cycles implications of this class of models are explored by Christiano, Eichenbaum and Evans (1997, 1998) and Fuerst (1992).

In these papers, firms can freely borrow from a frictionless short term loan market to finance their current production. By the end of the period, they are obliged to pay back not only the principal and the interest on the loan to the lenders but also profits, if any, to the owners so that no money is left in the vaults of the firms before the next period. Under these assumptions, the labor market equilibrium condition (Fuerst (1992)) states that real wage equals marginal product of labor divided by nominal interest rate. Taken together with the money market clearing condition, one can observe that nominal wages are unaltered, however goods prices carry a working capital premium in the long run equilibrium of such an economy, even under zero money growth.

Nevertheless, borrowing constraints do alter the aforementioned equilibrium conditions. Fuerst (1994) studied the implications of the presence of credit rationing schemes on the same type of cash-in-advance model. The presence of borrowing constraints has been justified by private information and costly monitoring (Stiglitz and Weiss (1981)).

We investigate here the omitted implications of equity financing in cash-in-advance economies. Our basic argument is that, even under severe borrowing limits, firms may choose to finance their working capital needs by owner's equity. Such an approach brings in the natural issue of corporate governance especially regarding the dividend decision, a hot issue in the corporate finance literature (See Miller and Modigliani (1961) for the irrelevance result and the follow-up literature collected in Jensen and Smith (1986)).

The main observation of the present paper is that regarding the size and nature of *working capital premium* on good prices, what matters is the sequencing of settlement of payments rather than the presence or lack of credit market imperfections appearing in the form of borrowing constraints. In addition we contribute to the theory of the demand for money by the firms and to the literature of optimum dividend decision in a macro general equilibrium environment.

The remaining part of the paper is organized as follows. Section 2 introduces the model and characterizes the set of stationary monetary competitive equilibria.

Section 3 gathers concluding remarks. The Appendix provides an extension of the classical Euler equation approach to dynamic optimization problems so as to allow for kinked objective functions and corner solutions along optimal paths.

2 The Model

The analysis is carried out in a production economy with two types of infinitely-lived agents. The cash-in-advance constraints are imposed on all markets, allowing the transaction of only one commodity with money at a time. Moreover, the market for short term loans is shut down in order to see implications of borrowing constraints.

The economy involves two commodities at each time: a factor of production, labor, and a nonstorable consumption good, apple. Time is indexed by $t = 1, 2, \dots$ and period t denotes the time interval between t and $t + 1$. There are two types of agents indexed by $i = 1, 2$. Both types are infinitely-lived and there exist finite numbers N_1 and N_2 of the first and second types, respectively. Neither of the types values leisure, and the preferences of both types over the lifetime consumption are in the same additively separable form given by $\sum_{t=0}^{\infty} \beta_i^t U_i(c_{it})$, where $\beta_i \in (0, 1)$ is the discount factor, c_{it} is the period- t apple consumption and $U_i(\cdot)$ is the instantaneous utility function of a representative agent of type i . We assume that U_i is twice continuously differentiable, $U_i'(\cdot) > 0$ and $U_i''(\cdot) < 0$.

Each type i agent has a labor endowment \bar{L}_i . We assume $\bar{L}_1 > 0$ and $\bar{L}_2 \geq 0$. Moreover, each type i agent has access to a constant returns technology $f_i(L) = \gamma_i L$ to convert labor into apples. Here, γ_i denotes the constant marginal product of labor in the plant of a type i agent. Type two agents are assumed to own a superior know-how, so we let $\gamma_2 = \gamma > 1$ and $\gamma_1 = 1$. Then, we will identify the type one and type two agents by the “low-tech” and the “high-tech” labels, respectively. Other than these production possibilities, there are no endowments of apples.

We denote and describe a *society* by $\mathcal{S} = \langle N_i, \bar{L}_i, U_i, \beta_i, f_i \mid i = 1, 2 \rangle$, provided that all the parameters listed obey the stated assumptions above. A *trade institution* for a given society is the description of choice variables for each type of agents, price variables, constraints on the given choice variables determined by given prices, and a feasibility requirement for the collective choices of agents.

We will consider two different trade institutions for the same society described above. The two trade institutions will both incorporate the same choice variables, price variables, and market clearing conditions. They will differ, however, in the constraints determined by prices. This difference will stem from the different sequencing of good and factor markets. The common choice variables and prices are listed below. The notational conventions that we use are also indicated.

Choice variables of a type i agent in period t :

- c_{it} : consumption,
- L_{it} : labor demand ((+) demand, (-) supply),
- q_{it} : apple demand ((+) demand, (-) supply),
- $M_{i,t+1}$: money carried over to period $t + 1$.

Prices in period t :

w_t : nominal wage rate,
 p_t : nominal apple price.

For simplicity in exposition, and without loss of generality, we will assume that trade institutions distribute to each agent of the same type the same amount of money balance at time zero, $M_{i,0}$, but the money held across types may differ (and in fact for the existence of a steady state equilibrium, we will show that they ought to differ). Let M denote the total quantity of money in the economy. We assume that there is no government intervention to the economy, so that total money stock does not change over time. For obvious reasons, we require that M is strictly positive, and $N_1 M_{1,0} + N_2 M_{2,0} = M$.

2.1 Labor Market First

The timing of transactions is as follows: Each type i agent starts a period t with a money balance of $M_{i,t}$. First the labor market opens where labor can be bought and sold at the nominal wage rate w_t . All wage bills must be paid before the good market opens. Then apple production takes place with the purchased and unsold labor. After the harvest of apples, good market opens and apples can be bought and sold at the nominal price p_t . These transactions determine the next period's money balance of each agent.

