Abstract—Since their first demonstration, modelocked thin-disk lasers have consistently surpassed other modelocked oscillator technologies in terms of achievable pulse energy and average power by several orders of magnitude. Surprisingly, state-of-the-art results using this technology have so far only been achieved in modelocking regimes where soliton pulse shaping is dominant (i.e., soliton modelocking with semiconductor saturable absorber mirrors or Kerr lens modelocking), in which only small nonlinear phase shifts are tolerable, ultimately limiting pulse energy scaling. Inspired by the staggering success of novel modelocking regimes applied to overcome these limitations in modelocked fiber lasers, namely the similariton (self-similarly evolving pulses) and dissipative soliton regimes, here, we explore these nonlinearity-resistant regimes for the next generation of high-energy modelocked thin-disk lasers, whereby millijoule pulse energies appear to be realistic targets. In this goal, we propose two possible implementations. The first is based on a passive multipass cell and designed to support dissipative solitons in an all-normal dispersion cavity. The second incorporates an active multipass cell and is designed to support similaritons. Our numerical investigations indicate that this is a very promising path to increase the pulse energy achievable directly from modelocked oscillators toward the millijoule level while additionally simplifying their implementation by eliminating the need for operation in cumbersome vacuum chambers.

Index Terms—Ultrafast lasers, modelocking, thin-disk lasers, high-power lasers, similariton.

I. INTRODUCTION

PROGRESS in the performance of ultrafast lasers continues to act as a catalyzer for many new exciting discoveries in various fields and sub-fields of science and technology. Access to state-of-the-art ultrafast laser systems enables researchers worldwide to explore and unveil ultrafast dynamics of matter as well as modify and control a large array of properties of various materials in unprecedented excitation regimes.

While the main workhorse of ultrafast laser technology has traditionally been Ti:sapphire laser technology, these systems suffer from well-known limitations in the achievable average power. A limited average power at a given pulse energy results in an unwanted compromise in repetition rate, which in many experiments limits signal-to-noise ratio, measurement times and/or speed. This has motivated large research efforts aiming at increasing the average power available from ultrafast laser sources, and immense progress has been achieved in this direction in the last decades. Most results have been demonstrated using multi-chain amplifier architectures based on diode-pumped Yb-doped gain materials in geometries with improved heat removal: fiber chirped pulse amplifiers [1], slab amplifiers (Innoslab) [2] and thin-disk regenerative and multi-pass amplifiers [3], [4] have all recently surpassed the kilowatt average power milestone.

A compelling approach is to achieve the desired high average power directly by the output of an oscillator. Among all the above-mentioned laser geometries, thin-disk lasers (TDLs) are the only technology where modelocking can be achieved at average powers comparable to their state-of-the-art multi-chain amplifier counterparts. Average power levels up to 275 W [5] and pulse energy levels up to 80 μJ [6] have been demonstrated from a simple one-box modelocked oscillator operating at MHz repetition rate, which is orders of magnitude higher than other modelocked oscillator technologies.

Further average power and pulse energy scaling to the kilowatt and millijoule regime appears feasible with this technology [7], however, several crucial challenges need to be overcome. One critical challenge is achieving stable pulse formation at extremely high intracavity pulse energies of hundreds of microwatt to millijoules, and beyond. State-of-the art TDLs with pulse energies exceeding the μJ level demonstrated so far...
operate in the negative dispersion regime, either in the soliton modelocking regime [8], started by a semiconductor saturable absorber mirror (SESAM) [9] or in the Kerr-lens modelocked (KLM) regime [10]. In both cases, soliton pulse formation imposes severe limits to the maximum nonlinear phase shift tolerable before wave-breaking occurs, and thus effectively limits the achievable intracavity pulse energy.

So far, mitigating strategies to avoid these limitations and be able to energy scale these oscillators have mostly targeted a reduction of intracavity nonlinearity, for example by operating in vacuum to minimize the nonlinearity of air [5], or by decreasing intracavity pulse energy (without sacrificing output) via an increase of the gain per roundtrip, which was obtained with a multiple gain pass geometry (the so-called active multispas cell, AMC) [11], [12]. Operation in the net positive dispersion regime has so far only been explored in very few proof-of-principle experiments in the case of TDLs [13], [14] and the results have been rather disappointing compared to soliton modelocking.

An alternative strategy is nonlinearity management, which is distinct from these previous approaches and can complement nonlinearity reduction [15]. In addition to the possibility of directly compensating nonlinear effects via self-defocusing nonlinear processes this concept has led to the exploration of special pulse propagation schemes that are highly tolerant towards strong nonlinearities. Following this approach, several breakthroughs have been demonstrated in fiber lasers, starting with the wave-breaking-free fiber lasers [16], the similariton laser [17], the soliton-similariton laser [18] and the all-normal dispersion lasers supporting dissipative solitons [19]. All these regimes can tolerate nonlinear phase shifts in the range of 10π per roundtrip. In addition, they all support, on average, linearly chirped pulses within the cavity. This leads to another order of magnitude increase in possible pulse energy due to the reduced peak powers for a given pulse energy. Consistent with these scaling arguments, these modelocking regimes have indeed scaled the achievable pulse energies from a fraction of 0.1 nJ for soliton fiber lasers in 1990’s [20] to the 10 nJ range [21], [22] without even using special, large-mode-area fibers or other forms of nonlinearity mitigation. The possible benefits of these regimes for high-power TDLs are extremely exciting and motivated us to bring our two fields of expertise (modelocked thin-disk lasers and modelocked fiber lasers) together in one piece of work. The primary purpose of this report is to bring this possibility to the attention of the TDL community, where these regimes have not been widely explored so far, presumably because the operation parameters of TDLs (in particular the small gain per roundtrip) appear at first glance unsuitable to support them.

