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IDENTIFICATION OF A PRINCESS UNDER INCOMPLETE
INFORMATION: AN AMARNA STORY

ABSTRACT. This article presents four analyses of an interaction between the middle-Bronze Age Pharaoh Nibmuarea and the Babylonian king Kadashman-Enlil as described in the Amarna letters (Moran [1992] *The Amarna Letters*, The Johns Hopkins University Press, Baltimore, Maryland). Intent on denying the Pharaoh his daughter in marriage, the Babylonian king was faced with the choice of sending messengers who could (“dignitaries”) or could not identify (“non-dignitaries”) his missing sister in the Pharaoh’s court. Intent on marrying the king’s daughter, the Pharaoh was faced with the choice of showing the sister or showing someone else. Based on the assumption of complete information (game 1), the analysis revealed a dominant-strategy equilibrium: Nibmuarea shows the sister and Kadashman-Enlil sends non-dignitaries. Based on the assumption of one-sided incomplete information (Pharaoh’s misperception; game 2), the analysis revealed that the Pharaoh had a dominant strategy of showing the sister irrespective of whether the king is keen or reluctant to learn about his sister’s fate. Based on the assumption of one-sided incomplete information (Kadashman-Enlil’s misperception; game 3), the analysis revealed that if non-dignitaries are sent, the Pharaoh prefers showing someone other than his sister. Based on the assumption of two-sided incomplete information (game 4), the Pharaoh finds it more beneficial to present the sister irrespective of whether his intentions are genuine or feigned. With incomplete information, it is difficult to judge the other’s intentions; the cost of being caught cheating by not showing the sister to knowledgeable messengers was quite high. These analyses highlight the strategic uncertainty that characterized this Bronze-Age interaction.

KEY WORDS: Dominant-strategy equilibria, Judging intentions, Middle-Bronze Age diplomacy, One and two-sided incomplete-information games, Strategic uncertainty

The Amarna letters dating to 1400–1350 B.C.E constitute a vivid demonstration of Bronze Age balance-of-power calculations and interactions in the Middle East. During that time, the kingdoms of Egypt, Hatti, Mitanni, and Babylon formed alliances, waged wars, and resisted threats. The 317 Amarna letters (written in Akkadian, the lingua franca during the bronze age) prove the existence of a triangle of powerful kingdoms, namely Egypt, Hatti, and Mitanni



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orbited by Babylon and a rising Assyria (Moran, 1992; Redford, 1992; Roux, 1992).

Alliances in those days were offered on the basis of “brotherhood” established by demands and offers of princesses, that is, dynastic marriages (Cohen, 1996). There is evidence in the Amarna letters of such diplomatic marriages linking Mitanni and Babylonian kings to Pharaohs of Egypt, the hegemonic rulers of that time (Moran, 1992). These rulers sent their daughters to Pharaohs to secure the mighty Egyptian alliance. Rarely would a Pharaoh send his daughter to be another ruler’s bride or would an Egyptian queen marry a foreign ruler.¹ In the first letter of the Amarna collection, EA1 (EA refers to “El Amarna”), the Pharaoh Nibmuarea (Amenophis III) replies to Kadashman-Enlil, the Babylonian king: “As for your writing me *in order to aggrandize yourself* (and) to put oil on the head of a girl, you for your part sent me one present. *Are we to laugh?*”²

This paper analyzes EA1 in an interactive setting where rulers misperceive each other. We propose first a complete-information game, and, then, one-sided and two-sided incomplete information games in strategic form. The latter develops the consequences of reciprocal suspicions implied in the letter between the sender, Pharaoh Nibmuarea, and the addressee, Kadashman-Enlil. A complete-information game model of the interaction at an ordinal level is presented by us elsewhere (Druckman and Güner, 2000).

In the game analyses to follow, we reason backward from known outcomes described in the letter to the processes leading to those outcomes. Each game is construed as a thought experiment in the sense of an “if ... then” simulation of a possible path that would have led to the decisions that were made: in this case, Nibmuarea’s decision on whether or not to present the princess and Kadashman-Enlil’s decision on whether or not to send messengers who were dignitaries. *If* we assume that the kings were acting strategically, *then* these are plausible depictions of the decision-making processes; however, it may not have been the actual processes about which the letter provides little information. The insights gained from the analyses illuminate the importance of assumptions made about the other’s intentions under conditions of strategic uncertainty.

The paper is organized into several parts. First, we describe the interaction as depicted in the first letter. Second, we analyze the interaction under the assumption of complete information. Third, we develop two games of one-sided incomplete information, one focusing on Nibmuarea's misperception of Kadashman-Enlil and the other on the latter's misperception of the former ruler. Fourth, we construct a two-sided information game where each ruler misperceives the other. The paper concludes with some implications for understanding the motives and perceptions of these Bronze Age rulers.

EL AMARNA LETTER 1 (EA1)

EA 1 focuses on the identification of a Babylonian princess at the Egyptian court.³ The princess is Kadashman-Enlil's sister given as a wife to Nibmuarea by his father. The letter is Nibmuarea's reply to Kadashman-Enlil's previous (unfound) letter that seemingly accused the Pharaoh of not presenting the princess to the Babylonian messengers. By using his ignorance about his sister's fate, Kadashman-Enlil refuses to give his daughter to Nibmuarea as another wife. Nibmuarea writes: "I have just heard what you wrote me about, saying, 'Here you are asking for my daughter in marriage, but my sister whom my father gave you was (already) there with you, and no one has seen her (so as to know) if now she is alive or if she is dead'" (Moran, 1992: 1).

Nibmuarea indicates to Kadashman-Enlil that the proper way of identifying the princess is by sending dignitaries as messengers: "... Did you, however, ever send here a dignitary of yours who knows your sister, who could speak to her and identify her? Suppose he spoke with her. The men whom you sent here are nobodies ... There has been no one among them who knows her, who was an intimate of your father, and who could identify her" (Moran, 1992: 1). He adds: "... And as for your writing me 'You addressed my messengers as your wives were standing gathered in your presence, saying: Here is your mistress who stands before you. But my messengers did not know her, (whether) it was my sister who was at your side' ... and still you say: 'Who is to identify her?' Why don't you send me a dignitary of yours who can tell you the truth, the well-being of

your sister here...?” (Moran, 1992: 1). Nibmuarea is therefore well aware that Kadashman-Enlil has the options of sending dignitaries and non-dignitaries as messengers to identify his sister.

