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Stable and Robust Controller Synthesis for Unstable Time Delay Systems via Interpolation and Approximation

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Abstract: In this paper, we study the robust stabilization of a class of single input single output (SISO) unstable time delay systems by stable and finite dimensional controllers through finite dimensional approximation of infinite dimensional parts of the plant. The plant of interest is assumed to have finitely many non-minimum phase zeros but is allowed to have infinitely many unstable poles in the open right half plane. Conservatism of the proposed methods is illustrated by numerical examples for which infinite dimensional strongly stabilizing controllers are derived in the literature.

Keywords: Robust stabilization, strong stabilization, stable controller, time delay systems, interpolation, finite dimensional approximation

1. INTRODUCTION

In this paper, we study the robust stabilization of single input single output systems, which have finitely many unstable zeros in the open right half plane, by stable controllers. Stable controllers are desired due to their robustness against sensor failures (Zeren and Özbay (1998)), saturation of the control input (Ünal and Iftar (2012b)) and other practical reasons, see e.g. Özbay and Garg (1995). Stabilization of a system by a stable controller is also known as strong stabilization, see Vidyasagar (1985) and Doyle et al. (1992) for details.

For finite dimensional case, there have been extensive research for robust stabilization by stable controllers using linear matrix inequalities, algebraic Riccati equations and non-convex optimization, see e.g. Petersen (2009), Gumussoy et al. (2008) and their references.

For infinite dimensional systems, sensitivity reduction by strong stabilization have been studied by Gumussoy and Özbay (2009), Özbay (2010), Wakaiki et al. (2012). Robust stabilization of infinite dimensional systems by stable controllers has also been studied by Wakaiki et al. (2013), considering only infinite dimensional controllers. In Wakaiki et al. (2013), upper and lower bounds for the maximum allowable uncertainty level have been obtained for robust and strong stabilization of infinite dimensional plants. To the best of our knowledge, strong and robust stabilization of infinite dimensional plants by stable and finite dimensional controllers is still an open research question.

In this study, first we concentrate on a simplified case in which we assume that the time delay system has finitely many unstable poles in the open right half plane. We propose a method to approximate the infinite dimensional and invertible part of the system by a finite dimensional transfer function. After that, using the error associated with this approximation, we introduce a sufficient condition under which it is possible to design a stable controller robustly stabilizing the time delay system. We additionally explain how to design the desired stable and finite dimensional controller when the problem is feasible. In the second part of the study, we deal with a more complicated case in which the time delay system has infinitely many unstable poles in the open right half plane. Similar to first part, by using the approximation error and the approximation itself, we introduce a sufficient condition under which the problem is feasible and outline how to design stable and finite dimensional controllers.

The rest of the paper is organized as follows: Section 2 defines the main problem of this paper together with the assumptions. In Section 3, we briefly point out the method defined in Wakaiki et al. (2013) for the sake of completeness in addition to a basic result about the feasibility of the modified Nevanlinna-Pick interpolation problem. Section 4 is about robust stabilization of time delay systems having finitely many unstable poles in the open right half plane. Section 5 considers the case where the plant has infinitely many unstable poles. Section 6 compares the effectiveness of the method of Wakaiki et al. (2013) and the methods given in Section 4 and 5 via numerical examples in order to present the conservatism of the proposed methods. Finally, Section 7 concludes the paper by some remarks.

2. PROBLEM STATEMENT

Throughout this study, we consider the linear, continuous time, single input single output unity feedback system given in Figure 1. The plant P is assumed to be a time

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delay system which has finitely many simple zeros in the open right half plane (denoted by $\mathbb{C}_+$).

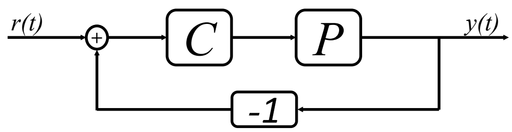

Fig. 1. Unity feedback system of interest

A controller C is said to stabilize P if S, PS and CS belong to $\mathcal{H}_\infty$, where $S = (1 + PC)^{-1}$ is the sensitivity function of the closed loop system. Let us denote the set of all stabilizing controllers for a specific plant P by $\mathfrak{C}(P)$, i.e. C stabilizes P if $C \in \mathfrak{C}(P)$. Then P is strongly stabilizable if $\mathfrak{C}(P) \cap \mathcal{H}_\infty \neq \emptyset$. It is essential to note that the set $\mathfrak{C}(P)$ may include infinite dimensional transfer functions as well as finite dimensional ones. Let us further define the set of all stabilizing and *finite dimensional* controllers that stabilize the plant P as $\mathfrak{C}_f(P)$.

