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Quantum computational gates with radiation free couplings

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Abstract. We examine a generic three level mechanism of quantum computation in which all fundamental single and double qubit quantum logic gates are operating under the effect of adiabatically controllable static (radiation free) bias couplings between the states. Under the time evolution imposed by these bias couplings the quantum state cycles between the two degenerate levels in the ground state and the quantum gates are realized by changing Hamiltonian at certain time intervals when the system collapses to a two state subspace. We propose a physical implementation of the mechanism using Aharonov-Bohm persistent-current loops in crossed electric and magnetic fields, with the output of the loop read out by using a quantum Hall effect aided mechanism.

PACS. 03.67.Lx Quantum computation – 03.67.-a Quantum information – 68.65.-k Low-dimensional, mesoscopic, and nanoscale systems: structure and nonelectronic properties – 68.65.Hb Quantum dots

1 Introduction

Quantum computation [1] is based on the realization of the logic gates by manipulating the quantum states *via* switching specific interactions in the course of the time evolution of a quantum system. Recent trends in the experimentation of the small scale quantum logic gates [2] suggest that the gate operations can be performed by efficient mechanisms based on superconducting states in Josephson tunneling junctions.

The fundamental differences between the well explored classical and the much less explored quantum computation arises primarily from the manifestations of the fundamental principles of quantum mechanics. From the information theory point of view, an ideal quantum computational algorithm is expected to make maximal use of the superposition and entanglement principles to implement what is often called the principle of quantum parallelism. Exponentially unsurpassed performances are expected as a result of this principle in tackling special algorithmic problems such as the quantum factorization algorithm due to Shor [3], the Grover's unstructured data search [4], or, more generally, quantum simulation of the many-body problems (*e.g.* Ref. [5]). On the other hand, and from the physical point of view, making use of the principles of quantum mechanics in a computational frame requires full control of the interacting quantum system with an *external* classical system which includes the input-output measurement devices and weakly controllable environmental agents. The decoherence arises from the susceptibility of the interacting quantum system to the interferences created by the environment. Crudely speaking, decoherence can be summarized in the computational terminology as the loss of computed information stored in the parameters of the quantum state. In turn, decoherence leaves a small room both spatially and temporally in the desirable manipulation of the quantum state and the conditions to fight decoherence may be very severe. In this context, finding new mechanisms and new experimental systems aiming to minimize all sources of decoherence is a major task of the current research efforts in the realm of quantum computation.

Most required interaction mechanisms in the manipulation of the quantum gates that are proposed in the literature are based on the coupling of the qubit states to some resonant external radiation which then becomes a major obstacle to control the environmental decoherence. In an opposite context, the coupling to the environment has also been suggested to keep the decoherence under control. Lately new theoretical mechanisms based on multi level quantum systems were proposed in which the environment strongly couples under certain conditions to the high levels of a multistate system but not to the qubit subspace in a direct way (dissipation free subspaces). The main requirement in these theories is therefore the preexistence of the dissipation free subspaces [6]. More recently Beige *et al.* [7] have examined this idea theoretically in a multi-atom three state model with a doubly degenerate ground state comprising the mentioned dissipationless subspace with the third levels of the atoms strongly coupled to each other and to a single cavity mode. Zanardi

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and Rasetti [8] discussed earlier a somewhat similar four-state model with a dissipation free subspaces. When all atoms are in the ground state and the cavity mode is unoccupied, the dissipative interaction is effectively switched off. As the result, the subspace comprising the doubly degenerate ground state is dissipation free if the cavity field is in the vacuum state.

In this article we propose a deceptively similar three state idea based on the Aharonov-Bohm persistent current rings. Persistent current states in normal (nonsuperconductive) metals have been found in [9] (see also a review paper [10]) and later in [11]. They appear as a manifestation of the Aharonov-Bohm effect [12] produced by a magnetic flux threading the loop, and are effected due to the so called 2-nd Aharonov-Bohm effect (also in [12]) by static electric field applied perpendicular to the magnetic field. Our model utilizes this principle in three island flux threaded rings in which the electron is allowed to hop from one site to the other. In fact, our system bears no analogy with the 3-state proposal of Beige *et al.* neither in the advantages of the latter nor in its disadvantages (the noise induced by repetitive measurements over the dissipative channel needed to keep system within the decoherence free subspace by a quantum Zeno effect [13]). The 3-state loop is the minimal Aharonov-Bohm system which allows accomplishing quantum transitions (the quantum logic gates) with static potentials.

