

# Locality of Quantum Electromagnetic Radiation

Alexander S. Shumovsky  
Physics Department, Bilkent University,  
Bilkent, Ankara, 06533, Turkey

We construct the local representation of the Weyl-Heisenberg algebra of multipole photons using the three-dimensional properties of polarization. It is shown that this representation is compatible with the operational approach to photon localization.

*Keywords:* quantum electrodynamics, electromagnetic radiation, localization

## 1. Introduction

In spite of the great success of quantum electrodynamics (QED), there remain a number of major unresolved problems (*e.g.*, see [1, 2, 15]). Leaving aside the detailed discussion of foundations of QED, we shall concentrate here on the problem of localization of photons, which has attracted a great deal of interest. The point is that the photon creation and annihilation operators are defined in QED as nonlocal objects. In other words, the photon number operator gives the total number of photons in the volume of quantization without specification of their space-time location [2, 15]. Moreover, it has been proven by Newton and Wigner [16] that no position operator can exist for the photon. There is a widespread belief that the maximum precise localization appears in the form of a wavefront [5]. At the same time, the specific fall-off of the photon energy density and photodetection rate can be interpreted as photon localization in space [6].

Perhaps, the most evident and best example of photon localization is provided by the photodetection process, when a photon is transmitted into an electronic signal in the sensor element of the detecting device [7]. This localization is usually described *operationally* (in terms of what can be measured by a macroscopic detector) by means of the so-called configuration number operator, which determines the number of photons in the cylindrical volume  $\sigma c \Delta t$ , where  $\sigma$  denotes the area of the sensor element,  $c$  is the light velocity, and  $\Delta t$  is detector exposure time [2, 7].

We now stress that, in the usual treatment of photon localization, the radiation field is considered to consist of the plane waves of photons [2, 15]. In reality, the quantum electromagnetic radiation emitted by the atomic and molecular transitions corresponds to *multipole photons* [8] represented by quantized spherical waves [9]. Although the classical plane and spherical waves are equivalent in the sense that

they both form complete orthogonal sets of solutions of the homogeneous Helmholtz wave equation [10], there is a strong qualitative difference between the two quantum representations. The plane waves of photons correspond to the running-wave solution in empty space with translational symmetry, which leads to states of photons with given linear momentum. In turn, the solution in terms of spherical waves assumes the existence of a singular point, corresponding to an atom (source or absorber of radiation) whose size is small with respect to the wavelength. In this case, the boundary conditions correspond to the rotational symmetry, and lead to states of photons with given angular momentum. Since the components of linear and angular momenta do not commute, the two representations of the quantum electromagnetic field correspond to physical quantities which cannot be measured at the same time.

The main objective of this paper is to show that the use of the multipole photon representation leads to an adequate description of localization in the atom-field interaction process. The paper is arranged as follows. In Section 2 we briefly discuss the difference between the spatial properties of plane and spherical waves of photons. In Section 3 we introduce the local representation of the multipole photon. Then, in Section 4, we discuss the problem of measurement and causality. A general conclusion and the implications of this work are presented in Section 5.

## 2. Plane and spherical waves of photons

An arbitrary free quantum electromagnetic field can be described by the operator vector potential whose positive-frequency part has the following form

$$\vec{A}^{(+)}(\vec{r}, t) = \sum_{\mu=-1}^1 (-1)^\mu \vec{\chi}_{-\mu} \sum_{k,\ell} V_{k\ell\mu}(\vec{r}) e^{-i\omega_k t} a_{k\ell}, \quad (1)$$

where the unit vectors

$$\vec{\chi}_{\pm} = \mp \frac{\vec{e}_x \pm i\vec{e}_y}{\sqrt{2}}, \quad \vec{\chi}_0 = \vec{e}_z, \quad (2)$$

form the so-called helicity or spin basis of the three-dimensional space [9, 11],  $V_{k\ell\mu}(\vec{r})$  is the mode function, and  $a_{k\ell}$  is the photon annihilation operator, which obey Weyl-Heisenberg commutation relations

$$[a_{k\ell}, a_{k'\ell'}^\dagger] = \delta_{kk'} \delta_{\ell\ell'}. \quad (3)$$

Here  $\ell$  is a cumulative index. By construction, the vector potential components  $A_{\mu=\pm 1}(\vec{r}, t)$  in (1) describe the circularly polarized transversal components of the field with positive and negative helicity respectively, while  $A_{\mu=0}(\vec{r}, t)$  gives the linearly polarized longitudinal component [11]. In the case of plane waves of photons

$$\vec{\chi}_0 = \frac{\vec{k}}{k},$$

and projection of spin of the photon on this axis is forbidden, so that there are only two transversal components of the field. In this case, index  $\ell \equiv \sigma = \pm$  describes the circular polarization of the field.

