Extension of an Anti-windup Scheme for Systems with Time Delay and Integral Action

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Abstract:
This study extends a recent anti-windup scheme by using Smith predictor based controller approach and by redesigning the transfer functions within the anti-windup structure. We present simulation studies for a system including time delay and integrator to illustrate that our extended structure successfully accomplish accurate tracking under the saturation nonlinearity.

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1. INTRODUCTION

The presence of actuator saturation frequently causes performance degradations or even instability and this phenomenon is called as windup see e.g. Kapila and Grigoriadis (2002) and Tarbouriech et al. (2011). Rich variety of anti-windup control mechanisms have been developed to deal with actuator saturation since the 1950's (Barbu et al., 2000; Lozier, 1956). Anti-windup architecture mainly focuses on the tracking error when controller operates at the actuator limits. One of the primary advantages of anti-windup scheme is that it helps to recover from saturation quickly.

The actuator saturation is ignored at first to design the stabilizing anti-windup controller. In other words, by eliminating the saturation, controller is designed in the linear phase and then the adverse effect of the saturation on system performance is reduced via anti-windup compensation. There have been highly promising anti-windup techniques depending on the performance requirements and system nonlinearities, see e.g. Kothare et al. (1994) for a review of early techniques.

Many existing anti-windup methods exclusively focus on eliminating the effect of saturation for the stable performance of control systems without considering specific tracking challenges (Galeani et al., 2006; Borisov et al., 2016). In this regard, internal model principle approach for the anti-windup compensator design is a significant technique for tracking and rejecting problems of the reference signal (Song et al., 2015). This approach is mainly based on a controller design to provide closed loop stability and to regulate the tracking error when specific system parameters are perturbed (Francis and Wonham, 1976). In contrast, there also exist internal model based solutions for the saturation control without aiming high performance tracking (Weston and Postlethwaite, 2000; Sornmo et al., 2013; Gayadeen and Duncan, 2016).

Another general approach to the anti-windup strategy is the conditioning technique which considers the controversy between actual input of a process and desired output of the controller under the actuator saturation (Hanus et al., 1987). Early applications of this technique have been introduced by Hanus et al. (1987) and Doyle et al. (1987). The improved version of conditioning scheme was also presented in Turner and Postlethwaite (2004) by proposing a low order anti-windup compensator and further developed by Turner et al. (2007) considering the robustness issue. An innovative anti-windup conditioning method for the time delay plants and controllers involving delayers in their structure is analyzed in Zitek et al. (2014). This technique is developed on internal model control loop with the delay operation by tuning the anti-windup scheme parameters and optimized on the basis of absolute error integral criterion (Zitek et al., 2014).

Recently, a unified anti-windup strategy to handle the input constraints such as magnitude or rate saturation for the dead-time plants has been developed and presented in Flesch et al. (2017). An anti-windup block proposed in this study does not address the time delay problem, instead Filtered Smith Predictor is used as a dead-time compensator. The proposed strategy is capable of updating the actual control action to prevent any violation due to input constraints for the time delay plants (Flesch et al., 2017).

Different than these approaches, anti-windup mechanism developed in Liu et al. (2016) includes saturation compensation blocks, internal model units as well as robust anti-windup compensator design and proven work well for finite dimensional plants. Compared to classical and more widely used anti-windup mechanisms, the one proposed in Liu et al. (2016) achieves better tracking of sinusoidal reference inputs, while taking robustness considerations in mind, see a recent application paper, Liu et al. (2018), for these claims. In this study, we extend their design to plants with time delay and integral action (i.e. unstable pole at
s = 0). For this purpose we use a modified Smith predictor form for the class of plants considered.

Brief description of the anti-windup scheme proposed in Liu et al. (2016) for delay-free systems is given in Section 2. Section 3 covers the basic steps of Smith predictor-based controller design to extend the anti-windup scheme and summary of the novel structure. We present the results of simulation studies in Section 4. Finally, concluding remarks and future directions are provided in the last section.

2. ROBUST ANTI-WINDUP SCHEME

The anti-windup structure introduced in Liu et al. (2016) includes parallel internal model units and robust anti-windup compensator for high precision trajectory tracking. The reference signal illustrated in Fig. 1 is defined based on the exogenous dynamical system equation in the form

\[ R(s) = \Lambda(s)^{-1} R_0(s) \]

where \( R_0(s) \) and \( R(s) \) represent Laplace transform of the signals \( r_0(t) \) and \( r(t) \), respectively, and \( \Lambda(s)^{-1} \) represents the dynamics of the exogenous system and typically \( \Lambda \) has roots on the Im-axis (see Section 3 and Section 4 for examples).

\[ R(s) = \Lambda(s)^{-1} R_0(s) \]

Fig. 1. The block diagram of a parallel internal model control structure.