From the perspective of quantum computation, the fundamental exception of the proposed model from the conventional approaches is that the mechanism utilizes radiation free static couplings to perform the quantum logic gates. Namely, the time evolution of the wavefunction is induced by static interactions between the first two levels (qubit) with the third (auxiliary) level. As the quantum state cycles in this three state Hilbert space, the gate operations are defined at the specific instants in this time evolution such that the desired qugate is obtained in the qubit subspace with no probability of occupation in the auxiliary state. Moreover in the proposed model the otherwise independent concepts of qubit and quantum gate are inseparably unified within the same quantum unit. It must therefore be stressed that in our model the leakage of the wavefunction into the third level is not avoided, on the contrary, the dynamical occupation of the third level is an essential part of the time evolution of the state. The advantages of the proposed model are that firstly the wavefunction never leaks out of the three state Hilbert space and secondly the quantum gates are obtained *via* radiationless mechanisms as they involve time-independent non-resonant interactions. To our knowledge, similar radiationless qugate operations have not been discussed yet and we expect that more physical realizations of the proposal here may be found. One of the clear advantages of the radiationless coupling is to suppress substantially the environmental dissipation in the case when, for instance, resonant coherent light pulse (as in the case of ion trap [14] and many other mechanisms) or magnetic rf-fields (as in the case of superconducting systems) are used to manipulate the quantum states.

The paper is organized as follows. In Section 2 we address the issue of the non-superconducting Aharonov-Bohm persistent current in the mesoscopic or nanoscopic system. In Section 3, the realization of the fundamental single and double quantum logic gates is demonstrated for these loops forming a full set of transformations needed for the universal quantum computation. Section 4 discusses the dissipation and decoherence effects in the Aharonov-Bohm qubits, and Section 5 suggests the physical implementation of persistent current loops, including also the usage of classical and quantum Hall effects for reading out the information from the qubits.

2 Persistent current qubit

The realization of the quantum computation schemes that use the concept of static flux, can be made with the use of macroscopic quantum interference effects in superconducting systems (the Josephson effect [15]), or the Aharonov-Bohm persistent-current states in nonsuperconducting structures of small size [10]. The latter structure naturally realizes the inverted double-degenerate ground state separated from the higher energy state(s) by a finite gap. Comparatively to this, the superconducting junctions in the macroscopic quantum regime [16] may suffer from the decoherence due to inavoidable admixture of gapless localized excitations near the barrier area activated at flip transitions between the degenerate states (this is seen in the broad resonances of the "Schrödinger Cat" states observed experimentally [17–19]. Much better resolution of such resonances in recent papers [20–22] still does not address the phases of superposition states when any switches are performed during the coherent evolution time).

In our paper we suggest the persistent current loops for the physical realization of qubits and qugates. The three-site loop is supplemented by a (macroscopic) nondemolition measuring device (the quantum Hall bar in this case), which performs both tasks in a coherent and decoherence-free fashion by coupling for a short time the qubit subspace to the third (auxiliary) level.

The three state system in our consideration is defined to be in a Λ-shaped configuration in Figure 1 under zero bias potential, *i.e.* the ground state is doubly degenerate and there is a third (auxiliary) state. One possible realization is *via* a three-sectional mesoscopic ring intersected by tunneling barriers (or consisting of overlapping metallic films separated by thin oxide layers) as shown in Figure 2. The isolated qubit structure can in principle be realized naturally as a three-island defect in an insulating crystal, similar to negative-ion triple vacancy (known as F_3-center) in the alkali halide crystal (see for instance established textbooks such as Ref. [23]). The gate manipulations can be performed *via* an Aharonov-Bohm flux perpendicular to the ring together with a constant electric field within the plane of the ring (Fig. 3). The information to be implemented into the computational basis of the quantum computer is stored in the form of parameters of the persistent-current states of the normal-state Aharonov-Bohm ring (the qubit), and processed *via* the