Thus, Kadashman-Enlil had two options, either sending dignitaries or sending non-dignitaries. Based on our interpretation of the letter, we assume that he had these options in the eyes of Nibruarea: The king could not know with certainty that his sister was alive or dead. However, even if Kadashman-Enlil could be certain that his sister was dead, the choice between sending dignitaries or non-dignitaries would serve another purpose, namely denying a princess to the Pharaoh. In the complete information game presented below, his dominant choice of sending non-dignitaries could be interpreted in this way. And, in the one-sided incomplete information game, also presented below, a *Reluctant* Babylonian king is not interested in the fate of his sister. On the other hand, if Nibruarea thought that the king was certain in his belief about his sister’s death, he would assume that Kadashman-Enlil only had the choice of sending non-dignitaries, leading him to maximize his own utilities in light of the king’s determined course of action.

As for Nibmuarea’s options, Kadashman-Enlil is aware that the Pharaoh could well present the princess or some other woman from the Court. We deduce this from the Pharaoh’s remarks about Kadashman-Enlil’s account of the story. Nibmuarea writes: “And as for your writing me: ‘Perhaps the one my messengers saw was the daughter of some poor man, or of some Kaskean, or the daughter of some Hanigalbatean, or someone from Ugarit.⁴ Who can believe them? The one who was at your side ..., she did not open her mouth. One cannot believe them at all.’ These are your words. But if your sister were dead, what reason would there be for one’s concealing her death and our presenting someone else?” (Moran, 1992: 1).

Kadashman-Enlil and Nibmuarea interact strategically: Kadashman-Enlil has to think about what Nibmuarea could choose and Nibmuarea likewise has to think about Kadashman-Enlil’s choices. Their separate decisions mutually affect them. Game theory is concerned with interactive decisions and could reveal strategic aspects of this particular interaction. These choices are revealed through the simultaneous choices of the players. In strategic form, the game consists of each player taking decisions without knowledge of the

other player's decisions. In this game, the Pharaoh does not know whether the messengers are dignitaries and the Babylonian king does not know whether the princess will be presented. A different game would be based on sequential choices. For example, the king would move first by choosing the type of messengers. The Pharaoh would then respond to that choice with his decision to present the princess or someone else. This kind of alternation of moves is referred to as a signaling game. The king signals his intentions by choosing the type of messengers in his move at the start of the game. (See Guner, 1998, for an example of a signaling game.)

An essential assumption of most game-theory applications is that players will make the same probability assessments if they have access to the same information. Referred to as the common knowledge assumption by Kreps (1990), it is a basis for our modeling. We assume that both Nibruarea and Kodashman-Enlil develop their beliefs about each other's intentions from the same body of information. Supporting this assumption is the evidence from the letters indicating that the two rulers are well acquainted with each other. Any attempt to relax this assumption would require sophisticated mathematics outside the scope of this article.

We address the following questions in this article: If both rulers are informed about their respective incentives, what rational decisions should they take? If they misperceive each other, should they still take the same decisions as if they were in a complete-information case? What game-theoretic advice can one provide for the interaction under different information conditions? Answers to these questions amount to the detection of strategic rationality at work many millennia ago as encapsulated in a clay tablet.

A COMPLETE-INFORMATION GAME

The players are Nibmuarea and Kodashman-Enlil. Nibmuarea can either show the princess (S) or show someone else ($-S$). Kodashman-Enlil can either send dignitaries as messengers to Nibmuarea's court (D) or send non-dignitaries ($-D$). These choices are shown in Figure 1.

For convenience, we set players' payoffs to zero when the identification outcome is reached, that is, when Nibmuarea presents

		Kadashman-Enlil	
		D	-D
Nibmuarea	S	$0, 0$	$-d, d$
	-S	$-d - c, d$	$-d, d$

Figure 1. Identification under complete information.

the princess and Kadashman-Enlil sends dignitaries. We define the players' payoffs for other outcomes by comparing them with those obtained from identification. This comparison is done by indicating the distance between the payoffs players get from the identification and those they obtain from each of the other outcomes.

We assume that Kadashman-Enlil cannot deny a daughter to Nibmuarea when the princess is identified. Nibmuarea suffers no cost of being caught presenting someone else to the dignitaries and can ask and get Kadashman-Enlil's daughter as his wife. The difference between the identification (0) and Nibmuarea presenting someone else to Kadashman-Enlil's dignitaries is that the latter can deny his daughter to Nibmuarea. So, Kadashman-Enlil obtains $d, d > 0$, the payoff of not sending his daughter to the Pharaoh. As for Nibmuarea, he cannot get a daughter from the Babylonian king, so obtains $-d$, and, in addition, he suffers the cost of being caught as a cheater which we denote as $-c$. When non-dignitaries are sent and the Princess is either shown or not, Nibmuarea suffers no cost of being caught as a cheater but he is denied Kadashman-Enlil's daughter. Kadashman-Enlil does not care about his sister; his aim is to deny a daughter to Nibmuarea and to get a bride from Egypt "to aggrandize himself". Thus, in the upper-right and the lower-right cells of Figure 1, Nibmuarea obtains $-d$ and Kadashman-Enlil d .

The strict opposition of interest indicated by d and $-d$ is intended to reflect Kadashman-Enlil's desire to aggrandize himself. Our interpretation of this letter suggests such a motive: By daring to deny the Pharaoh his daughter in marriage, the king is seen to assert his claim of equal status. This motive can be regarded to trump his concern for the welfare of his sister, as we note below. Further, the assumption of strict opposition of interest serves the goal of parsimony by limiting the number of possible motives considered. The impacts

of this assumption are reflected in all the equilibria we computed. Thus, there are two preference parameters: the dynastic marriage issue symbolized by d and the cost of cheating symbolized by c .

By showing the princess, Nibmuarea obtains 0 if Kadashman-Enlil sends dignitaries and $-d$ if non-dignitaries are sent. By not showing the princess, Nibmuarea obtains $-d - c$ if Kadashman-Enlil sends dignitaries and $-d$ if non-dignitaries are sent. Hence, showing the princess is better than showing someone else if dignitaries are sent and gives him the same payoff as showing someone else when the Babylonian king sends non-dignitaries. The strategy of showing the princess is therefore a dominant strategy.⁵ Kadashman-Enlil too has a dominant strategy: sending non-dignitaries. He gets a greater payoff if non-dignitaries are sent when the princess is shown ($d > 0$), and, if the princess is not shown, it gives an equal amount of utility as sending dignitaries, that is $d = d$. Hence, both players have dominant strategies in this game.

