Chapter 14

HIBERNATION DURATIONS FOR CHAIN OF MACHINES WITH MAINTENANCE UNDER UNCERTAINTY

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Abstract Maintenance of a machine and its replacements by newer ones in the course of a predetermined planning horizon with fixed intermediate dates for potential replacement opportunities is considered. Using the Kamien-Schwartz optimal control model for maintenance, allowance for ceasing of production until installation of a new machine is studied with respect to regeneration points.

1. Introduction

We consider a single machine and its possible replacements (allowed on a calendar of potential regeneration points) over time. The probability distribution of machine failure can be improved by predictive or preventive maintenance. The natural hazard rate for which the machine was designed for, can thus be reduced to a more favorable effective hazard rate.

If the retirement date of a machine is not required to be equal to the installment date of its successor, then the length of the hibernation duration for the production operations need to be determined. When capital expenditures of an organization are made at fixed points on a calendar (such as release of funds in first week of each quarter, or semi annually on first weeks of March and September), then new machine purchases may have to wait for these dates for the availability of the acquisition funds. In the meantime it is possible that the machine waiting for replacement may operate under potentially unprofitable circumstances.

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Selling the machine on hand and waiting idle may be more attractive than suffering unfavorable production costs, or a rapid deterioration in its resale/salvage value. In addition to such factors, constraints on delivery dates of the machine supplier can possibly prevent installation of a replacement at the retirement time of its predecessor. Hibernation can also be considered when buying the currently available machine yields negative expected net present value of cash flow, making it preferable to wait idle until the availability of profitable technologies.

We use the term hibernation to indicate such deliberate non-production periods where the system waits for the arrival of a new and profitable machine. If hibernation is allowed, when should they be scheduled? Answers to such questions may also put pressure for realignment of the calendar for the regeneration points, as well as company policies on borrowing versus use of internal funds. These in turn may raise considerations for the modification of machine replacement time windows.

2. The Model

The main model to be used is that of Kamien and Schwartz (1971) which was recently imbedded into a dynamic programming model by Dogramaci and Fraiman (2004) (in short D-F), for potential machine replacements at fixed intermediate dates over the planning horizon.

Notation:

T : Length of planning horizon consisting of T equal length periods. Starting point of each period constitutes a potential for the acquisition of a machine (a replacement opportunity), i.e. a regeneration point. Generalization of the model for periods of unequal lengths is straightforward and will not be addressed here.

j : Integer indicating a specific regeneration point in the planning horizon. Chronologically the one at the start of the terminal period of the planning horizon is set as $j = 1$, and earlier ones have higher values (in order to serve as index for computational backsweep operations.)

$F_j(t)$: Probability that a machine of vintage j (bought when there were j periods to go until the end of the planning horizon) fails at or before t units of time from its purchase date.

$h_j(t) = [dF_j(t)/dt]/[1 - F_j(t)]$: Natural hazard rate of a machine (of vintage j).

$u(t)$: intensity of maintenance effort at time t. $u(t) \in [\underline{U_j}, \overline{U_j}]$, $0 \leq \underline{U_j} < \overline{U_j} \leq 1$ where $\underline{U_j}$ and $\overline{U_j}$ denote minimum and maximum allowable intensities on a machine of vintage j.

$h_j(t)[1 - u(t)]$: Effective hazard rate of the machine.

$M_j(u(t))h_j(t)$: Cost of maintenance effort at time t. $M_j\,(u(t))$ is continuously differentiable with respect to $u(t)$, with $M_j^{'} > 0$, $M_j^{''} > 0$, and $M_j(0) = 0$.

r: Discount rate indicating time value of money.

D_j: Cost of acquiring and installing a machine of vintage j.

R_j: Revenue net of all costs except maintenance $u(t)$ generated by a machine of vintage j.

$S_j(t)$: Resale value at time t, of a working machine of vintage j. $0 \leq S_j(t) \leq R_j/r$.

L_j: Junk value of a failed machine costs due to in-service failure . $L_j < S_j(t)$.

$f_{(j)}$: Optimal dynamic programming value function at stage j of backward sweep. This is the net present value (with respect to node j) of an optimal regeneration and maintenance policy when there are j periods to go until the end of the planning horizon. It will be computed for $j = 1, 2, ..., T$ in that order. Subscripts in parentheses indicate stage number of dynamic programming calculations, rather than equipment vintage. $f_{(0)} = 0$.