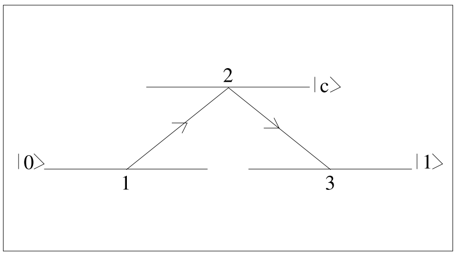

Fig. 1. Λ-shaped level configuration of the persistent-current normal-state Aharonov-Bohm qubit. 1, 2, 3 are the eigenstates of Hamiltonian (1) at $\alpha = \pi/3$ where 1 and 3 are the computational basis (qubit) registers $|0\rangle$, $|1\rangle$, whereas 2 is the qugate (control) register $|c\rangle$. Arrows show **virtual** transitions between the degenerate states through the excited state $|c\rangle$, effected by the potential biases applied to the metallic sites of the three sectional Aharonov-Bohm ring.

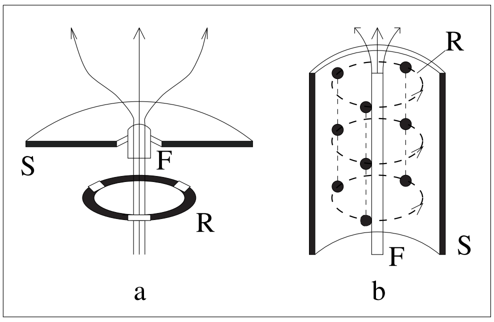

Fig. 2. (a) A sketch of magnetically focused lines of magnetic field (arrows) of the superconducting fluxon with flux $\Phi_1 = hc/2e$ making one half of the normal-metal flux quantum, $\Phi_0 = hc/e$, and effecting the ring **R** with three normal islands into a Λ-shaped configuration. The fluxon Φ_1 is trapped in the opening of superconducting foil (**S**) and further compressed by a ferromagnetic crystal to fit into the interior of the ring. (b) Schematic of the multi-ring qubit arrangement with the sites (circles) on the surface of cylindrical wall **R** surrounding ferromagnetic cylinder (**F**) which focuses lines of magnetic field in a cylindrical tube inside superconductor (**S**).

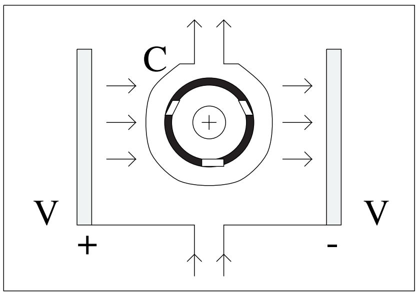

Fig. 3. A sketch of the electric field (shown by arrows) applied to the ring through the potential electrodes (**V**) in direction perpendicular to the direction of magnetic field. **C** is a coupling loop providing the connection to the nondemolition-measuring setup of the persistent current.

radiation free transitions between the states in an invariant subspace, effected by the static bias potentials on the sites of the ring (the qugates).

The system is expected to be robust with respect to (small) deviation from the perfect atomic structure, presence of small amount of defects or impurities or other imperfections. The reason is that the persistent-current state is not a one similar to the ohmic current states in classical metals but rather a thermodynamic equilibrium state [9–11] with a nonzero current persisting while the Aharonov-Bohm flux in a ring remains constant. Scattering by impurities doesn't affect the magnitude of the current if the effective mean free path of electron (l) is larger than the ring diameter (d), and monotonically decreases in amplitude at decreasing l. The current remains *finite* rather then infinite, even at $l = \infty$, counterintuitive to naive reasoning which may compare the persistent current to the Ohmic currents in classical metal.

In the absence of the bias potentials, the dynamics of the Aharonov-Bohm loop with 3 barriers is governed by the pure tunneling Hamiltonian

$$H_0 = -\tau \sum_{n=1}^{3} \left(a_n^\dagger \, a_{n+1} \, \mathrm{e}^{\mathrm{i}\alpha} + a_{n+1}^\dagger \, a_n \, \mathrm{e}^{-\mathrm{i}\alpha} \right) \qquad (1)$$

where τ is a real tunneling amplitude between the islands and α is a controllable phase. Equation (1) is represented in the diagonal basis by the eigenenergies $\epsilon_m = -2\,\tau \cos(\frac{2\pi}{3} m + \alpha)$ where $\alpha = 2\pi\Phi/3\Phi_0$ and Φ_0 is a normal-metal flux quantum hc/e. The eigenenergies form the Λ configuration at the symmetric point $\Phi = \Phi_0/2 = hc/2e$ with the energies $(-1, 2, -1)\,\tau$ for $m = 0, 1, 2$ respectively. In fact, the model Hamiltonian in equation (1) is an idealization of a realistic system where the higher excited states are far removed from the first three eigen levels. This condition is implied by $\tau \ll \Delta E$ where ΔE is the energy separation between the third and the next excitation. The physical realization of this condition is permitted by three sufficiently deep quantum wells localized in the corners of an equilateral triangle.