The game has a dominant-strategy equilibrium: Nibmuarea shows the princess and Kadashman-Enlil sends non-dignitaries. Thus, the equilibrium outcome is $(S, -D)$. Both players choose their dominant strategies even without reasoning that “if I choose not to show the princess, then if he sends dignitaries this will be my worst outcome”, or “if I send dignitaries but he does not show my sister then I will catch him trying to deceive me”, and so on. Such calculations due to strategic uncertainty simply do not exist, because neither Nibmuarea nor Kadashman-Enlil can think that dominated strategies will be chosen. Neither Nibmuarea nor Kadashman-Enlil can expect the other to choose his dominated strategy since each knows the other knows that each knows and ad infinitum that they are rational. Thus, the complete-information game explains the observed outcome as a result of dominant strategies, given that both rulers are aware of these options and that this is common knowledge.

ONE-SIDED INCOMPLETE-INFORMATION GAMES

The assumption of complete information is not realistic. Players could not have been sure that each had a unique preference ordering. We now relax this assumption. Accordingly, we propose two games of one-sided incomplete-information to understand the connections

		Kadashman-Enlil (Keen)	
		D	-D
Nibmuarea	S	0, 0	- d, d - 1
	-S	- d - c, d - 1	- d, d - 1
(p)			
		Kadashman-Enlil (Reluctant)	
		D	-D
Nibmuarea	S	0, 0	- d, d
	-S	- d - c, d	- d, d
(1 - p)			

Figure 2. Identification under Nibmuarea's misperception.

that can exist between belief systems and the observed outcome. The first game models Nibmuarea's misperception about Kadashman-Enlil and the second models Kadashman-Enlil's misperception about Nibmuarea. Each is an extension of the complete-information game taking these suspicions into account.

Nibmuarea argues that the observed outcome is the presentation of the princess to the non-dignitaries, and, contrary to this, Kadashman-Enlil pretends that someone other than his sister is presented to the non-dignitary messengers sent by him. To develop the incomplete-information game in the direction of such uncertainties, Nibmuarea's and Kadashman-Enlil's strategy choices are assumed to become meaningful given the private information they possess.

Pharaoh's misperception

Nibmuarea could have misperceived Kadashman-Enlil's intentions in two ways: one who is keen on learning his sister's fate and another who is not. The former type is called the "Keen" and the latter the "Reluctant". The matrices shown in Figure 2 represent the game where Nibmuarea is uncertain about Kadashman-Enlil's preferences but Kadashman-Enlil knows that the Pharaoh has no incentive to present someone else to either dignitaries or non-dignitaries, as he

affirms in EA1. The first game matrix represents Pharaoh's perception that Kadeshman-Enlil is the Keen on learning his sister's fate.⁶ The second one represents Pharaoh's perception that the Babylonian ruler is the Reluctant. If $p = 0$, the game reduces to the complete-information game previously discussed.

The Keen suffers a cost of $-l$, $l > 0$ in all outcomes other than the identification. This parameter measures the Keen's suffering of not learning whether his sister is well. It is assumed that $d < l$, that is, for the Keen learning his sister's fate represents a greater utility than the denial of a daughter to the Pharaoh insisting on a bride from the Egyptian Court. The Reluctant does not suffer that cost, in that case $l = 0$. The Reluctant has the same preference ordering as in the complete-information version. He cares only about aggrandizing himself. Thus, the preference parameter l helps to distinguish the two types of Kadeshman-Enlil. It indicates the private information that the Babylonian ruler possesses.

Nibmuarea's prior assessments about Kadeshman-Enlil's type are that he is Keen with probability p and Reluctant with $(1 - p)$. These probabilities are Pharaoh's prior assessments of Kadeshman-Enlil. They are assumed to be common knowledge. Pharaoh's assessments are known by the Babylonian ruler; Pharaoh knows that the Babylonian ruler is informed about his misperception about him; or the Babylonian ruler knows that the Pharaoh knows that the Babylonian ruler knows his misperception about him, and ad infinitum. Kadeshman-Enlil knows his own preference ordering.

Kadeshman-Enlil's payoffs in outcomes other than the identification are negative if he is the Keen: $d - l < 0$ as $d < l$. This implies that the Keen has a dominant strategy of D and the Reluctant has the dominant strategy of $-D$ as in the complete-information game. This forms a type-contingent strategy for Kadeshman-Enlil: "if I am the Keen, I send dignitaries, but, if I am Reluctant, I send non-dignitaries" denoted as $(D, -D)$. The other type-contingent strategies (D, D) , $(-D, D)$, $(-D, -D)$ are ruled out (the first letter refers to Keen's choice and the second one to that of the Reluctant) as none of the types should choose his dominated strategy. Nibmuarea has only one type, so he only has the choice of either showing the princess (S) or showing someone else ($-S$). He has his dominant strategy of S as in the game of complete information.

Equilibrium

The solution for incomplete-information games in strategic form is the Bayesian equilibrium. It is the counterpart of the Nash equilibrium for complete-information games.⁷ Players choose strategies that maximize their expected payoffs given their updated beliefs about each other. These strategies are also best replies against each other. Since one player has two types and the other only one, the posteriors do not differ from the prior beliefs.⁸

A Bayesian equilibrium concept is not needed to solve this game. The dominant-strategy equilibrium suffices to give the solution: both players choose their dominant strategies; they eliminate their dominated choices, and this holds for any misperception of Nibmuarea. Neither of the Kadashman-Enlil types has an incentive to switch from his strategy since, by choosing his dominant strategy, Nibmuarea shows the princess. The unique equilibrium is $\{S, (D, -D)\}$; for all $p, 0 \leq p \leq 1$. This is again an intersection between the dominant strategies.

Interpretation

The Pharaoh has the dominant strategy of showing the princess against both types of Kadashman-Enlil. The equilibrium does not ascribe two different actions for Nibmuarea depending on Kadashman-Enlil's type. Kadashman-Enlil also does not choose his dominated strategy depending on his type. Hence, the Bayesian equilibrium is similar to the equilibrium computed in the complete-information game.