It is well known in the literature that it is not possible to stabilize any P by a stable controller if P does not satisfy the parity interlacing property (PIP). In other words, $\mathfrak{C}(P) \cap \mathcal{H}_\infty \neq \emptyset$ if P has even number of poles between any pair of right half plane zeros on the extended positive real axis, see e.g. Ünal and Iftar (2012a).

Following assumption holds throughout the paper:

Assumption 1. Let us assume that the time delay system P is a ratio of two quasi-polynomials, i.e. $P(s) = q_n(s)/q_d(s)$ where $q_n(s)$ is retarded type with no direct I/O delay. The denominator quasi-polynomial $q_d(s)$ can be retarded or neutral type. Then, in this case, it has been shown that P has finitely many zeros in $\mathbb{C}_+$ and can be written in the form

$$P = \frac{M_n}{M_d} N_o \tag{1}$$

where M_n and M_d are inner and they hold zeros and poles of P in $\mathbb{C}_+$, respectively. Readers are directed to Bonnet and Partington (2002) and its references for further details on the analysis of delay systems of retarded and neutral type. We further assume that $q_n(s)$ and $q_d(s)$ do not have common roots in $\mathbb{C}_+$. Since the plant has finitely many zeros in $\mathbb{C}_+$, M_n is a finite dimensional transfer function. We also assume that the zeros of M_n are distinct and they are denoted by $z_1, \ldots, z_n$. Note that $N_o = PM_d/M_n$ is infinite dimensional and outer, for the sake of simplicity we assume that the relative degree of the plant is zero, in this case $N_o, N_o^{-1} \in \mathcal{H}_\infty$. When N_o has a relative degree greater than zero, then we need to make further assumptions on the uncertainty weight so that the resulting controller is proper, Doyle et al. (1992). Moreover, the above assumptions imply that the plant has finitely many poles within a sufficiently small neighborhood of the Im-axis, in particular this means that there is no chain of poles clustering the Im-axis. See also Gumussoy and Özbay (2018) for further technical discussions on this issue.

Assumption 1 does not declare the number of poles of the plant P in $\mathbb{C}_+$. If $q_d(s)$ is retarded type, or neutral type with all the asymptotic chains on the open left half plane, then P has finitely many poles in $\mathbb{C}_+$ (as it will be the case in Section 4), then M_d is a finite dimensional transfer function and all the infinite dimensionality of the plant is captured by invertible N_o. However, if $q_d(s)$ is neutral type with at least one asymptotic root chain in the open right half plane, then, the plant has infinitely many unstable poles in $\mathbb{C}_+$ (as it will be the case in Section 5), and M_d is infinite dimensional.

Let us further assume that P is the nominal model and the actual plant belongs to a set $\mathfrak{P}(P)$ with multiplicative uncertainty:

$$\mathfrak{P}(P) = \{P_\Delta = (1 + W\Delta)P : \|\Delta\|_\infty < 1, \Delta \in \mathcal{H}_\infty\} \tag{2}$$

The following assumption about the uncertainty weight W holds throughout the paper:

Assumption 2. Uncertainty weight W is a unit in $\mathcal{H}_\infty$, i.e. $W,\ W^{-1} \in \mathcal{H}_\infty$; moreover, it satisfies $\|W\|_\infty < 1$.

It can be shown that the controller C stabilizes all elements of the set P_Δ if it stabilizes the nominal plant model P and satisfies

$$\|WT\|_\infty \leq 1 \tag{3}$$

where $T = PC(1 + PC)^{-1}$.

Now, we can define the main problem as follows:

Problem 1. Find a **finite dimensional** controller $C \in \mathfrak{C}(P) \cap \mathcal{H}_\infty$ satisfying (3) under Assumptions 1 and 2.

Problem 1 is called the **R**obust **S**tabilization of Infinite Dimensional Plants by **S**table and **F**inite Dimensional **C**ontrollers (RSSFC).