Given the endowment structure described above, and a sequence of strictly positive prices $\{w_t, p_t\}_{t=0}^{\infty}$, a representative agent of type i faces the following problem:

$$(PL)_i \quad \max \sum_{t=0}^{\infty} \beta^t U_i(c_{it})$$

subject to, for all t

$$c_{it} = f_i(\bar{L}_i + L_{it}) + q_{it},$$

$$-\bar{L}_i \leq L_{it} \leq \frac{M_{i,t}}{w_t},$$

$$-f_i(\bar{L}_i + L_{it}) \leq q_{it} \leq \frac{M_{i,t} - w_t L_{it}}{p_t},$$

$$M_{i,t+1} = M_{i,t} - w_t L_{it} - p_t q_{it},$$

$$M_{i,0} \geq 0 \quad \text{is given.}$$

The upper bound on labor purchases, L_{it} , comes from the cash-in-advance requirement and the fact that labor market opens first. The lower bound (if multiplied by -1) shows the maximum amount of labor that can be sold. The constraints on apple purchases should be similarly read, taking into account that the apple market payments or receipts come after those of the labor market.

The constraints of $(PL)_i$ altogether describe the missing part of our trade institution. We would like to note that this institution treats every agent identically.¹ That is, the choice variables and constraints are the same for all agents regardless of their types. So in fact a “low tech” type, a priori has the opportunity to become an employer if he chooses to. However, this will turn out to be non-optimal under the equilibrium prices, and a low-tech type will choose to be a worker.

We call the trade institution that lets the labor market open first *financially constrained* by virtue of the fact that a producer is restricted in his labor purchases by the amount of money he holds at the beginning of each period. By a *financially constrained production economy* we mean a society \mathcal{S} operating under a financially constrained trade institution, and denote it by \mathcal{FCE} . We can now define our equilibrium concepts.

We say that $\{w_t, p_t, L_{it}, q_{it}, c_{it}, M_{i,t+1} \mid i = 1, 2\}_{t=0}^\infty$ is a *stationary monetary competitive equilibrium (SMCE)* of the financially constrained production economy \mathcal{FCE} , if $w_t, p_t > 0$ for all t , and

- (i) for all i , $\{L_{it}, q_{it}, c_{it}, M_{i,t+1}\}_{t=0}^\infty$ solves $(PL)_i$ under $\{w_t, p_t\}_{t=0}^\infty$,
- (ii) $N_1 L_{1t} + N_2 L_{2t} = 0$ for all t ,
- (iii) $N_1 q_{1t} + N_2 q_{2t} = 0$ for all t ,
- (iv) $N_1 M_{1,t} + N_2 M_{2,t} = M$ for all t ,
- (v) $\{w_{t+1}, p_{t+1}, L_{i,t+1}, q_{i,t+1}, c_{i,t+1}, M_{i,t+1}\} = \{w_t, p_t, L_{it}, q_{it}, c_{it}, M_{i,t}\}$ for all i and t .

A SMCE can also be called a steady state equilibrium. Since the institution has the equal treatment property and since all agents of the same type start with the same money balance, the above definition is stated in terms of the consumptions, labor demands, good demands and money demands per representative agent within each type. The first condition is lifetime utility maximization under perfect foresight of future prices and price taking behavior. The second, third, and fourth conditions state the labor, good, and money market clearing, respectively. The last condition is the stationarity of the optimal plan.

It should be noted that in the definition of SMCE, the initial money distribution over the two types is given as a part of the parameters of the trade institution. In propositions 1 and 2 below, it will be shown that the initial distribution of money, $(M_{1,0}, M_{2,0})$ matters regarding the existence of a stationary equilibrium.

The constant values of the choice and price variables in SMCE are denoted by the vector $\langle p, w, L_i, q_i, c_i, M_i \mid i = 1, 2 \rangle$. Now, to characterize SMCE, let us impose constant prices, and eliminate consumption, c_{it} , and quantity of good sold, q_{it} , using the equality constraints for an agent of type i . After the elimination, we can concentrate on the clearing of the labor and money markets only, since the third

¹ Equal treatment of agents within the same category is indeed one of the properties that an institution should satisfy in order to deserve that name according to Hurwicz (1994). A market economy is expected to provide the same *trade opportunities* to all of its participants, and hence treat them all in the same category.

one, the good market, will automatically clear as well, thanks to a version of Walras' law applicable to our case. The reduced form problem, $(PL)'_i$, of a type i agent can be expressed as,

$$(PL)'_i \quad \max \sum_{t=0}^{\infty} \beta_i^t U_i \left(f_i(\bar{L}_i + L_{it}) - \frac{w}{p} L_{it} + \frac{M_{i,t} - M_{i,t+1}}{p} \right)$$

subject to, for all t

$$-\bar{L}_i \leq L_{it} \leq \frac{M_{i,t}}{w},$$

$$0 \leq M_{i,t+1} \leq M_{i,t} - wL_{it} + pf_i(\bar{L}_i + L_{it}),$$

$$M_{i,0} \geq 0 \text{ is given.}$$

Lemma 1. *Given any path of money holdings $\{M_{i,t}\}_{t=0}^{\infty}$, period t labor demand of each type i agent satisfies*

$$L_{it}(w/p) = \begin{cases} -\bar{L}_i & \text{if } w/p > \gamma_i, \\ L_{it} \in [-\bar{L}_i, M_{i,t}/w] & \text{if } w/p = \gamma_i, \\ M_{i,t}/w & \text{if } w/p < \gamma_i. \end{cases} \quad (1)$$

Proof. In each possible range of real wage, one can argue that otherwise it would be possible for a type i agent to improve upon his lifetime utility by perturbing L_{it} only and keeping money holdings at all times and labor demands at all other times fixed. \square

From Lemma 1 it follows that $L_{it} = -\bar{L}_i$ if $w/p > \gamma$ and $L_{it} = M_{it}/w$ if $w/p < 1$. This observation brings us to the following.