In this equilibrium, there is the possibility that Kadashman-Enlil is indeed interested in his sister's fate and chooses to send dignitaries. Nibmuarea takes this intention into account in eliminating his dominated strategy: In the previous game the elimination was done solely against the Reluctant. Since Nibmuarea's misperception is common knowledge, Kadashman-Enlil cannot expect that Nibmuarea, who misperceives him, will show someone else as the princess. Nibmuarea could not expect that Kadashman-Enlil could expect that he will choose to show someone else because he knows that Kadashman-Enlil knows that he (Nibmuarea) misperceives him. Thus, Nibmuarea's misperception would not lead Kadashman-Enlil

to think that there is a chance that Nibmuarea will present someone else as the princess to his messengers.

The dominant-strategy equilibrium of the previous game and the equilibrium computed here give two different rationales for the observed outcome according to Nibmuarea. In the former, the (S, -D) outcome is due to the dominant strategies of both sides, with Pharaoh being informed of Kadashman-Enlil's preferences. In the latter, Nibmuarea again shows the princess and the sending of non-dignitaries indicates that Kadashman-Enlil is the Reluctant. Kadashman-Enlil maintains that (-S, -D) may be the outcome. However, if we assume Nibmuarea is genuine in his words, there is no support for Nibmuarea's presentation of someone else to the non-dignitaries. There is no possibility for the outcome (-S, -D) to be reached in any equilibrium of this game. Nevertheless, one may suspect that Nibmuarea was not genuine and perhaps prone to present someone else as the princess. We now turn to that aspect of the interaction.

Kadashman-Enlil's misperception

Kadashman-Enlil could possibly have misperceived Nibmuarea. When he is the Reluctant, he could have believed Nibmuarea had an incentive to present someone else as his sister to the messengers. In this version of the game two types represent Nibmuarea: one has a genuine interest in presenting the princess as he affirms in EA1. He is called the "Genuine". The other prefers to show someone else to non-dignitaries but shows the princess to the dignitaries. He is called the "Feigned". The first case occurs with probability q and the second one with $(1 - q)$. These probabilities are Kadashman-Enlil's prior assessments about Pharaoh's type. These probabilities are common knowledge, and Nibmuarea knows his own preference ordering.

In the first matrix of Figure 3, Kadashman-Enlil thinks the Pharaoh is genuine in his words. Nibmuarea shows the princess either to non-dignitaries or dignitaries, his dominant strategy. In the second matrix of the figure, the Feigned suffers a cost of $-h$, where $h > 0$, when the princess is presented to non-dignitaries. He therefore has no dominant strategy. Showing the princess to dignitaries is the best strategy. Showing someone else is best if non-dignitaries

		Nibmuarea (Genuine)	
		S	-S
Kadashman - Enlil	D	0, 0	d, -d - c
	-D	d, -d	d, -d
		[q]	

		Nibmuarea (Feigned)	
		S	-S
Kadashman - Enlil	D	0, 0	d, -d - c
	-D	d, -d - h	d, -d
		[1 - q]	

Figure 3. Identification under Kadashman-Enlil's misperception.

are sent. This condition adds another dimension to EA1: it contradicts the words of the Pharaoh but incorporates Kadashman-Enlil's suspicion.

Equilibrium

The Genuine has a dominant strategy of showing the princess (S) while the Feigned has no such strategy. Thus, we will never have $(-S, S)$ and $(-S, -S)$ but either (S, S) or $(S, -S)$. The first type-contingent strategy is: "I show the princess whether I am genuine or not" (S, S) ; the latter is: "I show the princess if I am the Genuine, and do not if I am the Feigned" $(S, -S)$. Kadashman-Enlil has only one type, so his strategies are D and $-D$. Similar to the previous analysis, Kadashman-Enlil has a dominant strategy of $-D$, and he will use it regardless of Nibmuarea's strategies (S, S) and $(S, -S)$.

There are two equilibrium candidates: $\{-D, (S, S); \text{ for all } q, 0 \leq q \leq 1\}$ and $\{-D, (S, -S); \text{ for all } q, 0 \leq q \leq 1\}$. However, since Kadashman-Enlil chooses $-D$ against (S, S) , the Feigned has an interest in deviating from showing the princess to showing someone else. So, $\{-D, (S, S); \text{ for all } q, 0 \leq q \leq 1\}$ is not a Bayesian equilibrium. The unique Bayesian equilibrium is $-D, (S, -S); \text{ for all } q, 0 \leq q \leq 1\}$. No player has an incentive to switch unilaterally from his equilibrium strategy when the other sticks to his.

Interpretation

Kadashman-Enlil should not send dignitaries due to his misperception of the Pharaoh. Similar to Nibmuarea's behavior in the previous game, either if $q = 0$ or $q = 1$ or for any q between 0 and 1, he has a dominant strategy of sending non-dignitaries. Nibmuarea's behavior, given Kadashman-Enlil's misperception, is more complex than Kadashman-Enlil's behavior given Nibmuarea's misperception: the Feigned would not find it beneficial to hide his preferences. Nibmuarea, knowing Kadashman-Enlil's uncertainty and dominant strategy of sending non-dignitaries as messengers, would not show the princess even if he would like to fool the Babylonian ruler by showing the princess to the non-dignitaries. Once Kadashman-Enlil is assumed to have a dominant strategy of sending non-dignitaries, the outcomes of the game should either be the presentation of the princess or the presentation of someone else to the non-dignitaries. Thus, while the Feigned has no dominant action, he reveals his preferences.

These games of incomplete information enrich the complete-information analysis by introducing beliefs. Equilibrium strategies remain the dominant ones. Both anticipate that the Genuine always shows the princess while the Feigned does not, and the Keen sends dignitaries while the Reluctant sends non-dignitaries. So, there is game-theoretic support for the observed outcome of presenting the princess to non-dignitaries. Pharaoh might have indeed been the Genuine and Kadashman-Enlil the Reluctant with the aim of aggrandizing himself. It may also be, as Kadashman-Enlil contends, that Nibmuarea was the Feigned and presented someone else to the non-dignitaries.

We could believe in Nibmuarea's words if Nibmuarea had the dominant strategy of showing the princess under both complete and incomplete information conditions, and, Kadashman-Enlil's suspicions are understood if Kadashman-Enlil indeed misperceived Nibmuarea. Yet, both rulers might have misperceived each other. Such compound misperceptions approximate the interaction better but the analysis becomes more complex.

A TWO-SIDED INCOMPLETE-INFORMATION GAME

The games matrices shown below in Figures 4 and 5 represent an interaction in which both Nibmuarea and Kadashman-Enlil misperceive each other. The probabilities p , q , r , and s are nature's choices of a particular type combination: they are respectively players' prior assessments about nature's choice of type combinations (G, K), (G, R), (F, K), and (F, R), where G denotes the Genuine, F the Feigned, K the Keen and R the Reluctant. The sum of these probabilities is equal to one. The four type combinations are mutually exclusive and exhaustive.