Unlike the plane waves of photons, the quantum multipole radiation has all three spatial components [9, 11], and index  $\ell = \{\lambda, j, m\}$  gives the parity  $\lambda = E, M$  (type of radiation, either electric or magnetic), angular momentum of photons  $j = 1, 2, \dots$ , and projection of the angular momentum on the quantization axis  $m = -j, \dots, j$ . It should be stressed that plane and multipole photons have different numbers of quantum degrees of freedom. In fact, a monochromatic radiation field has only two degrees of freedom, described by the polarization index  $\sigma = \pm$  in the case of plane photons. At the same time, a monochromatic multipole field of a given type  $\lambda$  at given  $j \geq 1$  is specified by  $(2j + 1) \geq 3$  degrees of freedom. Moreover, the polarization is not a quantum number and, thus, the global property of the multipole radiation changes from point to point [12].

The spatial properties of the field are described by the mode functions in (1). In the case of plane photons, the mode function has the simple form of plane waves (*e.g.*, see [2])

$$V_{k\sigma}(\vec{r}) = \gamma e^{i\vec{k}\cdot\vec{r}}, \quad \gamma = \sqrt{\frac{2\pi\hbar c}{kV}}, \quad (4)$$

where  $V$  is the volume of quantization. It is seen that this expression leads to the spatially homogeneous density of intensity of a monochromatic plane wave

$$\begin{aligned} I^{(plane)} &= \vec{E}^{(-)}(\vec{r}) \cdot \vec{E}^{(+)}(\vec{r}) \\ &= k^2 \vec{A}^{(-)}(\vec{r}) \cdot \vec{A}^{(+)}(\vec{r}) = (k\gamma)^2 \sum_{\sigma} a_{\sigma}^{\dagger} a_{\sigma}. \end{aligned} \quad (5)$$

In turn, the multipole radiation is specified by the mode functions [9, 11]

$$\begin{aligned} V_{Ejkj\mu} &= \gamma_{Ejkj} [\sqrt{j} f_{j+1}(kr) \langle 1, j+1, \mu, m-\mu | jm \rangle Y_{j+1, m-\mu}(\theta, \phi) \\ &\quad - \sqrt{j+1} f_{j-1}(kr) \langle 1, j-1, \mu, m-\mu | jm \rangle Y_{j-1, m-\mu}(\theta, \phi)], \\ V_{Mkj\mu} &= \gamma_{Mkj} f_j(kr) \langle 1, j, \mu, m-\mu | jm \rangle Y_{jm}(\theta, \phi) \end{aligned} \quad (6)$$

in the case of  $\lambda = E$  and  $\lambda = M$ , respectively. Here  $\langle \dots | jm \rangle$  denotes the Clebsch-Gordon coefficient of vector addition of spin and orbital parts of the angular momentum,  $Y_{\ell n}$  is the spherical harmonics, and

$$\gamma_{\lambda} = \begin{cases} \gamma / \sqrt{2j+1}, & \text{at } \lambda = E \\ \gamma & \text{at } \lambda = M. \end{cases}$$

The radial dependence in (6) is defined as follows [10]

$$f_{\ell}(kr) = \begin{cases} h_{\ell}^{(1)}(kr), & \text{outgoing spherical wave} \\ h_{\ell}^{(2)}(kr), & \text{incoming spherical wave} \\ j_{\ell}(kr), & \text{standing spherical wave,} \end{cases} \quad (7)$$

where  $h_\ell^{(1,2)}$  denotes the spherical Hankel function of the first and second kind respectively and  $j_\ell$  is the spherical Bessel function. It is clear that, unlike (5), the density of intensity of a monochromatic pure  $j$ -pole multipole radiation of a given type

$$I^{(multi)}(\vec{r}) = \sum_{\mu} \sum_{m, m' = -j}^j V_{\lambda k j m \mu}^*(\vec{r}) V_{\lambda k j m' \mu}(\vec{r}) a_{\lambda k j m}^+ a_{\lambda k j m'} \quad (8)$$

shows a certain position dependence with respect to the source location at the origin of the reference frame spanned by the helicity basis (2). This spatial inhomogeneity of the density of intensity of multipole radiation can be used to introduce the local representation of the Weyl-Heisenberg algebra of multipole photons [12].

