WHICH ALGEBRAIC K3 SURFACES DOUBLY COVER AN ENRIQUES SURFACE: A COMPUTATIONAL APPROACH

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

WHICH ALGEBRAIC K3 SURFACES DOUBLY COVER AN ENRIQUES SURFACE: A COMPUTATIONAL APPROACH

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M.S. in Mathematics
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The relationship between K3 Surfaces and Enriques Surfaces is known to mathematicians for the last 30 years. We examined this relationship from a lattice theoretical point of view by looking at transcendental lattice of a K3 surface in the case of Picard number 18 and 19. We established a better way of attacking this problem with the help of a computer assistance.

Keywords: K3 Surfaces, Picard Number, Enriques Surfaces, Lattice.

ÖZET

HANGİ CEBİRSEL K3 YÜZEYLERİ ENRIQUES YÜZEYİNİ ÖRTER: HESAPLAMALI YAKLAŞIM

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K3 Yüzeyleri ile Enriques Yüzeyleri arasındaki ilişki son 30 senedir biliniyor. Biz bu ilişkiyi örgü teorisi bakış açısını kullanarak, Picard sayısı 18 ve 19 olan K3 yüzeylerinin aşkın örgüsüne bakarak inceledik. Bu problemi çözmek için bilgisayar desteği ile daha hızlı bir çözüm yöntemi keşfettik.

Anahtar sözcükler: K3 Yüzeyleri, Picard Sayısı, Enriques Yüzeyleri, Örgü.

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Chapter 1

Introduction

1.1 Historical Background

K3 surface has its name coming from three important figures in Algebraic Geometry, namely, Kodeira, Kahler, Kummer. It was named after these three brilliant minds and the hill on Everest called K2. K2 is one of the hardest hill in Everest to climb. To emphasize the hardness of understanding K3 surfaces, this object is named to be K3, which indicates that it's even harder to climb K2 hill.

The problem has its roots coming from the discussion about classification of surfaces between Enriques and Castelnuovo. It was harder to classify algebraic surfaces since there were much to consider comparing to curves which has only one invariant, namely genus. Enriques and Castelnuovo has further investigated the problem and concluded that for a surface X, it is enough to consider the Kodeira dimension $\kappa(X)$, geometric genus $p_g(X)$ and irregularity $q(X)$ as surface invariants. From this point of view one first looks at the Kodeira dimension $\kappa(X)$ of the surface X. For $\kappa(X) = 0$, we have Enriques, K3, abelian and hyperelliptic surfaces. The difference between these four surfaces are coming the other invariants:

- The surface is called Enriques if $q(X) = p_g(X) = 0$,

- The surface is called K3 if $q(X) = 0,\ p_g(X) = 1$,
- The surface is called hyperelliptic if $q(X) = 1,\ p_g(X) = 0$,
- The surface is called abelian if $q(X) = 2,\ p_g(X) = 1$

It's known before Keum that every Enriques Surface is doubly covered by some K3 surface. A natural follow-up was whether every K3 surface doubly covers an Enriques Surface. Keum studied this problem and he concluded that it is not the case. However, he gave a full description of the criterion for a K3 surface to cover an Enriques Surface from a lattice theoretical point of view:

Theorem 1.1.1 (Keum's Criterion [1]). *Let X be an algebraic K3 surface over $\mathbb{C}$. Assume that $\ell(T_X)+2 \leq \rho(X)$, where $\ell(T_X)$ is the minimal number of generators of T_X and $\rho(X)$ is the Picard number of X. Then, the followings are equivalent:*

- *X admits a fixed point free involution.*
- *There exists a primitive embedding of T_X into Λ^- such that $\mathrm{Im}(T_X)^\perp$ doesn't contain any vector of self intersection -2, where $\Lambda^- = U \oplus U(2) \oplus E_8(2)$.*

Keum studied further with his criterion and he concluded that every algebraic K3 surface X that is Kummer doubly covers an Enriques surface when $17 \leq \rho(X) \leq 20$ [1]. Following Keum's lead Sertoz worked on the problem for the singular K3 surfaces X ($\rho(X) = 20$) and he fully resolved the problem by providing a clever lattice theoretical and quadratic form arguments. He worked on the parity of the entries of transcendental lattice [2]. Then following his ideas, Lee tried to solve the problem for the case $\rho(X) = 19$, he resolved all the case solely depending on the parity of the entries of the transcendental lattice but the problem still remains open for the cases that do not depends solely on them. There are still 7 cases open in when $\rho(X) = 19$.

1.2 Results

In this study, we essentially tried to figure out the relationship between different transcendental lattices of K3 surfaces having the same Picard number. Our main goal was to discover how parities behave if we apply a change of basis. We discovered that some parities can be transformed into one another and they form a nice equivalence class under the change of basis.

Definition 1.2.1. Let T_1 and T_2 be given as

$$T_1 = \begin{pmatrix} 2a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & 2a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & 2a_{n,n} \end{pmatrix}$$

$$T_2 = \begin{pmatrix} 2b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & 2b_{2,2} & \cdots & b_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n,1} & b_{n,2} & \cdots & 2b_{n,n} \end{pmatrix}$$

Where $a_{i,j}$'s and $b_{i,j}$'s are integer for $i, j = 1, 2, \ldots n$. Then, we say T_1 is $\mathbb{Z}$-equivalent to T_2 if $\exists B \in \{M \in Mat_n(\mathbb{Z}) : \det(M) = \pm 1\}$ such that $B^t T_1 B = T_2$.

We tried to characterize the problem in the light of $\mathbb{Z}$-equivalence classes of parities of $a_{i,j}$'s and we were able to reduce the cases to doable small portions of equivalence classes for $\rho(X) = 19, 18$.

In terms of parity we found all possible change of basis matrices(in terms of parities) and we applied this change of basis operation to each possible transcendental lattice(again, in terms of parity) of the same rank. We concluded that there are equivalence classes of such transcendental lattices under the action of change of basis. We have the following results for the case when $\rho(X) = 19$.

Theorem 1.2.2. *Let $\mathcal{L}$ be the set of all even lattices of rank 3. Let $L = \begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix} \in \mathcal{L}$. Then, the set consisting of lattices having different parities* *of a, b, c, d, e, f under the action of change of basis has 5 $\mathbb{Z}$-equivalence classes. These equivalence classes have number of elements $1, 7, 7, 21, 28$ among $2^6 = 64$ different.*

The Theorem 1.2.2 gives a good insight for resolving the case $\rho(X) = 19$.

Theorem 1.2.3. *Let X be an algebraic K3 surface over $\mathbb{C}$. Let*

$$T_X = \begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$$

be the transcendental lattice of X. If a, b, c, d, e, f all are even, then such a K3 surface can doubly cover an Enriques Surface.

Theorem 1.2.4. *For any algebraic K3 surface whose transcendental lattice has parities matching any of the parities given in the Table 4.2, doubly covers an Enriques Surface.*

Theorem 1.2.5. *For any algebraic K3 surfaces whose transcendental lattice has parities matching to any in the Table 4.3 doesn't cover an Enriques surface.*

Theorem 1.2.6. *For any algebraic K3 surfaces whose transcendental lattice has parities matching to any in the Table 4.4 doesn't cover an Enriques surface.*

For the remaining case that is given in the Table 4.5, we provided a family of K3 surfaces that cover an Enriques surface. However, we weren't able bring a closure to whole open part.

Theorem 1.2.7. *Let X be an algebraic K3 surface of Picard number 19 whose transcendental lattice T_X be of the form $T_X = \begin{pmatrix} 2a & a+2 & e \\ a+2 & 2b & e \\ e & e & 2c \end{pmatrix}$ where a, c, e are even, b is odd and $c < 0$, $b > a \geq 4$, e is nonzero. Then, X doubly covers an Enriques Surface.*

We also have some partial results when $\rho(X) = 18$. Even though the number of different parities grows exponentially, we used a similar trick that we employed for the case $\rho(X) = 19$.

Theorem 1.2.8. *Let $\mathcal{L}$ be the set of all even lattices of rank 4. Let* $L = \begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix} \in \mathcal{L}$. *Then, the set consisting of lattices having different parities of $a, b, c, d, e, f, g, h, i$ under the action of change basis has 7 $\mathbb{Z}$-equivalence classes. These equivalence classes have number of elements* $1, 15, 35, 105, 168, 280, 420$ *among* $2^{10} = 1024$ *different.*

The Theorem 1.2.8 gives a good insight for resolving the case $\rho(X) = 18$.

Theorem 1.2.9. *Any algebraic K3 surface of Picard number 18 whose transcendental lattice*

$$T_X = \begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix}$$

has parities matching to any of the parity in $\mathbb{Z}$-equivalence class having b, c, d are even and h, i, j are odd do not cover an Enriques surface.

Theorem 1.2.10. *Any algebraic K3 surface of Picard number 18 whose transcendental lattice*

$$T_X = \begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix}$$

has parities matching to any of the paritiy in $\mathbb{Z}$-equivalence class having a, b, e are odd do not cover an Enriques surface.

Chapter 2

Lattice Theory

Lattice Theory holds a key importance in dealing with some geometric objects. The main objective is to turn algebraic problems, which originally arise from the geometric problems, into arithmetic problems. The lattice theory is a very useful tool to be used in algebraic geometry, as we will see in the next chapter. In this section, for most of the definitions we will refer [3] and [4], [2], [5].

Definition 2.0.1. A symmetric bilinear form on $\mathbb{Z}$-module $\mathcal{X}$ is a map φ : $\mathcal{X} \,\mathrm{x}\, \mathcal{X} \to \mathbb{Z}$ where the following hold.

(1) $\forall a, b \in \mathcal{X}$, $\varphi(a, b) = \varphi(b, a)$

(2) $\forall a, b, c \in \mathcal{X}$, $\alpha \in \mathbb{Z}$, $\varphi(\alpha a + c, b) = \alpha\varphi(a, b) + \varphi(c, b)$

(3) $\forall a, b, c \in \mathcal{X}$, $\alpha \in \mathbb{Z}$, $\varphi(a, \alpha b + c) = \alpha\varphi(a, b) + \varphi(a, c)$

Definition 2.0.2. If $\exists a \in \mathcal{X}\backslash\{0_{\mathcal{X}}\}$ $\forall b \in \mathcal{X}$, $\varphi(a, b) = 0$, then this symmetric bilinear form is called degenerate, is called non-degenerate otherwise.

Definition 2.0.3. A free $\mathbb{Z}$-module L is called a lattice if it has a finite rank and is equipped with a non-degenerate symmetric bilinear form $\varphi : L \times L \to \mathbb{Z}$.

To avoid repetition of non-degenerate symmetric bilinear form, we will refer the φ in the above definition as symmetric bilinear form unless stated otherwise.

Definition 2.0.4. Let L be a lattice with a basis $\{e_1, e_2, \ldots, e_n\}$ and φ be the symmetric bilinear form on the lattice L. Then,

(1) For the rational number q, the lattice $L(q)$ refers to the lattice L with the new symmetric bilinear form $\varphi' = q \cdot \varphi$ with the condition that $\forall \ell, \ell' \in L$, we have $\varphi'(\ell, \ell') \in \mathbb{Z}$.

(2) The matrix defined by $(T_L)_{ij} = \varphi(e_i, e_j)$ is called the matrix representation of the lattice L. Note that $T_L^t = T_L$.