3. RELEVANT LITERATURE

In Wakaiki et al. (2013), a relaxed version of Problem 1 is considered where the controller is allowed to be infinite dimensional. According to them, this relaxed problem has a solution if it is possible to find a function U in $\mathcal{H}_\infty$ such that

- $U, U^{-1} \in \mathcal{H}_\infty$
- $U(z_i) = 1/M_d(z_i)$ for $i = 1, \ldots, n$ where $M_n(z_i) = 0$
- $\|W_s^{-1}U\|_\infty < 1$

where W_s is also a unit in $\mathcal{H}_\infty$ whose frequency response satisfies the following relation

$$|W_s(j\omega)| \leq \frac{1 - |W(j\omega)|}{|W(j\omega)|}, \quad \forall\omega \in \mathbb{R}. \tag{4}$$

If such U exists then the robustly stabilizing stable controller is given as

$$C = \frac{1 - M_dU}{M_nN_oU}. \tag{5}$$

As it is discussed in the previous section, N_o and possibly M_d are the infinite dimensional parts of the controller. Additionally, design of U may also lead to infinite dimensional transfer functions as it is described in Gumussoy and Özbay (2009) and Özbay (2010). Design of such U is also known as the modified Nevanlinna-Pick interpolation problem (mNPIP) or bounded unit interpolation problem in the literature. In Yücesoy and Özbay (2015) there was an attempt to find finite dimensional solutions of mNPIP by some iterative techniques for only real interpolation conditions. In Yücesoy and Özbay (2018b), we proposed a predetermined structure for the unit interpolating function and reduced the mNPIP to a classical Nevanlinna-Pick interpolation problem to analyse the feasibility of the mNPIP through the associated Pick matrix. When a feasible solution for mNPIP exists, it is calculated through the optimal strategy defined in Yücesoy and Özbay (2016) and Yücesoy and Özbay (2018a). In this study, we will make use of the proposed method of Yücesoy and Özbay (2018b) to solve mNPIP, where the solution is finite dimensional.

4. SOLUTION FOR THE CASE OF FINITELY MANY UNSTABLE POLES

When the plant has finitely many unstable poles in $\mathbb{C}_+$, the only infinite dimensional part of the controller is N_o. Following design method is based on finite dimensional approximation of N_o.

Proposition 1. RSSFC has a solution if there exists a rational transfer function R such that

- $R, R^{-1} \in \mathcal{H}_\infty$
- $R(z_i) = 1/M_d(z_i)$ for all $i = 1, \ldots, n$
- $\|K^{-1}R\|_\infty < 1$

for some $K, K^{-1} \in \mathcal{H}_\infty$ satisfying

$$|K(j\omega)| \leq \frac{1 - |W(j\omega)|}{|W(j\omega)| + |E(j\omega)|}, \quad \forall \omega \in \mathbb{R} \tag{6}$$

where $E = \hat{N}_o N_o^{-1} - 1$ is the error introduced by the approximation and $\hat{N}_o$ is a finite dimensional approximation of N_o.

Proof 1. Let us consider a **finite dimensional** controller of the form

$$C = \frac{1 - M_d R}{M_n \hat{N}_o R} \tag{7}$$

where $\hat{N}_o, \hat{N}_o^{-1} \in \mathcal{H}_\infty$ is a finite dimensional approximation of N_o. Note that if it is possible to find a rational transfer function $R \in \mathcal{H}_\infty$ such that $R^{-1} \in \mathcal{H}_\infty$ and R satisfies the following interpolation conditions for $z_i \in \mathbb{C}_+$ and $\forall i$

$$R(z_i) = 1/M_d(z_i)$$

where $M_n(z_i) = 0$ then $C \in \mathcal{H}_\infty$ and in case stabilization is obtained, it will be **Strong Stabilization**.

Next, let us derive the conditions under which the internal stability of the feedback loop is satisfied. To do so, we need to find the conditions which satisfy

$$S, PS, CS \in \mathcal{H}_\infty.$$

We can write S as

$$S = \frac{1}{1 + PC} = \frac{R M_d \hat{N}_o}{N_o \left(1 + \frac{R M_d (\hat{N}_o - N_o)}{N_o}\right)}. \tag{8}$$

Note that, if $\|ER\|_\infty < 1$ than $S \in \mathcal{H}_\infty$ by small gain theorem where

$$E = \frac{\hat{N}_o - N_o}{N_o} \tag{9}$$

since M_d is inner, i.e. $|M_d(j\omega)| = 1$ for all $\omega \in \mathbb{R}$. It is also easy to show that the aforementioned condition is sufficient to show PS, $CS \in \mathcal{H}_\infty$, hence **Internal Stability** for RSSFC is satisfied.