Returning to the ideal model in equation (1), the three sites interact by a bias *potential* loop with the site potential $V_n = V_0 \cos(2\pi\, n/3)$ where $n = 1, 2, 3$ is the site index. It is clear that for this choice, the potential can be obtained by a conservative field since the total potential around the loop vanishes, *i.e.* $V_1 + V_2 + V_3 = 0$. The total Hamiltonian is then the sum of equation (1) and the site-potential and is represented in the diagonal basis of H_0 by the matrix (in units of τ)

$$H_0 + H_1(V_0) = \begin{pmatrix} -1 & \nu & \nu \\ \nu & 2 & \nu \\ \nu & \nu & -1 \end{pmatrix} \qquad (2)$$

where $H_0 = \mathrm{diag}(-1, 2, -1)$ is the Hamiltonian (1) in the diagonal form, $\nu = V_0/2\tau$ is the dimensionless interaction parameter. The proposed mechanism is designed to be radiation free and the quantum gate operations are performed by adiabatically tuning the static potentials. The system is prepared in a particular ground state (no

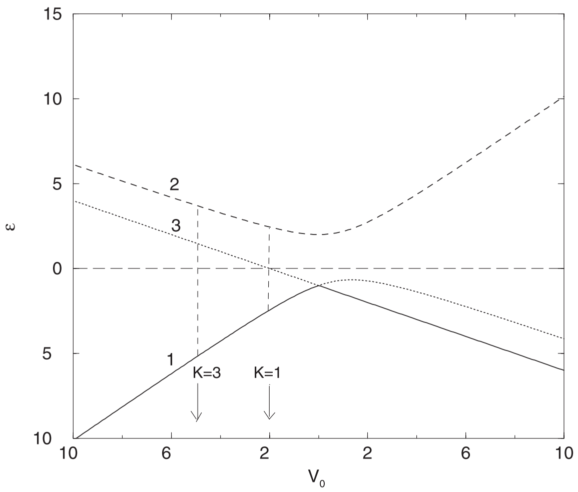

Fig. 4. Eigenenergies of the ring biased with a potential V_0. Eigenstates 1 and 3 are degenerate at $V_0 = 0$ where they form the qubit states. The commensurate situation, marked by arrows, appears at $K = 1$ and at $K = 3$ where it allows for the temporal (virtual) transition to a higher level 2 and back thus effecting the bit-flip transition (at $K = 1$) and the Hadamard-like gate (at $K = 3$).

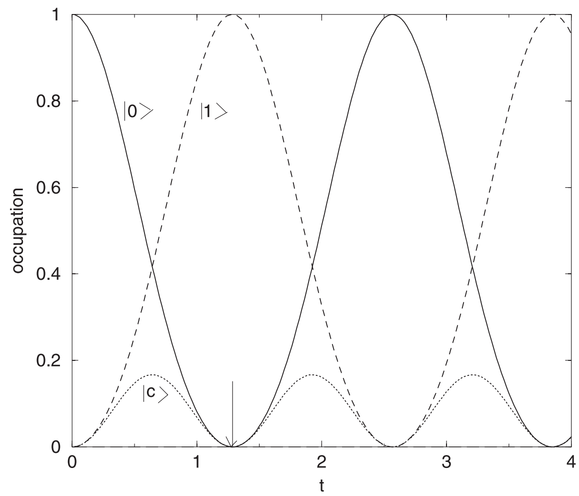

Fig. 5. Bit evolution at $K = 1$. At point indicated by an arrow ($t = t_1$), the population of control register ($|c\rangle$) vanishes whereas the populations of the computational registers of qubit ($|0\rangle$) and ($|1\rangle$), interchange.