(3) The signature of the lattice L is given by (s_+, s_-) where s_+ is the number of positive eigenvalues of the matrix T_L and s_- is the number of negative eigenvalues of the matrix T_L. An important observation is that such symmetric matrices are diagonalizable in $\mathbb{R}$ [5, p. 285]. Note that no eigenvalue is zero.

(4) The disciriminant of the lattice L, $\text{Discr}(L)$, is $\text{Det}(T_L)$.

(5) A lattice L is called unimodular if $\text{Discr}(L) = \pm 1$.

(6) A lattice L is called even if $\forall \ell \in L$, $\varphi(\ell, \ell)$ takes only even values, and is called odd otherwise.

(7) An element $L \ni \ell = \ell_1 e_1 + \ell_2 e_2 + \ldots + \ell_n e_n$ is called primitive if $\gcd(\ell_1, \ell_2, \ldots, \ell_n) = 1$.

Remark 2.0.5. Let L be a lattice with symmetric bilinear form φ and signature $\{s_+, s_-\}$ where $s_- > 0$. Then, $\exists \ell \in L$ s.t. $\varphi(\ell, \ell) < 0$. Otherwise, L becomes positive definite; hence, it contradicts the fact that $s_- > 0$.

Fact 2.0.6. Let $\ell \in L$ be a primitive element. Then ℓ can be completed into a basis [4, p. 23].

Now, we will give some examples of lattices which will be widely used in understanding K3 surfaces for the upcoming chapters.

Example 2.0.7. (1) U is called the hyperbolic unimodular even lattice of rank 2 with signature $(1, 1)$. The matrix representation of U with respect to the standard basis is given by the following matrix:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(2) E_8 is the unique unimodular even lattice of rank 8 with signature $(0, 8)$. The matrix representation of E_8 with respect to the standard basis is given by the following matrix:

$$\begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{pmatrix}$$

(3) For any $n \in \mathbb{Z}\backslash\{0\}$, $\langle n \rangle$ denotes the lattice $\mathbb{Z}e$ of rank 1 with the symmetric bilinear form φ defined on it by $\varphi(e, e) = n$. Notice that this lattice has signature $(0, 1)$ if n is negative, has signature $(1, 0)$ if n is positive.

Definition 2.0.8. Let L and M be two lattices with symmetric bilinear forms φ_L and φ_M defined on them, respectively, $\{e_1, e_2, \dots e_n\}$ is a basis for the lattice L. A $\mathbb{Z}$-module homomorphism $\phi : L \hookrightarrow M$ is an embedding if it is one to one and

$$\forall i, j \in \{1, 2, \dots n\} \qquad \varphi_L(e_i, e_j) = \varphi_M(\phi(e_i), \phi(e_j))$$

Remark 2.0.9. Since ϕ also preserves the structure, we see that it is also a $\mathbb{Z}$-linear map, thus images of basis elements uniquely determin e the embedding.

Definition 2.0.10. The dual of a lattice L is $L^* = \text{Hom}(L, \mathbb{Z})$, the set of all $\mathbb{Z}$-module homomorphisms from L to $\mathbb{Z}$ making an abelian group with the composition operation.

It is easy to see that a lattice is naturally embedded in its dual, which is called the canonical embedding, by the following map:

$$\phi : L \to \text{Hom}(L, \mathbb{Z}) = L^*$$
$$\ell \rightarrowtail \varphi(\ell, \cdot)$$

In the above mapping, φ is the symmetric bilinear form defined on the lattice L and for each $\ell \in L$, the map $\varphi(\ell, \cdot)$ is indeed a $\mathbb{Z}$-module homomorphism since φ is $\mathbb{Z}$-linear in the second component; thus, making the map $\varphi(\ell, \cdot)$ a $\mathbb{Z}$-module homomorphism.

Definition 2.0.11. Let L, φ, ϕ be the same as above. Then, the finite abelian group defined by the quotient $L^*/\phi(L)$ is called the discriminant group of the lattice L and $l(L)$ denotes the minimum number of generators of the discriminant group of the lattice L.

Remark 2.0.12. The above definition is well defined since the discriminant group is indeed finite abelian group and the order of this group actually divides $|\operatorname{Discr}(L)|$ [3].

Definition 2.0.13. An embedding of lattices $\phi : L \hookrightarrow M$ is called primitive if the quotient $M/\phi(L)$ is a free $\mathbb{Z}$-module. For the sake of simplicity, we will denote the quotient as M/L instead of $M/\phi(L)$.

Theorem 2.0.14. *An embedding $\phi : L \to M$ is primitive if and only if the greatest common divisor of the maximal minors of the matrix representation of the embeddeing is 1 with respect to any choice of basis.*

Proof. For the proof we will refer [2]. □

Notation 2.0.15. Let L and M be two lattices with symmetric bilinear forms φ_L and φ_M defined on them, respectively. By $L \oplus M$, we mean a new lattice with new symmetric bilinear form $\varphi_{L\oplus M}$ such that $\forall l, l' \in L\ \forall m, m' \in M$

$$\begin{aligned} \varphi_{L\oplus M}(l, m) &= 0 \\ \varphi_{L\oplus M}(l, l') &= \varphi_L(l, l') \\ \varphi_{L\oplus M}(m, m') &= \varphi_M(m, m') \end{aligned}$$

Definition 2.0.16. Let L and M be two lattices with symmetric bilinear forms φ_L and φ_M defined on them respectively such that there is a primitive embedding $\phi : L \hookrightarrow M$. Let $K := \{\alpha \in M : \varphi_M(\alpha, \phi(l)) = 0 \quad \forall l \in L\}$. Then K is called the orthogonal complement of L in M and is denoted by $K = (L)^{\perp}_{M}$ and usually M is omitted in the subscript if it is easily understood from the context.

Remark 2.0.17. It is easy to see that in Def. 2.0.16, K is a sublattice of M and we have $\phi(L) \oplus K \subset M$.

The tools we prepared in this section will come in handy for the next two chapters.

Chapter 3

K3 Surfaces and Enriques Surfaces

Before diving into what K3 surfaces and Enriques Surfaces are, we will first introduce some useful theorems that we will be using throughout the chapter. Our main references in this section will be [6], [7], [8], [1], [9], [10], [11].

Theorem 3.0.1 (Poincare Duality)**.** *Let M be an oriented n-manifold. Then we have*

$$H_k(M;\mathbb{Z}) \cong H^{n-k}(M;\mathbb{Z}) \tag{3.1}$$

a canonical isomorphism.

Proof. For the proof we will refer to [9, p. 9]. □

Theorem 3.0.2. *Let M be an oriented n-manifold, T_k be the torsion submodule of $H_k(M;\mathbb{Z})$ and F_k be a the free part of $H_k(M;\mathbb{Z})$ so that*

$$H_k(M;\mathbb{Z}) \cong F_k \oplus T_k$$

Then we have,

$$H^k(M,\mathbb{Z}) \cong F_k \oplus T_{k-1} \tag{3.2}$$

Proof. For the proof we will refer to [9, p. 15,16]. □

Corollary 3.0.3. *By combining the two isomorphism given in 3.1 and 3.2, we have the following:*

$$F_k \cong F_{m-k} \tag{3.3}$$
$$T_k \cong T_{m-k-1} \tag{3.4}$$

Corollary 3.0.4. *For the case when M is an oriented 4-manifold we have the following:*

$$H_0(M;\mathbb{Z}) \cong H^4(M;\mathbb{Z}) \cong \mathbb{Z} \tag{3.5}$$
$$H_1(M;\mathbb{Z}) \cong H^3(M;\mathbb{Z}) \cong F_1 \oplus T_1 \tag{3.6}$$
$$H_2(M;\mathbb{Z}) \cong H^2(M;\mathbb{Z}) \cong F_2 \oplus T_1 \tag{3.7}$$
$$H_3(M;\mathbb{Z}) \cong H^1(M;\mathbb{Z}) \cong F_1 \tag{3.8}$$
$$H_4(M;\mathbb{Z}) \cong H^0(M;\mathbb{Z}) \cong \mathbb{Z} \tag{3.9}$$

Remark 3.0.5. The only torsion that we have above is T_1 which originates from $H_1(M;\mathbb{Z})$. In the case of M is simply connected, we will have no torsion since $H_1(M;\mathbb{Z})$ is the abelianization of $\pi_1(M)$ and its trivial in the case of simply connectedness.

Definition 3.0.6. Let X be a complex manifold of dimension 2. The set of holomorphic line bundles on X forms a group with respect to the tensor product. This group is called the Picard group of X and is denoted by $\operatorname{Pic}(X)$. Also we say X.

Notation 3.0.7. $\rho(X)$ denotes the rank of the Picard group.

Definition 3.0.8. The exponential sequence of sheaves

$$0 \to \mathbb{Z}_X \to \mathcal{O}_X \to \mathcal{O}_X^* \to 0$$

gives rise to the exponential cohomology sequence

$$\ldots \to H^1(\mathbb{Z}_X) \to H^1(\mathcal{O}_X) \to H^1(\mathcal{O}_X^*) \xrightarrow{\delta} H^2(\mathbb{Z}_X) \to \ldots$$

by putting $\ker(\delta) = \operatorname{Pic}^0(X)$, we have $\operatorname{Pic}(X)/\operatorname{Pic}^0(X)$ is isomorphic to a subgroup $\operatorname{NS}(X)$ of $H^2(X;\mathbb{Z})$. $\operatorname{NS}(X)$ is called the Neron-Severi group. .

Definition 3.0.9. The orthogonal complement T_X of $\operatorname{NS}(X)$ inside $H^2(X;\mathbb{Z})$ is called the transcendental lattice of X.

3.1 K3 Surface

Definition 3.1.1. A K3 surface X is a compact, simply connected complex 2-dimensional manifold which has a nowhere vanishing holomorphic 2-form.

In the following chapters, all the K3 surfaces will be algebraic.

Remarks 3.1.2.

(1) Since X is connected, we have $H^0(X;\mathbb{Z}) = \mathbb{Z}$, by Poincare duality we also have $H_4(X;\mathbb{Z}) = \mathbb{Z}$ and since X is oriented, $H^4(X;\mathbb{Z}) = \mathbb{Z}$ as well. By again Poincare duality we have that $H_0(X,\mathbb{Z}) = \mathbb{Z}$.

(2) Since X is simply connected, by using Remark 3.0.5, we have

$$H_1(X;\mathbb{Z}) \cong H^3(X;\mathbb{Z}) = 0$$

(3) Since there is no torsion, $H_2(X;\mathbb{Z})$ has only free part:

$$H_2(X;\mathbb{Z}) \cong H^2(X;\mathbb{Z}) = \mathbb{Z}^k$$

for some $k \in \mathbb{N}$.

Theorem 3.1.3. *For K3 surface X we have that $H^2(M;\mathbb{Z})$ is of rank 22; when equipped with the cup product pairing, it is isomorphic $U^{\oplus 3} \oplus E_8^{\oplus 2}$.*

Proof. For the proof we refer to [6, p.241]. □

Notation 3.1.4. The lattice $U^{\oplus 3} \oplus E_8^{\oplus 2}$ is called the K3-lattice and it is denoted by Λ.

Remark 3.1.5. The K3-lattice Λ has signature $(3, 19)$ and it is unimodular and even.