In order to derive a condition for robust stability, let us first write T as

$$T = \frac{PC}{1 + PC} = \frac{1 - RM_d}{1 + RE}. \tag{10}$$

For robust stability due to multiplicative uncertainty, we need to satisfy (3). Since $\|W\|_\infty < 1$ then it is sufficient to simplify the condition as

$$|R(j\omega)| < \frac{1 - |W(j\omega)|}{|W(j\omega)| + |E(j\omega)|} \tag{11}$$

for all ω. Let us assume that there exists an outer function K such that

$$|K(j\omega)| \leq \frac{1 - |W(j\omega)|}{|W(j\omega)| + |E(j\omega)|}$$

and $K, K^{-1} \in \mathcal{H}_\infty$. With this assumption, we can simplify (11) to $\|K^{-1}R\|_\infty < 1$. If this is satisfied then **Robust Stability** condition of RSSFC is also satisfied. It is easy to show that $\|K^{-1}R\|_\infty < 1$ implies $\|ER\|_\infty < 1$.

5. SOLUTION FOR THE CASE OF INFINITELY MANY UNSTABLE POLES

When the plant has infinitely many unstable poles, M_d becomes infinite dimensional, in addition to N_o. We need to incorporate a finite dimensional approximation of M_d into the controller in order to design a finite dimensional one. Following proposition quantifies the effect of the error of this approximation on the controller design process when the plant has infinitely many unstable poles in $\mathbb{C}_+$.

Proposition 2. Consider Problem 1 under Assumptions 1 and 2. Additionally assume that the plant has infinitely many unstable poles, i.e. M_d is infinite dimensional. RSSFC has a solution if there exists a finite dimensional and rational transfer function H such that

- $H, H^{-1} \in \mathcal{H}_\infty$
- $H(z_i) = 1/\hat{M}_d(z_i)$ for all $i = 1, \ldots, n$
- $\|L^{-1}H\|_\infty < 1$

for some $L, L^{-1} \in \mathcal{H}_\infty$ satisfying

$$|L(j\omega)| \leq \frac{1 - |W(j\omega)|}{|W(j\omega)\hat{M}_d(j\omega)| + |E(j\omega)|}, \quad \forall \omega \in \mathbb{R} \tag{12}$$

where $\hat{N}_o$ and $\hat{M}_d$ are finite dimensional approximations of N_o and M_d, respectively. Note that, differently from Proposition 1, $E = \hat{M}_d - M_d \hat{N}_o N_o^{-1}$ is the error introduced by the finite dimensional approximations of both M_d and N_o.

Proof 2. Proof is omitted since it is very similar to the previous case, provided that the stable controller is taken to be

$$C = \frac{1 - \hat{M}_d H}{M_n \hat{N}_o H}. \tag{13}$$

Let us compare (4), (11) and (12): (4) is the bound on the interpolating unit function when the controller is assumed to be infinite dimensional. Note that (11) has an additional term in its denominator which is associated with the error of the finite dimensional approximation of N_o. As the approximation error increases the maximum allowable norm of the interpolating unit decreases, and the problem becomes harder to solve, as expected. In (12), we again observe the additional error term as the approximation error which is associated with the finite dimensional approximation of both N_o and M_d. However, additionally the finite dimensional approximation of M_d takes place in the denominator next to the plant's uncertainty bound W. As a result of (12), we can say that the deviation of the approximation of M_d from being inner is modelled within Proposition 2 as an extra uncertainty in the plant.

6. EXAMPLES

In this section, we compare the methods proposed in this study and the method proposed in Wakaiki et al. (2013) to present the conservatism caused by the finite dimensional approximation approach. We make use of three different numerical examples. First two examples are systems with time delay having finitely many unstable poles. Such plants are suitable to be analysed by the method defined in Proposition 1. Third one will also be a system with time delay, however, this time the plant has infinitely many unstable poles and is suitable for Proposition 2.