### 3. Local photon operators

In contrast to (5), the density of intensity (8) is represented by a non-diagonal form in the photon operators which can be represented as follows

$$I^{(multi)}(\vec{r}) = \sum_{m, m'} \mathcal{V}_{mm'}(\vec{r}) a_m^+ a_{m'} \quad (9)$$

where  $\mathcal{V}(\vec{r})$  is the Hermitian  $(2j+1) \times (2j+1)$  matrix with the elements

$$\mathcal{V}_{mm'}(\vec{r}) = k^2 \sum_{\mu=-1}^1 V_{m\mu}^*(\vec{r}) V_{m'\mu}(\vec{r}). \quad (10)$$

To simplify the notations, hereafter we omit the indexes  $\lambda$ ,  $k$ , and  $j$ . It is seen that

$$\text{tr} \mathcal{V}(\vec{r}) \equiv \sum_m \mathcal{V}_{mm}(\vec{r}) = k^2 [\vec{A}^{(+)}(\vec{r}), \vec{A}^{(-)}(\vec{r})], \quad (11)$$

so that the trace of (10) describes the electric-field contribution into the energy density of the zero-point oscillations [14] of the multipole field. Then

$$P_{\mu\mu}(\vec{r}) = k^2 \sum_{m=-j}^j |V_{m\mu}(\vec{r})|^2 \quad (12)$$

gives the contribution of spatial components with different polarization  $\mu$  into the zero-point energy of the multipole field. Since the polarization is the three-dimensional property of the multipole radiation [13, 14], it seems to be reasonable to define the spatial properties of multipole photons by means of polarization.

Consider for definiteness the electric type pure  $j$ -pole monochromatic radiation. Then, the operator polarization matrix takes the form [13]

$$P_{\mu\mu'}(\vec{r}) = E_{\mu}^{(-)}(\vec{r}) E_{\mu'}^{(+)}(\vec{r}) = k^2 A_{\mu}^{(-)}(\vec{r}) A_{\mu'}^{(+)}(\vec{r}). \quad (13)$$

By definition, this is the  $(3 \times 3)$  Hermitian matrix with the operator elements written in the normal order. In addition, one can define the anti-normal operator polarization matrix. Then, the difference between the anti-normal and normal matrices defines the zero-point oscillations of polarization [15] with the elements

$$P_{\mu\mu'}^{(0)}(\vec{r}) = k^2[A_{\mu}^{(+)}(\vec{r}), A_{\mu'}^{(-)}(\vec{r})] = k^2 \sum_m V_{m\mu}(\vec{r})V_{m\mu'}^*(\vec{r}). \quad (14)$$

It is easily seen that the diagonal elements of (14) coincide with (12). It is intuitively clear that the spatial properties of the zero-point oscillations of polarization described by (14) should be determined by distance  $r$  from the source *independent of the spherical angles*. In other words, the vacuum noise should have a homogeneous angular distribution, which can change with the distance.

The  $(3 \times 3)$  Hermitian matrix (14) can be diagonalized by a proper transformation of the reference frame spanned by the helicity basis (2)

$$U(\vec{r})P^{(0)}(\vec{r})U^+(\vec{r}) = \mathcal{P}^{(0)}(\vec{r}), \quad U^+(\vec{r})U(\vec{r}) = \mathbf{1}. \quad (15)$$

As a result of this transformation,

$$\vec{\chi}_0 \rightarrow \vec{\chi}'_0 = \vec{r}/r.$$

It is then a straightforward matter to arrive at the conclusion that

$$\mathcal{P}^{(0)}(r) = \begin{pmatrix} P_T(r) & 0 & 0 \\ 0 & P_L(r) & 0 \\ 0 & 0 & P_T(r) \end{pmatrix}, \quad (16)$$

where

$$\begin{aligned} P_T(r) &= k^2|V_{\mu\mu}(\vec{r})|^2, & \text{at } \mu = \pm 1, \\ P_L(r) &= k^2|V_{\mu\mu}(\vec{r})|^2, & \text{at } \mu = 0. \end{aligned} \quad (17)$$

In other words, the diagonal elements in (16) describe the transversal and longitudinal (with respect to  $\vec{\chi}'_0$ ) vacuum noise of polarization as a function of distance from the source.