The first cell in Figure 5 represents an interaction between the Genuine and the Keen. The second represents an interaction between the Genuine and the Reluctant. The third cell represents an interaction between the Feigned and the Keen, and the last one is an interaction between the Feigned and the Reluctant. The figure is useful in distinguishing between these combinations and introducing a notation for the priors in a two-sided incomplete-information game.

Players are assumed to be informed about the possible types of each other and the associated probability distribution p , q , r , s . We further assume that these types and the associated probability distribution are common knowledge. Nibmuarea and Kadashman-Enlil's payoffs depend on the strategy combinations and their own types.

Given their types, Nibmuarea and Kadashman-Enlil update their priors. Here, unlike in the one-sided incomplete-information games, updating matters. For example, given that he is the Genuine, Nibmuarea's posterior that Kadashman-Enlil is the Keen is:

$$\mu'_{ni}(K|G) = \mu(G, K)/[\mu(G, K) + \mu(G, R)] = p/(p + q).$$

We similarly calculate Nibmuarea's posteriors for each of the other combinations of intentions:

$$\mu'_{ni}(R|G) = \mu(G, R)/[\mu(G, R) + \mu(G, K)] = q/(q + p).$$

$$\mu'_{ni}(K|F) = \mu(F, K)/[\mu(F, K) + \mu(F, R)] = r/(r + s).$$

$$\mu'_{ni}(R|F) = \mu(F, R)/[\mu(F, R) + \mu(F, K)] = s/(s + r).$$

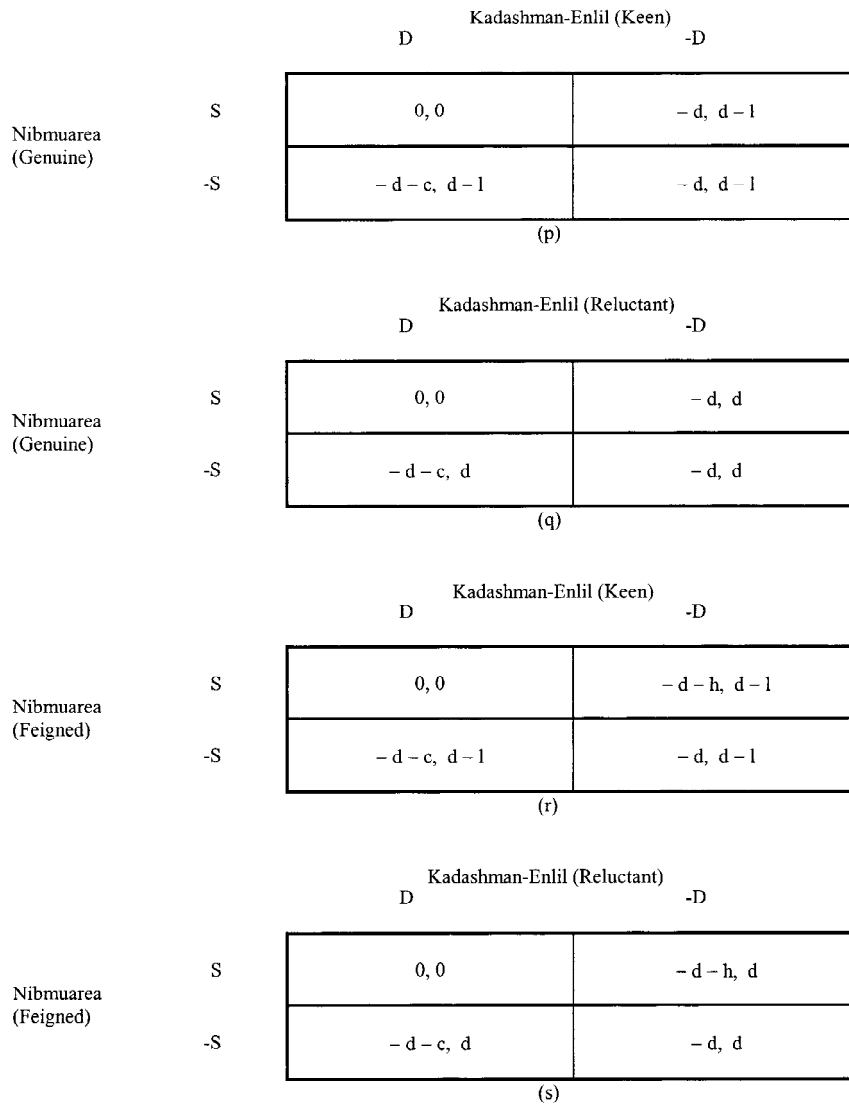


Figure 4. Identification under two-sided misperception.

Kadashman-Enlil's four posteriors are:

$$\mu'_{ke}(G|K) = \mu(G, K) / [\mu(G, K) + \mu(F, K)] = p / (p + r).$$

$$\mu'_{ke}(F|K) = \mu(F, K) / [\mu(F, K) + \mu(G, K)] = r / (r + p).$$

$$\mu'_{ke}(G|R) = \mu(G, R) / [\mu(G, R) + \mu(F, R)] = q / (q + s).$$

$$\mu'_{ke}(F|R) = \mu(F, R) / [\mu(F, R) + \mu(G, R)] = s / (s + q).$$

		Types of Kadashman-Enlil	
		Keen (K)	Reluctant (R)
Types of Nibmuarea	Genuine (G)	$\mu(G, K) = p$	$\mu(G, R) = q$
	Feigned (F)	$\mu(F, K) = r$	$\mu(F, R) = s$

Figure 5. Type combinations and priors.

Nibmuarea has four type-contingent strategies: (S, S), (S, -S), (-S, S), and (-S, -S). The first letter indicates the Genuine’s strategy choice, the second the Feigned’s. Kadashman-Enlil’s type-contingent strategies are similarly (D, D), (D, -D), (-D, D), and (-D, -D). Again, the first letter refers to the Keen’s strategy choice, the second one to the Reluctant. Nibmuarea and Kadashman-Enlil maximize their expected payoffs by choosing an appropriate strategy. The combinations of players’ type-contingent strategies make up a 4 × 4 matrix of expected payoffs for each player.

