

Chapter 12

Hub Location Problems: The Location of Interacting Facilities

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12.1 Introduction

O’Kelly’s (1986) classical paper started a new research stream by identifying a connection between spatial interaction models and location theory. The traditional spatial interaction theory applies models of travel behavior to investigate demand patterns between fixed locations. Location theory, on the other hand, takes demand as given, assumes a simple view of travel behavior, and focuses on finding the best location for facilities.

Spatial interaction theory focuses on the problem of locating centers of special interest, and observes that the selected locations have an effect on the evolution of the associated network. O’Kelly’s self-identified contribution in this context relates to the interaction effects between facility locations and spatial flows. He makes a distinction between endogenous and exogenous effects. In particular, he considers the given problem parameters as exogenous data, which are endogenously affected by the location of the hubs as well as the allocations. Hubs are special facilities acting as consolidation and dissemination points for the flows. Flows from the same origin with different destinations are consolidated enroute at a hub node where they are combined with flows from different origins with a common destination. The main idea is to keep the flow interactions in perspective at the design stage of the hub network. That is, the hubs need to be strategically located in view of their effects on the intensity and cost of the flow data. In general, the hub location problems are defined as analogous counterparts of the classical location problems with the addition of allocation decisions.

This chapter reviews and outlines the research on hub location problems that emerged as a new research stream led by O’Kelly’s (1986) seminal paper. Section 12.2 discusses the geographical applications leading the way to the ideas proposed by O’Kelly. Section 12.3 summarizes the major findings presented in

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O’Kelly’s original paper. Prominent theoretical developments that emerged from these findings are discussed in Sect. 12.4. Section 12.5 reviews some related application oriented studies. Finally, Sect. 12.6 concludes the chapter with highlights of the current and future trends for research in the area.

12.2 Before Hub Location

The identification of the importance of consolidation and dissemination points as well as their endogenous effects was well known in spatial interaction theory before O’Kelly’s work. For instance, the classical paper by Taaffe et al. (1963) discusses the issue in the context of formation of transportation infrastructure in third world countries. They observe that consolidation-dissemination points are located in administrative centers, political and military control centers, mineral exploitation areas, and areas of agricultural export production. Lines of penetration emerge between these points of demand concentration. Figure 12.1 illustrates a line of penetration between two fictitious centers of critical importance (centers A and B), resulting in indirect connections between the points previously connected to either one of these centers (i.e., points $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_m$). Once such penetration lines are formed, they have an impact on both the surrounding area along these lines and the initial centers in terms of local development. These local developments are analogous to the endogenous attraction proposed by O’Kelly, and they in turn manifest themselves as a factor that further supports the structure of the penetration lines. Once the development in the centers and along the penetration lines stabilizes, the formation of the backbone is completed.

In classical spatial theory, there are also examples of active strategic development of the transportation backbones. For instance, Miehle (1958) constructs a mechanical model to simulate alternative backbone structures enforcing the passage of flows through certain designated locations functioning as hubs. Goodchild (1978) mathematically considers the role of endogenous attraction. He assumes fixed locations and solves only the allocation problem, where attraction to a facility is modeled as a function of both distance and usage. Distance is an exogenously given

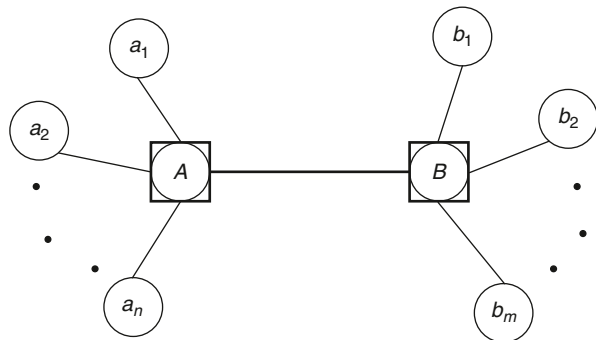


Fig. 12.1 Lines of penetration between centers A and B

factor, whereas attraction due to usage is endogenously or voluntarily determined based on the actual level of service that a facility provides.