In this paper, we numerically explore two modelocking mechanisms operating in the positive dispersion regime as a promising path to increase the available pulse energy of ultrafast oscillators based on TDLs: the dissipative soliton regime [23] and the similariton regime [17], [18], which have so far never been demonstrated with TDLs. We show that the advantages of these techniques – namely a longer average pulse duration within one resonator roundtrip and much higher tolerable nonlinear phase shifts before wave-breaking occurs – can be extended to the case of high energy modelocked TDLs, where the average power and pulse energy reachable is significantly larger. Rather than exploring the limits in terms of tolerable nonlinearity in idealized and unrealistic configurations [24], the focus of our numerical investigation is on a potential implementation of these regimes: therefore, we restrict our parameter exploration to stay within reach with the current state-of-the-art of TDL technology. With this in mind, we propose two layouts to apply these regimes to TDLs: on the one hand a passive multipass cell (MPC) (for instance a Herriott-type cell) approach with large normal overall dispersion, small gain and continuous nonlinearity enabling dissipative solitons; and on the other hand, an active multipass scheme with overall large positive dispersion (introduced in discrete steps), gain and continuous nonlinearity, which enables similariton-type modelocking in this laser geometry. We refer to these regimes as discrete dissipative soliton and discrete similariton because in TDLs gain, dispersion and much of the nonlinearity do not occur simultaneously as in the fiber geometry, but rather in discrete steps in the thin disk, the air and the dispersive mirrors of the multipass arrangements. We take this discrete aspect into account in our simulation and show that, despite this discretization and, consequently, the very different propagation compared to fibers, these regimes retain the essential nonlinearity-resistant characteristic of regular similaritons and dissipative solitons. To the best of our knowledge, these approaches are novel, and have never been thoroughly explored before. The results of our numerical investigation indicate that these regimes are very promising alternatives to commonly used soliton modelocked and KLM to increase the pulse energy achievable directly from state-of-the-art modelocked oscillators towards the millijoule level, while additionally simplifying their implementation by eliminating the need for vacuum or low-pressure gas atmosphere.

II. STATE-OF-THE-ART OF HIGH ENERGY MODELOCKED THIN-DISK LASERS

A TDL [25] consists of a laser in which the gain medium is shaped as a disk, with a thickness which is significantly smaller than the applied laser area (typical thicknesses are in the order of ≈100–200 μm and laser beam radii ≈1–10 mm). The disk is used in reflection as an active mirror in a resonator, and efficiently cooled through the back side. The resulting nearly one-dimensional heat flow allows for nearly unrestricted power scaling by a simultaneous increase of the pump power and the pumped area. As a result of this geometry, one reflection on the gain medium provides both very small absorption and gain. Reaching sufficient pump light absorption is commonly achieved with a multipass pumping scheme, using a parabolic and prism mirror arrangement. With respect to the low gain, efficient laser operation can be achieved either at very low loss level (with intracavity powers and pulse energies orders of magnitude larger than the output power, see Fig. 1) or by applying multiple gain pass schemes in similar fashion to the pump beam, to increase the overall gain per roundtrip.

The first demonstration of a modelocked TDL [26] was achieved using semiconductor saturable absorber mirror (SESAM) soliton modelocking [8], [27], and so far, this remains...
...the most commonly used technique for modelocking of this type of oscillators. In this regime, the slow saturable absorber is responsible for starting and stabilizing the pulses, whereas pulse formation is determined by the intracavity balance between negative group delay dispersion (GDD) and self-phase modulation (SPM). As a result of the correct balance between these two effects, transform-limited short soliton pulses circulate in the resonator and their pulse duration remains nearly constant within one resonator roundtrip. In simple resonator geometries with few-passes on the thin-disk (most commonly a standing-wave resonator is used in which two gain reflections are seen per roundtrip), intracavity powers largely exceed the output power by factors of ten to hundreds, therefore pulses with very high and constant peak power circulate (typically several tens to hundreds of MW). In the case of the AMC concept in the anomalous dispersion regime, deviations from standard soliton modelocking have been thoroughly explored via numerical simulations, and a modified soliton theorem and corresponding scaling laws were derived in [28]. More recently, the use of KLM for this type of oscillators was demonstrated [29] and proved very successful to achieve comparable record-high levels, at shorter pulse durations [30], [31]. In this case, the very fast saturable absorber starts the modelocking, but similarly to soliton modelocking, soliton formation is mostly responsible for shaping the pulses.

Both techniques have advantages and disadvantages, but they are very often limited by the small nonlinear phase shifts $\phi_{nl} = \gamma \cdot P_{pk}$ tolerable (typically, up to $\sim \pi$ rad). The fundamental limits in terms of tolerable nonlinearity have not been reached in these regimes with anomalous dispersion, and pulse energy could potentially still progress [7]. However, so far, crucial practical constraints have prevented reaching these limits given the current state of the art of the technology. Some examples are the low modulation depth of typical saturable absorbers compatible with very high average power modelocking, or thermal effects present in dispersive mirrors, that prevents introducing very large amounts of negative dispersion. In most cases, phase shifts $\ll \pi$ rad are only tolerated, and thus nonlinearity reduction is imperative, even at moderate pulse energies. Furthermore, when soliton pulses circulate in the resonator, the intracavity peak power $P_{pk,ic}$ is maximal because the pulses remain close to their transform-limited value along propagation. Therefore, reaching the highest peak powers requires to minimize intracavity nonlinearity (the $\gamma$ coefficient in rad/MW) or finding paths to minimize the ratio between intracavity and output levels by increasing the overall gain.

It was demonstrated early on that in many cases, the $\gamma$ parameter is dominated by the contribution of the air in long resonators [32], therefore, the first modelocked TDL with a pulse energy exceeding $10 \mu J$ was demonstrated in [33] by flooding the oscillator with helium. More recently both the highest average power [5] and pulse energy [6] so far achieved from a modelocked oscillator were also demonstrated by reducing the intracavity nonlinearity, this time using medium vacuum. In [5], 275 W of average power with 583 fs-long pulses and 17 $\mu J$ pulse energy were demonstrated with a SESAM modelocked Yb:YAG TDL, where the detrimental dominant nonlinearity of air was reduced by operating the oscillator <1 mbar of ambient pressure. In this result, the simple linear resonator was operated at an output coupling coefficient of 11%, resulting in an intracavity pulse energy of $\approx 150 \mu J$ and an intracavity average power of 2.5 kW. Further energy scaling using this technique was achieved in [6] reaching 80 $\mu J$ of output pulse energy at 242 W of average output power in 1-ps long pulses, resulting in 66 MW of peak power. In this case, a double pass arrangement on the disk allowed to double the roundtrip gain (to four reflections on the disk) and operate at 20% output coupling coefficient, thus relaxing the intracavity conditions: modelocking was achieved at $\approx 800$ W of intracavity average power and 240 $\mu J$ intracavity pulse energy. In [12], the AMC approach was used to demonstrate pulse energies in excess of 40 $\mu J$. In this result, increasing the number of passes on the disk (22 reflections on the disk per roundtrip), and operating at high output coupling coefficient enabled operation at an intracavity pulse energy of only 50 $\mu J$. In this result, the resonator could be operated in air because the intracavity pulse energy was significantly reduced. In the case of KLM TDLs, similar average powers of 270 W [30] and peak powers (60 MW) [31] as with SESAM modelocking were reached with shorter pulse durations (resp. 330 fs and 150 fs). In [31], where the peak power level is significantly higher, the oscillator was also operated in vacuum.

