Energy-Harvesting Irregular Repetition Slotted ALOHA with Unit-Sized Battery

Umut Demirhan  
School of ECEE, Arizona State University, Tempe, AZ  
Email: umderan@asu.edu

Tolga M. Duman  
EEE Dept., Bilkent University, Ankara, Turkey  
Email: duman@ee.bilkent.edu.tr

Abstract—We propose an irregular repetition slotted ALOHA (IRSA) based uncoordinated random access scheme for energy harvesting (EH) nodes. Specifically, we consider the case in which each user has a unit-sized battery that is recharged with energy harvested from the environment in a probabilistic manner. We analyze this scheme by deriving asymptotic throughput expressions and optimize the maximum throughput, resulting in IRSA schemes with a maximum asymptotic throughput close to the maximum value of 1 on the collision channel. As an extension, it is also shown that frame and slot asynchronous approaches with SIC provide significant improvements as well [5]–[7].

Study of MAC protocols including ALOHA variants with EH nodes are available in many existing papers. In [8], the authors examine the stability of slotted ALOHA in an EH environment. In [9], a scheme is proposed to optimize the sum throughput of slotted ALOHA in which dynamically selected transmission power is adopted depending on the EH rate and battery size. Various MAC protocols including frame ALOHA, frame slotted ALOHA (FSA) and dynamic FSA for an energy harvesting system are studied in [10]. The authors in [11] consider energy harvesting aware dynamic frame slotted ALOHA for M2M networks. Moreover, [12] examines reservation dynamic frame slotted ALOHA for M2M networks. On the other hand, there is no existing work proposing a scheme based on CRDSA or IRSA providing high throughputs with EH nodes. Given the drastically increasing number of machine type devices and their expected lifetime of operation, throughput maximization on uncoordinated RA schemes with EH nodes has a tremendous potential. Therefore, our interest in this paper is to investigate the applicability of IRSA for an EH system for the first time in the literature.

We propose a modified IRSA scheme accommodating EH nodes named as energy-harvesting irregular repetition slotted ALOHA (EH-IRSA). In the proposed scheme, sporadically activated users with EH capabilities are equipped with a unit-sized battery that can provide energy for one packet transmission. Users draw the number of replicas for their packets from a probability distribution and select specific time slots to send them across a frame, similar to IRSA. On the other hand, if a user has no energy in its selected slot, transmission cannot take place, hence the allocated slot is skipped. In our model, we assume that an empty battery is filled with a certain probability in each slot, independently of the other slots and users. We analyze the asymptotic throughput of the system for a fixed EH rate, which is defined as the expected number of energy arrivals in a frame, and we show that finite MAC frame length throughput performances conform with the theoretical (asymptotic) results. We utilize the derived results to find the optimal degree distributions for packet replicas via differential evolution for different EH rates. Our numerical results demonstrate the superiority of the proposed solutions.

I. INTRODUCTION

Recent developments on successive interference cancellation (SIC) enabled random access schemes have shown very promising results in terms of increasing the system throughput and providing a stable network to many users, particularly, for the emerging machine-to-machine (M2M) communications in 5G. Since it is of interest to devise systems that operate for a very long time, such sensing and communication nodes should be equipped with energy harvesting (EH) capabilities to work without the need for battery replacement. Therefore, it is essential to study ALOHA schemes providing uncoordinated random access (RA) with high throughputs using EH nodes.

Originating from diversity ALOHA [2], which utilizes time and frequency diversity for transmitting multiple copies of users’ packets, content resolution diversity slotted ALOHA (CRDSA) [3] considers grouping a number of slots as frames in which each user sends two copies of its packets referred to as replicas. SIC among slots may be adopted to resolve the collisions given the received signals in the entire frame. That is, a collision may be resolved by subtracting a decoded packet from the collision that includes its replica, resulting in a maximum throughput of $T \approx 0.55$, which is defined as successfully resolved packets per slot [3].

After representing the SIC process explicitly on a bipartite graph, Liva proposes to vary the number of copies according to a probability distribution resulting in irregular repetition slotted ALOHA (IRSA), and describes an iterative process to analyze the asymptotic performance of the system for a fixed repetition distribution [4]. He also shows that it is possible to optimize the repetition distribution, resulting in IRSA schemes with a maximum asymptotic throughput close to the maximum value of 1 on the collision channel. As an extension, it is also shown that frame and slot asynchronous approaches with SIC provide significant improvements as well [5]–[7].