Ducca and Wilson (1976) consider a similar problem in the context of the planned positioning of shopping centers. In their model, demand intensity is expressed as a simultaneous function of existing retail density, employment, and distances traveled. Allen and Sanglier (1979) develop a model of dynamically interacting urban centers. Each center has an associated attraction parameter, and once a center is located, the parameters of the original problem in terms of the demands generated at different locations are affected through immigration and emigration. In particular, there is a positive feedback loop due to the employment opportunities generated by a located center. In a subsequent paper, Allen and Sanglier (1981) improve their original model by also considering the negative feedback loop that reflects the crowding effect.

This brief discussion on inter-facility attraction shows that the fundamentals of the notion of endogenous attraction observed by O'Kelly date back to late 1950s. The formal definition of the problem in the context of location theory led to the development of a new field. The remainder of this chapter discusses this new field and ties its evolution back to O'Kelly's paper.

12.3 O'Kelly's Seminal Contribution

Genesis of location of interacting facilities as a new research area within location theory dates back to O'Kelly's paper, which was significantly impacted by the popular trend of simultaneous consideration of location and transportation decisions in spatial theory. In this paper, O'Kelly focuses on the interaction between hubs serving the United States inter-city air passenger streams, and studies the relevant data recorded in a Civil Aeronautics Board sample survey of 1970. He observes that, although airline companies in practice carefully consider the location of the hub facilities in view of their collective ability to efficiently connect the cities in their network, classical location research completely ignores these interactions. This perspective helps O'Kelly to identify a novel version of a location-allocation problem, in which the located facilities lie along the route between demand points.

O'Kelly studies both a single- and a two-hub version of this new problem in the 2-dimensional plane. The single-hub version is shown to be equivalent to the classical Weber least cost location problem. Regarding the economic advantages of building a single-hub network, O'Kelly points out that the only rational reason to justify such a system would be the potential savings in link costs due to the scale effects of routing the traffic through the hub. This issue in a problem with n demand points is mathematically expressed as

$$\sum_i \sum_j W_{ij} (C(p_i, Q) + C(Q, p_j)) + Kn < \sum_i \sum_j W_{ij} C(p_i, p_j) + 1/2n(n-1)K,$$

where the notation is defined as follows.

- p_i : Demand point $i, i=1, 2, \dots, n$
- W_{ij} : Flow between demand points p_i and $p_j, i=1, 2, \dots, n, j=1, 2, \dots, n$
- Q : Hub to be located at (x, y)
- $C(p_i, p_j)$: Cost per unit flow between points p_i and p_j measured in terms of the Euclidean distance
- K : Cost of intercity linkage (which may include the cost of using the transportation mode and the operational expenses such as fuel cost, driver wages, etc.)

Observe that if function C satisfies the triangular inequality, the savings result from the fewer links to operate when the hub is utilized. The expression indicates that the total transfer cost is greater when the traffic is routed through the hub. However, this difference is compensated by the smaller cost of operating fewer flow links in the hub version, i.e., n vs. $\frac{1}{2}n(n-1)$ in the hub and non-hub versions, respectively.

O’Kelly acknowledges the need for using multiple hubs to accommodate a large area and discusses also multiple-hub problems. In such a network, the inter-hub linkages can be specially designed to efficiently handle bulk flow. In this way, the unit transportation costs between hubs can be significantly reduced. The reduced cost of these flows in turn appears as an endogenous function of the hub locations.

O’Kelly proposes a simple approximation, and discounts the inter-hub costs by a factor α , such that $0 \leq \alpha < 1, \alpha \in \mathbb{R}$. Because of the special structure of the cost function, the multiple-hub problems involve a two-fold decision in the sense that both the location of the hubs and the assignment of the demand points to the hubs must be decided upon.

The paper particularly focuses on solving the two-hub version of the problem, which is significantly easier than the more general p -hub version. Using decision variables

$$X_{ik} = \begin{cases} 1, & \text{if demand point } p_i \text{ is assigned to hub } Q_k, k = 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

the cost function to be minimized is characterized as

$$\text{Min}_{Q_1, Q_2} \sum_i \sum_j W_{ij} R_{ij},$$

where R_{ij} is the routing and transportation cost between points i and j conditional upon the corresponding hub location decision. This cost is mathematically expressed as follows.