III. Targeted Nonlinearity-Resistant Modelocking Regimes

A. Modelocking of Solid-State Lasers With Net Normal Dispersion

An attractive, yet little explored route to potentially reaching significantly higher pulse energies and peak powers with TDLs is to use other modelocking mechanisms, where pulses can readily support nonlinear phase shifts of $10\pi$ or more per roundtrip. All these regimes happen to operate at net positive (normal) dispersion per roundtrip, despite having different characteristics (and set of advantages and disadvantages). Therefore, we aim here to clarify which aspects we target to exploit here for energy scaling of TDLs.

![Graph showing intracavity pulse energy versus intracavity average power demonstrated to date with modelocked TDLs with various Yb-doped materials.](image)
In general, modelocked oscillators operated with net positive dispersion share the common point of operating with chirped intracavity pulses, at least for most of the propagation through the resonator. One important consequence is that the pulses have much longer durations on average, thus, correspondingly lower average peak intensities compared to soliton modelocking. This fact already accounts for an approximately 10-fold potential increase in pulse energy. We would like to highlight that this is not the only reason and at least another factor of 10 is attained by the tolerance to higher nonlinear phase shifts. We also would like to note that our interest here is not the operating regime of small positive dispersion and weak spectral filtering and shaping, as in [13], [14], but in the full onset of the similariton and dissipative soliton regimes, which are inherently more robust with respect to strong nonlinearities.

These modelocking regimes are, by now, ubiquitously applied in fiber lasers, where they were adopted early on. This is due to the fact that the limitations of soliton modelocking were particularly severe in this geometry, as the $\gamma$ coefficients in long fibers with small core areas are significantly higher and only controllable up to a very small degree with the core size of the fiber. The corresponding increase in mode area results in decreased robustness due to the small numerical aperture. Additionally, the combination of large small signal gain, positive nonlinearity and normal dispersion that comes naturally in fiber at common operation wavelengths around 1 $\mu$m, makes these regimes a natural choice. In solid state lasers, these values are not inherent, in particular, a large small signal gain (SSG) is challenging to achieve in most cases. Large positive dispersion needs to be introduced, since the gain medium only provides very small amounts.

Most solid-state lasers to date have been demonstrated in the former category, an important (but not exclusive) example being the dispersion managed soliton (DM) regime [34]. In this case, sections of normal dispersion (with chirped pulses) and anomalous dispersion (supporting soliton propagation) coexist in the resonator, resulting in a pulse duration that is ‘breathing’ intracavity. The resulting longer average pulse duration throughout one cavity roundtrip is higher than for soliton modelocking, thus presenting an advantage in terms of achievable pulse energy. Typically, pulses are only weakly chirped and small positive dispersion is introduced per roundtrip. The regimes that fall under this category are ‘soliton’-type modelocking regimes, in the sense that it is mostly the balance of dispersion and nonlinearity that govern the pulse formation, and loss and gain are very weakly or not involved. Therefore, there is little to no advantage in terms of tolerable nonlinear phase shift compared to pure soliton modelocking: the advantage only stems from the reduced peak power of the intracavity pulses. Operation in this type of regime has been demonstrated both with SESAM-modelocked [13] and KLM TDLs [14]. In [13] an Yb:KLuW modelocked oscillator operating in the small normal dispersion regime was demonstrated, achieving 9.5 W and 450 fs pulses. The limitations in this experiment originated from a low modulation depth of the SESAM and a small amount of positive GDD introduced via glass plates. In [35], numerical simulations indicate the possibility of reaching 20 $\mu$J pulses or more, but this direction was never pursued, and millijoule pulse energy levels were not envisioned, potentially because in standard TDL layouts adding significantly large amounts of dispersion is not trivial. In [14], this regime was demonstrated for a KLM TDL, where 30 W of output power were reached, but further scaling was prevented by difficulties in starting modelocking, and the direction was also not further explored.

There is a clear distinction between the modelocking regimes that we target to investigate here, and that of these proof-of-principle results. In our case, spectral filtering of chirped pulses is a source of strong self-amplitude modulation (SAM) and is a key enabling benefit, which, to our knowledge, has not been exploited in any other technology than mode-locked fiber lasers. Therefore, we first discuss the operating principle behind it.

### B. Spectral Filtering as a Powerful Tool for Self-Amplitude Modulation

Spectral filtering of strongly chirped pulses is a powerful technique to provide SAM. SAM most commonly arises from some form of saturable absorber, like nonlinear polarization evolution (NPE) or SESAMs. However, when the cavity is supporting highly (linearly) chirped pulses, spectral filtering of such a pulse leads to strong temporal filtering. This effect arises because the higher and lower frequency components of the pulse reside predominantly at the leading and trailing edges of the pulse. Thus, spectrally removing them clips the edges of the pulse in the time domain. We illustrate this effect in Fig. 2. Spectral filtering can be introduced in a resonator by gain filtering, a separate filter
or a spectrally shaped output coupler. It is important to mention that the SAM due to filtering can far exceed the same from the saturable absorber [36]. However, it does not replace the effect of or need for a ‘real’ saturable absorber like SESAM or NPE, because its mechanism is only effective once the spectrum is already broad and is not capable of starting modelocking.