Let ω_X be the nowhere vanishing holomorphic 2-form on X and $\Lambda_{\mathbb{C}} = \Lambda \otimes \mathbb{C}$, with the extended symmetric $\mathbb{C}$-bilinearity. Choose an isometry $\psi : H^2(X;\mathbb{Z}) \to \Lambda$. Let $\psi_{\mathbb{C}}$ be the extension of ψ to mC. Then the $\psi_{\mathbb{C}}$ image of ω_X inside $\Lambda_{\mathbb{C}}$ determines a line, since the geometric genus of X is $p_g(X) = 1$.

Definition 3.1.6. Let $\mathbb{P}(\Lambda_{\mathbb{C}})$ be the projectivization of $\Lambda_{\mathbb{C}}$. Then the equivalence class $[\omega_X] \subseteq \Lambda_{\mathbb{C}}$ representes a line. The point $[\omega_X] \in \mathbb{P}(\Lambda_{\mathbb{C}})$ is called the period point of the marked K3 surface (X, ψ).

Fact 3.1.7. The set of all period points is denoted by Ω and we have

$$\Omega = \{[\omega] \in \mathbb{P}(\Lambda_{\mathbb{C}}) : (\omega, \omega) = 0, (\omega, \bar{\omega}) > 0\}$$

Theorem 3.1.8 (Weak Torelli Theorem)**.** *Two K3 surfaces X_1, X_2 are isomorphic if and only if there are markings for them ψ_1, ψ_2, respectively, such that the corresponding period points $[\omega_{X_1}]$, $[\omega_{X_2}]$, respectively, are the same.*

Proof. For the proof we refer to [6, p.239,240] □

Theorem 3.1.9. *Let X be a K3 surface, we have the following:*

- $\mathrm{Pic}(X) \cong \mathrm{NS}(X)$
- $\mathrm{NS}(X)$ *is a sublattice of* Λ.
- *The signature of* $\mathrm{NS}(X)$ *is given by* $(1, \rho(X) - 1)$.

Proof. For the proof we refer to [6]. □

Corollary 3.1.10. *Since the transcendental lattice T_X of K3 Surface X is the orthogonal complement of the Neron-Severi group of X inside Λ, we can say that it's signature is given by* $(3-1, 19-(\rho(X)-1)) = (2, 20-\rho(X))$ *and it is even.*

3.2 Enriques Surface

Definition 3.2.1. A smooth projective surface Y with irregularity $q(Y) = 0$ and $K_Y \neq 0$ but $2K_Y = 0$ is called an Enriques surface, where K_Y is the canonical divisor of Y and $q(Y) = h^{0,1} = H^1(Y; \mathcal{O}_Y)$.

Theorem 3.2.2. *Let Y be an Enriques surface. Then we have the followings:*

- $\pi_1(Y) \cong \mathbb{Z}_2$.
- $\mathrm{Pic}(Y) \cong \mathrm{NS}(Y) \cong H^2(Y, \mathbb{Z}) \cong \mathbb{Z}^{10} \oplus \mathbb{Z}_2$
- *The torsion free part $H^2(Y, \mathbb{Z})_f$ of $H^2(Y, \mathbb{Z})$ is isomorphic the lattice $U \oplus E_8$ when equipped with the intersection form.*

Proof. For the proof we refer to [6, p.270]. □

Remark 3.2.3. $H^2(Y, \mathbb{Z})_f$ has signature (1,9).

Theorem 3.2.4. *Let ι be an involution that is fixed point free on a K3 surface X. Then the quotient X/ι gives an Enriques surface.*

Proof. For the proof we will refer to [6] □

Remark 3.2.5. Actually all the Enriques surfaces are universal double covers of some K3 surface.

Let $\{u_1^{(1)}, u_2^{(1)}, u_1^{(2)}, u_2^{(2)}, u_1^{(3)}, u_2^{(3)}, e_1^{(1)}, e_2^{(1)}, \ldots, e_8^{(1)}, e_1^{(2)}, e_2^{(2)}, \ldots, e_8^{(2)}\}$ be a basis for Λ where $u_1^{(i)}, u_2^{(i)}$'s are the standard basis for U fo $i = 1, 2, 3$ and

$e_1^{(j)}, e_2^{(j)}, \ldots, e_8^{(j)}$ are the standard basis for E_8 for $j = 1, 2$. Define an involution $\theta : \Lambda \to \Lambda$ by

$$\begin{aligned} \theta(u_1^{(1)}) &= -u_1^{(1)} & \theta(u_2^{(1)}) &= -u_2^{(1)} \\ \theta(u_1^{(2)}) &= u_1^{(3)} & \theta(u_2^{(2)}) &= u_2^{(3)} \\ \theta(u_1^{(3)}) &= u_1^{(2)} & \theta(u_2^{(3)}) &= u_2^{(2)} \\ \theta(e_1^{(1)}) &= e_1^{(2)} & \theta(e_1^{(2)}) &= e_1^{(1)} \\ \theta(e_2^{(1)}) &= e_2^{(2)} & \theta(e_2^{(2)}) &= e_2^{(1)} \\ &\vdots & &\vdots \\ \theta(e_8^{(1)}) &= e_8^{(2)} & \theta(e_8^{(2)}) &= e_8^{(1)} \end{aligned}$$

We denote the θ-invariant sublattice of Λ to be Λ^+, which is isomorphic to $U(2) \oplus E_8(2)$ and the θ-anti-invariant sublattice of Λ to be Λ^- which is isomorphic $U \oplus U(2) \oplus E_8(2)$. Both are primitive sublattices of Λ and we have $(\Lambda^+)^\perp = \Lambda^-$.

Remark 3.2.6. If $\mathcal{P} : X \to Y$ is a covering projection from a K3 surface X to an Enriques surface Y, then we have,

$$\mathrm{Pic}(X) \supseteq \mathcal{P}^*(\mathrm{Pic}(Y)) \cong \Lambda^-.$$

Thus, we have $\rho(X) \geq 10$ since $\mathrm{NS}(Y) \cong \mathrm{Pic}(Y)$ has a free part of rank 10 and $\mathrm{Pic}(X) \cong \mathrm{NS}(X)$.

Lemma 3.2.7 ([7]). *Let X be the K3 cover of an Enriques surface Y. Let $\iota : X \to X$ be the involution such that $X/\iota \cong Y$. Then $\exists\, \psi : \Lambda \to \Lambda$ an isometry such that the following diagram*

$$\begin{array}{ccc} H^2(X;\mathbb{Z}) & \xrightarrow{\iota^*} & H^2(X;\mathbb{Z}) \\ \downarrow{\scriptstyle\psi} & & \downarrow{\scriptstyle\psi} \\ \Lambda & \xrightarrow{\theta} & \Lambda \end{array}$$

commutes. In particular the ψ induces an isomorphism $\bar{\psi} : \mathcal{P}^*(\mathrm{Pic}(Y) \to \Lambda^+$ where $\mathcal{P}$ is the covering projection. Also, ψ is unique up to

$$\Gamma = \{\gamma \in O(\Lambda) : \gamma \circ \psi = \psi \circ \gamma\}$$

where $O(\Lambda)$ denotes the self isometries on Λ.

Definition 3.2.8. A pair (Y, ψ) where Y is an Enriques surface and ψ is an isometry as in Lemma 3.2.7. Since we have that $\iota^* \omega_X = -\omega_X$, the period point $[\omega_X]$ of the marked K3 surface (X, ψ) is called the period point of (Y, ψ) and is contained in

$$\Omega^- = \left\{ [\omega] \in \mathbb{P}(\Lambda_{\mathbb{C}}^-) : (\omega, \omega) = 0, (\omega, \bar{\omega}) > 0 \right\}$$

where $\Lambda_{\mathbb{C}}^- = \Lambda^- \otimes \mathbb{C}$.

Now see that the assignment of an Enriques Surface Y to $[\omega_X] \in \Omega^-/\Gamma^-$ where $\Gamma^- = \{\gamma|_{\Lambda^-} : \gamma \in \Gamma\}$ is well defined, because by taking the quotient we are putting all the different possible isometries into the same equivalence class. The assignment is actually called the period map for Enriques Surfaces and is known to be injective by [7], [8]. Also the following theorem is due to [7], [8] and [6] is about the conditions which the assignment is surjective.

Theorem 3.2.9. *Let*

$$\Omega_0^- = \{[\omega] \in \Omega^- : < \omega, \delta > \neq 0, \forall \delta \in \Lambda^- \textit{ with } (\delta, \delta) = 2\}$$

$$\mathcal{D}_0 = \Omega_0^-/\Gamma^-.$$

Then, every point of $\mathcal{D}_0$ *is the period point of an Enriques Surface, thus making any point on* Ω_0^- *is the period point of some marked K3 surface.*

3.3 Criterion for Which K3 Surfaces Doubly Cover an Enriques Surface

After finding out that all the Enriques surfaces are doubly covered by a K3 surface, it is only natural to ask whether the converse is true, that is whether all K3 surfaces doubly cover an Enriques Surfaces. Unfortunately, that is not the case. In his article [1], Keum gives a criterion for which K3 surfaces doubly cover an Enriques Surface. In tis section we will closely follow his proof for the criterion.

Theorem 3.3.1 (Keum's Criterion [1]). *Let X be an algebraic K3 surface over $\mathbb{C}$. Assume that $\ell(T_X)+2 \leq \rho(X)$, where $\ell(T_X)$ is the minimal number of generators of T_X. Then, the followings are equivalent:*

(1) X admits a fixed point free involution.

(2) There exists a primitive embedding of T_X into Λ^- such that $\mathrm{Im}(T_X)^\perp$ doesn't contain any vector of self intersection -2, where $\Lambda^- = U \oplus U(2) \oplus E_8(2)$.

Corollary 3.3.2. *Using the fact that $\ell(T_X)+2 \leq \rho(X)$ is true when $\rho(X) \geq 12$ and by Theorem 3.2.4, we can say that when $\rho(X) \geq 12$, the theorem becomes a K3 surface X doubly covers an Enriques surface if and only if (1) is satisfied.*

We will prove the theorem when $\rho(X) \geq 12$.

Proof. Let X be an algebraic K3 surface over $\mathbb{C}$. Assume that X admits a fixed point free involution. Then we know by Theorem 3.2.4 that there is a fixed point free involution $\iota : X \to X$ such that X/ι corresponds to an Enriques Surface Y. Let $\mathcal{P} : X \to Y$ be the covering projection. Then by using the commutative diagram in Lemma 3.2.7, we have that $\psi(\mathrm{Pic}(X)) \supseteq \psi(\mathcal{P}(\mathrm{Pic}(Y))) = \Lambda^+$. Since $T_X \perp \mathrm{NS}(X) = \mathrm{Pic}(X)$, we have that $\psi(T_X) \subseteq \Lambda^-$. $\psi|_{T_X}$ is primitive since T_X is primitive and ψ is an isometry, thus preserves the primitivity. Now assume on the contrary that $\psi(T_X)^\perp$ has a vector ϑ such that $<\vartheta, \vartheta>= -2$. Then the class $d = \psi^{-1}(\vartheta)$ belongs to $\mathrm{Pic}(X)$. Using Riemann-Roch, we have that $h^0(\mathcal{O}_X(d)) + h^0(\mathcal{O}_X(-d)) = \frac{1}{2} < d, d > +2 + h^0(\mathcal{O}_X(-d)) = 1 + h^1(\mathcal{O}_X(-d))$, thus either d, or $-d$ is effective but an effective class cannot be ι^*-anti-invariant, this contradicts the fact that such a ϑ do exists. Now for the converse, now assume that there exists a primitive embedding $\psi : T_X \hookrightarrow \Lambda^-$. ψ can easily be extend to an isometry $\tilde{\psi}$ on Λ. Now the period point $[\omega_X] \in \Omega_0^-$ and thus by Theorem 3.2.9 we have that $[\omega_X]$ is a period point for the marked K3 surface $(X, \tilde{\psi})$. So, such K3 surface doubly covers an Enriques surfaces hence admits a fixed point free involution. $\square$

For the next chapter, we will use this theorem to determine which K3 surfaces of Picard rank 19 and 18 doubly cover an Enriques Surface.

