Reduction of Thermal Strain Induced Rate Error for Navigation Grade Fiber Optic Gyroscope

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Abstract: This paper presents an 8-fold reduction of thermal sensitivity of a navigation grade fiber optic gyroscope (FOG) coil. We used an advanced thermal modeling of a fiber optic gyroscope coil to obtain the strain fields along the coil and calculate the rate error. Strain field calculations are validated by optical time domain reflectometer (OTDR) measurements of a laboratory fiber optic gyroscope setup. We separated the strain analysis into axial and radial gradients along the fiber coil. We designed a new fiber coil with reduced axial strain sensitivity and run simulations to calculate its strain coefficients and the rate error. Simulations show more than 8-fold reduction in rate error along with experimental results which show nearly 4-fold reduction.

1. Introduction

Fiber optic gyroscopes are highly precise and widely used inertial sensors for angular rotation measurement. One of the main precision limits is the thermally induced rate error.\footnote{Thermal fluctuations create nonreciprocal phase shift between counterpropagating waves in the fiber coil. Nonreciprocal phase shift is defined by Shupe \cite{2}. Combining his equation with Sagnac relation, bias error $\Omega_S(t)$ for a FOG can be written as:

$$\Omega_S(t) = \frac{n}{LD} \left( \frac{\partial n}{\partial T} + n \alpha \right) \int_0^L \hat{T}(z,t)(L-2z)dz$$

where $L$ is the length of the fiber, $D$ is the diameter of fiber coil, $n$ is the refractive index of fiber core, $\frac{\partial n}{\partial T}$ is the temperature coefficient of $n$, $\alpha$ is the thermal expansion coefficient of fiber core and $\hat{T}(z,t)$ is the temperature field time derivative and $z$ is the fiber portion where temperature fluctuates.}

In literature, there are several works indicating the relation between strain and rate error \cite{3} and trying to reduce the total strain as a whole \cite{6}. In this paper, in order to reduce the elastooptic effects, we obtained the strain distribution of a fiber coil under temperature variation.

These two main bias error mechanisms are additive. So the total bias error is

$$\Omega_{Total}(t) = \Omega_S(t) + \Omega_{EO}(t)$$

In our previous work, we presented a model for thermally induced rate errors for a fiber optic gyroscope with quadrupole winding \cite{1}. In that work, we have developed a simulation environment for calculation of the errors of the fiber coil and verified the simulation environment with experimental results. We concluded that elastooptic effect is much stronger than the pure Shupe error.
2. Analysis and Simulation

In order to separate the analysis into axial and radial gradients, we represent the Shupe equation in cylindrical coordinates so we exchange the \( z \) variable to \( s \) to avoid any misunderstanding. The Shupe equation can be written as follows where \( \beta_0 = \frac{2\pi}{\lambda_0} \) is free space propagation constant and \( \lambda_0 \) is the wavelength of the light wave at vacuum.

\[
\Delta\phi(t) = \frac{\beta_0}{c} \left( \frac{\partial n}{\partial T} + \frac{n\alpha}{c} \right) \int_0^L \tilde{T}(s,t)(L-2s)ds \quad (4)
\]

In this equation, parameter \( dz \) states the location of the fiber segment, which is three dimensional in space. Fiber coil is cylindrical in space and each fiber segment can be represented in radius, azimuth and height \((r, \theta, z)\). Fiber coil usually consists of fiber core, cladding, coating and adhesive. Light travels only in fiber core so integral is taken through fiber core which is only continuous through azimuth and discrete through radius and height.

\[
\Delta\phi(t) = \frac{\beta_0}{c} \left( \frac{\partial n}{\partial T} + \frac{n\alpha}{c} \right) \sum_{i=1}^{N_{\text{Radius}}} \sum_{j=1}^{N_{\text{Height}}} \int_0^{2\pi} \tilde{T}(r_i, \theta, z_j, t)(L-2s)d\theta d\Delta s \quad (5)
\]

where \( \Delta s = \frac{L}{N} \), number of turns is \( N = N_{\text{Radius}} \times N_{\text{Radius}} \), the fiber segment distance is \( s = r_i \theta + s_{0ij} \) and \( s_{0ij} \) is the initial fiber segment distance of the \((i, j)\)th layer.

The fiber optic gyroscope works on the Sagnac Principle. Sagnac principle states that the rotation of the fiber loop creates a phase difference between the counter-propagating waves. The diameter parameter increases for every layer through radius. Therefore, using the Sagnac relation total bias error due to Shupe effect is

\[
\Omega(t) = \frac{n}{AN} N_{\text{Radius}} \frac{1}{\pi} \sum_{i=1}^{N_{\text{Radius}}} \sum_{j=1}^{N_{\text{Height}}} \int_0^{2\pi} \left( \frac{\partial n}{\partial T} + \frac{n\alpha}{c} \right) \tilde{T}(r_i, \theta, z_j, t)(L-2s)d\theta d\Delta s \quad (6)
\]

We can rewrite \( \Delta s = \frac{L}{N} = \frac{\pi dx}{N} = \pi d_i \).