Nevertheless, SAM from spectral filtering is inherently lossy and any consideration of its exploitation must also ensure adequate SSG within the cavity. Furthermore, the spectrum must be regenerated to its original width within a roundtrip, which requires SPM to be sufficiently large. This is a very welcome aspect, because it means that the cavity will not only tolerate large nonlinearities, thereby supporting large pulse energies, but that it will even thrive with strong nonlinearity, because with increasing spectral broadening, the SAM will become stronger. It should be noted that the pulse evolution also must be tolerant of large spectral broadening without becoming unstable, a feature that similariton and dissipative soliton pulses possess, in contrast to soliton-like pulses. These aspects make the investigation of these regimes for modelocked TDLs highly relevant.

C. Dissipative Soliton and Similariton Regimes

The combination of positive dispersion, positive (self-focusing) nonlinearity, together with gain, leads to the formation of parabolic self-similarly evolving pulses (similaritons), which are an asymptotic solution of the generalized nonlinear Schrödinger equation. Furthermore, being dissipative, they are strong attractors, meaning any initial pulse shape will evolve to such a pulse asymptotically, given enough propagation distance. However, they are continuously evolving pulses, and their temporal and spectral widths increase exponentially. This has led to strong initial doubts on their suitability for laser resonators, where any pulse evolution must satisfy the periodic boundary condition of a resonator by returning to their original state. It was shown that, with the combined action of spectral filtering, nonlinear gain and a limited amount of anomalous dispersion, periodic solutions supporting similaritons in the regions of the cavity where nonlinearity is strongest, can indeed exist [17], [18]. The SAM provided by the spectral filter is complemented either via a SESAM or nonlinear polarization evolution (NPE). The pulses remain positively chirped throughout the resonator, undergoing large breathing in temporal width in the different segments. The additional chirp, which is almost completely linear, can be removed extracavity, and pulses with good quality are typically achieved. This mechanism supports significantly larger nonlinear phase shifts (typically up to $\sim 10\pi$) than soliton modelocked lasers. Both the larger tolerable intracavity nonlinear phase shift and the longer intracavity average pulse duration allow for pulse energies several orders of magnitude higher than soliton modelocking.

In the dissipative soliton regime, modelocking is the result of a combined balance between nonlinearity, dispersion, spectral filtering, gain and loss. An important example that falls under this category is the all-normal dispersion regime, which significantly simplifies practical implementation (particularly in fiber lasers, but not in our case where dispersion is introduced externally in both then normal and anomalous regimes) by eliminating the need for a negative dispersion segment. In this type of lasers, the pulse remains very strongly chirped and undergoes much smaller variations throughout the laser resonator than in the similariton case. In this case, pulse energies exceeding 20 nJ have been demonstrated using regular fibers [21].

IV. IMPLEMENTATION AND NUMERICAL SIMULATIONS

We note in passing that we have designed the modelocking regimes described in this section using a new, intuitive and algorithmic method, which constructs and exploits an analogy between a modelocked oscillator and a thermodynamic engine, using tools and techniques of nonlinear dynamical systems analysis to meet the conditions of the design, including the periodic boundary conditions, while ensuring stability against fluctuations and convergence starting from noise fluctuations. This approach will be described in a dedicated forthcoming publication.

Applying the similariton and dissipative soliton regimes to modelocked TDLs appears to be a natural route to achieve energy scaling. However, several difficulties inherent to the thin-disk geometry make this challenging:

- TDLs have very small SSG (typical values are in the order of $< 1$ dB per active reflection on the disk). Large gain is in many cases required to support the strong action of spectral filtering, as discussed in III.B.
- Typical gain materials for TDLs are narrowband which is an inherent difficulty for the similariton laser, as self-similar pulse evolution is disrupted if it encounters a limitation to its bandwidth.
- The gain material in TDLs provides negligible normal dispersion and these regimes typically require large normal dispersion.
- TDLs operate with kilowatt intracavity average power, making thermal effects an extra difficulty to implement these regimes in classic schemes.

On the other hand, the highly energetic pulses achievable intracavity in TDLs provide a unique platform for benefiting from these regimes. In this section, we present two concepts that potentially allow us to overcome the above-mentioned difficulties commonly associated with the thin-disk geometry, and numerical simulations confirming that these regimes are promising for these laser systems.

In strong contrast to soliton modelocking, here, we follow the route of nonlinearity management: we try to make use of the existing sources of nonlinearity, as opposed to weakening them, as is typically done in modelocked TDLs. We specifically focus on demonstrating the suitability of this approach with typical gain and nonlinearity levels present in TDLs to scale the intracavity pulse energy achievable at the current state-of-the-art to the millijoule level and open a discussion on potential practical implementation of these schemes.

Both schemes share a general concept: in a comparable fashion to the continuous spatial distribution of nonlinearity and positive dispersion that occurs in a fiber in the normal dispersion regime, we suggest implementing two kinds of multipass
arrangements, in which the mirrors provide positive (normal) dispersion. In this way, overall large positive dispersion (introduced in discrete steps), and continuous nonlinearity work together to enable the targeted modelocking schemes in this laser geometry. We refer to these regimes as discrete because gain, dispersion and much of the nonlinearity do not occur simultaneously, but in discrete steps in the thin disk, the air in between and the dispersive mirrors of the MPC, respectively.

A. Numerical Model

We model pulse propagation through each optical element, comprising propagation through the laser cavity, using a generalized complex Ginzburg-Landau equation that accounts for Kerr nonlinearity, Raman scattering, dispersion and where appropriate gain, loss and spectral filtering. The gain model, albeit relatively simplistic, includes the effects of finite bandwidth and gain saturation. For our simulation, we choose a repetition rate of 3 MHz, which is typical for long-resonator high-energy TDLs. This allows us to model gain saturation as responsive to the aggregate effect of a large number of passes of the pulses, which effectively corresponds to saturation based on average power. The simulation code and the underlying model summarized here is based on the one described in [18]. A scheme of how the different elements are applied to the pulses in the numerical simulation is schematized in Figs. 4 and 7.