Some strategies will never be chosen, so we handle a simpler problem. The Keen has a dominant strategy of D and the Reluctant a dominant strategy of -D. Thus we will only observe (D, -D) in equilibrium. The choices (D, D), (-D, D), and (-D, -D) will not be made. Similarly, the Genuine will never play -S, but the Feigned has no dominant strategy; therefore we will not examine the strategy choices of (-S, S) and (-S, -S) but only (S, S) and (S, -S).

Nibmuarea’s expected payoff from (S, S) against (D, -D) is:

$$\begin{aligned}
 &P_{ni}[(S, D), G, K]\mu'_{ni}(K|G) + P_{ni}[(S, -D), G, R]\mu'_{ni}(R|G) \\
 &\quad + P_{ni}[(S, D), F, K]\mu'_{ni}(K|F) \\
 &\quad + P_{ni}[(S, -D), F, R]\mu'_{ni}(F|R) \\
 &= 0[p/(p + q)] + (-d)[q/(q + p)] + 0[r/(r + s)] \\
 &\quad + (-d - h)[s/(s + r)] \\
 &= -d[q/(q + p)] - (d + h)[s/(s + r)].
 \end{aligned}$$

		Kadashman-Enlil
		(D, -D)
Nibmuarea	(S, S)	$-d [q / (q + p)] - (d + h) [s / (s + r)]$
	(S, -S)	$-d [1 + q / (q + p)] - c [r / (r + s)]$

Figure 6. Nibmuarea’s expected payoff matrix.

Nibmuarea’s expected payoff from (S, -S) against (D, -D) is:

$$\begin{aligned}
 & P_{ni}[(S, D), G, K] \mu'_{ni}(K|G) + P_{ni}[(S, -D), G, R] \mu'_{ni}(R|G) \\
 & + P_{ni}[(-S, D), F, K] \mu'_{ni}(K|R) \\
 & + P_{ni}[(-S, -D), F, R] \mu'_{ni}(F|R) \\
 & = 0[p / (p + q)] + (-d)[q / (q + p)] + (-d - c)[r / (r + s)] \\
 & + (-d)[s / (s + r)] = -d[1 + q / (q + p)] - c[r / (r + s)].
 \end{aligned}$$

The expected payoff matrix for Nibmuarea is summarized in Figure 6.

Kadashman-Enlil’s expected payoff from (D, -D) against (S, S) is:

$$\begin{aligned}
 & P_{ke}[(D, S), G, K] \mu'_{ke}(G|K) + P_{ke}[(-D, S), G, R] \mu'_{ke}(G|R) \\
 & + P_{ke}[(D, S), F, K] \mu'_{ke}(F|K) \\
 & + P_{ke}[(-D, S), F, R] \mu'_{ke}(F|R) \\
 & = 0[p / (p + r)] + (d)[q / (q + s)] + (0)[r / (r + p)] \\
 & + (d)[s / (s + q)] = d.
 \end{aligned}$$

Kadashman-Enlil’s expected payoff from (D, -D) against (S, -S) is:

$$\begin{aligned}
 & P_{ke}[(D, S), G, K] \mu'_{ke}(G|K) + P_{ke}[(-D, S), G, R] \mu'_{ke}(G|R) \\
 & + P_{ke}[(D, -S), F, K] \mu'_{ke}(F|K) \\
 & + P_{ke}[(-D, -S), F, R] \mu'_{ke}(F|R) \\
 & = 0[p / (p + r)] + (d)[q / (q + s)] + (d - 1)[r / (r + p)] \\
 & + (d)[s / (s + q)] = d + (d - 1)[r / (r + p)].
 \end{aligned}$$

Kadashman-Enlil’s expected payoff matrix, then, is shown in Figure 7.

		Nibmuarea	
		(S, S)	(S, -S)
Kadashman- Enlil	(D, -D)	d	$d + (d - 1) [r / (p + r)]$

Figure 7. Kadashman-Enlil's expected payoff matrix.

For Nibmuarea we have the following best-reply conditions:

Condition 1: if $-d[q/(q+p)] - (d+h)[s/(s+r)] > -d[1+q/(q+p)] - c[r/(r+s)]$, then (S, S).

Condition 2: if $-d[q/(q+p)] - (d+h)[s/(s+r)] < -d[1+q/(q+p)] - c[r/(r+s)]$, then (S, -S).

Condition 3: if $-d[q/(q+p)] - (d+h)[s/(s+r)] = -d[1+q/(q+p)] - c[r/(r+s)]$, then, we are indifferent between (S, S) and (S, -S).

The type-contingent strategy (S, S) means that Nibmuarea shows the princess whether he is genuine or not (facing either the Keen or the Reluctant); in all four matrices Nibmuarea chooses S. Suppose that condition 1 is satisfied, $s/r < (d+c)/h$. Nibmuarea chooses (S, S) against (D, -D). Against this choice, Kadashman-Enlil has no incentive to deviate from (D, -D). The first Bayesian equilibrium is $\{(S, S), (D, -D); s/r < (d+c)/h\}$.

Suppose now that the second condition is satisfied. So, we have $s/r > (d+c)/h$. Nibmuarea's type-contingent best reply against (D, -D) is (S, -S). Hence, in the first and second matrices Nibmuarea chooses S, and in the third and fourth matrices he chooses -S. Against this choice, Kadashman-Enlil again has no incentive to deviate from (D, -D). The second Bayesian equilibrium is $\{(S, -S), (D, -D); s/r > (d+c)/h\}$.

If the third condition is satisfied, then Nibmuarea can choose either S in all four matrices or S in the first and second matrices and -S in the third and fourth ones. Kadashman-Enlil who has the unique type-contingent strategy made up of his dominant strategies should not deviate from (D, -D). So, such a mixture of (S, S) and (S, -S) does not lead to Kadashman-Enlil's indifference. Therefore, the third Bayesian equilibrium is: $\{(S, -S) \text{ or } (S, S), (D, -D); s/r = (d+c)/h\}$.