The aim is to minimize the tracking error, \( e(t) \), shown in Fig. 1 as much as possible while satisfying the conditions:

i. Considering zero tracking signal \( r(t) = 0 \), the unforced closed-loop system is asymptotically stable,

ii. Considering any initial conditions of the plant, the closed-loop system satisfies \( \lim_{t \to \infty} e(t) = 0 \).

The plant transfer function \( G(s) \) and internal model unit \( F(s) \) in Fig. 1 are defined as

\[ G(s) = \frac{B(s)}{A(s)}, \]

\[ F(s) = \frac{B(s)}{A(s)} \frac{P(s)}{Q(s)} = \frac{G(s)}{Q(s)}, \]

where \( A, B, P, Q \) are polynomials, and \( A \) has no roots in the closed right half plane. Note that the plant is assumed to be stable.

**Lemma 1.** (Liu et al., 2016) A stabilizing controller, \( C(s) \), achieves asymptotic tracking performance if the condition

\[ (1 + F(s)) = A(s)^{-1} \Lambda(s) \]

holds.

The unknown polynomials in the definition of internal model unit are \( P(s) \) and \( Q(s) \). Basically, choosing \( Q(s) = 1 \) gives the definition of numerator polynomial \( P(s) \) by using Lemma 1:

\[ P(s) = \frac{\Lambda(s) - A(s)}{B(s)}. \]

The anti-windup scheme with the internal model structure is designed based on a standard mixed sensitivity minimization \( H_\infty \) problem in order to optimize the stabilizer design with the performance requirements and robustness against uncertainties. Accordingly, the aim is to find a stabilizing controller \( K(s) \) for the mixed sensitivity minimization problem

\[ \inf_{K_{stab.G_A}} \| \begin{bmatrix} W_1S \\ W_2T \end{bmatrix} \|_\infty \]

where \( S = (1 + KG_A)^{-1} \) and \( T = 1 - S \) are denoted as the sensitivity and complementary sensitivity transfer functions of the augmented system \( G_A \), which is defined as

\[ G_A(s) = \frac{G(s)}{1 + F(s)}. \]

Here \( W_1(s) \) is denoted as the performance weighting function and poles of \( W_1(s) \) contain the poles of Laplace transform of the reference signal (i.e., typically poles of \( W_1 \) include roots of \( \Lambda \)). Besides, \( W_2(s) \) is the robustness weight and defined as the upper bound of the multiplicative plant uncertainty.

In addition to internal model unit, robust anti-windup compensator \( \theta_1(s) \) and \( \theta_2(s) \) are also designed based on a criteria to guarantee the stability of closed loop system. The anti-windup structure including all components is illustrated in Fig. 2.

![Fig. 2. Anti-windup tracking control architecture with the internal model structure.](image_url)

Note that \( sat(\cdot) \) is denoted as saturation operator which is expressed by the relationship between controller output \( u \) and plant input \( u_m \) as

\[ sat(u) := u_m = \begin{cases} \sigma_1, & \text{if } u \leq \sigma_1 \\ u, & \text{if } \sigma_1 < u < \sigma_2 \\ \sigma_2, & \text{if } u \geq \sigma_2 \end{cases} \]

where the limits \( \sigma_1 \) and \( \sigma_2 \) are determined based on the system specifications.

The filters \( \theta_1(s) \) and \( \theta_2(s) \) are defined by the following:

\[ \theta_1(s) = \tilde{\theta}_1(s)(1 + F(s)) \]

\[ \theta_2(s) = G(s) \left( \frac{\theta_1(s)}{1 + F(s)} \right) = G(s)(1 + \tilde{\theta}_1(s)) \]

(7)
Consider a plant transfer functions in the form
\[ R(s) = \frac{\theta(s)}{\gamma} \]
for \( \alpha > 0 \), \( \beta > 0 \) and \( \gamma \) is to be determined from the following.

Achieving robust stability and tracking the sinusoidal reference signal with the proper choices of \((\theta_1, \theta_2)\) are the main subjects in the design which can be described as
\[ f(\gamma) = \| W_a(\gamma + \tilde{\theta}_1(s)) \|_\infty \]
where \( W_a(s) \) represents the additive plant uncertainty. Liu et al. (2016) propose to minimize \( f(\gamma) \) by choosing the optimal values of \( \alpha, \beta \) and \( \gamma \).