The different parameters used in the simulations are presented in Table I. The laser cavity is modeled in a simplistic way: propagation through the air of the resonator is done with a beam of constant beam radius of 750 \( \mu \text{m} \). A constant beam radius is a good first approximation as long as the total nonlinear phase shift is not modified compared to a realistic resonator design. Here, we calculated this average beam radius in the following way: we used the total SPM-nonlinear coefficient (\( \gamma \)) per resonator roundtrip from air in the latest energy scaling result of TDLs as a reference value and averaged its effect assuming a constant beam radius throughout the entire resonator length. Whereas we believe this is a valid approximation for our investigation, a future implementation of the code will take into account the exact mode size distribution of a given laser resonator. The beam radius on the gain disk was set to a realistic value (1.7 mm radius) given the targeted average power and typical disk damage considerations. For our simulations, we used a small signal gain of 1 dB per bounce on the disk, i.e., 2 dB per roundtrip, based on a typical Yb:YAG disk. The air and the thin disk are modeled with effective dispersion and nonlinearity coefficients which were derived from recent experimental results on high-energy, ultrafast TDLs [6]. The linear resonator is considered by propagating the pulses forward and backwards through the different cavity elements. It is assumed that the major contribution to the nonlinear phase shift originates in the long propagation distance in the passive or active positive dispersion MPC, which is spread throughout the total length of the resonator, thus accounting for the correct amount of total nonlinear phase shift. This is a good approximation because propagation length inside both passive and active MPCs constitutes most of the total resonator length. In future implementations, the above-mentioned more general treatment can be made, where the exact resonator mode size distribution is used to calculate the effect of SPM via the \( B \)-integral.

In practice, the difference will be that the spectral broadening ‘steps’ between each dispersion bounce (see Fig. 9) will not be equal, as it is the case here, because the different lengths between dispersive bounces might not all contain the same mode size distributions, and thus accumulate more or less nonlinear phase. The total resulting spectral broadening per roundtrip is, however, identical assuming a given \( \gamma \) parameter.

The dispersive mirrors (DM) inside both active and passive MPCs provide positive (normal) dispersion via for example GTI-type mirrors. A simplification was made when introducing the steps of positive dispersion in both passive and active MPCs in our model: we assume that the dispersion bounces are regularly spaced throughout the resonator length. This is most likely the case in a Herriott-type MPC but can be slightly different in an AMC depending on the layout chosen. In addition, in the case of the AMC, we assume that every time the pulse circulates through the MPC, there is one bounce on a positive dispersion mirror, placed exactly between two gain bounces. In practice, one could potentially add multiple dispersion bounces per AMC pass with lower value per bounce, as this is typically beneficial in high average power lasers. In fact, the higher the dispersion value per bounce, the stronger the thermal effects at high average power [37]. Additionally, GDD bandwidth typically scales inversely proportionally with the GDD value, therefore small values are preferred to support larger bandwidths.

We also include when necessary the action of DMs providing negative (anomalous) dispersion, a saturable absorber in transmission (modeled as a SESAM) and a reflective output coupler mirror, which includes a narrow Gaussian filter for the pulse circulating in the cavity (a spectrally shaped output coupler). The negative dispersion section is modeled as an ‘action’ – meaning without considering any propagation distance between the mirrors, and thus no SPM. This is a reasonable approximation as one can in most cases arrange negative DM at the end of a resonator in a compact multi-bounce approach, for example using rectangular mirrors, or simply place the negative dispersion arrangement at a location of the resonator with large mode areas, where SPM is negligible.

In the following sections, we present two designs and the corresponding numerical simulations, in which we could achieve stable operation with promising performance based on the two suggested layouts. We would like to highlight that we focused our exploration on parameter ranges with realistic targets, considering the current state-of-the-art of thin disk technology. In particular, we imposed a limit on the intracavity average power to approximately 5 kW, which is two times the maximum intracavity power achieved so far in a mode-locked TDL [5], in the goal of proposing realistic yet impactful targets.

B. Discrete Dissipative Soliton TDL Using Passive MPC

We start our investigation by evaluating possible regimes where low gain per roundtrip is tolerable – i.e., regimes where the spectral filtering induced losses and other various sources of loss are small enough to be tolerable by a SSG such as the one
provided in a standard thin-disk oscillator with one single pass through the gain medium. We found solutions in the dissipative soliton regime, more precisely in the all-normal dispersion regime, that converge with typical TDL parameters as described above. In this result, large amounts of positive dispersion are required: we suggest below a possible implementation of such concept based on a Herriott-type MPC that is compatible with the TDL geometry.

1) Implementation: As indicated by our numerical simulations, reaching the dissipative soliton regime can be achieved with low gain, but large amounts of positive dispersion are required. We suggest to implement a passive Herriott-type MPC [38], in which the mirrors are dispersive (DMs) that provide positive (normal) dispersion. MPCs are commonly used in high-energy TDLs to extend resonators and maximize pulse energy [6], [33], most typically using highly-reflective (HR) mirrors without dispersion, or negative (anomalous) dispersive mirrors to achieve soliton modelocking [6]. In our case, we suggest instead to use positive dispersion DMs. In this way, the energetic pulses see continuous nonlinearity (SPM in the air inside the long propagation distance), and discrete steps of normal dispersion, that are equally spread out in space: the SPM-broadened spectrum accumulates a strong linear chirp in semi-discrete steps during this propagation. After this, the combined effect of the gain, filter, and saturable absorber restores the pulse to its initial state. This effect compliments that of the saturable absorber to achieve modelocking. In this case, the spectral filtering effect is relatively small, and thus the resonator can still be operated with low loss. Therefore, the SSG provided by one reflection on the thin-disk is sufficient to support the pulse formation. A schematic illustration of the suggested implementation is presented in Fig. 3.

2) Numerical simulation: We present here one particularly promising simulation run, in which 1.8 mJ intracavity pulse energy was achieved. Given the linear losses of 10% applied with a spectrally shaped output coupler as mentioned above, 180 μJ would be extracted in this case. Considering the additional rejected light by the filter, we could potentially extract a total pulse energy of 210 μJ. In Table I, the complete set of parameters used for this simulation are presented. In this case, our results converged using a SESAM, with parameters compatible with high-power designs [39] (in this case a 5% modulation depth). In particular, it is worth noting that the absorber used has a long recovery time of 100 ps, which typically implies a low loss level from SESAMs, even at the proposed relatively high modulation depth [40].