This work is based on Umut Demirhan’s M.S. Thesis [1] completed at Bilkent University.
both asymptotically and through extensive finite frame length simulations for the case with EH nodes over the distributions of SA, CRDSA and the optimized distributions of IRS, which do not take into account the EH process.

The paper is organized as follows. In Section II, the system model is given. The proposed EH-IRSA scheme is described and its convergence analysis is conducted in Section III. In Section IV, several optimized packet replica distributions are obtained and their performances are compared with those of CRDSA and IRS (optimized without the EH considerations), and superiority of the proposed solution within the EH framework is demonstrated. Conclusions are provided in Section V.

II. SYSTEM MODEL

We consider a slotted ALOHA system in which the time slots are grouped as MAC frames (simply referred to as frames). Each frame consists of $N$ equal length slots. The number of total users that are sporadically activated (that send messages to a common receiver) is denoted by $M_t$. The users are synchronized across the time slots and frames. Each user is activated with a probability $\pi$ for a given frame independently of activations in other frames and other users. The number of active users in a frame is $M_a$. The expected channel load $G$ is defined as the expected number of active users per slot given by

$$G = \frac{E[M_a]}{N} = \frac{\pi M_t}{N}.$$

The transmitting nodes are capable of harvesting energy from a renewable energy source and each is equipped with a battery of capacity $\delta$. We assume that transmission of each packet replica consumes $\delta$ energy. The batteries of the users are fully recharged with probability $p_{rc}$, independently, in any given time slot $t^1$. The probability of no energy arrival in a particular slot is $p_{nrc} = 1 - p_{rc}$. The arriving energy is lost if the battery is full. We simply take $\delta = 1$ without any loss of generality.

Let $E_s$ denote the amount of energy in the battery at the beginning of the $s$-th slot in a frame. If there is a transmission attempt in that slot, $E_{s+1} = 0$, independent of the energy arrival and success of the transmission. If there is no transmission attempt in that slot,

$$E_{s+1} = \begin{cases} 1 & \text{with probability } p_{rc}, \\ E_s & \text{with probability } p_{nrc}. \end{cases}$$

We use the common channel model utilized in the analysis of RA schemes as in [3], [4], namely, the collision channel, and make similar assumptions throughout this paper. Particularly, we assume that the receiver is able to recognize slots with a single replica, collisions and no transmission. Slots with a single replica are always successfully decoded. On the other hand, collisions are considered as non-resolvable, i.e., no information can be obtained from the collision in the absence of any additional knowledge. Each packet carries the

$^1$The adopted energy harvesting and battery models are common in the communications literature as an abstraction and simplification of more general systems (e.g. [13], [14]).

$^2$To eliminate the probable signaling burden, the random number generators of the users may be synchronized with the receiver.

![Fig. 1. An illustration of the proposed scheme with 4 users and 5 slots on a bipartite graph. User nodes (circles) represent users and sum nodes (squares) represent slots. Edges connect the users to their pre-determined slots.](image-url)
A. Convergence Analysis

We are interested in the asymptotic throughput behavior for a load level $G$ and an EH rate $\alpha$, which is defined as the expected number of energy arrivals in a frame given by $\alpha = N_p \epsilon$. We let $M, N \to \infty$ and $p_c \to 0$ while keeping the EH rate $\alpha = N_p \epsilon$ a constant, and analyze the SIC process from a probabilistic perspective. In other words, we analyze the effects of different energy arrival rates asymptotically. The density evolution analysis is similar to that of IRSA in [4], hence the details are omitted.

We define user node (UN) and sum node (SN) distributions from the edge perspective as

$$
\lambda(x) \triangleq \sum_i \lambda_i x^{i-1}, \quad \rho(x) \triangleq \sum_i \rho_i x^{i-1},
$$

where $\lambda_i$ and $\rho_i$ correspond to the probability of an edge being connected to a degree-$i$ UN and a degree-$i$ SN, respectively. Let $q_i$ and $p_i$ denote the probability of an edge not being revealed after the $i$-th iteration from a UN and an SN, respectively. These quantities can be calculated by iterating through $q_i = \lambda(p_{i-1})$ and $p_i = 1 - \rho(1 - q_i)$.