Lemma 2. *SMCE of a FCE exists only if $w/p \in [1, \gamma]$.*

Proof. Trivial by contradiction, once one recalls that $\bar{L}_i > 0$ for each i and $M_{1,0} + M_{2,0} = M > 0$. \square

Lemma 3. *SMCE of a FCE exists only if $M_{1,0} = 0$ and $M_{2,0} = M/N_2$.*

Proof. Suppose there exists SMCE with $M_1 = M_{1,0} > 0$. The first step is to show that for a "low-tech" agent, the constraint $L_1 \leq M_1/w$ can never bind in SMCE. This is clear for $w/p > 1$. If $w/p = 1$, then $L_1 = M_1/w$, which violates market clearing for $M_1 > 0$ since we also have $L_2 = M_2/w$. The second step is the substitution of a constant value of $L_1 < M_1/w$ for $L_{1,t}$ in $(PL)'_i$. Then it is a standard exercise to show that $M_1 > 0$ is not consistent with optimality of the stationary plan.² Finally, $M_2 = M/N_2$ follows from money market clearing. \square

² Stokey and Lucas's (1989) exercise 5.17 studies such a problem and guides the reader to show that the optimal policy is to bring the money stock to down to zero in *finite* time.

The following proposition characterizes the set of SMCE over the parameter space of (β_2, γ) .

Proposition 1. *SMCE of a FCE exists if and only if $(M_{1,0}, M_{2,0}) = (0, M/N_2)$ and $\beta_2\gamma \geq 1$. Moreover the set of SMCE is characterized by (2)-(11):*

$$w = \begin{cases} M/(N_1\bar{L}_1) & \text{if } \beta_2\gamma > 1 \\ \bar{w} \in [M/(N_1\bar{L}_1), \infty) & \text{if } \beta_2\gamma = 1 \end{cases} \quad (2)$$

$$p = \frac{w}{\beta_2\gamma} \quad (3)$$

$$L_1 = -\frac{M}{N_1w} \quad (4)$$

$$L_2 = \frac{M}{N_2w} \quad (5)$$

$$q_1 = -\frac{w}{p}L_1 \quad (6)$$

$$q_2 = -\frac{w}{p}L_2 \quad (7)$$

$$c_1 = \bar{L}_1 + (1 - \beta_2\gamma)L_1 \quad (8)$$

$$c_2 = \gamma\bar{L}_2 + (1 - \beta_2)\gamma L_2 \quad (9)$$

$$M_1 = 0 \quad (10)$$

$$M_2 = M/N_2 \quad (11)$$

Remark 1. In case a SMCE exists, the real wage is strictly below the marginal product of labor in the “high-tech” production plant, and is given by $w/p = \beta_2\gamma < \gamma$. We also observe that when the firm owners are less patient, the equilibrium real wage turns out to be lower.

Remark 2. SMCE does not exist if $\beta_2\gamma < 1$. If $\beta_2\gamma > 1$, then SMCE exists and is unique. If $\beta_2\gamma = 1$, then there exists a continuum of SMCE. In that case, workers are indifferent over the set of SMCE, since the real wage is equal to workers’ reservation rate, “one”, when $\beta_2\gamma = 1$. But firms have a strict preference over the set of SMCE, since the lower the nominal wage, the higher their lifetime consumption.

Remark 3. Even if a type 2 agent has zero labor endowment, he can still consume a positive amount of apples, $c_2 = (1 - \beta_2)\gamma L_2$, forever in equilibrium. This clearly shows that the financial constraints arising from the requirement of making factor payments first do not allow competition to wipe out such pure profits in equilibrium.

After these remarks, we can proceed with the proof of the Proposition 1.

Proof. Let $\{w_t, p_t, L_{it}, q_{it}, c_{it}, M_{i,t+1} \mid i = 1, 2\}_{t=0}^\infty$ be a SMCE. Also let $\langle p, w, L_i, q_i, c_i, M_i \mid i = 1, 2 \rangle$ denote the constant values corresponding to the SMCE.

We have $w/p \in [1, \gamma]$, from Lemma 2. From Lemma 3 and the stationarity of money holdings it follows that $M_1 = 0$ and $M_2 = M/N_2$.

The first type's money holding plan is feasible since $L_1 = -\bar{L}_1 < 0$. The feasibility condition for the second type's money holding is also satisfied since

$$\frac{M}{N_2} \leq \frac{M}{N_2} - wL_2 + pf_2(L_2) \leq \frac{M}{N_2} - wL_2 + pf_2(\bar{L}_2 + L_2)$$

by the fact that $w/p \leq \gamma$.

The equality $w/p = \gamma$ contradicts the optimality of type 2's stationary plan, because, given $w/p = \gamma$, the plan $M_{2,t} = M/N_2$ implies $c_2 = \gamma\bar{L}_2$ forever, and hence yields minimal lifetime utility. But higher utility can be obtained by choosing, for example, $M_{2,t+1} = 0$ for all t , since this plan yields $c_{2,0} = \gamma\bar{L}_2 + \gamma N_1 \bar{L}_1/N_2$ and $c_{2,t} = \gamma\bar{L}_2$ for all $t \geq 1$. Therefore, SMCE exists only if $w/p < \gamma$. Under such prices, the labor demand of a type 2 agent for an arbitrary sequence of money holdings, $\{M_{2t}\}_{t=0}^\infty$, is $L_{2t}(M_{2t}) = M_{2t}/w$. Substituting for this necessary condition in $(PL)_2$, we obtain

$$(PL)_2'' \quad \max \sum_{t=0}^\infty \beta_2^t U_2 \left(f_2 \left(\bar{L}_2 + \frac{M_{2t}}{w} \right) - \frac{M_{2,t+1}}{p} \right)$$

subject to, for all t

$$0 \leq M_{2,t+1} \leq pf_2 \left(\bar{L}_2 + \frac{M_{2t}}{w} \right),$$

$$M_{2,0} = M/N_2 \quad \text{is given.}$$

Since constant money holdings constitute an interior optimal plan, the Euler equation

$$U_2'(c_{2,t}) = \frac{p}{w} \gamma \beta_2 U_2'(c_{2,t+1})$$

must hold for all t . Clearly for any stationary plan $c_{2,t+1} = c_{2,t}$ to be optimal for type 2, it must be true that $w/p = \beta_2 \gamma$. In that case, $w/p < \gamma$ is always satisfied.