3) Results: In the type of modelocking regime where the pulse duration and spectral width undergo strong variations within one roundtrip, it is interesting to visualize the evolution of these parameters along the resonator. We present this in Fig. 9 (left), where we plot the evolution of the spectral width as a function of the ratio of the pulse duration and the corresponding transform-limited pulse duration (i.e., a measure of the chirp level of the pulses). In this way, we can more easily distinguish between the particular aspects of each modelocking mechanism.

We recognize several characteristic features of this type of modelocking regime. On the way forward along the MPC, the spectral width and the pulse duration increase continuously due to the accumulated effect of SPM, in this case mostly from the air in the resonator, and the positive dispersion bounces. The discrete aspect of this regime is seen in the step-wise variation of the pulse duration, which increases due to the bounces on positive dispersion mirrors. The combined effects of spectral filtering (due to the gain and the filter), and the SESAM then dramatically reduce the spectral bandwidth as well as the pulse duration. On the way backwards, the pulses see the same components again, and a similar behavior is observed until the pulses are restored to complete a resonator roundtrip. It is worth noting that the pulse duration and spectral width do not undergo strong variations and remain strongly chirped throughout the resonator (in this case around 17 ps), which distinguishes this regime from the similariton regime, that we will describe below. The pulses can for example, be extracted before the spectral filtering occurs, where the bandwidth is broadest. In this simulation, this spectral width supports 374 fs pulses, which can be obtained by externally de-chirping the pulses (Fig. 5). The pulse duration reachable is of course dependent on the gain bandwidth available and would have to be studied in more details on a case to case basis. However, it is worth to note that it is not uncommon in these modelocking regimes to obtain bandwidths that match or even exceed the gain bandwidth, as the pulse propagation is strongly driven by SPM-induced broadening. This makes this concept even more interesting for the case of TDLs, because most Yb-doped gain materials suitable for high-power operation in this geometry are relatively narrowband.
The intracavity pulse energy reached largely exceeds the typical values in high energy soliton modelocked TDLs, and in strong contrast, this oscillator is operated in an air environment, which is one very attractive aspect we want to exploit here. Throughout one resonator roundtrip, the pulses in this case accumulate approximately $\pi$ nonlinear phase shift, which is at the limit of what soliton modelocking can tolerate, but significantly below the typical maximum amount that could be achieved in this regime. Nevertheless, these results already correspond to an intra-cavity average power of 5.4 kW, which is already very challenging, but possible to achieve. The fact that the laser design is primarily limited by achievable average power is an additional indication of the potential of these regimes for high-energy TDL modelocking: as the technology matures, we believe both regimes could support pulse energies of several tens of millijoules. However, this would require significant advances in TDL and SESAM technology to be achieved and is out of the scope of this paper.

Whereas this approach is potentially very simple to implement, it suffers from the low gain of the oscillator: in fact, although high pulse energies are reached intracavity, only relatively moderate energies can be extracted. This difficulty can be circumvented by applying a scheme with more gain passes on the disk per resonator roundtrip. We exploit such a scheme in the following section and achieve significantly higher output pulse energy. It is also worth mentioning that higher intracavity pulse energies can potentially be obtained by increasing the amount of positive dispersion. However, the target of this manuscript was to remain within realistic boundaries of TDL technology, as mentioned above.

C. Discrete Similariton TDL Using Active MPC

In the case of similariton modelocking in fiber lasers, self-similar pulse propagation is typically achieved in a fiber where large nonlinearity, gain and positive dispersion are simultaneously and continuously present. Contrary to the case described above, in this case the spectral filter (here added as a spectrally shaped output coupler) has a more dramatic effect on the pulse, and thus additionally brings a high level of loss. Therefore, self-similar propagation mandates substantial gain per roundtrip, and thus requires a different approach than the one described in the previous section. We describe this approach in the next paragraph, which consists of using an AMC such as the one demonstrated in [12]. This allowed us to run the simulation this time with a total gain per roundtrip of 20 dB, and thus to find solutions in the similariton regime that converge with typical TDL parameters as described above.

1) Implementation: We propose the use of an AMC such as the one demonstrated in [12] for soliton modelocking with negative dispersion, but in this case with positive dispersive mirrors.
Fig. 9. Illustration of the evolution of pulse width and spectral bandwidth in (a) the dissipative soliton (all-normal dispersion, implemented using a passive MPC) and (b) the discrete similariton (implemented using an active MPC) within one TDL roundtrip. The color code refers qualitatively to the pulse duration throughout the resonators. The spectral and temporal widths were extracted from Gaussian fits to the electric field intensity along the propagation. With this illustration, the characteristic behavior of each mode-locking regime can clearly be seen. In the dissipative soliton case the pulses remain strongly chirped throughout one resonator roundtrip, with only small variations and experiencing negligible temporal and spectral breathing. In the case of the similariton TDL, the pulse parameters vary significantly within one resonator roundtrip, and the strong effect of spectral filtering is evident. DM: Dispersive mirror with positive dispersion, DMa: dispersive mirror with anomalous dispersion.

A section outside of the positive dispersion AMC, this time with negative dispersion, complements the effect of spectral filtering to restore the pulse to complete a roundtrip and support similariton mode-locking. The overall cavity would operate with small net positive dispersion, and would have large gain (Fig. 6). We numerically investigate this implementation in the following section.

2) Result of numerical simulations: We present here one particularly promising simulation run, in which high pulse energy (1.27 mJ intracavity pulse energy, 0.95 output pulse energy) was achieved. The specific parameters of this simulation run are summarized in Table I.

In the phase diagram in Fig. 9 (right) we distinguish several characteristic features of the similariton regime. Along the AMC, the spectral width increases due to the accumulated effect of SPM due to propagation in air and the thin-disk, and at every bounce on a positive DM the pulse duration increases discretely. In this case, as we multiply the gain passes, the nonlinearity of the disk is not negligible anymore, and stronger spectral broadening is achieved. The pulse shape becomes parabolic during propagation along the long AMC, acquiring the typical shape of a similariton. After this propagation, a combination of negative dispersion, a Gaussian spectral filter (incorporated as a spectrally shaped output coupler) and the saturable absorber, narrows down the bandwidth significantly. We notice that in contrast to the previous regime, the pulse undergoes much larger variations in its spectral width and pulse duration, which is typical of this type of mode-locking mechanism. Throughout one resonator roundtrip, the pulses in this case accumulate approximately $2\pi$ of nonlinear phase shift, which is in this case higher than what soliton mode-locking could tolerate. As pointed out in the DS case, much larger pulse energies could be feasible as indicated by the relatively moderate nonlinear phase shift per roundtrip.