Interpretation

If Nibmuarea cheats and messengers detect the cheating, his reputation as the greatest among Middle-Eastern rulers of that time would be severely tarnished: Why should a Pharaoh double-cross the messengers sent by the ruler of a lesser kingdom? If he cheats, he is also sure that Kadashman-Enlil will break the Egypt-Babylon dynastic alliance and will deny his daughter to Nibmuarea. The Babylonian alliance is not as important as is the loss of his reputation. This cost of reputation loss is higher or at least equal to the cost of being denied a daughter by the Babylonian ruler. We assume that $c > d$. Nibmuarea suffers the cost h when he prefers to present the princess to dignitaries. When the princess is presented to non-dignitaries, Kadashman-Enlil can only pretend, but cannot be sure, that the Pharaoh is cheating, thus denying his daughter to Nibmuarea. This cost is less than the cost of reputation loss. Thus, we also assume that $c > h$.⁹

Therefore, given that the Keen sends dignitaries and the Reluctant sends non-dignitaries, both types of Nibmuarea (the Genuine and the Feigned) find the presentation of the princess as being more beneficial than the strategy “present the princess if I am genuine and do not if I am feigned.” The Feigned’s cost of cheating is greater than any cost suffered from presenting the princess to non-dignitaries. If the Feigned assesses that it is more probable that he will face the Keen than the Reluctant, he presents the princess. The presentation of someone else gives a smaller payoff.

Even if $s > r$, c is sufficiently greater than h , the condition of presenting the princess is still satisfied. Thus, while the Feigned judges that there is a greater probability that he faces the Reluctant, he still chooses to present the princess since the cost of cheating is quite high. Both Nibmuarea types then choose the same strategy of presenting the princess under these conditions.

This equilibrium implies that Kadashman-Enlil’s claim is not grounded: the one presented to the messengers was indeed the princess. This holds true even if he perceived Nibmuarea as the Feigned. For the same reason, the first equilibrium provides support for Nibmuarea’s words in EA1. Nibmuarea should indeed have no reason to conceal the death of the princess and to present someone else,

because the loss of reputation loomed large whether the Keen sends dignitaries or the Reluctant sends non-dignitaries.

The second condition gives $d[s/(s+r)] + h[s/(s+r)] > d + c[r/(r+s)]$. This implies that $hs > r(d+c)$. As $c > h$, and $d > 0$, if $r > s$, the condition cannot be satisfied. Given that the Keen sends dignitaries and the Reluctant sends non-dignitaries, the Genuine presents the princess. The Feigned also does not find that his best reply is the presentation of someone else. This holds if the Feigned thinks that it is equally probable that he is likely to face the Keen or the Reluctant, or that it is more probable that Kadashman-Enlil is the Keen.

This result contrasts with the equilibrium of the one-sided incomplete-information game that models Kadashman-Enlil's misperception. In that game, the Feigned shows someone else to non-dignitaries even though he can suffer the cost of reputation loss, because he knows that Kadashman-Enlil is the Reluctant. In the first equilibrium, however, the Feigned prefers to show the princess because there is a chance of facing the Keen.

Against Kadashman-Enlil's type-contingent strategy, the Feigned prefers to show someone else if he assesses that he is more likely to face the Reluctant than the Keen, that is, if $s > r$. Under such a strict inequality where r is sufficiently small there is a chance for the second condition to hold. Therefore, Nibmuarea's types take different actions if it is more likely that the game is played between the Feigned and the Reluctant rather than between the Feigned and the Keen.

This rationale is similar to the equilibrium computed in the one-sided incomplete-information game which models Kadashman-Enlil's misperception. Given that the Reluctant Kadashman-Enlil sends non-dignitaries, the Feigned Nibmuarea prefers to show someone else. Hence, Nibmuarea shows the sister if he perceives that Kadashman-Enlil is sufficiently keen to learn of his sister's fate. Assuming that Nibmuarea indeed showed the princess, he must have thought that Kadashman-Enlil was the Keen and was surprised to see non-dignitaries at his Court.

The third condition implies $hs = r(d+c)$. The indifference means that Nibmuarea can show the princess regardless of his and Kadashman-Enlil's types. Since the Genuine shows the prin-

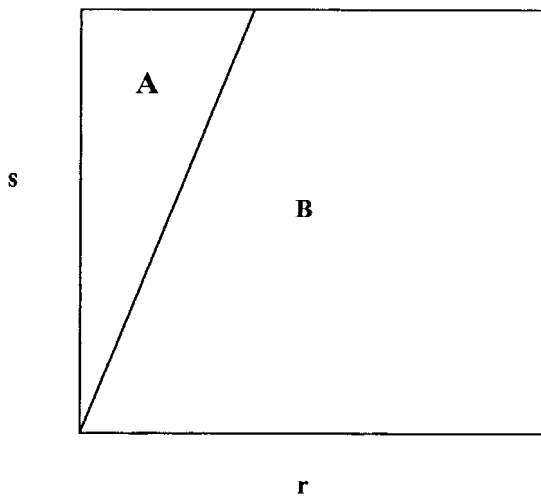


Figure 8. Location of equilibria under two-sided misperception.

cess anyway, this condition implies that the Feigned is indifferent between showing the princess or someone else. Again, if $s = r$ or $s < r$, the condition cannot be satisfied. Given Kadashman-Enlil's type-contingent strategy, the Feigned cannot be indifferent between presenting the princess or someone else *if* he thinks that Kadashman-Enlil can be either the Keen or the Reluctant, or that Kadashman-Enlil is in fact interested in his sister's fate (the Keen).

Geometric interpretation

All three equilibria can be presented on the unit square taking the parameters h , d , and c as constants and the priors r and s as variables with values between 0 and 1. Condition 3 is the equation of a line, $s = [(d + c) / h] r$, passing through the origin with a positive slope of $(d + c) / h$. The slope is greater than 1 as $d + c > h$ under the payoff assumptions. The line in Figure 8 divides the unit square into two regions. The unit square contains all pairs of (r, s) , that is, priors indicating the interaction between the Feigned and the Keen, and the one between the Feigned and the Reluctant respectively. The diagonal line contains all pairs (r, s) satisfying the equality $s h = r (d + c)$.

Hence, the pairs of (r, s) leading to Nibmuarea's indifference satisfy the equation of the line $s = [(d + c) / h] r$. The line gives the third equilibrium. In the triangular region A, the condition $s h >$

$r(d + c)$ holds; likewise the condition $sh < r(d + c)$ holds in region B. Thus, in A, Nibmuarea's best reply is to "show the princess if I am the Genuine and someone else if I am the Feigned", and this gives the second equilibrium. The first equilibrium is given by B where Nibmuarea's best reply is "show the princess whether I am the Genuine or the Feigned".

The area of A is $h/2(d + c)$, and the area of B is equal to $1-h/2(d + c)$. Since c and d increase while h is constant, A shrinks. The slope of the line becomes steeper and the line approaches the ordinate. Consequently, r diminishes while s remains constant in A. In contrast, A gets larger if h increases while c and d are constant.