$V(j, K)$: Optimal expected net present value for a vintage j machine acquired at time $T - j$, in other words at node j, at cost of D_j dollars with the intention of keeping it for K periods ($K \leq j$) and subsequent replacements (if any). Present value is computed with respect to the time when the machine is introduced to the production system ($T - j$. Maximum value of K is j. However, managerial considerations can dictate it to be shorter.

Z_j: Hibernation time (measured in terms of machine age): Planned retirement age of machine of chosen at node j. If hibernation is not allowed, $Z_j = K$. Otherwise, $0 \leq Z_j \leq K$.

K_{Z_j}: Closest regeneration point downstream of Z_j. ($0 \leq Z_j \leq K_{Z_j} \leq K$). K_{Z_j} is the smallest integer larger than or equal to Z_j.

$V(j, K)$ shall be determined after $f_{(j-1)},,f_{(0)}$ are obtained, and will in turn feed into the computation of $f_{(j)}$ as follows:

$$f_{(j)} = \max_{K=1,..,j_K} [V(j, K)]\,, \quad j = 1, 2, ..., T;\ j_K \leq j. \tag{14.1}$$

j_K is the upper bound on intended machine life for vintage j, as dictated by technical, safety, and managerial considerations. If there is no such limit, then one can set $j_K = j$. At node j different types of machines may be available, (and hibernation times of each of these alternatives may be different.) If there are alternative models, i.e. a variety of technologies available at time $T - j$, then $V(j, K)$ can be solved for each and the alternative with largest expected net present value may be chosen.

Consider any point in time t, during the time span addressed by any $V(j, K)$. With probability $1 - F_j(t)$ the machine has not yet failed implying a cash flow rate of $R_j - M_j(u(t))h_j(t)$. On the other hand failure of the machine at time t is associated with probability density $dF_j(t)/dt = [1 - u(t)]h_j(t)[1 - F_j(t)]$ and cash flow of L_j right away, as well as $f_{(j-\tau-1)}$ which with respect to time t, is the nearest downstream optimal dynamic programming value function. The index number of the nearest downstream regeneration point is $j - \tau - 1$. In case machine fails at time t a new one is bought at this node. (Values of τ, $\tau + 1$, $\cdots$ are chosen to target such nodes.) Thus $V(j, K)$ is obtained by solving the following problem.

$$
\begin{aligned}
& V(j,K) = \\
& = \max_{u(t), Z_j} \sum_{\tau=0}^{K_{Z_j}-1} \int_{\tau}^{\min[(\tau+1), Z_j]} \{e^{-rt}\{[R_j - M_j(u(t))h_j(t)][1 - F_j(t)] \\
& \quad + L_j[1 - u(t)]h_j(t)[1 - F_j(t)]\} \\
& \quad + e^{-r(\tau+1)} \, f_{(j-\tau-1)}[1 - u(t)]h_j(t)[1 - F_j(t)]\}dt \\
& \quad + [1 - F_j(Z_j)] \, [e^{-rZ_j} S_j(Z_j) + e^{-rK_{Z_j}} f_{(j-K_{Z_j})}] - D_j \qquad (14.2)
\end{aligned}
$$

subject to

$$\frac{dF_j(t)}{dt} = [1 - u(t)]h_j(t)[1 - F_j(t)] \qquad (14.3)$$

with

$$0 \leq \underline{U_j} \leq u(t) \leq \overline{U}_j \leq 1, \quad F_j(0) = 0, \quad t \in [0, Z_j] \text{ and } 0 \leq Z_j \leq K_{Z_j} \leq K.$$

If solution of (14.2)-(14.3) above yields $V(j, K) < 0$, then managerial policies allowing, we can set $V(j, K) = 0$ (implying that an imaginary machine of zero costs and revenues) and stay idle from time $T - j$ until $T - j + K$.

In the objective function (14.2), jumps from $f_{(j-1)}$ to $f_{(j-2)}$ to $f_{(j-\ 3)}\cdots$ are addressed by breaking the problem into K unit period segments and imbedding each into the adjacent upstream one.