6.1 Example 1

Let us consider the plant $P = M_n N_o / M_d$, given as

$$\begin{aligned} P &= \frac{(e^{-s} + 0.1s - 2)(s+1)(s-z_1)}{(e^{-s} + 0.3s + 0.2)(s-0.6)(s-1.5)} \\ M_n &= \frac{(s-z_1)(s-z_2)}{(s+z_1)(s+z_2)} \\ M_d &= \frac{(s-0.6)(s-1.5)(s^2 - 0.7488s + 4.3109)}{(s+0.6)(s+1.5)(s^2 + 0.7488s + 4.3109)} \\ N_o &= P M_d / M_n \\ W &= K \frac{s+1}{s+10} \end{aligned} \tag{14}$$

where $K > 0$ and $z_2 \approx 20$ is the only root of the quasi-polynomial $(e^{-s} + 0.1s - 2)$ in $\mathbb{C}_+$. Figure 2 illustrates the maximum allowable uncertainty level K for which a solution can be found for Problem 1, for the values of z_1 between 1.5 and 7. Note that, when $z_1 < 1.5$, the plant P does not satisfy PIP, hence it is not possible to stabilize it by a stable controller. As z_1 becomes larger than 1.5, the plant relaxes (i.e. it becomes far from violating PIP) and according to Smith and Sondergeld (1986), it becomes easier to find a finite dimensional and stable controller to stabilize the plant. This effect is clear in Figure 2 as the maximum allowable uncertainty bound (i.e. K) under which RSSFC is feasible gets larger as z_1 gets larger for all methods. Figure 2 also shows the effect of the conservatism caused by the finite dimensional approximation of N_o. Matlab built-in function `pade` is used to approximate N_o by finite dimensional functions of 13 and 21 degrees and results in Proposition 1 are used to derive the bounds in Figure 2. Throughout this study, all finite dimensional approximations of each N_o is conducted via Pade, however, it is not compulsory to use Pade. Any approximation method can be used to generate $\hat{N}_o$ provided that the resulting transfer function is a unit in $\mathcal{H}_\infty$. To satisfy this requirement, each delay element in N_o is replaced by its Pade approximation and an approximate right half plane pole-zero cancellation is used to have a unit approximation in $\mathcal{H}_\infty$.

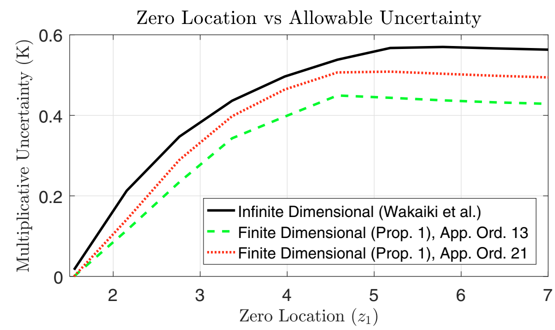

Fig. 2. Maximum allowable multiplicative uncertainty level with respect to the location of the unstable zero z_1 in Example 1

Figure 3 represents an example case where $z_1 = 7$ and the approximation order is 13. In the figure, the pole-zero map of the approximating finite dimensional transfer function ($\hat{N}_o$) is shown.

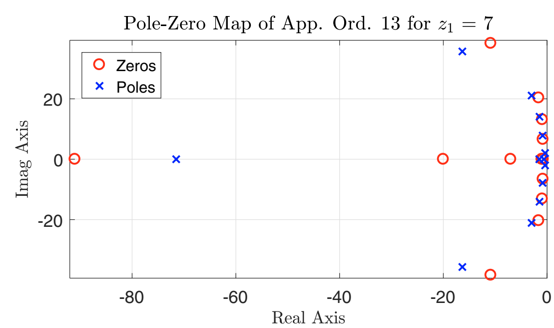

Fig. 3. Pole-zero map of the finite dimensional approximation of $\hat{N}_o$ given in (14). Maximum approximation error ($\max_{\omega \in \mathbb{R}} |N_o(j\omega) - \hat{N}_o(j\omega)|$) is -14.15 dB.

6.2 Example 2

Let us consider the plant $P = M_n N_o / M_d$, given as

$$\begin{aligned} P &= \frac{(e^{-0.1s} + 0.1s - 1.25)(s^2 - 2s + (1+\omega_1))}{(e^{-s} + 0.3s + 0.2)(s-2)(s+1)} \\ M_n &= \frac{(s-p)(s^2 - 2s + (1+\omega_1))}{(s+p)(s^2 + 2s + (1+\omega_1))} \\ M_d &= \frac{(s-2)(s^2 - 0.7488s + 4.3109)}{(s+2)(s^2 + 0.7488s + 4.3109)} \\ N_o &= P M_d / M_n \\ W &= K \frac{s+1}{s+10} \end{aligned} \tag{15}$$

where $K > 0$ and $p \approx 8.0122$ is the only root of the quasi-polynomial $(e^{-0.1s} + 0.1s - 1.25)$ in $\mathbb{C}_+$.