There are three ranges of interest for parameters β_2 and γ :

If $\beta_2 \gamma > 1$, then type 1 agents supply all their labor endowments at all times, so that $L_{1t} = -\bar{L}_1$ for all t . Observing $L_{2t} = M/(N_2 w)$ for all t , and using the labor market-clearing condition, we obtain

$$w = \frac{M}{N_1 \bar{L}_1}.$$

If $\beta_2 \gamma = 1$, then $L_{1,t} \in [-\bar{L}_1, 0]$ and $L_{2t} = M_{2,t}/w$ for all t . So, labor market clears for a continuum of wages given by

$$w \in [M/\bar{L}_1 N_1, \infty).$$

If $\beta_2\gamma < 1$ then $w/p = \beta_2\gamma < 1$ is not consistent with labor market clearing. Therefore there exists no SMCE for this range of parameter values.

So far, we have checked some necessary conditions for optimality together with the market-clearing and stationarity conditions. To make sure that both agents optimize under the proposed prices and plans of action, we will make use of the sufficiency result, Theorem A.3.

Whenever $w/p \geq 1$, a type 1 agent faces the reduced form problem

$$(PL)''_1 \quad \max \sum_{t=0}^{\infty} \beta_1^t U_1 \left(\frac{w}{p} \bar{L}_1 + \frac{M_{1,t} - M_{1,t+1}}{p} \right)$$

subject to, for all t

$$0 \leq M_{1,t+1} \leq M_{1,t} + w\bar{L}_1,$$

$$M_{1,0} = 0 \text{ is given.}$$

For this problem, the modified Euler equation

$$U'_1(c_{1,t}) > \beta_1 U'_1(c_{1,t+1})$$

and the transversality condition

$$\lim_{t \rightarrow \infty} \beta_1^t \left(-\frac{1}{p}\right) U'_1(c_{1,t}) M_{1,t+1} = 0$$

are satisfied for the consumption plan $c_{1t} = (w/p)\bar{L}_1$ and the money holding plan $M_{1,t} = 0$. Therefore by Theorem A.3, this plan, which is a corner solution, is optimal.

Similarly, whenever $w/p = \beta_2\gamma$, a type 2 agent faces the reduced form problem

$$(PL)''_2 \quad \max \sum_{t=0}^{\infty} \beta_2^t U_2 \left(f_2 \left(\bar{L}_2 + \frac{M_{2,t}}{w} \right) - \frac{M_{2,t+1}}{p} \right)$$

subject to, for all t

$$0 \leq M_{2,t+1} \leq p f_2 \left(\bar{L}_2 + \frac{M_{2,t}}{w} \right),$$

$$M_{2,0} = M/N_2 \text{ is given.}$$

For this agent, the Euler equation

$$U'_2(c_{2,t}) = \frac{p}{w} \gamma \beta_2 U'_2(c_{2,t+1})$$

and the transversality condition

$$\lim_{t \rightarrow \infty} \beta_2^t \left(-\frac{1}{p}\right) U'_2(c_{2,t}) M_{2,t+1} = 0$$

are satisfied for the consumption plan $c_{2t} = \gamma\bar{L}_2 + \gamma(1 - \beta_2)M/(N_2w)$ and the money holding plan $M_{2,t} = M/N_2$ for all t . Again by Theorem A.3., the plan, which in this case is an interior solution for all t , is optimal. \square

2.2 Good Market First

Here we assume that it is possible to buy and sell commodity contracts for commodities to be produced in the current period. In each period t , the good market opens first. Here apple contracts can be transacted with money. Next opens the labor market which also operates with money. Then, apple production takes place. After production is complete, commodity contracts are fulfilled by the delivery of the promises. Whatever is left in hand after all these transactions, is consumed at the end of period t .

Given the endowment structure described above, and the strictly positive prices $\{w_t, p_t\}_{t=0}^{\infty}$, a representative agent of type i faces the following the problem:

$$(PG)_i \quad \max \sum_{t=0}^{\infty} \beta^t U_i(c_{it})$$

subject to, for all t

$$c_{it} = f_i(\bar{L}_i + L_{it}) + q_{it},$$

$$-f_i(\bar{L}_i + L_{it}) \leq q_{it} \leq \frac{M_{i,t}}{p_t},$$

$$-\bar{L}_i \leq L_{it} \leq \frac{M_{i,t} - p_t q_{it}}{w_t},$$

$$M_{i,t+1} = M_{i,t} - p_t q_{it} - w_t L_{it},$$

$$M_{i,0} \geq 0 \text{ is given.}$$

We say that $\{w_t, p_t, L_{it}, q_{it}, c_{it}, M_{i,t+1} \mid i = 1, 2\}_{t=0}^{\infty}$ is a *stationary monetary competitive equilibrium (SMCE)* of the financially non-constrained production economy $\mathcal{FNC}\mathcal{E}$, if $w_t, p_t > 0$ for all t , and

- (i) for all i , $\{L_{it}, q_{it}, c_{it}, M_{i,t+1}\}_{t=0}^{\infty}$ solves $(PG)_i$ under $\{w_t, p_t\}_{t=0}^{\infty}$,
- (ii) $N_1 L_{1t} + N_2 L_{2t} = 0$ for all t ,
- (iii) $N_1 q_{1t} + N_2 q_{2t} = 0$ for all t ,
- (iv) $N_1 M_{1,t} + N_2 M_{2,t} = M$ for all t ,
- (v) $\{w_{t+1}, p_{t+1}, L_{i,t+1}, q_{i,t+1}, c_{i,t+1}, M_{i,t+1}\} = \{w_t, p_t, L_{it}, q_{it}, c_{it}, M_{i,t}\}$ for all i and t .