3. EXTENDED ANTI-WINDUP COMPENSATOR VIA SMITH PREDICTOR-BASED CONTROLLER DESIGN

The goal in this section is to extend the above design to systems with time delay and integrator action.

3.1 Smith Predictor-Based Controller

The main advantage of the Smith predictor-based design for the dead-time systems is that time delay is effectively taken outside the characteristic equation of the closed loop system and also every stabilizing controller can be expressed in terms of a predictor structure (Mirkin and Raskin, 2003).

Consider a plant transfer functions in the form
\[ P(s) = \frac{K}{s} R_0(s) e^{-T_d s} \]
where \( K \) is the gain of the nominal plant, \( T_d > 0 \) is the time delay in the system and \( R_0(s) \) represents the minimum phase transfer function which has the form
\[ R_0(s) = \prod_{k=1}^{n} \left( \frac{s^2/\omega_k^2}{s^2/\omega_k^2 + 2\zeta_k s/\omega_k + 1} \right) \]
where \( 0 < \omega_k < \omega_k \) are the resonant and anti-resonant frequencies, and \( \zeta_k, \zeta_k \) are the damping factors which take values between 0 and 1.

Proposed Smith predictor-based model controller structure is illustrated in Fig. 3-A as well as the controller itself is given in Fig. 3-B. Using the structure in Fig. 3-B, the Smith predictor-based controller can be defined as
\[ C_1(s) = \frac{\hat{R}_0(s)^{-1}}{K} \left( \frac{C_0(s)}{1 + C_0(s) e^{-T_d s}} \right) \]
where \( \hat{R}_0(s)^{-1} \) includes the estimated values of the parameters \( \omega_i, \zeta_i, \omega_i, \zeta_i \) for \( i = 0, 1, \ldots, n \) whereas \( R_0(s) \) consists of the real values of these parameters. We define \( C_0(s) \) as the free part of the controller which is designed based on delay free part of the plant. In the stability analysis of the closed loop feedback system, typically \( H(s) \) is chosen as 1 since it does not contribute to the system stability.

In the design of Smith predictor controller, we consider that the system successfully follows ramp and sinusoidal reference input \( r(t) \), since our aim is to achieve perfect steady-state tracking. In order to satisfy this, \( C_1(s) \) must have poles at \( s = 0 \) and at the periodic signal frequencies \( s = \pm j\omega_d \).
approaches in order to redesign the proposed anti-windup structure described in Section 2.

The known parameters in the design are the plant transfer function \( P(s) \), additive uncertainty bound \( W_a(s) \), saturation limits of the actuator and desired sinusoidal reference \( r(t) \). Based on these parameters, we mainly focus on the redesign of internal model unit \( F(s) \), robust stabilizer \( K(s) \), augmented system transfer function \( G_A(s) \) and anti-windup compensators \( \theta_1(s) \) and \( \theta_2(s) \) given in Fig. 2 via Smith predictor-based design.

The relationship is established by analyzing closed loop transfer functions of these two approaches. The transfer function \( T(s) \) for the Smith predictor design is divided into two parts by applying inner-outer factorization. As stated above, the controller to be designed must have poles at \( s = 0 \) and \( s = \pm j\omega_d \) where \( \omega_d \) is the frequency of the reference signal. With the same strategy, anti-windup controller must also have poles at these desired locations. By using this idea, controller in Fig. 1 is rewritten to eliminate the exogenous term by incorporating the integrator to the controller structure. To include the other roots \( (s = \pm j\omega_d) \), remaining part of the controller is formulated as a function, which resembles the outer part of \( T(s) \).

The detailed calculations for the new definitions of these transfer functions are provided in Öztürk (2017), here we mainly describe the final forms. Internal model unit is redefined as

\[
F(s) = \frac{1}{s} W_{aw}(s)e^{-T_{ds}}. \tag{15}
\]

\( W_{aw}(s) \) is a stable transfer function defined as \(-sT_{so}(s)\) where \( T_{so}(s) \) is the outer part of the Smith predictor-based design closed loop transfer function. The stabilizer is divided into two parts, \( K_0(s) \) and \( K_1(s) \), which are determined as

\[
K(s) = K_0(s)K_1(s) = \frac{C_0(s)}{1 + \frac{1}{2}C_0(s)}(K^{-1}R_0(s)^{-1}). \tag{16}
\]

For the augmented system transfer function \( G_A(s) \), novel definition of the internal model unit (15) is used in (6). Similarly, the compensator \( \theta_1(s) \) and \( \theta_2(s) \) are described using (15) in the definitions provided in (7). Finally, the new anti-windup controller (equivalent of \( C(s) \) in Fig. 1 using the new definitions) has the form

\[
C_{aw}(s) = \frac{K(s)}{1 - T_{so}(s)e^{-T_{ds}}} \tag{17}
\]

where \( T_{so}(s) = \frac{C_0(s)}{1 + C_0(s)/2} \) represents the outer closed-loop Smith predictor-based transfer function, which implies that the design of \( C_0 \) is such that it does not contain any zeros in the open right half plane. Note that \( C_0(s) \) is a stabilizing controller for \( 1/s \) and an example for its design will be provided in Section 4.