3. A Solution Procedure

The procedure proposed here builds upon the D-F approach with the added complexity of checking for hibernation possibilities. We first investigate the (potentially) last period of usage to check whether $K_{Z_j} =$

K. Hence, the machine of vintage j , to be used for K periods is studied from $t = K - 1$ to K.

$$
\begin{aligned}
&J_{j,K-1,F_j(K-1)} = \\
&= \max_{u(t)} \int_{t=K-1}^{K} \{e^{-rt}\{[R_j - M_j(u(t))h_j(t)][1 - F_j(t)] \\
&\quad + L_j[1 - u(t)]h_j(t)[1 - F_j(t)] \\
&\quad + e^{-rK} f_{(j-K)}[1 - u(t)]h_j(t)[1 - F_j(t)]\}dt \\
&\quad + e^{-rK}[S_j(K) + f_{(j-K)}][1 - F_j(K)]
\end{aligned}
\tag{14.4}
$$

subject to

$$
\frac{dF_j(t)}{dt} = [1 - u(t)]h_j(t)[1 - F_j(t)] \tag{14.5}
$$

with $0 \leq \underline{U_j} \leq u(t) \leq \overline{U}_j \leq 1$, $F_j(K-1)$ given, and $F_j(K)$ free.

The probability that the machine would still be up and running is reflected in the value of the state variable at time (in this context, time=age) $K - 1$: $F_j(K - 1)$. The optimal value of this problem, $J^*_{j,K-1,F_j(K-1)}$, feeds in as a salvage value to the adjacent optimal control problem from $K - 2$ to $K - 1$. D-F showed that for $\tau = 1, ..., K$, $J^*_{j,\tau-1,F_j(\tau-1)}$ is a linear function of the starting value of the state variable $F_j(\tau - 1)$. Thus the problem starting at $\tau - 1$ needs only to be solved for a starting state variable value of $F_j(\tau - 1) = 0$. Its optimal value will be imbedded into the adjacent earlier problem on the left (i.e. into the model that starts at time $\tau - 2$) as salvage value term, in the form: $[1 - F_j(\tau - 1)\ ]\ J^*_{j,\tau-1,0}$. Thus the objective function in (14.4) can be stated for $F_j(K - 1) = 0$ as:

$$
\begin{aligned}
&J_{j,K-1,0} = \\
&= \max_{u(t)} \int\limits_{t=K-1}^{K} \{e^{-rt}\{[R_j - M_j(u(t))h_j(t)][1 - F_j(t)] \\
&\quad + L_j[1 - u(t)]h_j(t)[1 - F_j(t)]\} \\
&\quad + e^{-rK} f_{(j-K)}[1 - u(t)]h_j(t)[1 - F_j(t)]\}dt \\
&\quad + e^{-rK}[S_j(K) + f_{(j-K)}][1 - F_j(K)] \\
&= \max_{u(t)} \int\limits_{t=K-1}^{K} \{e^{-rt}\{[R_j - M_j(u(t))h_j(t)][1 - F_j(t)] \\
&\quad + L_j[1 - u(t)]h_j(t)[1 - F_j(t)]\}dt \\
&\quad + \int\limits_{t=K-1}^{K} \{e^{-rK} f_{(j-K)} \frac{dF_j(t)}{dt}\}dt \\
&\quad + e^{-rK}[S_j(K) + f_{(j-K)}][1 - F_j(K)] \\
&= \max_{u(t)} \int\limits_{t=K-1}^{K} \{e^{-rt}\{[R_j - M_j(u(t))h_j(t)][1 - F_j(t)] \\
&\quad + L_j[1 - u(t)]h_j(t)[1 - F_j(t)]\}\}dt \\
&\quad + e^{-rK} f_{(j-K)}[F_j(K) - F_j(K-1)] \\
&\quad + e^{-rK}[S_j(K) + f_{(j-K)}][1 - F_j(K)]
\end{aligned}
$$

Since $F_j(K-1) = 0$, the objective function of the problem becomes,

$$
\begin{aligned}
&J_{j,K-1,0} = \\
&= \max_{u(t)} \int\limits_{t=K-1}^{K} \{e^{-rt}\{[R_j - M_j(u(t))h_j(t)][1 - F_j(t)] \\
&\quad + L_j[1 - u(t)]h_j(t)[1 - F_j(t)]\}\}dt \\
&\quad + e^{-rK}[S_j(K)][1 - F_j(K)] + e^{-rK} f_{(j-K)}
\end{aligned}
\tag{14.6}
$$