Let us now define the packet loss ratio (PLR) as the ratio between the number of active users whose packets are not resolved and the number of active users, which is an important metric in the evaluation of the system performance. We can obtain the PLR by converting the edge perspective probability to the node perspective probability using $PLR = \Lambda(p_m)$ for a selected arbitrarily large number of SIC iterations, $m$, representing $m \to \infty$.

The system throughput is defined as $T = G(1 - PLR)^3$. By utilizing a differential evolution based optimization algorithm [15], we obtain RDs that provide the maximum channel throughput $T^*$. We also consider the classical approach that maximizes the offered channel load $G$ whilst achieving an arbitrarily chosen target PLR that is close to 0 in our numerical results [4].

To calculate the resulting PLR, we need to obtain $\rho(x)$ and $\lambda(x)$ polynomials, which are derived in [4] as $\lambda(x) = \lambda_0(x) \lambda_1(x)$ and $\rho(x) = e^{-G \lambda_1(1-x)}$. Differently from [4], we utilize the ERDs instead of the RDs in the density evolution analysis of the decoding process. To clarify, we only use the active packet replicas, i.e., the absent packet replicas have no effect on the SIC process. We further assume that the receiver is able to obtain the knowledge of active and absent edges and it can figure out when a replica is skipped due to lack of energy [5].

$$
\tilde{\lambda}(x) = \sum_{k=0}^{k_{max}} \Lambda_k \Phi^k(x), \quad \Phi^k(x) = \sum_{l=0}^{k} \Phi_l^k x^l,
$$

which simply results in $\tilde{\lambda}_1 = \sum_{k=0}^{k_{max}} \Lambda_k \Phi_1^k$.

Due to the sporadic activity of users, slots after the last transmission trial should be considered to calculate the probability of a user’s battery being recharged or not at the beginning of a frame. Let $Pr\{E_0 = 0\}$ denote the probability of not having energy and $Pr\{E_0 = 1\} = 1 - Pr\{E_0 = 0\}$ be the probability of the battery being full at the beginning of a frame. We split $\Phi^k_1$ into two parts based on its battery state at the beginning of a frame as

$$
\Phi^k_1 = Pr\{E_0 = 1\} \phi^k_1 + Pr\{E_0 = 0\} \tilde{\phi}^k_1,
$$

where $\phi^k_1$ is the probability of $l$ active replicas over $k$ replicas when the first replica is successfully transmitted using the energy at the beginning of the frame, and similarly, $\tilde{\phi}^k_1$ is the probability of successful transmission of $l$ replicas over $k$ replicas when the user has no energy at the beginning of the frame.

To derive $\phi^k_1$ and $\tilde{\phi}^k_1$, we average across the possible selections of $k$ slots and $l$ successful transmissions by using the separations (referred to as distances) among the selected slots denoted by the vector $d = \{d_1, d_2, ..., d_k\}$. Note that $d_1$...
is the position of the first selected slot, while $d_i$ for $i > 1$ is the slot distance between the $(i - 1)$-th and $i$-th selected slots. These distances represent the number of slots for energy arrivals before transmission as shown in Fig. 2.

We use $| \cdot |_1$ to denote the $l_1$-norm, and $[k]$ as the set of integers from 1 to $k$. For every $k \in \mathbb{Z}_+ \text{ and } l \in \mathbb{N} \text{ with } l \leq k$, $D = \{d \in \mathbb{Z}_+^k : |d|_1 \leq N\}$ is the set of all possible distance selections, $B^k_l = \{b \in [k]^l : b_i \neq b_j \forall i, j \in [l] \text{ with } i \neq j\}$ is the set of all possible selection of indices of $l$ active replicas. With this notation, we can write

$$
\hat{\phi}_k^l = \lim_{N \to \infty} \sum_{p_n \to 0} \sum_{N p_n = \alpha} P(d) \prod_{j \in b} (1 - p_n^{d_j}) \prod_{i \in [k] \setminus b} (p_n^{d_j}).
$$

(3)