Chapter 4

Which K3 Surfaces Doubly Covers an Enriques Surface

4.1 The Case When Picard Number of the K3 Surface is 20

This case has been handled by Sertoz in his paper [2]. Sertoz did realize that the problem can be solved by using the parity argument. Sertoz fully characterized which K3 surfaces doubly cover an Enriques surface with the help of quadratic forms. His techniques were followed by Lee in his paper [12], and he solved nearly all of the possible cases and only a small portion of the problem was left open. In this chapter, we will mainly focus on the case when Picard number is 19 and 18. We will offer a better way of attacking the problem in terms of equivalences of parities of entries.

4.2 The Case When Picard Number of the K3 Surface is 19

4.2.1 Transcendental Lattice of a K3 Surface of Picard Number 19

Let X be a K3 surface with $\rho(X) = 19$. Then the transcendental lattice of X, denoted by T_X is given by

$$T_X = \begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix} \tag{4.1}$$

where a, b, c, d, e, f are integers and T_X has signature $(2, 1)$. Here, observe that the matrix T_X can have 64 different form regarding to the parities of a, b, c, d, e, f. By changing the basis of the lattice, we can reach to some other parities of a, b, c, d, e, f. Remember the change of basis can be given by:

$$B^t T_X B$$

where B is a 3 by 3 integral matrix whose determinant is ± 1. So by using an appropriate change of basis matrix, we can easily change the parity of a given transcendental lattice. However, this is not an easy task and we certainly may not change every given transcendental lattice to any given parity.

A quick look at the matrix B will tell us that there are $2^9 = 512$ different possibilities for the parity of its entries but we should also force the fact that any given B has to have determinant ± 1. A quick computer search in the set $\{M \in \mathrm{Mat}_{3x3}(\{0,1\})$ will give us 168 different matrices whose determinant is odd and have different parities of entries. Luckily all of these 168 matrices having different parities have determinants ± 1 (For further information on those matrices refer to Appendix A). So, now all is left to check how these 168 matrices can change the parities of the entries of a given transcendental lattice. With further computer assistance we were able to find 5 equivalent classes of parities of transcendental lattices under the action of change of basis:

Theorem 4.2.1. *Let $\mathcal{L}$ be the set of all even lattices of rank 3. Let* $L = \begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix} \in \mathcal{L}$. *Then, the set consisting of lattices having different parities of a, b, c, d, e, f under the action of change basis has 5 $\mathbb{Z}$-equivalence classes. These equivalence classes have number of elements $1, 7, 7, 21, 28$ among $2^6 = 64$ different.*

We will reconstruct some of Lee's work [12] and follow his steps. Then, we will take a different turn and reduce the number of cases regarding parities. Let X be a K3 surface whose transcendental lattice T_X ids given by

$$\begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$$

and has a basis $\{x, y, z\}$ and $c < 0$. Let $\{u_1, u_2\}$ be a basis for U, $\{v_1, v_2\}$ be a basis for $U(2)$. We will now prove a lemma that c can be made negative.

Lemma 4.2.2. *Any matrix M of signature $(2, 1)$ with entries* $\begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$ *can be made into* $\begin{pmatrix} 2a' & d' & e' \\ d' & 2b' & f' \\ e' & f' & 2c' \end{pmatrix}$ *where $c' < 0$ by using change of basis.*

Proof. If $c < 0$, we are done. If not, let $\{x, y, z\}$ be a basis for the given matrix. If $a < 0$ we use the change of basis

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

and we will get the new matrix

$$\begin{pmatrix} 2c & f & e \\ f & 2b & d \\ e & d & 2a \end{pmatrix}.$$

If $b < 0$ we use the change of basis with

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

and we get the new matrix

$$\begin{pmatrix} 2a & e & d \\ e & 2c & f \\ d & f & 2b \end{pmatrix}.$$

If $a, b, c \geq 0$ then take $m \in M$ such that $m \cdot m < 0$. Such element is possible because of the signature. So $m = \alpha x + \beta y + \gamma z$. Let $\theta = \gcd(\alpha, \beta, \gamma)$. See that $m' = \dfrac{\alpha}{\theta} x + \dfrac{\beta}{\theta} y + \dfrac{\gamma}{\theta} z$ is a primitive element in M, thus m' can be extended to be a basis for M [5]. Let $\{k, l, m'\}$ be the new basis for M. Now we will have Gramm Matrix of the lattice M

$$\begin{pmatrix} <k,k> & <k,l> & <k,m'> \\ <k,l> & <l,l> & <l,m'> \\ <k,m'> & <l,m'> & <m',m'> \end{pmatrix}$$

where $<\cdot,\cdot>$ is the symmetric bilinear form on M, hence making $c' < 0$. □

Lemma 4.2.3. *Any matrix M of signature $(2, 1)$ with entries* $\begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$ *where $c < 0$ can be made into* $\begin{pmatrix} 2a' & d' & e' \\ d' & 2b' & f' \\ e' & f' & 2c' \end{pmatrix}$ *where $b', c' < 0$ and $a' = a$ by using change of basis.*

Proof. For the proof we refer to [12]. □

Lemma 4.2.4. *Any matrix M of signature $(2, 1)$ with entries* $\begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$ *where $b, c < 0$ can be made into* $\begin{pmatrix} 2a' & d' & e' \\ d' & 2b' & f' \\ e' & f' & 2c' \end{pmatrix}$ *where $a', b', c' < 0$ by using change of basis.*

Proof. For the proof we refer to [12]. $\square$

4.2.2 Which K3 Surfaces Doubly Covers an Enriques Surface

Lemma 4.2.5. *Let* $T = \begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$ *be a matrix of signature (2,1) with* $a, b, c < 0$ *ad matches to a parity given in the Table 4.2. Then* T *is* $\mathbb{Z}$*-equivalent to a matrix* $T' = \begin{pmatrix} 2a' & d' & e' \\ d' & 2b' & f' \\ e' & f' & 2c' \end{pmatrix}$ *where* a', b', c', d', e' *are even and* f' *is odd. Moreover,* $c < 0$.

Proof. All the necessary transformations are explicitly given in Appendix B. $\square$

a	b	c	d	e	f
Even	Even	Even	Even	Even	Even

Table 4.1: First $\mathbb{Z}$-Equivalence Class

Theorem 4.2.6. *Let* X *be an algebraic K3 surface over* $\mathbb{C}$*. Let*

$$T_X = \begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$$

be the transcendental lattice of X*. If* a, b, c, d, e *are even, and* $c < 0$ *then such a K3 surface can doubly cover an Enriques Surface.*

Proof. Consider the following mapping from T_X to $U \oplus U(2) \oplus E_8(2)$.

$$\varphi(x) = \frac{a}{2}v_1 + v_2 \tag{4.2}$$

$$\varphi(y) = bu_1 + u_2 + \frac{d}{2}v_1 \tag{4.3}$$

$$\varphi(z) = fu_1 + +\frac{e}{2}v_1 + \omega \tag{4.4}$$

where $\omega \in E_8(2)$ is primitive(this is ensured by [3]) and $\omega^2 = 2c$ First check that it is indeed an embedding;

$$\varphi(x) \cdot \varphi(x) = 2\frac{a}{2}v_1v_2 = 2a \tag{4.5}$$

$$\varphi(y) \cdot \varphi(y) = 2bu_1u_2 = 2b \tag{4.6}$$

$$\varphi(z) \cdot \varphi(z) = \omega^2 = 2c \tag{4.7}$$

$$\varphi(x) \cdot \varphi(y) = \frac{d}{2}v_1v_2 = d \tag{4.8}$$

$$\varphi(x) \cdot \varphi(z) = \frac{e}{2}v_1v_2 = e \tag{4.9}$$

$$\varphi(y) \cdot \varphi(z) = fu_1u_2 = f \tag{4.10}$$

So we verified that φ is an embedding into $U \oplus U(2) \oplus E_8(2)$. Now we need to prove the primitiveness of the embedding. Observe that the matrix representation of the embedding is

$$\begin{pmatrix} 0 & b & f \\ 0 & 1 & 0 \\ a/2 & d/2 & e/2 \\ 1 & 0 & 0 \\ 0 & 0 & \omega_1 \\ \vdots & \vdots & \vdots \\ 0 & 0 & \omega_8 \end{pmatrix}$$

where $\omega = \sum_{i=1}^{8} \omega_i e_i$ and $\{e_1, \ldots, e_8\}$ is a basis for $E_8(2)$. Primitiveness of ω implies that $\gcd(\omega_1, \ldots, \omega_8) = 1$. See also that the maximal minors consisting of the second, forth and another row from fifth to twelfth row is $\det \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \omega_i \end{vmatrix}$ $= -\omega_i$, forcing the gcd of the maximal minors to be 1. Thus, the embedding is primitive by Theorem 2.0.14. Now using Keum's criterion we should look for (-2)-self intersection vector in the orthogonal complement of the image. Let $\theta = \theta_1 u_1 + \theta_2 u_2 + \theta_3 v_1 + \theta_4 v_2 + \omega' \in \varphi(T_X)^\perp \subseteq U \oplus U(2) \oplus E_8(2)$. Since θ is in orthogonal to the image of φ, we must have $\varphi(y) \cdot \theta = 0$. That is, $(bu_1 + u_2 + dv_1) \cdot (\theta_1 u_1 + \theta_2 u_2 + \theta_3 v_1 + \theta_4 v_2 + \omega') = b\theta_2 + \theta_1 + 2d\theta_4 = 0$. Which implies that $b\theta_2 + \theta_1$ is even, hence, θ_1 is even since b is even. Thus, $\theta^2 = 2\theta_1\theta_2 + 4\theta_3\theta_4 + (\omega')^2$ is divisible by 4, since θ_1 is even, and $(\omega')^2$ is divisible by 4. Hence for any such θ it can not have square equals -2. So, by Keum's criterion, we can conclude that such K3 surface doubly covers an Enriques surface. □

Theorem 4.2.7. *A K3 surface whose transcendental lattice has parities matching any of the parities given in the Table 4.1 doubly covers an Enriques Surface.*

Proof. There is only one parity in the Table 4.1, and it is a, b, c, d, e, f are even. So if $c < 0$, then we are done. If not, we apply the Lemma 4.2.2 and we will still have the same parities but we will have a new c which will be negative. Then by Theorem 4.2.6, we say that such a K3 surface doubly covers an Enriques surface. □

Theorem 4.2.8. *A K3 surface whose transcendental lattice has parities matching any of the parities given in the Table 4.2 doubly covers an Enriques Surface.*

Proof.

- **Case:1** $a, b, c < 0$
  Let
$$T_X = \begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$$
  with $a, b, c < 0$. Then, by using Lemma 4.2.5, we have that T_X can be transformed into
$$T'_X = \begin{pmatrix} 2a' & d' & e' \\ d' & 2b' & f' \\ e' & f' & 2c' \end{pmatrix}$$
  where a', b', c', d', e' are even, f' is odd and $c < 0$. So by Theorem 4.2.6 we are done.