4. NUMERICAL RESULTS ON THE CASE STUDY

This section discusses the application of anti-windup controller structure on a plant including time delay and integral term. We present the simulation studies we performed with and without the extended structure.

4.1 Design of \( C_0 \)

In order to apply the new scheme, we have to design the stabilizing controller \( C_0(s) \), and calculate the transfer functions in Fig. 2. As described, \( C_0(s) \) must be designed to stabilize \( 1/s \). If we assign \( P_1(s) = 1/s \), then the set of controllers stabilizing the plant \( P_1(s) \) can be parametrized as

\[
C_0 = \frac{X + D_P Q}{Y - N_P Q} \tag{18}
\]

where \( N_P(s) = \frac{1}{s+a} \) and \( D_P(s) = \frac{a}{s+a} \). The parameter \( a > 0 \) is determined based on the desired pole locations of the closed loop system.

In this system \( X(s) = a \) and \( Y(s) = 1 \) solve the Bezout equation. Consequently, the stabilizing controller can be rewritten in the following form

\[
C_0(s) = \frac{a + \frac{s}{s+a} Q(s)}{1 - \frac{1}{s+a} Q(s)} \tag{18}
\]

and the problem reduces to designing a stable \( Q(s) \) satisfying tracking requirements. In order to achieve high performance tracking, the plant or controller should include poles at the periodic signal frequencies. As in (12) and (13), in the design of \( C_0(s) \), we determine two interpolation conditions

\[
\begin{align*}
C_0(0) &= -\frac{1}{T_d}, & C_0(j\omega) &= \frac{-j\omega}{1 - e^{-T_d j\omega}}, \tag{19}
\end{align*}
\]

where \( \omega \) is the frequency of periodic reference signal of interest and using (18), the interpolation conditions are translated to

\[
\begin{align*}
Q(0) &= a(1 + aT_d), & Q(j\omega) &= \frac{j\omega + a - ae^{-jT_d}(j\omega + a)}{j\omega e^{-jT_d}}. \tag{20}
\end{align*}
\]

Hence, the problem can be redefined as designing a stable \( Q(s) \) which satisfies the conditions in (20) and then designing the appropriate \( C_0(s) \) described in (18).

By considering the known roots \((0, \pm j\omega)\), minimum degree of \( Q(s) \) is postulated as two. In order to guarantee the stability of \( Q(s) \), roots of the denominator polynomial are chosen to place the closed loop system poles at the desired locations based on the given input signal. We determine that \( Q(s) \) has the form

\[
Q(s) = \frac{bs^2 + cs + d}{s^2 + es + f} \tag{21}
\]

where \( e, f > 0 \) are free parameters. Once these are chosen, the other parameters are determined by employing the interpolation conditions defined in (20). Also, one has to check that the resulting \( C_0 \) does not have zeros in the right half plane. Otherwise, the free parameters can be changed or the order of \( Q \) can be increased to gain more freedom.

4.2 Simulation Results

Simulation studies are performed both using the extended anti-windup structure and using only controller and plant without anti-windup scheme. We simulate the following plant transfer function,
where $\zeta_n = 0.08$, $\omega_n = 175$, $\zeta_d = 0.02$, $\omega_d = 285$ and $h = 8.1 \text{ m/s}$. Note that the transfer function has the form described in (10) and saturation limits of the actuator are $[-1, 1] (V)$. The additive upper bound $W_a(s)$ is described by considering the cumulative error differences between frequency response tests conducted on the real hardware and designed plant transfer function:

$$W_a(s) = \frac{0.011 (1 + s/20)}{(1 + 2 \zeta_{d,a} (s/\omega_{d,a}) + (s/\omega_{d,a})^2)}$$

where $\zeta_{d,a} = 0.01$ and $\omega_{d,a} = 280 \text{ rad/sec}$. The aim using additive upper bound is to calculate optimal values of $\alpha$, $\beta$ and $\gamma$ given in (8).