The use of the same unitary transformation (15) allows the operator polarization matrix (13) to be cast in the form

$$\mathcal{P}(\vec{r}) = U(\vec{r})P(\vec{r})U^+(\vec{r}), \quad (18)$$

where

$$\mathcal{P}_{\mu\mu'}^{(E,n)}(\vec{r}) = k^2 \mathcal{A}_{Ekj\mu}^+(\vec{r})\mathcal{A}_{Ekj\mu'}(\vec{r}), \quad (19)$$

and

$$\mathcal{A}_{Ekj\mu}(\vec{r}) = \sum_{\mu'=-1}^1 U_{\mu\mu'}^*(\vec{r}) \sum_{m=-j}^j V_{Ekjm\mu'}(\vec{r}) a_{Ekjm}. \quad (20)$$

In view of (3), the operators (20) obey the commutation relations

$$[\mathcal{A}_{\lambda kj\mu}(\vec{r}), \mathcal{A}_{\lambda' k' j' \mu'}^+(\vec{r})] = \delta_{\lambda\lambda'} \delta_{kk'} \delta_{jj'} \delta_{\mu\mu'} \times \begin{cases} P_T(r) & \text{at } \mu = \pm 1 \\ P_L(r) & \text{at } \mu = 0 \end{cases} \quad (21)$$

where  $P_L, P_T$  are the diagonal elements (17). Similar results can also be obtained in the case of the magnetic multipole radiation.

We now note that the only difference between (3) and (21) is the presence of position-dependent factors in the right-hand side of (21). It seems to be tempting to introduce the normalized local operators

$$b_{\lambda kj\mu}(\vec{r}) = \frac{\mathcal{A}_{\lambda kj\mu}(\vec{r})}{\sqrt{P_{\mu}(\vec{r})}}, \quad (22)$$

which obey the standard Weyl-Heisenberg commutation relations

$$[b_{\lambda kj\mu}(\vec{r}), b_{\lambda' k' j' \mu'}^+(\vec{r})] = \delta_{\lambda\lambda'} \delta_{kk'} \delta_{jj'} \delta_{\mu\mu'} \quad (23)$$

at any point  $\vec{r}$ . Hence, the transformation (15) can be interpreted as a local Bogolubov canonical transformation [17], conserving the Weyl-Heisenberg commutation relations. In fact, the equations (15) and (22) describe the transformation of global multipole photon operators  $a_{\lambda kjm}$  with given  $m = -j, \dots, j, j \geq 1$ , into the local photon operators  $b_{\lambda kj\mu}(\vec{r})$  with given polarization  $\mu$  at any point of the space.

## 4. Measurement and locality

In the operational approach to photon localization [7] (also see [2, 15]), the local absorption operator

$$\vec{a}(\vec{r}, t) = \gamma \sum_{k, \sigma} e^{i(\vec{k} \cdot \vec{r} - kct)} \vec{e}_{k\sigma} a_{k\sigma} \quad (24)$$

is defined in the case of plane waves of photons. Here summation is taken over a finite set of modes to which a detector responds. Then, the so-called *configuration space number operator* is defined by the relation

$$\begin{aligned} \mathcal{N}(\mathcal{V}, t) &= \int \vec{a}^+(\vec{r}, t) \cdot \vec{a}(\vec{r}, t) d^3r \\ &= \gamma^2 \sum_{k, \sigma} \sum_{k', \sigma'} \vec{e}_{k\sigma} \cdot \vec{e}_{k'\sigma'} e^{-i(\vec{k} - \vec{k}') \cdot \vec{r}} e^{i(k - k')ct} a_{k\sigma}^+ a_{k'\sigma'}, \end{aligned} \quad (25)$$

where the integral is taken over the volume of photon localization (cylinder with base corresponding to the sensitive area of the detector and height proportional to

the exposure time). It is clear that the operators (24) and (25) obey the following commutation relations

$$[\mathcal{N}(\mathcal{V}, t), \mathcal{N}(\mathcal{V}', t)] = 0 \tag{26}$$

and

$$[\vec{a}(\vec{r}, t), \mathcal{N}(\mathcal{V}, t)] \approx \begin{cases} -\vec{a}(\vec{r}, t), & \text{if } \vec{r} \in \mathcal{V} \\ 0, & \text{otherwise,} \end{cases} \tag{27}$$

where  $\mathcal{V}$  denotes the volume of localization (detection). Let us stress that (27) has an approximate sense.

There is a principal difference which makes difficult the direct use of the operational approach to the problem of localizing photons in the case of multipole radiation. The point is that the multipole photons are in the state with given angular momentum, and therefore they have no well defined direction of propagation. At the same time, these photons are localized initially inside the source.