Imposing constant prices, and eliminating c_{it} and q_{it} using the equality constraints in $(PG)_i$, we obtain the reduced form problem of a type i agent to be

$$(PG)'_i \quad \max \sum_{t=0}^{\infty} \beta^t U_i \left(f_i(\bar{L}_i + L_{it}) - \frac{w}{p} L_{it} + \frac{M_{i,t} - M_{i,t+1}}{p} \right)$$

subject to, for all t

$$\begin{aligned}
 &-\bar{L}_i \leq L_{it}, \\
 &-M_{i,t+1}/w \leq L_{it}, \\
 &0 \leq M_{i,t+1} \leq M_{i,t} - wL_{it} + pf_i(\bar{L}_i + L_{it}), \\
 &M_{i,0} \geq 0 \text{ is given.}
 \end{aligned}$$

Lemma 4. *Given any path of money holdings $\{M_{i,t}\}_{t=0}^\infty$, period t labor demand of each type i agent satisfies*

$$L_{it}(w/p) = \begin{cases} \max\{-\bar{L}_i, -M_{i,t+1}/w\} & \text{if } w/p > \gamma_i, \\ L_{it} \geq \max\{-\bar{L}_i, -M_{i,t+1}/w\} & \text{if } w/p = \gamma_i, \\ \infty & \text{if } w/p < \gamma_i. \end{cases} \quad (12)$$

Proof. Lemma must be true, for otherwise it would be possible for a type i agent to improve upon his lifetime utility by perturbing L_{it} only and keeping money holdings at all times and labor demands at all other times fixed. \square

From Lemma 4, the following result follows.

Lemma 5. *SMCE of a \mathcal{FNCE} exists only if $w/p = \gamma$.*

Proof. To show Lemma 5 must hold is straightforward after recalling that in equilibrium, $M_{1,t} + M_{2,t} = M > 0$ for all t and $\bar{L}_1 > 0$, and then studying the implications of Lemma 4 on labor market clearing at each time. The main observation to make is that, since there are no financial constraints on the labor demand of a type 2 agent, for any real wage with $w/p < \gamma$, there would be infinite labor demand, but supply is restricted contradicting market clearing. \square

Lemma 6. *SMCE of a \mathcal{FNCE} exists only if $M_{1,0} = M/N_1$ and $M_{2,0} = 0$.*

Proof. Suppose there exists SMCE with $M_2 = M_{2,0} > 0$. Since there are no real profits under the wage $w/p = \gamma$, constant money holding forever yields minimal consumption and hence minimal lifetime utility to a firm type, while higher utility could be obtained by choosing, for example, $M_{2t} = 0$ for all $t \geq 1$. This alternative plan yields maximal consumption in period zero, and minimal consumption afterwards, hence a higher lifetime utility, contradicting optimality of the first plan. So, SMCE exists only if $M_{2,0} = 0$. Then, whenever SMCE exists, $M_{1,0} = M/N_1$ follows from money market clearing. \square

Now, we are ready to write the main result of this subsection.

Proposition 2. *SMCE of a \mathcal{FNCE} exists if and only if $(M_{1,0}, M_{2,0}) = (M/N_1, 0)$ and $\beta_2\gamma \geq 1$. Moreover the set of SMCE is characterized by (13)-(22):*

$$w = \begin{cases} M/(N_1\bar{L}_1) & \text{if } \beta_1\gamma > 1 \\ \bar{w} \in [M/(N_1\bar{L}_1), \infty) & \text{if } \beta_1\gamma = 1 \end{cases} \tag{13}$$

$$p = w/\gamma \tag{14}$$

$$L_1 = -\frac{M}{N_1w} \tag{15}$$

$$L_2 = \frac{M}{N_2w} \tag{16}$$

$$q_1 = -\frac{w}{p}L_1 \tag{17}$$

$$q_2 = -\frac{w}{p}L_2 \tag{18}$$

$$c_1 = \bar{L}_1 + (1 - \gamma)L_1 \tag{19}$$

$$c_2 = \gamma\bar{L}_2 \tag{20}$$

$$M_1 = M/N_1 \tag{21}$$

$$M_2 = 0 \tag{22}$$

Remark 4. In the case SMCE exists, the real wage is equal to the marginal product of labor in the “high tech” production plant, and is given by $w/p = \gamma$. So competition wipes out profits. This should be contrasted with Remark 1 above related to the “labor market first” case.

Remark 5. If $\beta_1\gamma > 1$, then SMCE exists and is unique. If $\beta_1\gamma = 1$ then there exists a continuum of SMCE. In that case, firms are indifferent over the set of SMCE; but workers are better off, the lower are the wages. SMCE does not exist if $\beta_1\gamma < 1$.

Remark 6. If a type 2 agent has zero labor endowment, he can consume nothing in equilibrium. That is, the absence of financial constraints that arise from the possibility of selling good before making factor payments allows competition to wipe out pure producers’ profits in equilibrium. This should be contrasted with Remark 3 above related to the “labor market first” case.

Proof of Proposition 2. Let $\{p_t, w_t, L_{it}, q_{it}, c_{it}, M_{i,t+1} \mid i = 1, 2\}_{t=0}^\infty$ be a SMCE. Also let $\langle p, w, L_i, q_i, c_i, M_i \mid i = 1, 2 \rangle$ denote the constant values corresponding to the SMCE.