This implies that if both the cost of reputation loss and the value of the dynastic marriage increase, a smaller prior that the Feigned faces the Keen for the same value of s can satisfy the condition $sh > r(d + c)$. For the Feigned to present someone else and the Genuine to present the princess, a smaller prior that the Feigned faces the Keen suffices. This is consistent with the previous observation that against Kadashman-Enlil's type-contingent strategy, the Feigned prefers to show someone else if he thinks that he faces the Reluctant rather than the Keen, that is, if $s > r$.

In contrast to the previous result, if the Feigned's cost h increases while c and d are constant, then A gets larger. If the Feigned suffers a relatively greater cost when presenting the princess to non-dignitaries, then a pair (r, s) previously not in A can be included in A. A greater prior that the Feigned faces the Keen for the same value of s can satisfy the condition $sh > r(d + c)$. Hence, if the costs of presenting the princess to non-dignitaries are sufficiently high, then even if the Feigned thinks that he will eventually face the Keen, he presents someone else.

CONCLUSION

These equilibria indicate that if Nibmuarea was the Feigned he must have thought that Kadashman-Enlil was interested in his sister's fate, since he presented the princess even to the non-dignitaries. But, if as the Feigned he thought that Kadashman-Enlil was primarily interested in aggrandizing himself, he would have presented someone else to the non-dignitaries. Our analysis of the complete-information

case supports Nibmuarea's statements in EA1. The same applies in the case of one-sided incomplete-information: the princess is always presented by Nibmuarea under that condition. In the two-sided incomplete-information game, both types of Nibmuarea show the princess. If the Feigned thinks that Kadashman-Enlil is in fact interested in his sister's fate, the cost of cheating is quite high. Nevertheless, Kadashman-Enlil's suspicions that Nibmuarea preferred to show the princess *only* to non-dignitaries would prove correct *if* Nibmuarea, in his role as the Feigned, attributed self-centered, aggrandizing motives to his opposite number.

This game-theoretic analysis of rational choices under different information conditions reveals the interactive reasoning that may have taken place between the two rulers. Choice of strategy is contingent on the player's beliefs about the other's intentions. Of course, we do not have the possibility of confirming these mutual expectations, suspicions, and choices. Nevertheless, both rulers' claims find support in these equilibria. Consequently, strategic uncertainty characterized the interactions depicted in EA1.

The analysis of strategic uncertainty is an important contribution of game theory. Alternative approaches to the analysis of history provide limited insights into the implications of such uncertainty including the consequences of unrevealed intentions and suspicions, as well as the implementation of interactive choices. We have, in this article, reconstructed a bronze-age story in terms of contemporary strategic thinking. Our reconstruction demonstrated the power of "what . . . if" reasoning: Plausible outcomes were derived from assumptions about available choices as revealed in the Amarna letters. We assumed first that the Pharaoh's account of the situation was correct, namely that he had a dominant strategy and that Kadashman-Enlil did not send non-dignitaries. We then relaxed these assumptions, one-by-one, in the incomplete-information games, and discovered that the Pharaoh's account may not have been correct. The Feigned Pharaoh could well have presented someone else to the dignitaries sent by the Keen king. On the other hand, when d (marriage value) and c (cost of cheating) increase, it is more likely that both types make the same choice: present the princess. This analysis is particularly significant in light of missing information in other letters about the sequel to these interactions.

NOTES

1. Ankhesenamun, Egyptian queen and daughter of the Pharaoh Amenophis IV was a childless widow; thus, she asked the powerful Hittite king Suppiluliumas to send her one of his sons to rule Egypt (Gurney, 1990: 24–25).
2. Italics in original. To put oil on the head of a girl means a step towards marriage in the Bronze Age (like an engagement).
3. Moran (1992: xviii) indicates the exceptional quality of the EA1: it is a copy of the translation from Egyptian into Akkadian and kept in the foreign language section of the Egyptian archives. As a rule, the Egyptian versions are filed in the archives and the translated letters are sent (Moran, xvii).
4. Kaska, a region in Anatolia's northwest Black Sea coast, was the land of ferocious tribes. They regularly attacked Hittites of Anatolia, the competitor of Egypt during the Middle-Bronze Age. Hanigalbat was a Mitanni kingdom based in upper-eastern Mesopotamia, a part of southeast Anatolia. Ugarit was a city-kingdom on the Syrian coast under the influence of Egypt.
5. Showing the princess dominates showing someone else if showing her is as good as not showing her to non-dignitaries and better than not showing her if dignitaries are sent.
6. For convenience, row players are chosen to represent those who have incomplete information.
7. Precisely, Bayesian equilibrium is a strategy choice that maximizes players' expected payoffs against others' equilibrium strategies. Under the belief conditions, players gain nothing or suffer utility losses when they deviate unilaterally from their equilibrium strategies given that others stick to their strategies. For illustrative examples see Eichberger 1993: 123–151 and Friedman 1986: 48–67.
8. Since we have one type for the Pharaoh but two types for Kadashman-Enlil, there are two type combinations (T, K) and (T, R) where T denotes Pharaoh's type, K the Keen, and R the Reluctant. The probabilities p and $(1 - p)$ respectively represent the selection of (T, K) and (T, R), the selection of a particular type-pair. Nibmuarea updates his priors about Kadashman-Enlil's type given his type is T. These updated beliefs are Nibmuarea's posterior beliefs about Kadashman-Enlil. Pharaoh's posterior about Kadashman-Enlil of type K given that he is of type T is calculated with the help of the Bayes' theorem: $\mu(K|T) = \mu(T, K)/(\mu(T, K) + \mu(T, R)) = p/(p + 1 - p) = p$. Similarly, Pharaoh's posterior about Kadashman-Enlil of type R given that he is of type T is: $\mu(R|T) = \mu(T, R)/(\mu(T, R) + \mu(T, K)) = (1 - p)/(1 - p + p) = 1 - p$.
9. The first condition implies $(d + h)[s/(s + r)] < d + c[r/(r + s)]$. This gives $d[s/(s + r) + h[s/(s + r)]] < d + c[r/(r + s)]$. So, we have $(d + c)r > h s$. As $0 < s/(s + r) < 1$, we have $d[s/(s + r)] < d$, for the inequality to be satisfied it suffices to have $h[s/(s + r)] \leq c[r/(r + s)]$. The inequality in turn implies $h s \leq c r$. Under the assumption of $c > h$, the condition $h s \leq c r$ is satisfied if $r \geq s$.

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