Despite the very large gain, we could obtain stable pulses with a SESAM with relatively low modulation depth of 5%. As we mentioned in the case of the DS, this value is potentially compatible with high-power SESAM designs, making the similariton approach very promising.

In this case, we extract the pulses via an output coupler with 50% transmission. Additionally, the filter (incorporated into the output coupler) rejects an additional 25%, meaning that a total of 0.95 mJ pulse energy could be extracted from the resonator. The extracted pulses are shown in Fig. 8. The pulses extracted here are also linearly chirped, supporting sub-100 fs pulses (95 fs).

D. Discussion on Possible Practical Implementation

Our numerical simulations seem to indicate that the active multipass approach supporting similaritons is the most promising one for extracting highest energy levels, albeit a slightly higher complexity of implementation. Furthermore, by providing more gain, it also is more tolerant to potentially lossy intracavity elements. This is particularly important at the targeted average powers of several kilowatts. We therefore focus most of the discussion in this paragraph to the similariton case: we discuss what we believe will be the main challenges in the practical implementation of this regime. Most of the discussion can also
be applied to the case of the dissipative soliton modelocking. It is worth noting, that the AMC approach could potentially also be explored in the dissipative soliton (all-normal dispersion) regime described above, however, we focus here on showing the simplest implementations possible for each regime.

1) Saturable absorber: One crucial challenge will be to design SESAMs or other saturable absorbers that are able to provide the required modulation depth to support these regimes, while simultaneously tolerating extremely high intracavity powers (in the case of the similariton example above, we obtain 3.8 kW of intracavity average power, and in the case of the dissipative soliton 5.4 kW of intra-cavity average power). While we believe that 5% modulation depth is a realistic and ongoing target, most high-power designs so far had modulations depths in the order of 1% [39], [41]. One interesting possibility for this regime is to explore other alternative saturable absorbers, that can potentially provide much higher modulation depth, while still tolerating high intracavity average power. This topic has recently received significant attention in the community, as it is well-known that even for soliton modelocking, higher modulation depths are desired to reach shortest pulses at high power [42]. Several directions are currently being explored [43], and we believe a potential route to achieve this could be the implementation of an all-reflective NPE.

2) Nonlinearity of optics: In our simulations, we assumed that the nonlinearity originated exclusively from the thin-disk and the air inside the resonator. In practice, it is known that parasitic nonlinearities arising from the mirrors [44], particularly at the targeted high energy levels, can also be present. In spite of the longer pulse duration in these regimes, the maximum intracavity peak power at certain positions in the resonator is still considerable, with 100 MW in the dissipative soliton case and 250 MW in the similariton case. We believe this should not pose problems. In fact, the targeted modelocking regime is significantly better suited to tolerate these nonlinearities than soliton modelocking. Potentially, they can even be beneficial to

---

**TABLE I**

<table>
<thead>
<tr>
<th>Parameters Used for Numerical Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td><strong>Gain parameters</strong></td>
</tr>
<tr>
<td>Central gain wavelength (nm)</td>
</tr>
<tr>
<td>Net roundtrip unsaturated gain (dB)</td>
</tr>
<tr>
<td>Gain saturation energy (mJ)</td>
</tr>
<tr>
<td>Gain bandwidth (nm)</td>
</tr>
<tr>
<td>$n_{2}$ of disk (10$^{-16}$ m$^2$/W)</td>
</tr>
<tr>
<td>GVD of disk (fs$^2$/mm)</td>
</tr>
<tr>
<td>Beam radius on disk (mm)</td>
</tr>
<tr>
<td><strong>Nonlinearity and dispersion</strong></td>
</tr>
<tr>
<td>Average beam radius in air (mm)</td>
</tr>
<tr>
<td>SPM coefficient of air (1/kW-m)</td>
</tr>
<tr>
<td>GVD of air (fs$^2$/m)</td>
</tr>
<tr>
<td>Cavity length (roundtrip, m)</td>
</tr>
<tr>
<td>GDD per bounce dispersive mirrors (fs$^2$)</td>
</tr>
<tr>
<td>Mirror separation (m)</td>
</tr>
<tr>
<td>Number of passes MPC (roundtrip)</td>
</tr>
<tr>
<td><strong>Saturable absorber</strong></td>
</tr>
<tr>
<td>Modulation depth (%)</td>
</tr>
<tr>
<td>Saturation energy (mJ)</td>
</tr>
<tr>
<td>Recovery time (ps)</td>
</tr>
<tr>
<td><strong>Loss, filter</strong></td>
</tr>
<tr>
<td>Filter center wavelength (nm)</td>
</tr>
<tr>
<td>Filter bandwidth (nm)</td>
</tr>
<tr>
<td>Output coupling rate (%)</td>
</tr>
<tr>
<td>Filter rejection (%)</td>
</tr>
<tr>
<td><strong>Total dispersion per roundtrip</strong></td>
</tr>
<tr>
<td>Total normal GDD (fs$^2$)</td>
</tr>
<tr>
<td>Total anomalous GDD (fs$^2$)</td>
</tr>
<tr>
<td>Net cavity GDD (fs$^2$)</td>
</tr>
<tr>
<td><strong>Laser parameters</strong></td>
</tr>
<tr>
<td>Intra-cavity pulse energy</td>
</tr>
<tr>
<td>Output pulse energy (mJ) (OC and filter reject)</td>
</tr>
<tr>
<td>Intra-cavity average power (kW)</td>
</tr>
<tr>
<td>Intra-cavity peak power (MW)</td>
</tr>
<tr>
<td>Pulse duration (ps) *</td>
</tr>
<tr>
<td>Spectral width (nm)</td>
</tr>
<tr>
<td>Nonlinear phase shift per roundtrip (rad)</td>
</tr>
<tr>
<td>Dechirped pulse duration (fs)</td>
</tr>
</tbody>
</table>

*approximate — varies along resonator
the modelocking mechanism, provided that these do not result in damage.