Note that, as $\omega_1 \to 0$, the plant P gets closer to violating PIP since when $\omega_1 = 0$ PIP does not hold because of

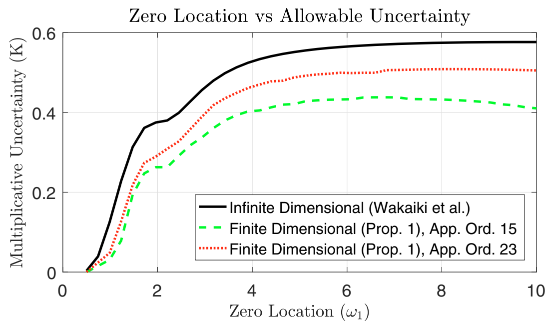

Fig. 4. Maximum allowable multiplicative uncertainty level with respect to the location of the real part of the unstable zero (ω_1) in Example 2

the pole at 2 staying in between the zeros at 1 and p. Similar to discussions in Example 1, according to Smith and Sondergeld (1986), the strong stabilization problem becomes harder and requires higher degrees of interpolating functions as the plant comes closer to violate PIP. Because of this phenomena, problem relaxes and becomes feasible for larger uncertainty levels as ω_1 gets larger.

As an example, the pole-zero map of the 15^{th} order finite dimensional approximation ($\hat{N}_o$) is given in Figure 5 for $\omega_1 = 10$.

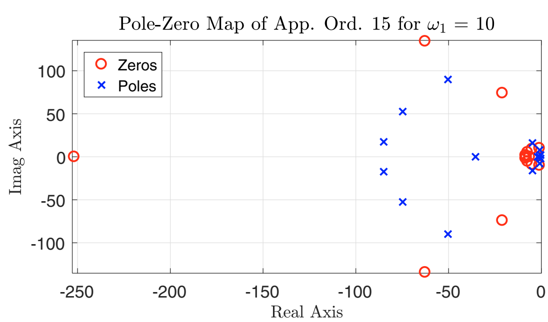

Fig. 5. Pole-zero map of the finite dimensional approximation of $\hat{N}_o$ given in (15). Maximum approximation error $(\max_{\omega\in\mathbb{R}} |N_o(j\omega) - \hat{N}_o(j\omega)|)$ is -21.69 dB.

It is important to note that in Figures 2 and 4, the multiplicative uncertainty bounds under which RSSFC is feasible (i.e. red and green dotted lines) are the unattainable upper bounds, i.e. it is not possible to achieve these bounds by finite dimensional controllers because it is not possible to solve the bounded unit interpolation problem by finite dimensional interpolating functions at that level. However, as described in detail in Yücesoy and Özbay (2018b), it is always possible to get closer to these bounds by increasing the order of the finite dimensional unit interpolating function. These bounds are calculated by utilizing $\hat{N}_o$, the finite dimensional approximation of N_o, and than solving the infinite dimensional mNPIP as described in Gumussoy and Özbay (2009) and Özbay (2010).

6.3 Example 3

Let us consider the infinite dimensional system example from Wakaiki et al. (2013) as follows:

$$\begin{aligned} P &= \frac{(s-2)(s-4e^{-s}+1)}{(s-10)(s-15)(2e^{-s}+1)} \\ M_n &= \frac{(s-2)(s-p)}{(s+2)(s+p)} \\ M_d &= \frac{(s-10)(s-15)(2e^{-s}+1)}{(s+10)(s+15)(e^{-s}+2)} \\ N_o &= PM_d/M_n \\ W &= K\frac{s+1}{s+10} \end{aligned} \tag{16}$$

where $K > 0$ and $p \approx 0.799$ is the only root of the quasi-polynomial $(s - 4e^{-s} + 1)$ in $\mathbb{C}_+$. It is shown in Wakaiki et al. (2013) that for $K < 0.47$ it is possible to find an infinite dimensional and stable controller to robustly stabilize the given plant P in (16). They have additionally designed a controller when $K = 0.468$.