$$\begin{aligned} R_{ij} = & X_{i1}X_{j1}(C(p_i, Q_1) + C(p_j, Q_1)) \\ & + X_{i2}X_{j2}(C(p_i, Q_2) + C(p_j, Q_2)) \\ & + X_{i1}X_{j2}(C(p_i, Q_1) + \alpha C(Q_1, Q_2) + C(p_j, Q_2)) \\ & + X_{i2}X_{j1}(C(p_i, Q_2) + \alpha C(Q_2, Q_1) + C(p_j, Q_1)) \end{aligned}$$

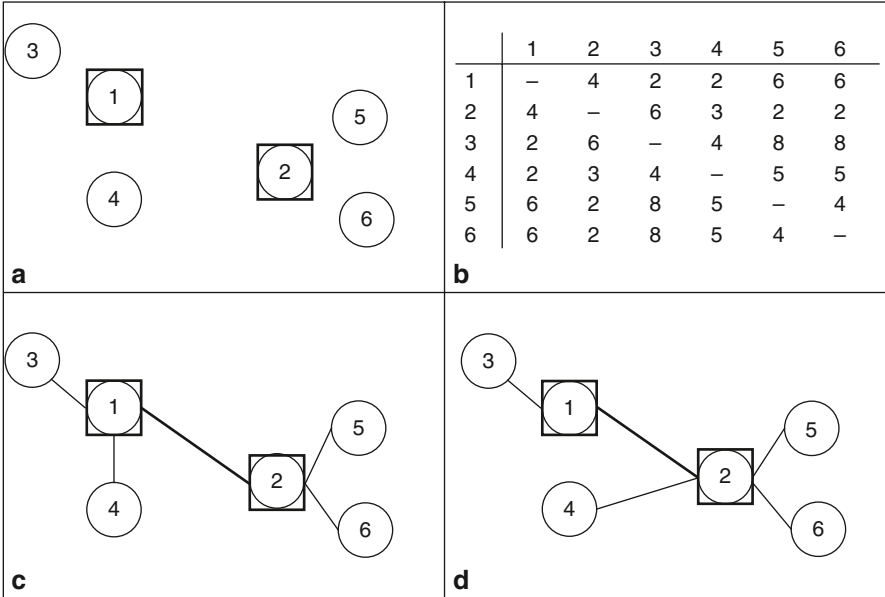


Fig. 12.2 Two different allocation schemes on an example network. **a** Network configuration. **b** Distance matrix. **c** Allocation scheme 1. **d** Allocation scheme 2

Due to the binary nature of the decision variables, for each origin destination pair only one of the four possible components of the objective function will take on a positive value. The possibilities involved are the cost of flow from origin to destination via the same hub (either hub 1 or hub 2), and the transfer cost from origin to destination via both hubs (either from hub 1 to hub 2 or the reverse). Obviously, when both hubs are used, the cost of inter-hub transfer is discounted by factor α .

O’Kelly observes that due to the quadratic term and the discounting effect, assignment to the nearest hub may turn out to be suboptimal. We develop an example to illustrate this phenomenon in Fig. 12.2. The network configuration and corresponding distance matrix are shown in Fig. 12.2a and b, respectively. The magnitude of symmetric flows between point 4 and points 5 and 6 are equal to 10. Flow densities between all other pairs have a much smaller value of 1. The discount factor α is set equal to 0.60. Figure 12.2c shows allocation scheme 1, in which all points are allocated to their nearest hub. This scheme results in a total cost value of 353.60. On the other hand, allocation scheme 2, shown in Fig. 12.2d, assigns point 4 to the more distant hub, and gives a smaller total cost value of 308.40.

The proposed approach to solving the two-hub problem is to minimize the discounted cost function by simply taking the first order derivatives with respect to the location coordinates and setting them equal to zero. In this problem, however, the cost function is minimized for different partitions of demand points corresponding to the hubs. A partition refers to the set of demand points assigned to a given hub. The partitions whose convex hulls are non-overlapping are defined as non-overlap-