- **Case:2** At least one of a, b, c is greater than or equal to 0

  (i) If $c \geq 0$, apply Lemmas 4.2.2, 4.2.3, 4.2.4 consecutively.

  (ii) If $c < 0$ and $b \geq$, apply Lemmas 4.2.3, 4.2.4 consecutively.

  (iii) If $c, b < 0$ and $a \geq$, apply Lemma 4.2.4.

  In each (i), (ii), (iii) we will end up with a matrix

  $$T''_X = \begin{pmatrix} 2a'' & d'' & e'' \\ d'' & 2b'' & f'' \\ e'' & f'' & 2c'' \end{pmatrix}$$

  where $a'', b'', c'' < 0$. Thus, reducing the problem to Case 1.

□

a	b	c	d	e	f
Even	Even	Even	Even	Even	Odd
Even	Even	Even	Even	Odd	Even
Even	Even	Even	Even	Odd	Odd
Even	Even	Even	Odd	Even	Even
Even	Even	Even	Odd	Even	Odd
Even	Even	Even	Odd	Odd	Even
Even	Even	Odd	Even	Even	Odd
Even	Even	Odd	Even	Odd	Even
Even	Even	Odd	Even	Odd	Odd
Even	Even	Odd	Odd	Odd	Odd
Even	Odd	Even	Even	Even	Odd
Even	Odd	Even	Odd	Even	Even
Even	Odd	Even	Odd	Even	Odd
Even	Odd	Even	Odd	Odd	Odd
Even	Odd	Odd	Odd	Odd	Even
Odd	Even	Even	Even	Odd	Even
Odd	Even	Even	Odd	Even	Even
Odd	Even	Even	Odd	Odd	Even
Odd	Even	Even	Odd	Odd	Odd
Odd	Even	Odd	Odd	Even	Odd
Odd	Odd	Even	Even	Odd	Odd

Table 4.2: Second $\mathbb{Z}$-Equivalence Class

4.2.3 Which K3 Surfaces Don't Doubly Cover an Enriques Surface

Let X be a K3 surfaces with $\rho(X) = 19$. Here, we want to find for which parities of a, b, c, d, e, f there will be no embedding of T_X into $U \oplus U(2) \oplus E_8(2)$.

Let $\{x, y, z\}$ be a basis for T_X, $\{u_1, u_2\}$ be a basis for U, $\{v_1, v_2\}$ be a basis for $U(2)$ and ω_1, ω_2, $\omega_3 \in E_8(2)$. To show that, assume on the contrary that $\varphi : T_X \to U \oplus U(2) \oplus E_8(2)$ is an embedding given by

$$\varphi(x) = a_1u_1 + a_2u_2 + a_3v_1 + a_4v_2 + w_1 \tag{4.11}$$

$$\varphi(y) = b_1u_1 + b_2u_2 + b_3v_1 + b_4v_2 + w_2 \tag{4.12}$$

$$\varphi(z) = c_1u_1 + c_2u_2 + c_3v_1 + c_4v_2 + w_3 \tag{4.13}$$

where a_i's, b_i's and c_i's are integers and $\omega_1, \omega_2, \omega_3 \in E_8(2)$. Since φ is an embedding we must have the followings;

$$\varphi(x) \cdot \varphi(x) = 2a_1a_2 + 4a_3a_4 + w_1^2 = 2a \tag{4.14}$$

$$\varphi(y) \cdot \varphi(y) = 2b_1b_2 + 4b_3b_4 + w_2^2 = 2b \tag{4.15}$$

$$\varphi(z) \cdot \varphi(z) = 2c_1c_2 + 4c_3c_4 + w_3^2 = 2c \tag{4.16}$$

$$\varphi(x) \cdot \varphi(y) = a_1b_2 + a_2b_1 + 2a_3b_4 + 2a_4b_3 + w_1w_2 = d \tag{4.17}$$

$$\varphi(x) \cdot \varphi(z) = a_1c_2 + a_2c_1 + 2a_3c_4 + 2a_4c_3 + w_1w_3 = e \tag{4.18}$$

$$\varphi(y) \cdot \varphi(z) = b_1c_2 + b_2c_1 + 2b_3c_4 + 2b_4c_3 + w_2w_3 = f \tag{4.19}$$

Theorem 4.2.9. *For any algebraic K3 surfaces whose transcendental lattice has parities matching to any in the Table 4.3 doesn't cover an Enriques surface.*

Proof. We will prove for the case a, b, c is even and d, e, f is odd. The rest in the Table 4.3 can be transformed into this case by using change of basis. Now assume a, b, c is even and d, e, f is odd and assume that we have an embedding given us by the Equations 4.11, 4.12 and 4.13 ,where $w_1, w_2, w_3 \in E_8(2)$ Now since a, b, c are even, we must have by Equations 4.14, 4.15, 4.16

$$2a_1a_2 + 4a_3a_4 + w_1^2 \equiv 0 \mod 4$$

$$2b_1b_2 + 4b_3b_4 + w_2^2 \equiv 0 \mod 4$$

$$2c_1c_2 + 4c_3c_4 + w_3^2 \equiv 0 \mod 4$$

So, we have a_1a_2, b_1b_2, c_1c_2 is divisible by two. Without loss of generality say that a_1 is even. Then by using Equation 4.17 and 4.18 we have

$$a_2b_1 \equiv 1 \mod 2$$

$$a_2c_1 \equiv 1 \mod 2$$

Thus, we have a_2, b_1, c_1 are odd and that makes c_2, b_2 even. But this is a contradiction since by Equation 4.19, f must be odd but

$$b_1c_2 + b_2c_1 + w_2w_3 \equiv 0 \mod 2$$

So, there is no such an embedding, hence such a K3 surface cannot doubly cover an Enriques surface. □

Theorem 4.2.10. *For any K3 algebraic surfaces whose transcendental lattice has parities matching to any in the Table 4.4 doesn't cover an Enriques surface.*

Proof. Assume that a, d, e are even and b, c, f are odd. Then by Equation 4.15, 4.16 we have

$$2b_1b_2 \equiv 0 \mod 4$$

$$2c_1c_2 \equiv 0 \mod 4$$

So, a_1a_2 is even and b_1b_2, c_1c_2 are odd, making b_1, b_2, c_1, c_2 odd. Hence, we have $b_1c_2+b_2c_1$ is even. But we have $b_1c_2+b_2c_1+w_2w_3$ is odd since f is odd by Equation 4.19. So this is a contradiction, hence, a K3 surface having such parities in its transcendental lattice doesn't cover an Enriques surface. The rest of the Table 4.4 can be transformed into the this case by using change of basis. This completes the proof. □

a	b	c	d	e	f
Even	Even	Even	Odd	Odd	Odd
Even	Even	Odd	Odd	Even	Even
Even	Even	Odd	Odd	Even	Odd
Even	Even	Odd	Odd	Odd	Even
Even	Odd	Even	Even	Odd	Even
Even	Odd	Even	Even	Odd	Odd
Even	Odd	Even	Odd	Odd	Even
Even	Odd	Odd	Even	Odd	Even
Even	Odd	Odd	Even	Odd	Odd
Even	Odd	Odd	Odd	Even	Even
Even	Odd	Odd	Odd	Even	Odd
Even	Odd	Odd	Odd	Odd	Odd
Odd	Even	Even	Even	Even	Odd
Odd	Even	Even	Even	Odd	Odd
Odd	Even	Even	Odd	Even	Odd
Odd	Even	Odd	Even	Even	Odd
Odd	Even	Odd	Even	Odd	Odd
Odd	Even	Odd	Odd	Even	Even
Odd	Even	Odd	Odd	Odd	Even
Odd	Even	Odd	Odd	Odd	Odd
Odd	Odd	Even	Even	Even	Odd
Odd	Odd	Even	Even	Odd	Even
Odd	Odd	Even	Odd	Even	Odd
Odd	Odd	Even	Odd	Odd	Even
Odd	Odd	Even	Odd	Odd	Odd
Odd	Odd	Odd	Even	Even	Odd
Odd	Odd	Odd	Even	Odd	Even
Odd	Odd	Odd	Odd	Even	Even

Table 4.3: Third $\mathbb{Z}$-Equivalence Class

a	b	c	d	e	f
Even	Odd	Odd	Even	Even	Odd
Odd	Even	Odd	Even	Odd	Even
Odd	Odd	Even	Odd	Even	Even
Odd	Odd	Odd	Even	Odd	Odd
Odd	Odd	Odd	Odd	Even	Odd
Odd	Odd	Odd	Odd	Odd	Even
Odd	Odd	Odd	Odd	Odd	Odd

Table 4.4: Fourth $\mathbb{Z}$-Equivalance Class

a	b	c	d	e	f
Even	Even	Odd	Even	Even	Even
Even	Odd	Even	Even	Even	Even
Even	Odd	Odd	Even	Even	Even
Odd	Even	Even	Even	Even	Even
Odd	Even	Odd	Even	Even	Even
Odd	Odd	Even	Even	Even	Even
Odd	Odd	Odd	Even	Even	Even

Table 4.5: Fifth $\mathbb{Z}$-Equivalence Class

For the remaining case there is not a solution at the moment. Lee hasn't found any solution regarding the cases but only showed that all them are equal [12]. We will give a family of transcendental lattices that will correspond to K3 surfaces that can doubly cover an Enriques surface. The next theorem does show that there are some family of cases whose transcendental lattice has parity matches to one in the Table 4.5.

4.2.4 Missing Cases

Theorem 4.2.11. *Let X be a K3 surface of Picard number 19 and whose transcendental lattice T_X is of the form* $T_X = \begin{pmatrix} 2a & a+2 & e \\ a+2 & 2b & e \\ e & e & 2c \end{pmatrix}$ *where a, c, e are even, b is odd and $c < 0$, $b > a \geq 4$, e is nonzero. Then, X doubly covers an Enriques Surface.*

Proof. Let U be the even unimodular lattice whose signature is (1,1) and E_8 be the even unimodular lattice whose signature is (0,8) and $\{x, y, z\}$ be a basis for T_X, $\{u_1, u_2\}$, $\{v_1, v_2\}$ be the standard basis of U and $U(2)$, respectively. Define the map $\varphi : T_X \to U \oplus U(2) \oplus E_8(2)$ by

$$\varphi(x) = v_1 + \frac{a}{2}v_2$$
$$\varphi(y) = (b-2)u_1 + u_2 + v_1 + v_2$$
$$\varphi(z) = \frac{e}{2}v_2 + \omega$$

where $\omega^2 = 2c$ and ω is a primitive element in $E_8(2)$. Existence of such primitive element is ensured by the corollary 1.12.3 in Nikulin's paper [3]. Now, firstly we will show that this is an embedding, then show that it is a primitive embedding. See that,

$$\varphi(x)\varphi(x) = 2 \cdot \frac{a}{2}v_1v_2 = 2a$$
$$\varphi(y)\varphi(y) = 2(b-2)u_1u_2 + 2v_1v_2 = 2b - 4 + 4 = 2b$$
$$\varphi(z)\varphi(z) = \omega^2 = 2c$$
$$\varphi(x)\varphi(y) = v_1v_2 + \frac{a}{2}v_1v_2 = 2 + a$$
$$\varphi(x)\varphi(z) = \frac{e}{2}v_1v_2 = e$$
$$\varphi(y)\varphi(z) = \frac{e}{2}v_1v_2 = e$$