In this study, we show that it is possible to design finite dimensional and stable controllers for the same plant in (16) when $K < 0.375$ by using Proposition 2. Additionally, as an example, we design a controller when $K < 0.25$. For this design, approximation of N_o is also obtained through its Pade approximation as it was described in prior examples. As it is given in (18), we designed a 7^{th} order $\hat{N}_o$ to approximate N_o in (16) and the pole-zero map of $\hat{N}_o$ is depicted in Figure 6.

For the finite dimensional approximation of M_d, finitely many unstable zeros are utilized among its infinitely many zeros. Let us say that the zeros of M_d in $\mathbb{C}_+$ are $z_k = 0.6931 + j2\pi k$ and their complex conjugates (i.e. $\bar{z}_k$) for all $k \in \{1, 3, 5, \dots\}$ in addition to 10 and 15. In the light of this parametrization, we can generate N^{th} dimensional finite approximation of M_d for even $N > 2$ as follows

$$\hat{M}_d = \frac{(s-10)(s-15)}{(s+10)(s+15)} \prod_{k=1}^{\frac{N-2}{2}} \frac{(s-z_k)(s-\bar{z}_k)}{(s+z_k)(s+\bar{z}_k)}. \tag{17}$$

We used an approximation of M_d where $N = 26$ in (17) for the numerical example in (16). All other elements of the designed controller are given numerically in (19). Note that $L(s)$ in (19) is generated by Matlab built-in function `fitmagfrd` and the interpolating part of $H(s)$ is calculated by the method that is proposed in Yücesoy and Özbay (2018b). When all the elements are combined to form the controller in (13), a 44^{th} order finite dimensional and stable controller is obtained which robustly stabilizes the infinite dimensional plant given in (16) for $K < 0.25$.

7. CONCLUSION

We considered the robust stabilization of a class of unstable time delay systems by finite dimensional and stable controllers. We divide the problem into two subclasses and derived similar sufficient conditions under which the associated problems are feasible. For the subclass of systems having finitely many unstable poles in $\mathbb{C}_+$, we propose a method to reduce the robust and strong stabilization problem to a mNPIP through the finite dimensional approximation of the infinite dimensional part of the plant, which is both stable and invertible. With this interpretation and via numerical examples, we show that as the

$$\hat{N}_o(s) = \frac{(s+30.01)(s+2)(s+0.7989)(s^2+0.423s+23.81)(s^2+5.362s+158.9)}{(s+86.47)(s+15)(s+10)(s^2+1.386s+10.35)(s^2+2.144s+101.4)} \quad (18)$$

$$L(s) = \frac{0.25787(s+86.95)(s^2+2.475s+110.3)}{(s+0.9844)(s^2+12.09s+77.58)}, \quad H(s) = \frac{0.98787(s+0.0002641)^{10}}{(s+0.2032)^{10}} L(s) \quad (19)$$

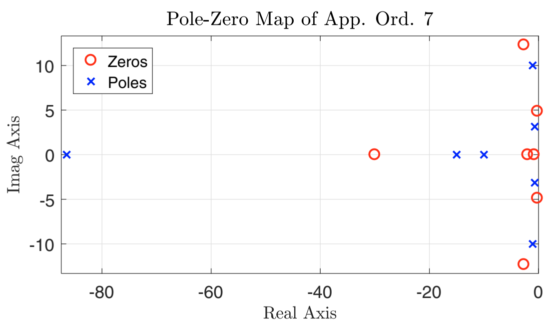

Fig. 6. Pole-zero map of the finite dimensional approximation of $\hat{N}_o$ given in (16). Maximum approximation error ($\max_{\omega\in\mathbb{R}} |N_o(j\omega) - \hat{N}_o(j\omega)|$) is -3.52 dB.

dimension of the approximation increases, and as the error of the approximation decreases, it is possible to solve the problem for larger multiplicative uncertainty levels. We also compare the results of the proposed methods to the results of the method of Wakaiki et al. (2013) and concluded that we can design finite dimensional and stable controllers for satisfactory levels of uncertainty.

For the second subclass of systems having infinitely many unstable poles in $\mathbb{C}_+$, we propose another finite dimensional approximation scheme to reduce the original problem to a mNPIP. Since the infinite dimensional part of the plant is not invertible this time, we divide the approximation process into two parts. We approximate the inner part of the infinite dimensional plant by finitely many unstable roots. The approximation of the invertible part is done as it is explained in the first subclass. We use a numerical example from the literature in order to discuss the conservatism of the proposed method.

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