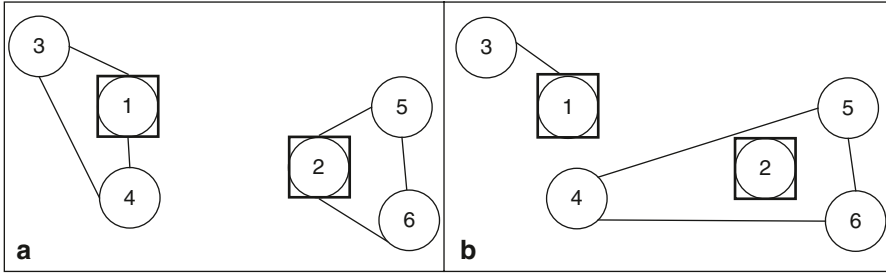


Fig. 12.3 Two different non-overlapping partitions of the example network of Fig. 12.2. **a** Partition 1. **b** Partition 2

ping partitions. Figure 12.3 shows two non-overlapping partitions on the previous example network. Partitions 1 and 2 correspond to allocation schemes 1 and 2, respectively. Motivated by the fact that consideration of only the non-overlapping partitions yields the optimum solution for the two-center location-allocation problem (Ostresh 1975), O’Kelly relies on the simplifying assumption that the assignment of demand points can be considered only for non-overlapping partitions, though he acknowledges that this approach may not necessarily yield the true optimum solution in the current problem.

Another of O’Kelly’s observations relates to the effect of the hub network structure on the intensity of flow between demand points. He proposes the following function that updates the revised flow.

$$W_{ij} = \frac{O_i D_j \exp(-\beta R_{ij})}{\sum_{k=1}^n D_k \exp(-\beta R_{ik})}$$

Recall that the R_{ij} values are the routing and transportation costs considering all hub assignment possibilities. This function revises the flow density between each pair as a decreasing function of the relevant transportation costs. The sensitivity of the flow volume to the cost is governed by coefficient $\beta \geq 0$, where a larger value of this coefficient leads to a more significant effect. O’Kelly presents some computational analysis on the Civil Aeronautics Board data in which the effects of using different parameters (α and β) for modeling endogenous attraction are investigated.

The most significant contribution of this classical paper remains the identification of the hub location problem as a version of the p -median location-allocation problem involving interactions. The solution techniques for the multiple-hub problems are later improved by various researchers including O’Kelly himself (e.g. 1987, 1992). In addition, a multitude of studies focusing on hub-location counterparts of different classical location problems emerged, and the next section presents an overview of these studies within the framework of a new proposed taxonomy.

12.4 Theoretical Developments in the Hub Location Literature

This section starts out by identifying the connections between hub location problems with their counterparts in the classical location literature. After observing the factors that result in different types of hub location problems, the authors propose a new taxonomy that serves for a convenient classification of the relevant developments. These developments are presented for the cost minimization and minmax type of objectives in Sects. 12.4.3 and 12.4.4, respectively.

12.4.1 Analogies with Location Theory

Having identified hub-location problems involving interacting facilities, O'Kelly (1987) formulates a general version of the problem where flow between demand points is to be transferred via p hubs to be cited at a subset of the nodes corresponding to origins and/or destinations. The following additional notation is needed.

N : Set of nodes

C_{ij} : Transportation cost for a unit flow between nodes i and j

Note that the transportation cost is redefined to highlight its correspondence to the network distance of the shortest path linking the two nodes. Although this cost is defined here as related in some way only to the distances involved, it is important to acknowledge that there may be a multitude of different factors affecting the magnitude of the cost and the discount factor. The only decision variable X_{ik} is now redefined for $i, k=1, 2, \dots, n$. Note that if $i=k$ and $X_{ii}=1$, node i is a hub.

The proposed formulation is as follows.

$$\text{Min } z = \sum_i \sum_j W_{ij} \left(\sum_k C_{ik} X_{ik} + \alpha \sum_k \sum_m C_{km} X_{ik} X_{jm} + \sum_m C_{jm} X_{jm} \right) \quad (12.1)$$

$$\text{s.t. } (n-p+1)X_{jj} - \sum_i X_{ij} \geq 0 \quad \forall j \quad (12.2)$$

$$\sum_j X_{ij} = 1 \quad \forall i \quad (12.3)$$

$$\sum_j X_{jj} = p \quad (12.4)$$

$$x_{ij} \in \{0,1\} \quad \forall i, j \quad (12.5)$$

The objective function, shown in (12.1), minimizes the total cost comprising the origin-to-hub, discounted inter-hub transportation, and hub-to-destination cost components. The inter-hub transportation cost in this basic formulation includes a quadratic term to account for the origin-destination pairs connected through their designated hubs. Note that the discount factor works on the transportation costs, not the distances. Constraint set (12.2) ensures that no switching is allowed through a non-hub node. Constraint sets (12.3) and (12.5) enforce allocation of each node to exactly one hub. Finally constraint (12.4) sets the number of hubs equal to p . Note that this initial formulation is subsequently considered as the “basic formulation” in the hub location literature.