We have $w/p = \gamma$ from Lemma 5. From Lemma 6 and stationarity of money holdings, we have $M_{1,t} = M/N_1$ and $M_{2,t} = 0$, for all t . We will check the feasibility of these money holdings later.

The period t labor demand of a type 1 agent for an arbitrary sequence of money holdings $\{M_{1,t}\}_{t=0}^\infty$, is $L_{1t}(M_{1,t}) = -\min\{\bar{L}_1, M_{1,t+1}/w\}$. Substituting for L_{1t} in $(PG)_1'$, we obtain

$$(PG)_1'' \max \sum_{t=0}^\infty \beta^t U_1 \left(\bar{L}_1 + \left(\frac{w}{p} - 1 \right) \min\left\{ \bar{L}_1, \frac{M_{1,t+1}}{w} \right\} + \frac{M_{1,t} - M_{1,t+1}}{p} \right)$$

subject to, for all t

$$0 \leq M_{1,t+1} \leq w\bar{L}_1 + M_{1,t},$$

$$M_{1,0} = M/N_1 \text{ is given.}$$

In the above problem, the objective function is concave but not differentiable. For constant money holdings to constitute an interior optimal plan, the Euler equation must be satisfied in a modified form which takes into consideration the kinks in the instantaneous utility function as discussed in the Appendix. First we will show that in SMCE

$$\bar{L}_1 < \frac{M}{N_1 w}$$

is not possible, because in this case the Euler equation

$$U'_1(c_{1t}) = \beta_1 U'_1(c_{1,t+1})$$

is implied, but it is not consistent with stationarity. The inequality, $\bar{L}_1 > M/(N_1 w)$ holds in SMCE only if

$$\left(\frac{1}{p} - \frac{1}{w} - \frac{1}{p}\right)U'_1(c_{1t}) + \beta_1 \frac{1}{p}U'_1(c_{1,t+1}) = 0,$$

that is, if $\beta_1 w/p = 1$. Since $w/p = \gamma$, this condition can be rewritten as $\beta_1 \gamma = 1$. For parameter values satisfying the above condition, the nominal wage and the initial money holding of a type 1 agent determines his labor supply as $L_{1t} = -M/(N_1 w)$, from Lemma 4. Then, the labor demand of a type 2 agent follows from the market-clearing condition to be $L_{2t} = M/(N_2 w)$. It can be verified that given any wage rate $w > M/(N_1 \bar{L}_1)$, the sufficiency conditions (listed in Theorem A.3) for maximization regarding the problems of both types as well as the market-clearing conditions are satisfied. Therefore, there exists a continuum of SMCE if $\beta_1 \gamma = 1$.

The remaining case is the equality, $\bar{L}_1 = M/(N_1 w)$, which tells us that the maximum is placed at the *kink* in problem $(PG)_1''$. Under this stationary money holding plan, increasing $M_{1,t+1}$ above M/N_1 for some t will not improve welfare. On the other hand, decreasing $M_{1,t+1}$ below M/N_1 for an arbitrary t will not improve welfare if and only if $\beta_1 w/p \geq 1$, i.e., $\beta_1 \gamma \geq 1$.

Finally, to check that $M_1 = M/N_1$ and $M_2 = M_{2,0} = 0$ are feasible, note that the money holding of a type 2 agent is always feasible. The money holding of a type 1 agent is also feasible, since $w/p = \gamma$ and $L_1 = -M/(N_1 w) \geq -\bar{L}_1$ in the equilibrium, which imply

$$\frac{M}{N_1} \leq \frac{M}{N_1} - wL_1 + pf_1(\bar{L}_1 + L_1).$$

Similar to the proof of Proposition 1, the last step is to make sure that the proposed plan indeed maximizes the problem $(PG)_i$ for all i . This can be done by using Theorem A.3 of the Appendix again. But this time one should be aware of the fact that reduced form problem, $(PG)_1''$, of a type 1 agent exhibits a *kink* in the instantaneous utility function, and moreover, the optimum in a generic stationary equilibrium (i.e., when $\beta_1 \gamma > 1$) is placed exactly at the kink. \square

3 Concluding Remarks

The ‘working capital premium’ on commodity prices has previously been linked to the nominal rate of interest in a model where production costs are financed by short term loans (Fuerst (1992)). In models with borrowing limits and credit rationing, this connection is broken (Fuerst (1994)). In the present paper, we argue that the main reason for the presence or absence of such a ‘working capital premium’ is the sequence in which payments are settled in a production cycle. We show that the presence of borrowing constraints does not alter this conclusion since self-financing through owner’s equity is always an option available to firms. This option was ignored in Fuerst (1992, 1994).

Our model, for simplicity, goes to the extreme form of credit rationing and assumes that short term loans are simply not available. In such a case cash, in the form of equity capital, turns out to be held by the firm at the beginning of each production cycle. Then the firm is naturally assumed to maximize its owner’s lifetime utility, rather than the present discounted value of profits, by choosing an appropriate real dividend sequence. If real dividends in a period are chosen too high, by means of lowering current real sales, the working capital and hence production in the next period become too low and vice versa. Due to this trade off, the resulting steady state equilibrium prices carry a working capital premium which is positively linked to the subjective discount rate of the firm owner. The working capital premium also drives a wedge between marginal productivity of labor and the real wage.

In such an economy, however, if firms have the ability of selling their goods in advance, then the working capital premium vanishes. This is simply because in such a case the need for money as working capital disappears. The firms carry no cash balances from one production cycle to the next, since they are able to finance their labor costs by their sales proceeds collected in advance. In such a case, all cash is demanded and carried over by consumers as is the case in the more traditional cash-in-advance models of Lucas and Stokey (1983, 1987) and Svensson (1985).