The first summation in (3) is over all the possible $l$ active slot selections out of $k$ total slots. The set $D$ is symmetric for each dimension, i.e., the $i$-th and $j$-th elements for any $i, j$ can be interchanged over $D$ without changing $D$. Therefore, for any $b \in B^k_l$, the inner summation is the same. Hence, we obtain

$$
\hat{\phi}_k^l = \lim_{N \to \infty} \sum_{p_n \to 0} \sum_{N p_n = \alpha} \frac{k}{N} \prod_{j \in b} \sum_{i \in [k] \setminus b} (1 - p_n^{d_j}).
$$

(4)

We modify (4) by further exploiting this symmetry of $D$ and change $d_i$ and $d_j$ terms for some $i, j$ inside the summation over $D$ and write

$$
\hat{\phi}_k^l = \lim_{N \to \infty} \sum_{p_n \to 0} \sum_{N p_n = \alpha} \frac{k}{N} \prod_{j \in b} \sum_{i \in [k] \setminus b} (1 - p_n^{d_j}).
$$

Defining

$$
f(k, r) = \lim_{N \to \infty} \sum_{p_n \to 0} \sum_{N p_n = \alpha} \frac{k}{N} \prod_{j \in b} \sum_{i \in [k] \setminus b} (1 - p_n^{d_j}),
$$

(6)

and by using (5) and (6), we obtain

$$
\hat{\phi}_k^l = \left(\frac{k}{l}\right) \sum_{r = 0}^{l} \left(\begin{array}{c} l \\ r \end{array}\right) (-1)^r f(k, k - l + r).
$$

(7)

We can numerically calculate $f(k, r)$ for all possible $k$ and $r$ values, and we can compute any $\hat{\phi}_k^l$ term from a given RD. Moreover, the complexity of the numerical calculations can be decreased with the following recursive equations

$$
-\frac{df(k, r)}{d\alpha} \frac{k + 1}{r} = f(k + 1, r + 1),
$$

(8)

$$
-\frac{df(k, r)}{d\alpha} \frac{k + 1}{k + 1 - r} = f(k + 1, r),
$$

(9)

whose proofs can be found in [1].

For the case with full battery at the beginning of a frame, we consider $\hat{k} = k - 1$ and $\hat{l} = \hat{l} - 1$, and ignore the first replica that is certainly transmitted since battery is full at the start of the frame. Thus, in a complementary manner to (4)-(7), we obtain

$$
\phi_k^l = \lim_{N \to \infty, p_n \to 0} \sum_{N p_n = \alpha} \frac{k}{N} \prod_{j \in b} \sum_{i \in [k] \setminus b} (1 - p_n^{d_j}) \prod_{j \in b} (p_n^{d_j}).
$$

(11)

To sum up, we can compute the $\phi_k^l$ and $\hat{\phi}_k^l$ terms corresponding to any given RD and any EH rate $\alpha$, which can be utilized to obtain the asymptotic throughput performance of the system.

In our analysis, we also need the probabilities $\text{Pr}\{E_0 = 0\}$ and $\text{Pr}\{E_0 = 1\}$, which are computed under the same asymptotic assumptions. These probabilities depend on two different variables: the last slot that a user tries to send a replica and the number of frames between two consecutive frames that the user is active. We denote the average probability of not being charged after the last trial of sending the replica in that frame as $P_{\text{slots}}$ and the average probability of being not charged during the frames that the user is not active as $P_{\text{frames}}$. We write

$$
\text{Pr}\{E_0 = 0\} = P_{\text{slots}}^0 P_{\text{frames}}^0,
$$

as these two events are independent. We form $P_{\text{slots}}^0$ as

$$
P_{\text{slots}}^0 = \lim_{N \to \infty} \sum_{k = 1}^{k_{\text{max}}} \sum_{i = 1}^{N - k + 1} \left(\begin{array}{c} N - i \\ k - 1 \end{array}\right) p_n^{i}.
$$

(13)

which is the average probability of the battery not being charged after the last trial of transmission. We note that the inner summation is equivalent to $f(k, 1)$, and simplify the expression as

$$
P_{\text{slots}}^0 = \sum_{k = 1}^{k_{\text{max}}} \Lambda_k f(k, 1). \tag{14}
$$