O’Kelly (1987) reiterates that interaction is the factor differentiating this new problem from the p -median and multi-facility Weber problems, both of which are widely considered in the classical location literature. The novelty is that the locations of the hubs have a direct effect on the magnitude of the inter-hub flows and the associated linkage costs.

An immediate consequence of this endogenous effect is in the allocation of nodes to hubs. In classical location theory with uncapacitated facilities, once the locations are given, the allocation subproblem can be optimally solved by assigning a node to its nearest facility. In hub location problems, on the other hand, the assignment of a node to a facility is impacted also by that facility’s ability to service the interaction pattern. Therefore, proximity of the hub to a node ceases to be the sole factor dictating the allocation.

The logical connections between hub location problems and the location theory literature were outlined by Campbell (1994a). Campbell defines location analogous versions of the hub-location problem, namely the p -hub median, hub location with fixed costs, p -hub center, and hub covering problems. The basic problem defined in O’Kelly (1987) is a p -hub median problem. A detailed discussion of the other problems defined by Campbell (1994a) will follow below. Campbell’s principle contribution to the expansion of the hub location literature relates to the consideration of alternative criteria for objectives. On the constraint side, researchers identified the following three major factors to produce alternative versions of this basic problem:

1. Single- vs. multi-allocation (Campbell 1990),
2. Full vs. partial hub network (Chou 1990), and
3. Presence/absence of direct connectivity between non-hub nodes (Aykin 1995).

Recall that the basic model assumes each node is served by exactly one hub, all hubs are connected to each other, and any transfer between two non-hub nodes must be via at least one hub. O’Kelly and Miller (1994) suggest that different combinations of these three factors result in eight alternative versions of the basic problem.

Based on the alternative objective functions and possible variations in the constraint set, the present authors propose a taxonomy in the next section to facilitate a convenient and systematic discussion of the emerging literature.

12.4.2 A Taxonomy of Hub Location Problems

We observe that the factors that determine the nature of the problem can be considered in four categories. To also accommodate other problem-specific restrictions,

we propose a five-fold taxonomy in the following form $\varepsilon/\varphi/\kappa/\lambda/\omega$. The fields in this short-hand notation correspond to the following specific factors.

- ε : Objective criterion
- φ : Allocation structure
- κ : Capacity
- λ : Inter-hub connectivity
- ω : Other restrictions.

The alternative objective criteria corresponding to the p -hub median, hub location with fixed cost, p -hub center and hub covering problems will be denoted shortly as *pH-median*, *fixH-cost*, *pH-center* and *H-cover*, respectively. The allocation structure refers to the degree of flexibility in terms of the number of hubs to which a node can be assigned. The corresponding parameter φ thus in turn may be either single or multi. Various types of capacities may be imposed upon the flow handled by the hubs and the transportation lines. The uncapacitated version of the problem is denoted by U , whereas the presence of node and arc capacities is indicated by *node* and *arc*, respectively. Finally, the underlying network topology appearing in the λ field may range from full to different partial structures such as path, tree, ring, and star. Since the other restrictions are expected to vary depending on the circumstances of a specific problem, the notation to be used in the ω field is left to the discretion of other authors. Note that the basic problem can be denoted as *pH-median/single/U/full*.

We remark here that Campbell et al. (2002) also provide a taxonomy to help classify the hub location problems. The alternative proposed herein is based on the review and synthesis provided in O’Kelly and Miller (1994) as well as the objective criteria discussed in Campbell (1994a).

After the basic problem was identified by O’Kelly (1986), for almost a decade researchers worked on mathematical formulations that would efficiently solve it. The initial formulation provided in O’Kelly (1987) was quadratic. Linear formulations were given in Aykin (1990), Campbell (1996), and Skorin-Kapov et al. (1996), among others. In these formulations, single- and/or multiple-allocation versions of the problem were considered under the cost objective. For the *pH-median* problems, the objective is the minimization of the total transportation cost. Conversely, in the *fixH-cost* problem a fixed cost associated with opening a new hub was considered alongside the transportation cost. We first discuss several important studies on the *pH-median* and *fixH-cost* problems. Then we proceed with the *pH-center* and *H-cover* versions of the problem investigated in the more recent literature.