Hence, showing that φ is an embedding. Now, see that the embedding matrix of

φ is of the form

$$\begin{pmatrix} 0 & b-2 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ a/2 & 1 & e/2 \\ 0 & 0 & \omega_1 \\ \vdots & \vdots & \vdots \\ 0 & 0 & \omega_8 \end{pmatrix}$$

where $\omega = \sum_{i=1}^{8} \omega_i e_i$ and $\{e_1, \ldots, e_8\}$ is a basis for $E_8(2)$. Primitiveness of ω implies that $\gcd(\omega_1, \ldots, \omega_8) = 1$. See also that the maximal minors consisting of the second, third and another row from fifth to twelfth row is $\det \begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \omega_i \end{vmatrix} = -\omega_i$, forcing the greatest common divisors of the maximal minors of the embedding matrix to be 1. Hence, using the Theorem 2.0.14 [2], we can say that φ is a primitive embedding. Now take any element $s = x_1u_1 + x_2u_2 + x_3v_1 + x_4v_2 + \omega' \in \varphi(T_X)^{\perp} \subset U \oplus U(2) \oplus E_8(2)$ where $\omega' \in E_8(2)$, we have the following equations:

$$s \cdot \varphi(x) = 2x_4 + ax_3 = 0 \tag{4.20}$$

$$s \cdot \varphi(y) = (b-2)x_2 + x_1 + 2x_3 + 2x_4 = 0 \tag{4.21}$$

$$s \cdot \varphi(z) = ex_3 + \omega \cdot \omega' = 0 \tag{4.22}$$

Now, assume that $\omega \cdot \omega' = 0$. Then, we have, $x_3 = 0$ (since e is nonzero), $x_4 = 0$,$x_1 = (2-b)x_2$. So, $s^2 = 2x_1x_2 + 4x_3x_4 + (\omega')^2 = 2(2-b)x_2^2 + (\omega')^2 < -2$ since $b > 4$ by assumption and $(\omega')^2 = -4n$ for non-negative integer n.

Now, let $\omega \cdot \omega' \neq 0$. By equation 4.22 have $x_3 = \dfrac{-\omega \cdot \omega'}{e}$, since $x_3 \in \mathbb{Z}$ we must have $e \mid -\omega \cdot \omega'$, thus $\omega \cdot \omega' = ek$ for some nonzero integer k. So, $x_3 = -k$. Putting this in Equation 4.20 we get $2x_4 = ak$, $x_4 = \dfrac{ak}{2} \in \mathbb{Z}$ since a is even by the assumption. Continuing with Equation 4.21 and putting all the info we have we get $(b-2)x_2 + x_1 + ak - 2k = 0$, hence; $x_1 = (2-b)x_2 + k(2-a)$.

Define $m := (2-b)$ and $n := k(2-a)$. See that $m < 0$ since $b > a \geq 4$ and $n > 0$ if $k < 0$ and $n < 0$ if $k > 0$. So we have the following graphs that explains the relation between x_1 and x_2.

for $k < 0$

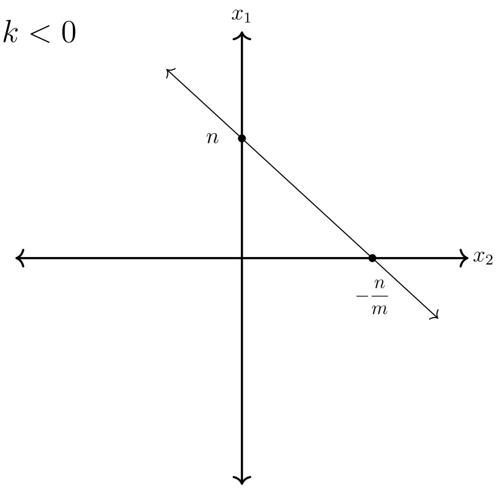

Figure 4.2.1: $k < 0$

for $k > 0$

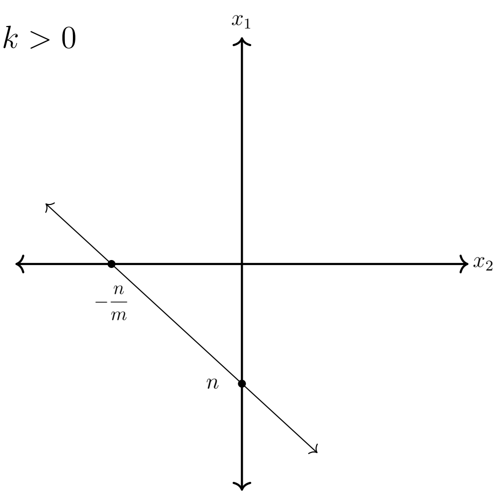

Figure 4.2.2: $k > 0$

Now, we will show that for both cases $s^2 < -2$. First, assume that $k < 0$. See that $n \geq 2$ because $a \geq 4$ and $k < 0$. If $x_1 \geq n$ we have $x_2 \leq 0$. So, $s^2 = 2x_1x_2 + 4x_3x_4 + (\omega')^2 < -2$ because $s^2 \leq 4x_2 - 2ak^2 + (\omega')^2 < -2$ since a is even and is greater than 4, k is nonzero and $x_2 \leq 0$. Similarly, if $x_2 \geq -\dfrac{n}{m}$ we have $x_1 \leq 0$. So $s^2 < -2$ since $-2ak^2 \leq -8$, $2x_1x_2 \leq 0$ and $(\omega')^2 \leq 0$. Now, consider only the points (x_2, x_1) on the first quadrant and on the line in Figure 4.2.1, see that $2x_1x_2 \leq -\dfrac{2n^2}{m}$. So $s^2 = 2x_1x_2 + 4x_3x_4 + (\omega')^2 \leq -\dfrac{2n^2}{m} - 2ak^2 + (\omega')^2 \leq$

$-\dfrac{2n^2}{m} - 2ak^2$ since $(\omega')^2 \leq 0$. Now putting $m = (2-b)$ and $n = k(2-a)$ in the inequality, we have $s^2 \leq \dfrac{2k^2(a-2)^2}{b-2} - 2ak^2 \leq \dfrac{2k^2(a-2)^2}{a-2} - 2ak^2 = 2ak^2 - 4k^2 - 2ak^2 = -4k^2 \leq -4$. Now we showed that $s^2 < -2$ if $k < 0$.

Assume that $k > 0$, we will have the graph in Figure 4.2.2. Observe that we have the same inequality for the points (x_2, x_1) are on the third quadrant as in the above case when the points (x_2, x_1) are on the first quadrant(the sign of k doesn't change anything because we have k^2 in both cases.). Again using a symmetric argument as above it can be shown that $s^2 < -2$ for the rest as well. Hence, by using Keum's Criterion [1] we have that such K3 surfaces doubly covers an Enriques surface. □

4.3 The Case When Picard Number of the K3 Surface is 18

4.3.1 Transcendental Lattice of a K3 Surface of Picard Number 18

Let X be a K3 surface with $\rho(X) = 19$. Then the transcendental lattice of X, denoted by T_X is given by

$$T_X = \begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix} \tag{4.23}$$

where $a, b, c, d, e, f, g, h, i, j$ are integers and T_X has signature $(2, 2)$. Here, again observe that the matrix T_X can have 1024 different form regarding to the parities of $a, b, c, d, e, f, g, h, i, j$. By changing the basis of the lattice, we can reach to some other parities of $a, b, c, d, e, f, g, h, i, j$. Running the similar computer search for this case as we did with the case 19, we find 7 $\mathbb{Z}$-Equivalence Classes:

Theorem 4.3.1. *Let $\mathcal{L}$ be the set of all even lattices of rank 4. Let* $L = \begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix} \in \mathcal{L}$. *Then, the set consisting of lattices having different parities of $a, b, c, d, e, f, g, h, i$ under the action of change basis has 7 $\mathbb{Z}$-equivalence classes. These equivalence classes have number of elements* $1, 15, 35, 105, 168, 280, 420$ *among* $2^{10} = 1024$ *different.*

Lemma 4.3.2. *Any integral matrix*

$$\begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix}$$

can be made into

$$\begin{pmatrix} 2a' & e & 0 & 0 \\ e' & 2b' & h' & 0 \\ 0 & h' & 2c' & j' \\ 0 & 0 & j' & 2d' \end{pmatrix}$$

by using change of basis.

Proof. We refer to [13] for the proof. □

Remark 4.3.3. This is actually quite a known result in quadratic forms for centuries.

It becomes harder to solve the problem as the Picard number gets lower, because the variables in the transcendental lattice grows exponentially. Here we will only show the $\mathbb{Z}$-equivalence classes that do not cover an Enriques surface.

4.3.2 Which K3 Surfaces of Picard Number 18 Do Not Cover an Enriques Surface

Theorem 4.3.4. *Any algebraic K3 surface of Picard number 18 whose transcendental lattice*

$$T_X = \begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix}$$

has parities matching to any of the paritiy in $\mathbb{Z}$-equivalence class having b, c, d are even and h, i, j are odd do not cover an Enriques surface.

Remark 4.3.5. The $\mathbb{Z}$-equivalence class having a, b, c, d, e, f, g are even, h, i, j are odd has 420 different parities in the class and the $\mathbb{Z}$-equivalence class having a, b, c, d, e, f are even, g, h, i, j are odd has 280 different parities in the class and the $\mathbb{Z}$-equivalence class having a, b, c, d are even, e, f, g, h, i, j are odd has 168 different parities in the class Thus, this theorem singlehandedly solves 868 cases.