Fiber coil can be approximated to be axially symmetric. Fiber coil mechanical structure generally designed to be axial symmetric for thermal variations. For the approximation generally holds for the thermal models. For vibrational inputs axial asymmetry should be taken into account. With the approximation integral over \( \theta \) vanishes. However, another parameter to represent the midpoint of each turn should be added to weighting function.

\[
\Omega(t) = \frac{n}{N} \sum_{i=1}^{N_{\text{Radius}}} \int_0^{2\pi} \left( \frac{\partial n}{\partial T} + \frac{n\alpha}{c} \right) \tilde{T}(r_i, \theta, z_j, t) \left[ L - 2 \left( s + \frac{l_i}{2} \right) \right] \quad (7)
\]

Refractive index temperature dependence, \( \frac{\partial n}{\partial T} \), is a parameter defined by silica inside the fiber. Thermal expansion coefficient, \( \alpha \), is determined by whole fiber coil. Fiber core, cladding, coating and adhesive determines the strain inside the fiber core, which should be taken as the thermal expansion coefficient.

Equation should be rewritten to cover the elastooptical bias drift.

\[
\Omega(t) = \frac{n}{N} \sum_{i=1}^{N_{\text{Radius}}} \frac{1}{d_i} \sum_{j=1}^{N_{\text{Height}}} \left( \hat{n}(r_i, z_j, t) + n\hat{\varepsilon}_r(r_i, z_j, t) \right) (L - 2s - l_i) \quad (8)
\]

where \( \hat{n} \) is determined by both temperature and the strain derivatives.

Finally,

\[
\Omega(t) = \frac{n}{N} \sum_{i=1}^{N_{\text{Radius}}} \frac{1}{d_i} \sum_{j=1}^{N_{\text{Height}}} \left( \frac{\partial n}{\partial T} \tilde{T}(r_i, z_j, t) + \frac{\partial n}{\partial \varepsilon_r} \hat{\varepsilon}_r(r_i, z_j, t) + \frac{\partial n}{\partial \varepsilon_z} \hat{\varepsilon}_z(r_i, z_j, t) \right) (L - 2s - l_i) \quad (9)
\]

\[
\frac{\partial n}{\partial T} \tilde{T}(r_i, z_j, t) + \frac{\partial n}{\partial \varepsilon_r} \hat{\varepsilon}_r(r_i, z_j, t) + \frac{\partial n}{\partial \varepsilon_z} \hat{\varepsilon}_z(r_i, z_j, t) \] is the phase error component due to refractive index change where \( n\hat{\varepsilon}_r(r_i, z_j, t) \) is the phase error component due to path length elongation.

We obtained strain temperature coefficients of each fiber turn of the coil as the simulation output. We do the simulations for different materials of the fiber coil mandrel. We choose aluminium and titanium. Titanium has a temperature expansion coefficient much closer to fiber. Moreover, we choose the fiber cross-section more like a square form.
3. Experiment

Fiber length is measured using an OTDR (optical time domain refractometer). The measurements are based on the assumption that the refractive index of fiber changes with $10^{-5}/^\circ C$ depending on the temperature. (Fig. 1)

![Graph showing comparison between OTDR measurement and simulation results.](image)

Fig. 1: Comparison OTDR measurement with simulation results.

A closed-loop fiber optic gyroscope is built for laboratory experiments (Fig. 2a). This set-up consists of one (amplified spontaneous emission) ASE light source, two multifunctional integrated optical chip (MIOC) and fiber coil pairs, and digital and analog electronic boards. To the best of our knowledge, FOG bias can be changed by many different effects due to temperature ($T$), but there is no effect due to temperature change ($\dot{T}$) except for Shupe effect and elastooptic effect which we have investigated. For this reason, the entire setup was placed in a temperature chamber during temperature tests (Fig. 2b).

![Images of the test bench and climatic chamber.](images)

Fig. 2: Testing of fiber coils.

We do the experiments with different fiber coils with spool material aluminium and titanium. We showed that although the total strain of the fiber coil stays the same, the average axial strain coefficients are reduced, resulting in lower rate error Fig. 3.

Rate error measurement of the newly designed fiber coil is carried under a temperature profile which is ranging from $-40^\circ C$ up to $+60^\circ C$. Measurement values along with the simulation results are given in Tab. 1. We concluded
that results are consistent and the newly designed coil performs nearly 4 times better under thermal variation.

Table 1: Rate error coefficient for two different coil designs.

<table>
<thead>
<tr>
<th>Thermal Sensitivity Coef.</th>
<th>Aluminium Spool</th>
<th>Titanium Spool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>2.71</td>
<td>0.32</td>
</tr>
<tr>
<td>Experiment</td>
<td>3.01</td>
<td>0.78</td>
</tr>
</tbody>
</table>

4. Conclusion

Strain temperature coefficient versus fiber turn location is a serrated line, where serration indicates the axial strain and line indicates the radial strain. Although the change in the strain temperature coefficient is higher in the radial direction than the axial, practical quadrupole asymmetry of the Shupe coefficient is much higher for the axial direction. Our simulations show that the axial strain gradient is the dominant source of rate error of our fiber coil.

We designed a new fiber coil with spool material titanium, which has a temperature expansion coefficient much closer to fiber. Secondly, we changed the fiber cross-section and made it more like a square form.

Acknowledgments

The authors acknowledge the financial support of ASELSAN Inc.

References