12.4.3 Minisum Objectives

In the early 1990s, due to the quadratic nature of the formulation, researchers attempted to solve the single allocation version of the *pH-median* problem with heuristic approaches. Three important examples of such attempts can be found in Klinecicz (1991 and 1992), as well as Skorin-Kapov and Skorin-Kapov (1994).

The first solvable exact formulation dates to back to Campbell (1996), who studies *pH-median/single/U/full* and *pH-median/multi/U/full*. Campbell gives linear mathematical programming formulations for the two problems by defining a four-indexed binary variable X_{ijkm} , which takes on a value of 1 only if the flow between nodes i and j is routed via hubs k and m . He observes that the integrality of these variables can be conveniently relaxed when solving *pH-median/multi/U/full* without forgoing optimality. He also remarks that solution to the multiple allocation version of the problem constitutes a lower bound for the single allocation version.

The single allocation version of the problem with the Civil Aeronautics Board data was optimally solved for the first time in Skorin-Kapov et al. (1996) by using a branch-and-bound algorithm utilizing a tight lower bound obtained from the linear programming relaxation of their original formulation. Ernst and Krishnamoorthy (1996) provide an efficient network flow formulation to solve the same problem. This formulation relies on modeling flows generated by each node as a different commodity which results $O(n^3)$ binary integer variables as opposed to $O(n^4)$ in the previous formulations. Ernst and Krishnamoorthy (1998a) embed this notion in a branch-and-bound algorithm, which to the best knowledge of the present authors is the most efficient solution algorithm for this problem to date. Ernst and Krishnamoorthy (1998b) apply the network flow notion also to the multi-allocation version of the problem and obtain optimum solutions to large instances. In the same paper, they observe that the problem can be solved polynomially by an all-pairs shortest path algorithm when the hub locations are fixed.

Following O'Kelly (1986), all research until O'Kelly (1992) considered solely the transportation costs in the objective function. O'Kelly (1992) incorporates this fixed cost into the problem and addresses the capacitated version of the problem, *fixH-cost/single/node/full*. He uses a modified version of his basic formulation with the addition of the total hub cost, $\sum_j F_j X_{jj}$, in the quadratic objective function. The multi-allocation version of this problem with additional arc costs, *fixH-cost/multi/node/full/ {direct, arc-costs}* was studied by Aykin (1994). Recall that the term "direct," in this context, implies that a non-stop connection between non-hub nodes is permissible. He proposed a branch and bound algorithm utilizing a Lagrangian-based lower bound. This problem, with the only difference of not allowing direct connections, was studied by Ernst and Krishnamoorthy (1999), who proposed efficient integer programming formulations. Recent exact solution approaches exploit the polyhedral structure of the hub location problems. Labbé and Yaman (2004) derive facet-defining inequalities for *fixH-cost/single/U/full*. For the multiple allocation version of the problem, Hamacher et al. (2004) propose valid inequalities by modifying the facet defining inequalities for the uncapacitated facility location problem. Similarly, Marin (2005) exploits the polyhedral structure of the set packing problem to develop valid inequalities for *fixH-cost/multi/U/full* with Euclidian distances.

12.4.4 Minmax Objectives

An inherently different class of hub location problems is of the minmax type. The 1-hub center problem was originally defined by O’Kelly and Miller (1991) to minimize the maximum cost incurred by any origin-destination pair. This problem was motivated by a desire to achieve equity between user nodes in terms of the transportation costs incurred. Following the remarks made in O’Kelly and Miller’s (1991) conclusion, the minmax objectives in later studies focus on the service time concerns rather than the cost issues. In the p -hub center problem, the objective is to minimize the worst service time between any origin destination pair. Alternatively, the objective of the hub cover problem is to serve all node pairs with the minimum possible number of hubs while keeping the travel times below a predetermined threshold level. These problems received attention in the literature partly due to their practical applications in such systems as perishable goods transfer and overnight delivery.

The first paper which fully defines and classifies different versions of these problems is Campbell (1994a). In addition to providing integer programming formulations for pH -center/single/U/full, pH -center/multi/U/full, H -cover/single/U/full, H -cover/multi/U/full, Campbell identifies different types of service time restrictions. In particular, he additionally defines separate service times for the segments constituting a path between origin-destination pairs. He also proposes integer programming formulations for the pH -center and H -cover problems based on these new service time definitions. These alternative versions of the two problems are still open areas that require further investigation.