Let us now note that the operators (22) describe the local properties of the multipole radiation, and that the density of intensity operator (9) can be represented by

$$I^{(multi)}(\vec{r}) = \sum_{\mu=-1}^1 b_{\mu}^{+}(\vec{r})b_{\mu}(\vec{r}). \tag{28}$$

Under the condition that the we have a strictly monochromatic field, the operator (28) can be considered as an analog of (25) at a given point  $\vec{r}$ , while (22) is similar to (24). The principal difference between the two local representations is that

$$[b_{\lambda k j \mu}(\vec{r}), b_{\lambda' k' j' \mu'}^{+}(\vec{r}')] = \delta_{\lambda \lambda'} \delta_{k k'} \delta_{j j'} f_{\mu \mu'}(\vec{r}, \vec{r}'), \tag{29}$$

where  $f_{\mu \mu'}(\vec{r}, \vec{r}')$  is, perhaps, a sharp function but  $f_{\mu \mu'}(\vec{r}, \vec{r}') \neq \delta_{\mu \mu'} \delta(\vec{r} - \vec{r}')$ . Such a violation of the Weyl-Heisenberg commutation relations reflects the causal dependence between the multipole radiation fields at different points.

Nevertheless, the operators (22) can be used for description of a real measurement. Consider a model of a Hertz-type experiment on emission and detection of multipole photons in the system of two identical atoms separated by a distance  $d$ . If we assume that a photon is first emitted by the atom number one (source) and then absorbed by the atom number two (detector), it is most natural to consider the field as a superposition of outgoing and incoming spherical waves focused on the source and detector respectively. This superposition should obey the boundary conditions for the real radiation field, so that only one multipole photon can exist in the space. Then, in direct analogy to (24), we can construct a configuration space photon absorption operator

$$\vec{a}(\vec{r}, t) = \sum_{\mu} (-1)^{\mu} \vec{\chi}_{-\mu} \sum'_{\lambda k j m} V_{\lambda k j m \mu}(\vec{r}) e^{-ikc} a_{\lambda k j m}. \tag{30}$$

Here the sum is taken over the modes allowed by the selection rules for the atom-field interaction under consideration. The volume of detection is defined in this case as follows

$$\mathcal{V} = \frac{4\pi}{3} [(c\Delta\tau)^3 - r_a^3],$$

where  $\Delta\tau$  is the atomic "exposure time" defined by the natural breadth of the spectral line, and  $r_a$  denotes the atomic radius. Then, the configuration space multipole photon number operator takes the form

$$\mathcal{N}(\mathcal{V}, t) = \sum_{\mu} \int_{\mathcal{V}} b_{\mu}^{+}(\vec{r}, t) b_{\mu}(\vec{r}, t) P_{\mu}(\vec{r}) d^3r, \quad (31)$$

where the definition of  $b_{\mu}(\vec{r}, t)$  differs from (22) by summation over all allowed modes, which induces the time dependence. It is straightforward to show that the operators (30) and (31) obey commutation relations of the type (26) and (27). Thus, the picture of measurement in the source-detector system of two identical atoms expressed in terms of the local operators (22) is compatible with Mandel's operational approach to the photon localization.

The above picture, based on the superposed state of outgoing and incoming waves of photons, completely eliminates an enquiry concerning the trajectory of photons between the atoms. In fact, the quantum mechanical path of a photon is not a well-defined notion [17]. The most that we can state about the path of a quantum particle in many cases is that it is represented by a nondifferentiable, statistically self-similar curve [17]. For example, the path of a tunneling electron and time spent in the barrier are not still defined unambiguously [18]. Moreover, recent experiments on photonic tunneling and transmission information show the possibility of superluminal motion of photons inside an opaque barrier [19].

We now note that, according to the principles of quantum theory, not the path, but causality in the transmission of information from one object to another, is important. In the above considered Hertz experiment with two atoms separated by empty space, this means that the detecting atom cannot be excited earlier than  $d/c$  seconds after the emission of a photon by the first atom. Such a causality has been proven in [20].