Proof. Let X be a K3 surface of Picard number 18 whose transcendental lattice $T_X = \begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix}$ and T_X has basis x, y, z, t. Assume on the contrary that there exist an embedding $\varphi : T_X \hookrightarrow U \oplus U(2) \oplus E_8(2)$ given by

$$\varphi(x) = a_1u_1 + a_2u_2 + a_3v_1 + a_4v_2 + w_1 \tag{4.24}$$

$$\varphi(y) = b_1u_1 + b_2u_2 + b_3v_1 + b_4v_2 + w_2 \tag{4.25}$$

$$\varphi(z) = c_1u_1 + c_2u_2 + c_3v_1 + c_4v_2 + w_3 \tag{4.26}$$

$$\varphi(t) = d_1u_1 + d_2u_2 + d_3v_1 + d_4v_2 + w_4 \tag{4.27}$$

Since by assumption φ is an embedding, we have the followings

$$\varphi(x) \cdot \varphi(x) = 2a_1a_2 + 4a_3a_4 + w_1^2 = 2a \tag{4.28}$$

$$\varphi(y) \cdot \varphi(y) = 2b_1b_2 + 4b_3b_4 + w_2^2 = 2b \tag{4.29}$$

$$\varphi(z) \cdot \varphi(z) = 2c_1c_2 + 4c_3c_4 + w_3^2 = 2c \tag{4.30}$$

$$\varphi(t) \cdot \varphi(t) = 2d_1d_2 + 4d_3d_4 + w_4^2 = 2d \tag{4.31}$$

$$\varphi(x) \cdot \varphi(y) = a_1b_2 + a_2b_1 + 2a_3b_4 + 2a_4b_3 + w_1w_2 = e \tag{4.32}$$

$$\varphi(x) \cdot \varphi(z) = a_1c_2 + a_2c_1 + 2a_3c_4 + 2a_4c_3 + w_1w_3 = f \tag{4.33}$$

$$\varphi(x) \cdot \varphi(t) = a_1d_2 + a_2d_1 + 2a_3d_4 + 2a_4d_3 + w_1w_4 = g \tag{4.34}$$

$$\varphi(y) \cdot \varphi(z) = b_1c_2 + b_2c_1 + 2b_3c_4 + 2b_4c_3 + w_2w_3 = h \tag{4.35}$$

$$\varphi(y) \cdot \varphi(t) = b_1d_2 + b_2d_1 + 2b_3d_4 + 2b_4d_3 + w_2w_4 = i \tag{4.36}$$

$$\varphi(z) \cdot \varphi(t) = c_1d_2 + c_2d_1 + 2c_3d_4 + 2c_4d_3 + w_3w_4 = j \tag{4.37}$$

By inspecting Equation 4.29 we have that b_1 or b_2, by Equation 4.30 c_1 or c_2, by Equation 4.31 d_1 or d_2 are even. Assume that b_1 is even(for the second case follows in parentheses b_2 is even). Then by Equation 4.35 we have that b_2, c_1 are odd (b_1, c_2 are odd) since h is odd. Thus making c_2 even (c_1 even). By Equation 4.37 d_2 is odd (d_1 is odd) since j is odd, making d_1 even (d_2 is even). But now we have $b_1d_2 + b_2d_1$ is even in both cases, which contradicts the fact that i is even by Equation 4.36. Thus, such embedding do not exists. □

Theorem 4.3.6. *Any algebraic K3 surface of Picard number 18 whose transcendental lattice*

$$T_X = \begin{pmatrix} 2a & e & f & g \\ e & 2b & h & i \\ f & h & 2c & j \\ g & i & j & 2d \end{pmatrix}$$

has parities matching to any of the parity in $\mathbb{Z}$-equivalence class having a, b, e are odd do not cover an Enriques surface.

Remark 4.3.7. This $\mathbb{Z}$-equivalence class has 35 different parities.

Proof. By observing Equation 4.28 and 4.29, we have that a_1, a_2, b_1, b_2 are all odd, thus making the sum $a_1b_2 + a_2b_1$ even. But this contradicts the fact that e is odd, hence such an embedding is not possible. □

After showing the ones that doesn't have any embedding we have left with the $\mathbb{Z}$-equivalence classes with 1,15,105 elements. Sonel partially solved some parts of the remaining cases in his thesis [14]. For those cases we will not offer any new solutions. Curious readers can have a look at Sonel's thesis.

4.3.3 Closing Discussion

The problem can be reduced by using computer search on possible $\mathbb{Z}$-Equivalence classes as we did in the two cases. This drastically reduces the time to spend on proving the cases as it grows exponentially without reducing them. It stays linear -at least by the observation we made with cases 20,19,18- and gives better possibilities for solving it for the other cases.

Bibliography

[1] J. Keum, "Every algebraic kummer surface is the k3-cover of an enriques surface," *Nagoya Math J*, pp. 99 – 110, 1990.

[2] S. Sertoz, "Which singualar k3 surfaces cover an enriques surface," *Proc. Amer. Math. Soc.*, pp. 43 – 50, 2005.

[3] V. V. Nikulin, "Integral symmetric bilinear forms and some of their applications," *Math. USSR Isvetija*, pp. 103 – 167, 1980.

[4] C. Lekkerkerker and P. Gruber, *Geometry of Numbers*. North Holland, 1987.

[5] G. Strang, *Linear Algebra and Its Applications*. Wellesley Cambridge Press, 2009.

[6] W. Barth, C. Peters, and A. v. d. Ven, *Compact complex surfaces / W. Barth, C. Peters, A. Van de Ven*. Springer-Verlag Berlin ; New York, 1984.

[7] E. Horikawa, "On the periods of enriques surface," *Math. Ann.*, vol. 234, 1978.

[8] E. Horikawa, "On the periods of enriques surface," *Math. Ann.*, vol. 235, 1978.

[9] A. Scorpan, *The Wild World of 4-manifolds*. American Mathematical Soc., 2005.

[10] D. Huybrechts, *Lectures on K3 Surfaces*. Cambridge Studies in Advanced Mathematics, Cambridge University Press, 2016.

[11] A. Hatcher, *Algebraic Topology*. Cambridge: Cambridge Univ. Press, 2000.

[12] K. Lee, "Which k3 surfaces with picard number 19 cover an enriques surface," *Bull. Korean Math. Soc.*, vol. 49(1), 2012.

[13] N. Jacobson, *Lectures in abstract algebra: Basic Concepts*. Springer, 1976.

[14] S. Sonel, "Which algebraic k3 surfaces cover an enriques surface," Sept. 2018.

Appendix A

Change of Basis Transformation Matrices

Here is the list of change of basis matrices used in Chapter 4:

$$T_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad T_2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad T_3 = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad T_4 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_5 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad T_6 = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad T_7 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad T_8 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_9 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{10} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{11} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{12} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_{13} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{14} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{15} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{16} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_{17} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{18} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{19} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{20} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_{21} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{22} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{23} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{24} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_{25} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{26} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{27} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{28} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T_{29} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{30} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{31} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{32} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T_{33} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{34} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{35} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{36} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T_{37} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{38} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{39} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{40} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T_{41} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{42} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{43} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{44} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T_{45} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{46} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{47} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{48} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T_{49} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{50} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{51} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{52} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$$T_{53} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{54} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{55} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{56} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$$T_{57} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{58} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{59} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{60} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$T_{61} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{62} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{63} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{64} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$T_{65} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{66} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{67} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{68} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$T_{69} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{70} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{71} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad T_{72} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$T_{73} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{74} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{75} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{76} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_{77} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{78} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{79} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{80} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_{81} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{82} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{83} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{84} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_{85} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{86} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{87} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{88} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_{89} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{90} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{91} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{92} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_{93} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{94} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{95} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{96} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_{97} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{98} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{99} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{100} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$T_{101} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{102} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{103} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{104} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$T_{105} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{106} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{107} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{108} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$T_{109} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{110} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{111} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{112} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$T_{113} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{114} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{115} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{116} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$T_{117} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{118} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{119} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad T_{120} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$T_{121} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{122} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{123} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{124} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$T_{125} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{126} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{127} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{128} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$T_{129} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{130} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{131} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{132} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$T_{133} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{134} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{135} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{136} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$T_{137} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{138} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{139} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{140} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$T_{141} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{142} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{143} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad T_{144} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$T_{145} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{146} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{147} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{148} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$T_{149} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{150} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{151} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{152} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$T_{153} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{154} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{155} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{156} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$T_{157} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{158} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{159} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{160} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$T_{161} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{162} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{163} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{164} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$T_{165} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{166} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{167} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad T_{168} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Appendix B

Making c Even and Negative

Let T_X be given by

$$\begin{pmatrix} 2a & d & e \\ d & 2b & f \\ e & f & 2c \end{pmatrix}$$

where a, b, c are all negative and for any change of basis matrix T with $\det(T) = \pm 1$ let T'_X be

$$\begin{pmatrix} 2a' & d' & e' \\ d' & 2b' & f' \\ e' & f' & 2c' \end{pmatrix}$$

the matrix after applying change of basis T to T_X. In this part we will give the exact method of how to change the parities given in Table 4.2 to the parities given in top position of the Table while preserving the negativity of the entry at (3,3).

- **a is even, b is even, c is even, d is odd, e is even, f is even**

  Using the change of basis $T = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = c$, $b' = b$, $c' = a$, $d' = f$, $e' = e$, $f' = d$.

- **a is odd, b is even, c is even, d is odd, e is even, f is even**

Using the change of basis $T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = c$, $b' = a + b + d$, $c' = b$, $d' = e + f$, $e' = f$, $f' = 2b + d$.

- **a is even, b is even, c is even, d is even, e is odd, f is even**

  Using the change of basis $T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = b$, $b' = c$, $c' = a$, $d' = f$, $e' = d$, $f' = e$.

- **a is odd, b is even, c is even, d is even, e is odd, f is even**

  Using the change of basis $T = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = b$, $b' = a+c+e$, $c' = c$, $d' = d+f$, $e' = f$, $f' = 2c+e$.

- **a is even, b is even, c is even, d is odd, e is odd, f is even**

  Using the change of basis $T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = b + c + f$, $b' = b$, $c' = a$, $d' = 2b + f$, $e' = d + e$, $f' = d$.

- **a is odd, b is even, c is even, d is odd, e is odd, f is even**

  Using the change of basis $T = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = b + c + f$, $b' = a + b + d$, $c' = b$, $d' = 2b + d + e + f$, $e' = 2b + f$, $f' = 2b + d$.

- **a is even, b is odd, c is even, d is odd, e is even, f is even**

  Using the change of basis $T = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = c$, $b' = a + b + d$, $c' = a$, $d' = e + f$, $e' = e$, $f' = 2a + d$.

- **a is even, b is even, c is even, d is odd, e is even, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + c + e$, $b' = b$, $c' = a$, $d' = d + f$, $e' = 2a + e$, $f' = d$.

- **a is even, b is even, c is even, d is even, e is odd, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + b + d$, $b' = c$, $c' = a$, $d' = e + f$, $e' = 2a + d$, $f' = e$.

- **a is odd, b is even, c is even, d is odd, e is odd, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + b + c + d + e + f$, $b' = a + b + d$, $c' = b$, $d' = 2a + 2b + 2d + e + f$, $e' = 2b + d + f$, $f' = 2b + d$.

- **a is even, b is odd, c is even, d is even, e is even, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a$, $b' = b + c + f$, $c' = c$, $d' = d + e$, $e' = e$, $f' = 2c + f$.

- **a is even, b is odd, c is even, d is odd, e is even, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + c + e$, $b' = a + b + d$, $c' = a$, $d' = 2a + d + e + f$, $e' = 2a + e$, $f' = 2a + d$.

- **a is odd, b is odd, c is even, d is even, e is odd, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + b + d$, $b' = a + c + e$, $c' = c$, $d' = 2a + d + e + f$, $e' = e + f$, $f' = 2c + e$.

- **a is even, b is odd, c is even, d is odd, e is odd, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + b + c + d + e + f$, $b' = a + b + d$, $c' = a$, $d' = 2a + 2b + 2d + e + f$, $e' = 2a + d + e$, $f' = 2a + d$.

- **a is even, b is even, c is odd, d is even, e is odd, f is even**

  Using the change of basis $T = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = b$, $b' = a + c + e$, $c' = a$, $d' = d + f$, $e' = d$, $f' = 2a + e$.

- **a is even, b is odd, c is odd, d is odd, e is odd, f is even**

  Using the change of basis $T = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = b + c + f$, $b' = a + b + d$, $c' = a$, $d' = 2b + d + e + f$, $e' = d + e$, $f' = 2a + d$.

- **a is even, b is even, c is odd, d is even, e is even, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a$, $b' = b + c + f$, $c' = b$, $d' = d + e$, $e' = d$, $f' = 2b + f$.

- **a is odd, b is even, c is odd, d is odd, e is even, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + c + e$, $b' = a + b + d$, $c' = b$, $d' = 2a + d + e + f$, $e' = d + f$, $f' = 2b + d$.