The reference input is described as $r(t) = 50 \sin(\omega t + \pi/2) (mm)$ for $\omega = 1.5 \text{ rad/sec}$ and free parameters in $Q(s)$ and $C_0(s)$ are determined as $a = 2$, $e = 4$, $f = 1$. Free part of the controller $C_0(s)$ can be calculated using stable $Q(s)$ and Bezout equation polynomials $X(s)$, $Y(s)$, $N_p(s)$ and $D_p(s)$:

$$C_0(s) = \frac{-123.46 (1 + s/0.253)}{(1 - s/0.014)} \times \frac{(1 + 1.471(s/1.392) + (s/1.392)^2)}{(1 - 0.09 (s/1.494) + (s/1.494)^2)}.$$  

Finally, internal model unit $F(s)$ is calculated using (15) and found as

$$F(s) = \frac{-1.0588 (1 + 1.471(s/1.392) + (s/1.392)^2)}{(1 + s/3.732)(1 + s/2^2)} e^{-hs}.$$ 

Time delay in this expression is replaced with its rational equivalent obtained via second order Padé approximation in order to solve the $H_\infty$ control problem (5) (though there are direct $H_\infty$ design methods for systems with delays see e.g. Foias et al. (1996)). The aim here is to obtain low order stabilizers; for this reason we are choosing a finite dimensional approximation (Padé is widely used and readily available in Matlab, there are various other methods as well, see e.g. Michiels et al. (2011) and references therein).

Stabilizer parameters $K_0(s)$ and $K_1(s)$ are also computed from equation (16):

$$K_0(s) = \frac{s (1 + s/0.253) (1 + 1.471(s/1.392) + (s/1.392)^2)}{(1 + s/3.732)(1 + s/2^2)(1 + s/0.268)},$$

$$K_1(s) = \frac{0.14 (1 + 0.04 (s/285) + (s/285)^2) (1 + 0.016 (s/175) + (s/175)^2)}{(1 + s/175)(1 + s/175)^2}.$$ 

We further recall equation (6) to calculate the augmented transfer function $G_A(s)$:

$$G_A(s) = -\frac{128.8 (1 + s/3.732)(1 + s/2^2)}{s (1 - s/0.36)(1 + s/1.49)(1 + s/1.49)^2} \times \frac{(1 + 0.016 (s/175) + (s/175)^2)}{(1 + 0.04 (s/285) + (s/285)^2)} e^{-hs}.$$ 

Anti-windup compensators $\theta_1(s)$ and $\theta_2(s)$ are also derived with the corresponding definitions as

$$\theta_1(s) = \frac{-5.35 (1 - s/0.36)(1 + 0.06 (s/1.49) + (s/1.49)^2)}{(1 + s/40)(1 + s/20)(1 + s/3.732)(1 + s/2^2)},$$

$$\theta_2(s) = \frac{706.27 (1 + 0.016 (s/175) + (s/175)^2)}{s (1 + s/40)(1 + s/20)} e^{-hs}.$$ 

In the simulation analysis, we first examine the system behavior in the effect of input saturation without anti-windup controller structure. The resulting system output together with the reference sinusoidal signal is illustrated in Fig. 4. Note that there is a significant difference between the output and desired input which can be seen clearly in the second graph. Tracking error is approximately 24 mm.

Fig. 4. System output under the effect of input saturation when there is no anti-windup structure. The tracking error is also represented in the second graph.

Fig. 5. System output under the effect of input saturation when extended anti-windup structure is operating. The tracking error is also represented in the second graph.

Fig. 5 illustrates the system output when we use the proposed anti-windup architecture. The output recovers from nonlinearity after around 7.28 seconds and tracking error converges to zero accurately.
By comparing the system output results depending on the anti-windup and without anti-windup studies, the system successfully recovers nonlinearity after a time and minimizes the tracking error when we apply the extended architecture. Measured system output follows the desired sinusoidal reference with the acceptable performance despite the saturation nonlinearity and time delay.

5. CONCLUSION AND FUTURE WORK

The proposed anti-windup mechanism in Liu et al. (2016) including internal model structure together with the robust anti-windup compensator is used to allow high tracking performance, however, this method is not applicable for the dead-time systems. The present work fills this gap by focusing on how the adverse effects of actuator saturation can be suppressed independently of time delay in the system. Motivated by the Smith predictor-based design strategy of Taşdelen and Özbay (2013), we employed a new anti-windup mechanism applicable for the dead-time systems by extending the anti-windup architecture. Robustness to parameter mismatch in the plant and internal structure of the controller is analyzed and stability conditions are determined in Öztürk (2017). The longer term goal of this study is to design a Smith predictor-like controller based on the extended anti-windup scheme for the plants including more than one pole at $\mathbb{C}_+$. 

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