The effect of the frames between the consecutive frames that a user is active can be expressed as the probability of no energy arrival in those frames, i.e.,

$$
P_{\text{frames}}^0 = \lim_{N \to \infty} \pi p_n^0 + \pi(1 - \pi) p_n^N + \pi(1 - \pi)^2 p_n^{2N} + \ldots
$$

$$
= \frac{\pi}{1 - (1 - \pi)e^{-\alpha}}.
$$

(15)

Finally, by combining (12), (14) and (15), we obtain

$$
\text{Pr}\{E_0 = 0\} = \frac{\pi}{1 - (1 - \pi)e^{-\alpha}} \sum_{k = 1}^{k_{\text{max}}} \Lambda_k f(k, 1). \tag{16}
$$
### IV. Numerical Results

We now present several numerical examples to illustrate the results of our analysis along with the simulations of the EH-IRSA scheme. For fixed $\alpha$ and $\pi$ values, we search for RDs providing the largest $T^*$ values via differential evolution using the proposed convergence analysis. We limit the maximum repetition degree $k_{\text{max}}$ to 8 in our design. The resulting maximum asymptotic throughputs of the optimized distributions for $\alpha = 5$ and $\pi = 0.1, 1.1$ are presented in Table 1.8 For the finite length simulations, we normalize the number of total users $M_t$ to change the expected load $G$ while keeping the activity probability $\pi$ and number of slots $N$ the same.

Figs. 3 and 4 depict the resulting throughputs as a function of $G$. In Fig. 3 (with $\pi = 0.1$), we observe that the maximum throughput provided by $\Lambda_L(x)$ is about 0.58 while the newly optimized distributions can offer a maximum asymptotic throughput of 0.87, which is a substantial gain. In Fig. 4 depicting the optimization results with $\pi = 1$, we make similar observations, namely, the expected throughputs are significantly higher with the newly optimized distributions. Also note that finite size simulation results conform with the results obtained via the asymptotic analysis, hence they verify the accuracy of the approximation used. The classical waterfall effect of the optimization results with density evolution in the PLR graphs is also observed.

To illustrate the system performance with different energy harvesting rates, we provide Fig. 5, which depicts the maximum throughputs of the newly optimized distributions with all active users ($\pi = 1$) as a function of $\alpha$. The results show that by using the EH rate in the optimization process we can achieve higher throughputs compared to other distributions, namely, the distribution of slotted ALOHA, CRDSA and the 8-th order optimized distribution of IRSA [4]. Also note that as $\alpha$ increases (approaches $N$), the energy scarcity is alleviated, and plain IRSA optimization ($\Lambda_L$) offers good results. However, our optimized distributions reach to a high throughput for considerably wide range of $\alpha$ values while $\Lambda_L(x)$ requires a much higher $\alpha$ value to approach such values as it can be seen from Fig. 5.

$\Lambda_L(x)$ is the 8-th order optimized distribution of the IRSA in [4] and $\Lambda_C(x)$ is the regular-2 distribution of the classical CRDSA.

![Graph](image1)

**Table I**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\pi$</th>
<th>Distribution, $\Lambda(x)$</th>
<th>$T^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.1</td>
<td>$\Lambda_1(x) = 0.25x^3 + 0.13x^2 + 0.62x$</td>
<td>0.8701</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Lambda_L(x) = 0.5x^2 + 0.28x^3 + 0.22x$</td>
<td>0.5791</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Lambda_C(x) = x^2$</td>
<td>0.4633</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>$\Lambda_2(x) = 0.02x^4 + 0.02x^5 + 0.34x^7 + 0.62x$</td>
<td>0.8682</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Lambda_L(x)$</td>
<td>0.5521</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Lambda_C(x)$</td>
<td>0.4542</td>
</tr>
</tbody>
</table>

*Fig. 3. The results of the simulation and analysis for energy harvesting rate of 5, frame length of 300 and activity probability of 0.1.*

*Fig. 4. The results of the simulation and analysis for energy harvesting rate of 5, frame length of 300 and activity probability of 1.*
critical changes on the density evolution, and can potentially be extended to more complex channel models, e.g., using the recent works on erasure channels in [16], [17] and fading channels in [18]. Furthermore, it can potentially be extended to the cases with higher battery capacity and different models on energy arrivals as well.

REFERENCES