The use of cash-in-advance constraints in macroeconomic models was first proposed by Clower (1967) and operationalized especially in papers by Lucas (1980, 1984, 1990) and Lucas and Stokey (1983, 1987). In these papers, cash-in-advance constraints are imposed on the consumers’ purchases of a subset of commodities or assets.³ However in these papers the firm, if introduced at all, is taken as an artificial entity which has a constant return to scale production function and does not face any finance constraints. In that case, the classical results of zero pure profits and marginal products being equal to factor returns follow, and (since there are no profits to distribute), the ownership issue and the dividend distribution problem, which are both quite important in an incomplete markets setup (Magill and Quinzii (1996)), can be safely ignored.

³ The analysis of asset market equilibrium under cash-in-advance constraints of various forms is an active research area. Examples from this literature that study alternative sequencing possibilities for goods and asset markets are Stockman (1980), Lucas (1984), Svensson (1985), Nicolini (1998). Altuğ and Labadie (1994, Ch.5) provide a survey.

In the present paper, we have established the striking difference in the stationary competitive equilibrium prices and allocations that results from a change in the sequencing of the good and labor market. In case the labor market opens first, money is useful in financing the wage bill, so that it is demanded by the “high-tech” type who plays the role of a firm in our scenario. The presence of a cash-in-advance requirement in the labor market limits the demand for labor, so that an equilibrium with the real wage being lower than the marginal productivity of labor can be sustained.

If the good market opens first, a firm can sell, in advance, the amount of good to be produced within the current period, thus there remains no financial constraint for the firm. Any amount of labor can be hired by simply selling more commodity contracts and using the proceeds in wage payments. In this case, however, worker-consumers need money at the beginning of every period to buy apple contracts.

Economies with a finite number of infinitely-lived agents in a *complete markets* setup are known to exhibit Pareto efficiency under quite weak assumptions. However, if there are cash constraints on transactions, so that fiat money is valued in equilibrium, one does not expect to observe efficiency of equilibrium allocations. Grandmont and Younes (1973) has an example for a monetary economy which exhibits inefficient equilibrium allocations. Woodford (1990) gives examples from the literature of cash-in-advance models for both efficient and inefficient allocations that may arise under careful monetary policy. For example, under the institutional setups studied by Sargent (1987, Chapters 5 and 6), which involve no credit goods, it turns out to be possible to restore efficiency via a *deflationary* monetary policy.

In contrast, both of the institutional setups that we studied exhibit unique Pareto efficient allocations for almost all parameter values supporting an equilibrium. This is despite the fact that money supply is *fixed* over time. The more interesting observation is that one of our institutions leads to a deviation from an Arrow-Debreu equilibrium. Although both are efficient, it is easy to see that the two allocations under the two different trade institutions are not Pareto ranked. A worker type, rather paradoxically, would prefer to live in an economy where the wages are paid *after* the good market is closed, simply because the equilibrium wage is *higher* in that case. In contrast, an entrepreneur type would prefer the sequencing to be the other way around.

Regarding neutrality of money, a recent paper by Christiano, Eichenbaum and Evans (1997) evaluates two alternative theoretical approaches in the light of some empirical stylized facts. They identify the two theoretical approaches as the *limited participation* and the *sticky price* models. The former approach imposes short term financial needs on the firm side as exemplified by the work of Fuerst (1992). The latter introduces *menu costs* of changing prices in monopolistically competitive models, as partially surveyed by Romer (1996, Ch.5).

Both of our trade institutions could be considered in the class of *limited participation* models since the firms must somehow obtain cash for their wage payments. In the “good market first” case, they achieve this by selling the good in advance. In the “labor market first” case, however, they need to hold their own currency as there are no borrowing opportunities.

The labor supply of the worker type has a very specific and simple form. The elasticity is zero for $w/p > 1$ and is infinity for $w/p = 1$. Therefore, for parameter values that lead to $w/p > 1$ in a steady state equilibrium, money is neutral in both of our setups. That is, an unexpected increase in money supply at the beginning of any period, by increasing every agents money holding in the same proportion, only yields a proportionate increase in nominal wages and prices.

However if the parameter values lead to $w/p = 1$ in equilibrium, then the labor supply is infinitely elastic and money may not be neutral. Since there is a continuum of equilibria in this case, the precise effects of an unexpected monetary expansion are indeterminate. To say something conclusive about non-neutrality, we need to specify the reaction of wages and prices to an increase in demand.

For instance, suppose that a “labor market first” economy is in a stationary monetary competitive equilibrium with (voluntary) underemployment. At the beginning of period t , a proportionate money injection to all “high-tech” agents takes place. If all agents believe that current and future wages will remain fixed at their old levels, at and after time t , more labor will be demanded and supplied, more production will take place and the “high tech” type will enjoy more consumption forever.

Similarly, suppose a “good market first” economy goes through the same experiment, with the same static (but also rational) expectations for nominal wages and prices. Then the result is that at time t and onwards, more apples will be demanded and supplied, more labor will be demanded and supplied, more production will take place, but in this case the “low-tech” type will be better-off from enjoying more consumption forever.

Although they look very exciting, to be valid in a market-clearing model, the results of the above two paragraphs necessitate an infinitely elastic labor supply. However, if one gives up the labor market-clearing condition, by introducing a sluggish wage adjustment dynamics instead, the rather unrealistic infinite (long-run) labor supply elasticity assumption could be dispensed with in restoring similar non-neutrality arguments to the ones made above. This is in line with the observation of Christiano, Eichenbaum and Evans (1997) on the inadequacy of limited participation models, with labor market clearing but also with a *reasonable* labor supply elasticity, in explaining some of the stylized facts related to the non-neutrality of money.