Since 14.6 subject to 14.5 is structurally a standard K-S model, any hibernation possibility in this period can be studied in the context of a free terminal time problem. Keeping j and $K-1$ fixed, and calling the terminal time Z_j , the terminal condition for

$$\begin{aligned} J_{j,K,0}(Z_j) \; = \\ = \max_{u(t)} \int\limits_{t=K-1}^{Z_j} \{e^{-rt}\{[R_j - M_j(u(t))h_j(t)][1 - F_j(t)] \\ + L_j[1 - u(t)]h_j(t)[1 - F_j(t)]\}\}dt \\ + e^{-rZ_j}[S_j(Z_j)][1 - F_j(Z_j)] + e^{-rK} f_{(j-K)} \end{aligned} \tag{14.7}$$

involves the evaluation of

$$\begin{aligned} e^{-rZ_j}\,(1 - F_j(Z_j))\,[R_j - M_j\,(u^*(Z_j))\,h_j(Z_j) + L_j\,(1 - u^*(Z_j))\,h_j(Z_j) \\ -\,(r + (1 - u^*(Z_j))\,h(Z_j))\,S_j(Z_j) + dS_j(Z_j)/dZ_j] \end{aligned} \tag{14.8}$$

where $u^*(Z_j)$ denotes the optimal value of the control at the optimal hibernation time. (See for example Kamien and Schwartz (1971) or Sethi and Thompson (2000) ch. 9.)

$u^*(Z_j)$ is chosen so as to maximize the following:

$$\max_{0 \leq \, u(Z_j) \, \leq \, 1} \; \{(S_j(Z_j) - L_j)\,u(Z_j) - M_j[u(Z_j)]\} \tag{14.9}$$

The expression in square brackets in (14.8) determines sign of the marginal benefit (negative if cost) of an infinitesimal increase in terminal time and shall be denoted by $B(Z_j)$.

$$\begin{aligned} B\,(Z_j)\; = R_j - M_j\,(u^*(Z_j))\,h_j(Z_j) + \;\, L_j\,(1 - u^*(Z_j))\,h_j(Z_j) \\ -\,(r + (1 - u^*(Z_j))\,h(Z_j))\,S_j(Z_j) + \frac{dS_j(Z_j)}{dZ_j} \end{aligned} \tag{14.10}$$

and can be numerically evaluated for any candidate terminal time. It is clear that at optimal Z_j, we must have $B\,(Z_j) \geq 0$. Otherwise for some $\varepsilon > 0$, $Z_j - \varepsilon$ (which may be less than $K - 1$) may be more profitable.

Since all the expressions can now be numerically evaluated, the procedure involves the following:

1 If $B(Z_j) \geq 0$ for all $Z_j \in [K - 1, K]$ then we can set $Z_j := K$, implying no hibernation.

2 If $B(Z_j) \leq 0$ for all $Z_j \in [K - 1, K]$ then one can set $K := K - 1$ and if the new $K \geq 1$, solve this one-period-shorter problem for hibernation possibility.

3 Otherwise, using numerical search, find the values of Z_j for which $B(Z_j) = 0$ and compute the corresponding values of $J_{j,Z_j,0}$ as well

as for $Z_j = K - 1$, and $Z_j = K$. Pick the Z_j for which $J_{j,Z_j,0}$ is largest. (If this $J_{j,Z_j,0} \leq 0$ then set $K := K - 1$ and if the new $K \geq 1$, solve this one-period-shorter problem for hibernation possibility.)

4. Implications for Realigning the Calendar for Regeneration Points.

Allowance for hibernation relaxes the D-F model to ensure non- negative expected net present values for a machine and in particular, for the cash flow towards the end of its life.

If optimal value of hibernation time does not turn out to be an integer, the management may be advised to evaluate the allowance of shorter periods between regeneration points. Numerical experiments of D-F had indicated that reduction of such granularity increases the computational time as a polynomial function of the number of regeneration points. This evaluation also needs to take into account other considerations including whether acquisitions (or deliveries) of machines at the newly proposed times are feasible. While the optimal control model cannot comprise the non-quantifiable factors of managerial decisions, it nevertheless can serve as a useful tool for providing some of the basic building blocks that feed into the final decision.

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