After being defined by Campbell (1994a), the pH -center and H -cover problems were not studied until Kara and Tansel (2000). They provide a proof of NP-hardness for pH -center/single/U/full and develop an efficient integer programming formulation with n^2 binary variables. Ernst et al. (2002) give a more efficient formulation for the same problem by using auxiliary variables. They also show that the multiple allocation version of the problem is NP-hard, and propose a modification of their original formulation for its solution. Baumgartner (2003) analyzes these two formulations, develops facet defining valid inequalities, and proposes a branch-and-cut algorithm based on these inequalities.

For the covering version of the problem with single allocation, Kara and Tansel (2003) provide an NP-hardness proof along with an efficient integer programming formulation. Ernst et al. (2005) present formulations for both the single and multiple allocation versions of this problem. Their formulation for the single allocation case outperforms that of Kara and Tansel (2003). Polyhedral properties of these H -cover problems are studied by Hamacher and Meyer (2006).

A variant of this problem is motivated by real life applications based on the observation that trucks are synchronized at the hub nodes by occasionally delaying their departures. Kara and Tansel (2001) call this variant the latest arrival hub location problem, defining the pH -median, pH -center and H -cover versions of the problem. They propose a formulation for pH -center/single/U/full/latest-arrival which

can efficiently solve all Civil Aeronautics Board instances. The *H-cover* version of this problem was investigated in a similar way by Tan and Kara (2007) who test the performance of their formulation based on a new data set of Turkish highway travel times.

Special cases of the *pH-center* problems with fixed hubs are investigated by Iyer and Ratliff (1990) and Campbell et al. (2007). Iyer and Ratliff (1990) consider a “guaranteed time distribution” problem, which is in fact equivalent to the uncapacitated *p*-hub center problem with a tree type network structure. They propose a polynomial time exact algorithm to solve this problem. Their algorithm was later modified by Campbell et al. (2007) to solve the *2H-center/single/U/path*, *pH-center/single/U/tree*, *2H-center/single/U/full*, *pH-center/multi/U/full* problems. Campbell et al. prove additionally that problems *pH-center/single/arc/full* and *pH-center/single/node/full* are NP-hard.

12.5 Application-Oriented Studies

In addition to the theoretical investigations discussed in the previous section, the hub location problem identified by O’Kelly (1986) has been widely studied in the past two decades regarding other practical applications than airline passenger streams. These different practical applications, which occasionally lead to alternative versions of the problem, can be broadly classified as telecommunication networks and cargo delivery practices. This section discusses major findings in these two areas.

In the context of telecommunication networks, data packets are transferred between user nodes through concentrators (servers, switches, multiplexers, etc.) which function as hubs. The user nodes are connected to the concentrators via access networks, whereas the concentrators are connected to each other and/or to a central root node through a backbone network. Different topologies of backbone/access networks such as clique, star, tree, path, ring, and their hybrids, are possible. Objectives considered in the design of telecommunication networks include equipment installation and routing cost as well as reliability (survivability), capacity, and expandability concerns. Klincewicz (1998) provides an extensive review of the literature in this area. More recent works on telecommunication network design are discussed in Gourdin et al. (2002) and Labbé et al. (2005). Motivated by ongoing technological developments, there has been extensive research in this area in the past few years, and this trend is expected to continue for the foreseeable future.

As discussed in the theoretical aspects presented in Sect. 12.4.2, in cargo delivery practice time issues overshadow the cost concerns, resulting in minimax objectives. Cargo delivery networks are designed and managed mostly either with a constraint on the delivery times or with the objective of minimizing the delivery times. Hall (1989) identifies the issues of critical concern in the design of cargo networks as the number of hub terminals, the routing strategies of the transportation modes serving these terminals, and the synchronization of the inflow and outflow at a terminal.