3) Thermal effects: Our simulations rely on introducing high levels of both positive and negative dispersion inside of the resonator. DMs typically exhibit stronger thermal effects than pure quarter-wave high reflectors, due to internal field resonances inside the mirror structure that are required to achieve the desired dispersion values. As efforts are being carried out to improve the quality of such highly dispersive mirrors [37], [44], a potential solution is to decrease the absolute amount of dispersion per bounce and increase the number of bounces per roundtrip, which is a viable option in our case by folding the MPC and adding additional bounces. This would be bringing us closer to the case of continuous dispersion, thus we believe would not affect the results.

4) Gain bandwidth: As discussed in III.B, spectral filtering is a crucial technique in these modelocking regimes, therefore the gain material needs to be carefully chosen to support the desired regime. In particular, self-similar pulse evolution is known to be disrupted if the bandwidth encounters a strong disturbance in its propagation. This means that broadband gain materials will in this case be preferred. Furthermore, a broader gain bandwidth will support shortest possible pulses. However, it is a well-known ongoing challenge to find broadband materials for high-power operation in the TDL geometry, and this continues to be a topic of investigation [45], [46]. Progress in the area will simplify advances of our approach. Other approaches, such as the dual-gain TDL [47] will also be potentially interesting to explore in this regime.

V. CONCLUSION AND OUTLOOK

We investigated, for the first time, the possibility of implementing the all-normal-dispersion dissipative soliton and the similariton modelocking regimes to high-energy modelocked TDLs. These regimes are known to support several orders of magnitude higher pulse energies than the commonly applied soliton and Kerr-lens modelocking regimes. We proposed two layouts to implement these regimes in TDLs in a practical way: one based on a passive Herriott-type MPC with large positive (normal) dispersion and small gain, supporting dissipative solitons in the all-normal dispersion regime, and one with an AMC with large positive (normal) dispersion and multiple thin-disk passes, thus operating with large gain.

Our numerical simulations indicate that these regimes are feasible with common parameters obtained with TDLs and provide a viable path to achieve significantly higher energies from modelocked oscillators. We believe the most promising approach is to implement the similariton regime, where our simulations indicate that intracavity (resp. output) pulse energies exceeding (resp. approaching) the millijoule level may be possible while still operating the oscillator in air. Overall, the simulations indicate that using either of these modelocking regimes, TDLs will cease to be limited by nonlinear phase shifts, and the main challenge appears to be successful experimental implementation. For this reason, the focus of this study was placed on the investigation of practically achievable parameters within the reach of current TDL technology. Given that the technology evolves rapidly, potentially much higher pulse energies can be demonstrated using these nonlinearity resistant modelocking regimes, maybe even in combination with nonlinearity mitigation techniques such as operation in low pressure environments or using of high-modulation-depth saturable absorption mechanisms, including NPE. In future numerical simulations, we will investigate a larger parameter range to further explore the limits of the proposed techniques.

We target to implement these layouts in first proof-of-principle experiments in the near future, as well as further explore the parameter range using our numerical model. We further note that another very interesting modelocking regime is that of a soliton-similariton laser [18]. Our calculations indicate that this regime allows even superior performance and potentially easier modelocking due to its strong attractor dynamics, but it was not considered in this contribution since its implementation requires a cavity incorporating both a passive and an active MPC. We believe the ideas put forward here will be key to take modelocked TDLs and more generally ultrafast oscillators to the next pulse energy milestone with simple setups.

REFERENCES


**Fatih Ömer Ilday** was born in Istanbul, Turkey, in 1976. He received the B.S. degree from Boğaziçi University, Istanbul, Turkey, in 1998, and the Ph.D. degree from Cornell University, Ithaca, NY, USA, in 2003. From 2003 to 2006, he was a Postdoctoral Scientist and later a Research Scientist with Massachusetts Institute of Technology, Cambridge, MA, USA. In 2006, he joined the Faculty with Bilkent University, Ankara, Turkey, where he is currently a Professor with the Department of Electrical and Electronics Engineering, the Department of Physics, National Nanotechnology Research Center, and Institute of Materials Science and Nanotechnology. Among various other awards and recognitions, he was the recipient of a European Research Council Consolidator Grant and a TÜBİTAK Science Award, the most prestigious award for science in Turkey.

**Denizhan Koray Kesim** was born in Konya, Turkey, in 1991. He received the B.S. and M.S. degrees in electrical and electronics engineering in 2013 and 2016, respectively, from Bilkent University, Ankara, Turkey, where he has been working toward the Ph.D. degree in electrical and electronics engineering since 2016. He works on fiber lasers and laser material interactions.

**Martin Hoffmann** was born in Dessau, Germany, in 1979. He received the Diploma in physics and the Ph.D. degree, both from ETH Zurich, Zurich, Switzerland, in 2007 and 2011, respectively. He was a Researcher with Cyber Laser, Inc., Tokyo, Japan, from 2012 to 2013, and a Senior Scientist with the Time and Frequency Laboratory, University of Neuchâtel, Neuchâtel, Switzerland, from 2013 to 2016. Since 2016, he has been a Researcher with the Photonics and Terahertz Technology Chair, Ruhr University Bochum, Bochum, Germany.

**Clara Jody Saraceno** was born in Buenos Aires, Argentina, in 1983. She studied with the Institut d’Optique, Palaiseau, France. After completing her studies, she went into industry from 2007 to 2008, working for a laser manufacturer in the USA (Coherent, Inc.). She then continued her academic training in Switzerland, and received the Doctorate degree in physics from ETH Zurich, Zurich, Switzerland, in 2012, which brought her, amongst others, the 2013 QEDO Thesis Prize, awarded by the Electronics and Optics Division of the European Physical Society. After graduation, she has worked at ETH Zurich and the University of Neuchatel as a Postdoctoral Researcher. In 2016, she became a Professor with the Faculty of Electrical Engineering and Information Technology, Ruhr University Bochum, Germany, where she currently works on various aspects of ultrafast laser science. Most recently, her research work on high-power ultrafast lasers earned her the Sofja Kovalevskaja Award of the Alexander von Humboldt Foundation (2015).