## 5. Conclusion

Let us briefly discuss the results obtained here. It has been shown that the clear-cut distinction between the properties of plane and spherical waves leads to a qualitative difference in the spatial behaviour of the corresponding photons as well as of the zero-point oscillations. The successive use of the spatial inhomogeneity of multipole radiation permits us to construct a local representation of the Weyl-Heisenberg algebra of multipole photons based on the properties of polarization. Let us stress once more that the polarization defined to be the spin state of photons has a one-to-one correspondence with the spatial properties of radiation. The local representation



of multipole photons obtained in Sec. 3 is compatible with Mandel's operational definition of photon localization [7]. It permits us to describe a complete Hertz-type experiment with two identical atoms used as the emitter and detector. The two-atom Hertz experiment can be realized for the trapped Rydberg atoms [21]. Finally, we stress the fact that this measurement is closely connected with the problem of engineered atomic entanglement discussed in [22].

## References

- [1] E.A. Power, *Introductory Quantum Electrodynamics* (Longman, London, 1964); I. Bialynicki-Birula and Z. Bialynicka-Birula, *Quantum Electrodynamics* (Pergamon Press, Oxford, 1976); H.A. Kramers, *Quantum Mechanics* (North-Holland, Amsterdam, 1985); M.W. Evans and J.-P. Vigiér, *The Enigmatic Photon*, Vol. 1 and 2 (Kluwer, Dordrecht, 1994); M.W. Evans, J.-P. Vigiér, S. Roy, S. Jeffers, and G. Hunter, *The Enigmatic Photon*, Vol. 3 and 4 (Kluwer, Dordrecht, 1997).
- [2] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).
- [3] M.O. Scully and M.S. Zubairy, *Quantum Optics* (Cambridge University Press, New York, 1997).
- [4] T.D. Newton and E.P. Wigner, *Rev. Mod. Phys.* **21**, 400 (1949).
- [5] R. Acharya and E.C.G. Sudarshan, *J. Math. Phys.* **1**, 532 (1960).
- [6] I. Bialynicki-Birula, *Acta Phys. Pol. A* **86**, 97 (1994); I. Bialynicki-Birula, *Phys. Rev. Lett.* **80**, 5247 (1998).
- [7] L. Mandel, *Phys. Rev.* **144**, 1071 (1966).
- [8] E.U. Condon and G.H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, New York, 1937).
- [9] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954); C. Cohen-Tannouji, J. Dupont-Roc, and G. Grinberg, *Atom-Photon Interaction* (Wiley, New York, 1992).
- [10] H. Bateman, *The Mathematical Analysis of Electric and Optical Wave-Motion* (Dover, New York, 1955); J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1978).
- [11] A.S. Davydov, *Quantum Mechanics* (Pergamon Press, Oxford, 1976).
- [12] A.S. Shumovsky, in *Electronic Proc. of 7<sup>th</sup> Central European Workshop on Quantum Optics, April 28 - May 1, 2000* <http://optics.szfki.kfki.hu/cewqo2000>.
- [13] A.S. Shumovsky and Ö.E. Müstecaplıoğlu, *Phys. Rev. Lett.* **80**, 1202 (1998).
- [14] A.S. Shumovsky, *J. Phys. A* **32**, 6589 (1999).
- [15] W.H. Louisell, *Radiation and Noise in Quantum Electronics* (McGraw-Hill, New York, 1964).
- [16] N.N. Bogolubov *Selected Works, Part II, Quantum and Classical Statistical Mechanics*, edited by N.N. Bogolubov, A.M. Kurbatov, and A.S. Shumovsky (Gordon and Breach, New York, 1991).
- [17] R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
- [18] E.H. Haug and J.A. Støvneng, *Rev. Mod. Phys.* **61**, 917 (1989); R. Landauer, *Nature* **341**, 567 (1989); R. Landauer and Th. Martin, *Rev. Mod. Phys.* **66**, 217 (1994).
- [19] A. Enders and G. Nimtz, *J. Phys. I(France)* **2**, 1693 (1992); A. Ranfagni, P. Fabeni, G. Pazzi, and D. Mugnai, *Phys. Rev. E* **48**, 1453 (1993); Ch. Spielmann, R. Szipöcs, A. Stingl, and F. Kraus, *Phys. Rev. Lett.* **73**, 2308 (1994); D. Mugnai, A. Ranfagni, and R. Ruggeri, *Phys. Rev. Lett.* **84**, 4830 (2000).
- [20] D. Kaup and V.I. Rupasov, *J. Phys. A* **29**, 6911 (1996).
- [21] S. Haroche, *AIP Conf. Proc.* Vol. **464**, Issue 1, p. 45 (1999).
- [22] M.B. Plenio, S.F. Huelga, A. Beige, and P.L. Knight, *Phys. Rev. A* **59**, 2468 (1999).