An application to the postal delivery systems was described by Ernst and Krishnamoorthy (1996) based on a data set obtained from the Australian Post. Due to the

possibility of having different modes of transfer in the collection and distribution segments, the cost structure in postal delivery services is different from that in the airline data. To model these differences, Ernst and Krishnamoorthy (1996) propose the use of two additional parameters apart from α . In particular, parameters χ and $\delta \geq \alpha$ correspond to differences in transportation costs in collection and distribution processes, respectively. Use of different factor coefficients allows for the consideration of possible differences in the collection, transportation, and distribution costs that may result due to the use of different transportation modes. Note that this problem is equivalent to the basic problem when $\chi = \delta = 1$.

Nickel et al. (2001) relax the assumption of all hubs being interconnected, and study a public transportation problem that can be denoted as *fixH-cost/multi/U/incomplete/hub-arc*. The authors are the first to address the incomplete hub network version of the problem. They propose four-indexed mixed-integer programming formulations for the single and multiple hub versions. Campbell et al. (2005a, b) exploit this same idea to address the *pH-median/single/U/incomplete/hub-arc* problem. They introduce a new perspective for the solution of this problem. In particular, instead of locating hubs, they locate discounted hub arcs. They develop mixed-integer programming models and two exact algorithms for four different versions of the problem accommodating different objective criteria. They give exact solutions for the Civil Aeronautics Board data.

Motivated by a Federal Express application, Kuby and Gray (1993) model the practical case, in which feeder links consolidate local flows at a convenient node. In their problem, transportation media serving the regional hub are allowed to make multiple stops along their way. This problem can be denoted as *IH-median/single/arc/full/stopover-feeder*. Kuby and Gray considered a single, fixed hub air network problem, and developed a path-based mixed-integer programming formulation to explore the savings provided by the consideration of stopovers and feeders. Later, Yaman et al. (2007) provide integer programming models for *H-cover/single/U/full/{latest-arrival, stopovers}*. The authors propose a different mixed-integer programming formulation, which is strengthened by valid inequalities and lifting. They test the performance of the model on the Turkish highway travel time data. Wasner and Zapfel (2004) suggest that the stopovers can be modeled in the form of a vehicle routing problem.

The modeling complications necessitated by these practical observations suggest that the basic problem proposed by O'Kelly (1986) has implications in a variety of real life applications. The specific needs of these applications provide many ideas that continuously support the evolution of research in this area.

12.6 Conclusion

O'Kelly's classical 1986 paper led to the emergence of a new research area by identifying a connection between location theory and spatial interaction theory. This connection mainly manifests itself in the form of an endogenous interaction that has an impact on both the intensity and cost of flow to be routed through the facili-

ties that are selected as hubs. The problem has been widely studied in the past two decades both from a theoretical and a practical perspective. Theoretical papers in this new area investigated various objectives including *pH-median*, *fixH-cost*, *pH-center* and *H-cover* problems as well as network topologies with fully and partially connected structures. On the practical side, many researchers modeled and solved various real life applications observed in airflow streams, telecommunication networks, cargo delivery systems, and urban transit.

This chapter discussed the most prominent research relevant to both the theoretical and practical aspects of the problem within the framework of a proposed new taxonomy. The interested reader is referred to the excellent review papers written by Campbell (1994b), Klincewicz (1998), Bryan and O'Kelly (1999), Campbell et al. (2002), and Alumur and Kara (2008) for more in-depth coverage of the area.

The authors would like to note that hub location is still a very active research area with many potentially fruitful extensions. One of these extensions is identified by Marianov et al. (1999), who study a multi-allocation hub location problem in the presence of competitors. This interesting problem offers an avenue for further research, as it has not received much attention since. Another important extension is observed by O'Kelly and Bryan (1998) on the fundamental assumption that characterizes the endogenous attraction via the constant scaling factor α . They propose a nonlinear cost function to more accurately model this attraction. Although a few other researchers later improved or modified this function, further research is necessary in this regard. Recall that O'Kelly (1986) proposed two different types of endogenous attraction. In the first type, cited hubs affect the cost of flow, whereas in the second category, the affected parameter is the intensity of flow. The entire literature stemming from this idea focused on the former type and investigated the hub location problem in view of the cost advantages provided by the economies of scale. The latter aspect, which requires modeling of the impact on the intensity of flow, received no attention other than O'Kelly's original proposal.

The authors would like to conclude by emphasizing that these are just a few avenues for future research in this area led by O'Kelly's classical paper (1986). The relevance of the problem to a number of application areas and the wide interest received from many researchers are expected to trigger further developments in the future.

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