Appendix

Here we present a theorem for the sufficiency of a modified set of Euler inequalities and a transversality condition for a class of discrete time dynamic optimization problems where the contemporary payoff functions are concave but not necessarily differentiable. The result also allows for a specific type of corner solutions along optimal paths. We build on and extend a result in Stokey and Lucas (1989, Thm 4.17).

A.1 The Problem

Let $F : \mathfrak{R}_+ \times \mathfrak{R}_+ \rightarrow \mathfrak{R}$ be a concave function that is *decreasing* in its *second argument*. Notice that F is *not* required to be differentiable here. Also let $0 < \beta < 1$.

The problem is to

$$\text{maximize } \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1})$$

subject to

$$x_{t+1} \in [l, b(x_t)],$$

$$x_0 > l \text{ given,}$$

over all *admissible* sequences. By an admissible sequence, we mean a sequence, x , that starts with x_0 , makes the infinite sum converge, and obeys $x_{t+1} \in [l, b(x_t)]$ for each $t \geq 0$. We let $l \geq 0$ be a common lower bound on possible states in all periods and $b : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ be a function putting an upper bound on the future state and assume that $b(x) \geq l$ for all $x \in [0, \infty)$.

We will assume throughout that F is differentiable in its first argument but not necessarily so in its second argument. For any given $x \in \mathfrak{R}_+$, we know from the *subdifferentiability theorem* that $F(x, \cdot)$ is subdifferentiable, so that the following right and left derivatives exist.

$$F_2^+(x, y) = \lim_{\epsilon \rightarrow 0^+} \frac{F(x, y + \epsilon) - F(x, y)}{\epsilon}$$

$$F_2^-(x, y) = \lim_{\epsilon \rightarrow 0^-} \frac{F(x, y + \epsilon) - F(x, y)}{\epsilon}$$

Since F is concave, we have for any x and y in the domain of F ,

$$F_2^+(x, y) \leq F_2^-(x, y),$$

with equality holding only on differentiable points of F in its domain.

A.2 Modified Euler and Transversality Conditions

Modified Euler conditions (MEC):

$$\left. \begin{aligned} F_2^+(x_t, x_{t+1}) + \beta F_1(x_{t+1}, x_{t+2}) &\leq 0 && \text{if } x_{t+1} = l \\ F_2^+(x_t, x_{t+1}) + \beta F_1(x_{t+1}, x_{t+2}) &\leq 0 \\ F_2^-(x_t, x_{t+1}) + \beta F_1(x_{t+1}, x_{t+2}) &\geq 0 \end{aligned} \right\} \text{otherwise}$$

Transversality condition (TVC):

$$\lim_{T \rightarrow \infty} \beta^T F_2^+(x_T, x_{T+1})x_{T+1} = 0$$

Notice that TVC implies

$$\lim_{T \rightarrow \infty} \beta^T F_2^-(x_T, x_{T+1})x_{T+1} = 0,$$

since F is concave and decreasing in its second argument.

A.3 Sufficiency Result

Theorem A.3. *Let x be an admissible sequence satisfying the transversality condition and for each t the modified Euler conditions. Moreover, suppose $l \leq x_{t+1} < b(x_t)$ for all t . Then, x solves the maximization problem.*

Proof. Let x be an admissible sequence satisfying the transversality condition and for each t the modified Euler equations and $l \leq x_{t+1} < b(x_t)$. Also let y be any other admissible sequence. Then for an arbitrary t , we have

$$F(x_t, x_{t+1}) - F(y_t, y_{t+1}) \geq F_1(x_t, x_{t+1})(x_t - y_t) + F_2^*(x_t, x_{t+1}, y_{t+1})(x_{t+1} - y_{t+1}),$$

where F_2^* is a function artificially formed using the given y and x values through the rule

$$F_2^*(x_1, x_2, y) = \begin{cases} F_2^+(x_1, x_2) & \text{if } y \geq x_2, \\ F_2^-(x_1, x_2) & \text{if } y < x_2. \end{cases}$$

Now, letting D denote the difference between the lifetime utilities of x and y , we can follow steps analogous to those in Stokey and Lucas' (1989) proof of their Theorem 4.15.

$$\begin{aligned} D &= \lim_{T \rightarrow \infty} \sum_{t=0}^T \beta^t [F(x_t, x_{t+1}) - F(y_t, y_{t+1})] \\ &\geq \liminf_{T \rightarrow \infty} \sum_{t=0}^T \beta^t [F_1(x_t, x_{t+1})(x_t - y_t) + F_2^*(x_t, x_{t+1}, y_{t+1})(x_{t+1} - y_{t+1})] \\ &= \liminf_{T \rightarrow \infty} \left\{ \sum_{t=0}^{T-1} \beta^t [F_2^*(x_t, x_{t+1}, y_{t+1}) + \beta F_1(x_{t+1}, x_{t+2})](x_{t+1} - y_{t+1}) \right. \\ &\quad \left. + \beta^T F_2^*(x_T, x_{T+1}, y_{T+1})(x_{T+1} - y_{T+1}) \right\} \\ &\geq \liminf_{T \rightarrow \infty} \beta^T F_2^*(x_T, x_{T+1}, y_{T+1})(x_{T+1} - y_{T+1}) \\ &\geq \liminf_{T \rightarrow \infty} \beta^T F_2^*(x_T, x_{T+1}, y_{T+1})x_{T+1} \\ &\geq 0. \end{aligned}$$

The second line follows from the concavity of F as discussed above. Recognizing the possibility that the limit of right hand side series may not exist, we use \liminf . The third and fourth lines altogether are obtained by rearranging the second line after substituting for $x_0 = y_0$. The fifth line follows from modified Euler conditions, (MEC). The sixth line one follows from $F_2^* \leq 0$ and $y_t \geq 0$ for each t . The last line follows from the transversality condition (TVC). Therefore, since $D \geq 0$ for each admissible y, x must be maximal in the admissible set. \square

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