- **a is even, b is even, c is odd, d is even, e is odd, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + b + d$, $b' = a + c + e$, $c' = a$, $d' = 2a + d + e + f$, $e' = 2a + d$, $f' = 2a + e$.

- **a is even, b is even, c is odd, d is odd, e is odd, f is odd**

  Using the change of basis $T = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, we can transform T_X to the matrix T'_X where $a' = a + b + c + d + e + f$, $b' = b$, $c' = a$, $d' = 2b + d + f$, $e' = 2a + d + e$, $f' = d$.

Appendix C

Equivalence Classes of Picard Number 18

C.0.1 Equivalence Class for the Case 18 of 1 Element

a	b	c	d	e	f	g	h	i	j
Even	Even	Even	Even	Even	Even	Even	Even	Even	Even

C.0.2 Equivalence Class for the Case 18 of 105 Elements

a	b	c	d	e	f	g	h	i	j
Even	Even	Even	Even	Even	Even	Even	Even	Even	Odd
Even	Even	Even	Even	Even	Even	Even	Even	Odd	Even
Even	Even	Even	Even	Even	Even	Even	Even	Odd	Odd
Even	Even	Even	Even	Even	Even	Even	Odd	Even	Even
Even	Even	Even	Even	Even	Even	Even	Odd	Even	Odd
Even	Even	Even	Even	Even	Even	Even	Odd	Odd	Even
Even	Even	Even	Even	Even	Even	Odd	Even	Even	Even
Even	Even	Even	Even	Even	Even	Odd	Even	Even	Odd
Even	Even	Even	Even	Even	Even	Odd	Even	Odd	Even
Even	Even	Even	Even	Even	Even	Odd	Even	Odd	Odd
Even	Even	Even	Even	Even	Odd	Even	Even	Even	Even
Even	Even	Even	Even	Even	Odd	Even	Even	Even	Odd
Even	Even	Even	Even	Even	Odd	Even	Odd	Even	Even
Even	Even	Even	Even	Even	Odd	Even	Odd	Even	Odd
Even	Even	Even	Even	Even	Odd	Odd	Even	Even	Even
Even	Even	Even	Even	Even	Odd	Odd	Odd	Odd	Even
Even	Even	Even	Even	Odd	Even	Even	Even	Even	Even
Even	Even	Even	Even	Odd	Even	Even	Even	Odd	Even
Even	Even	Even	Even	Odd	Even	Even	Odd	Even	Even
Even	Even	Even	Even	Odd	Even	Even	Odd	Odd	Even
Even	Even	Even	Even	Odd	Even	Odd	Even	Even	Even
Even	Even	Even	Even	Odd	Even	Odd	Odd	Even	Odd
Even	Even	Even	Even	Odd	Odd	Even	Even	Even	Even
Even	Even	Even	Even	Odd	Odd	Even	Even	Odd	Odd
Even	Even	Even	Even	Odd	Odd	Odd	Even	Even	Even
Even	Even	Even	Odd	Even	Even	Even	Even	Even	Odd
Even	Even	Even	Odd	Even	Even	Even	Even	Odd	Even
Even	Even	Even	Odd	Even	Even	Even	Even	Odd	Odd
Even	Even	Even	Odd	Even	Even	Even	Odd	Odd	Odd

a	b	c	d	e	f	g	h	i	j
Even	Even	Even	Odd	Even	Even	Odd	Even	Even	Even
Even	Even	Even	Odd	Even	Even	Odd	Even	Even	Odd
Even	Even	Even	Odd	Even	Even	Odd	Even	Odd	Even
Even	Even	Even	Odd	Even	Even	Odd	Even	Odd	Odd
Even	Even	Even	Odd	Even	Odd	Odd	Even	Even	Odd
Even	Even	Even	Odd	Even	Odd	Odd	Odd	Odd	Odd
Even	Even	Even	Odd	Odd	Even	Odd	Even	Odd	Even
Even	Even	Even	Odd	Odd	Even	Odd	Odd	Odd	Odd
Even	Even	Even	Odd	Odd	Odd	Odd	Even	Odd	Odd
Even	Even	Odd	Even	Even	Even	Even	Even	Even	Odd
Even	Even	Odd	Even	Even	Even	Even	Odd	Even	Even
Even	Even	Odd	Even	Even	Even	Even	Odd	Even	Odd
Even	Even	Odd	Even	Even	Even	Even	Odd	Odd	Odd
Even	Even	Odd	Even	Even	Odd	Even	Even	Even	Even
Even	Even	Odd	Even	Even	Odd	Even	Even	Even	Odd
Even	Even	Odd	Even	Even	Odd	Even	Odd	Even	Even
Even	Even	Odd	Even	Even	Odd	Even	Odd	Even	Odd
Even	Even	Odd	Even	Even	Odd	Odd	Even	Even	Odd
Even	Even	Odd	Even	Even	Odd	Odd	Odd	Odd	Odd
Even	Even	Odd	Even	Odd	Odd	Even	Odd	Even	Even
Even	Even	Odd	Even	Odd	Odd	Even	Odd	Odd	Odd
Even	Even	Odd	Even	Odd	Odd	Odd	Odd	Even	Odd
Even	Even	Odd	Odd	Even	Even	Even	Odd	Odd	Even
Even	Even	Odd	Odd	Even	Odd	Odd	Even	Even	Even
Even	Even	Odd	Odd	Even	Odd	Odd	Odd	Odd	Even
Even	Even	Odd	Odd	Odd	Odd	Odd	Odd	Odd	Even
Even	Odd	Even	Even	Even	Even	Even	Even	Odd	Even
Even	Odd	Even	Even	Even	Even	Even	Odd	Even	Even
Even	Odd	Even	Even	Even	Even	Even	Odd	Odd	Even
Even	Odd	Even	Even	Even	Even	Even	Odd	Odd	Odd
Even	Odd	Even	Even	Odd	Even	Even	Even	Even	Even

a	b	c	d	e	f	g	h	i	j
Even	Odd	Even	Even	Odd	Even	Even	Even	Odd	Even
Even	Odd	Even	Even	Odd	Even	Even	Odd	Even	Even
Even	Odd	Even	Even	Odd	Even	Even	Odd	Odd	Even
Even	Odd	Even	Even	Odd	Even	Odd	Even	Odd	Even
Even	Odd	Even	Even	Odd	Even	Odd	Odd	Odd	Odd
Even	Odd	Even	Even	Odd	Odd	Even	Odd	Even	Even
Even	Odd	Even	Even	Odd	Odd	Even	Odd	Odd	Odd
Even	Odd	Even	Even	Odd	Odd	Odd	Odd	Odd	Even
Even	Odd	Even	Odd	Even	Even	Even	Odd	Even	Odd
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C.0.3 Equivalence Class for the Case 18 of 420 Elements

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C.0.4 Equivalence Class for the Case 18 of 280 Elements

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C.0.5 Equivalence Class for the Case 18 of 168 Elements

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C.0.6 Equivalence Class for the Case 18 of 15 Elements

a	b	c	d	e	f	g	h	i	j
Even	Even	Even	Odd	Even	Even	Even	Even	Even	Even
Even	Even	Odd	Even	Even	Even	Even	Even	Even	Even
Even	Even	Odd	Odd	Even	Even	Even	Even	Even	Even
Even	Odd	Even	Even	Even	Even	Even	Even	Even	Even
Even	Odd	Even	Odd	Even	Even	Even	Even	Even	Even
Even	Odd	Odd	Even	Even	Even	Even	Even	Even	Even
Even	Odd	Odd	Odd	Even	Even	Even	Even	Even	Even
Odd	Even	Even	Even	Even	Even	Even	Even	Even	Even
Odd	Even	Even	Odd	Even	Even	Even	Even	Even	Even
Odd	Even	Odd	Even	Even	Even	Even	Even	Even	Even
Odd	Even	Odd	Odd	Even	Even	Even	Even	Even	Even
Odd	Odd	Even	Even	Even	Even	Even	Even	Even	Even
Odd	Odd	Even	Odd	Even	Even	Even	Even	Even	Even
Odd	Odd	Odd	Even	Even	Even	Even	Even	Even	Even
Odd	Odd	Odd	Odd	Even	Even	Even	Even	Even	Even

C.0.7 Equivalence Class for the Case 18 of 35 Elements

a	b	c	d	e	f	g	h	i	j
Even	Even	Odd	Odd	Even	Even	Even	Even	Even	Odd
Even	Odd	Even	Odd	Even	Even	Even	Even	Odd	Even
Even	Odd	Odd	Even	Even	Even	Even	Odd	Even	Even
Even	Odd	Odd	Odd	Even	Even	Even	Even	Odd	Odd
Even	Odd	Odd	Odd	Even	Even	Even	Odd	Even	Odd
Even	Odd	Odd	Odd	Even	Even	Even	Odd	Odd	Even
Even	Odd	Odd	Odd	Even	Even	Even	Odd	Odd	Odd
Odd	Even	Even	Odd	Even	Even	Odd	Even	Even	Even
Odd	Even	Odd	Even	Even	Odd	Even	Even	Even	Even
Odd	Even	Odd	Odd	Even	Even	Odd	Even	Even	Odd
Odd	Even	Odd	Odd	Even	Odd	Even	Even	Even	Odd
Odd	Even	Odd	Odd	Even	Odd	Odd	Even	Even	Even
Odd	Even	Odd	Odd	Even	Odd	Odd	Even	Even	Odd
Odd	Odd	Even	Even	Odd	Even	Even	Even	Even	Even
Odd	Odd	Even	Odd	Even	Even	Odd	Even	Odd	Even
Odd	Odd	Even	Odd	Odd	Even	Even	Even	Odd	Even
Odd	Odd	Even	Odd	Odd	Even	Odd	Even	Even	Even
Odd	Odd	Even	Odd	Odd	Even	Odd	Even	Odd	Even
Odd	Odd	Odd	Even	Even	Odd	Even	Odd	Even	Even
Odd	Odd	Odd	Even	Odd	Even	Even	Odd	Even	Even
Odd	Odd	Odd	Even	Odd	Odd	Even	Even	Even	Even
Odd	Odd	Odd	Even	Odd	Odd	Even	Odd	Even	Even
Odd	Odd	Odd	Odd	Even	Even	Odd	Even	Odd	Odd
Odd	Odd	Odd	Odd	Even	Odd	Even	Odd	Even	Odd
Odd	Odd	Odd	Odd	Even	Odd	Odd	Odd	Odd	Even
Odd	Odd	Odd	Odd	Even	Odd	Odd	Odd	Odd	Odd
Odd	Odd	Odd	Odd	Odd	Even	Even	Odd	Odd	Even
Odd	Odd	Odd	Odd	Odd	Even	Odd	Odd	Even	Odd
Odd	Odd	Odd	Odd	Odd	Even	Odd	Odd	Odd	Odd
Odd	Odd	Odd	Odd	Odd	Odd	Even	Even	Odd	Odd

a	b	c	d	e	f	g	h	i	j
Odd	Odd	Odd	Odd	Odd	Odd	Even	Odd	Odd	Odd
Odd	Odd	Odd	Odd	Odd	Odd	Odd	Even	Even	Even
Odd	Odd	Odd	Odd	Odd	Odd	Odd	Even	Odd	Odd
Odd	Odd	Odd	Odd	Odd	Odd	Odd	Odd	Even	Odd
Odd	Odd	Odd	Odd	Odd	Odd	Odd	Odd	Odd	Even