occupation of the auxiliary level) and the time evolution is continued until the period $t = t^*$ at which the auxiliary level cycles back to its initial configuration. The other parameters are adjusted so that the desired single-qubit gate is realized at the end of the single cycle of the auxiliary level. The unitary time evolution at $t = t^*$ is then given by

$$e^{it^*(H_0+H_1(V_0))} = \begin{pmatrix} A & 0 & B \\ 0 & X & 0 \\ C & 0 & D \end{pmatrix} \qquad (3)$$

where A, B, C, D are complex, X is a pure phase and, other than the unitarity condition, no other restrictions apply on the matrix elements. The form of the unitary matrix in equation (3) leaves the qubit subspace invariant irrespectively of the value of X as long as the initial wavefunction is confined to the same subspace. The instantaneous vanishing of the certain matrix elements in (3) is due to the destructive interference in the transition amplitudes between the auxiliary level and the qubit subspace. Using the exact expressions describing the absolute level amplitudes, it can be inferred that the destructive interference condition at $t = t^*$ can be satisfied if the transition energies are commensurate. One way to express this condition is

$$E_3 - E_1 = K\,(E_2 - E_3)\ , \qquad K = integer \qquad (4)$$

where $E_i = E_i(V_0)\ ,\quad i = 1, 2, 3$ are the eigenenergies of (2) plotted against V_0 in Figure 4. In fact, equation (4) is a condition on the static potential. Solving the eigenvalues of equation (2) we find that the potential is allowed to take a discrete set of values determined by

$$V_0^{(K)} = -\frac{2}{3K}\left[K^2 + K + 1 + (K-1)\sqrt{K^2+4K+1}\right]. \qquad (5)$$

Mention that similar transformation may also apply to a three-Josephson junction qubit as discussed in [24].

3 Qugate operations

We now demonstrate that different values of the integer K performs different qubit gates. In particular, among the fundamental single qubit gates the bit flip and the Hadamard-like gates can be realized by the time evolution of the Hamiltonian alone in equation (2) at certain instants and at specifically tuned values of V_0. Among the elementary qubit gates, the phase gate requires a control on the relative phase between the degenerate states. In order to induce a relative phase, the otherwise degenerate states in the qubit subspace are made nondegenerate by a shift in their eigenlevels by turning on a degeneracy breaking interaction. This is a relatively well known method in the case of Aharonov-Bohm rings, for instance, by slightly shifting the dc-flux away from the value where doubly degenerate configuration is defined. The net effect of this shift in the adiabatic limit is represented by a diagonal, degeneracy breaking effective term in the total Hamiltonian

$$H_2 = \mathrm{diag}(\Delta\epsilon_1, \Delta\epsilon_2, \Delta\epsilon_3). \qquad (6)$$

The diagonal form of equation (6) implies that the phase shift can be obtained independently from the other gates since, due to the diagonal form, the time evolution is manifestly adiabatic. One other advantage in this diagonal form is that the transformation leaves the qubit subspace invariant and thus it can be conveniently used for phase correction. We demonstrate below the realization of the different single qubit operations by a mere change of the integer K and letting the system time-evolve. The populations of the three eigenstates of the Hamiltonian in equation (2) are plotted in Figure 5 as functions of time

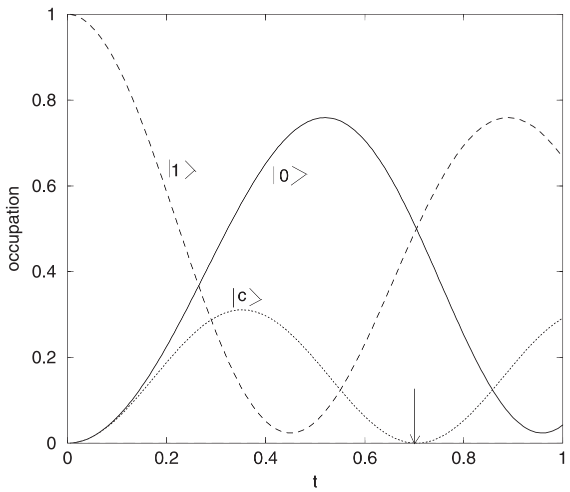

Fig. 6. Bit evolution at $K = 3$. At point indicated by an arrow ($t = t_3$), the population of control register ($|c\rangle$) vanishes whereas the computational basis of the qubit, originally in a state ($|1\rangle$, equally populates to states $|0\rangle$ and $|1\rangle$.

and for $K = 1$. The first observation is that the maximal occupation of the auxiliary state is 20% of the total unit probability. At periodic time intervals, of which period t_1 is indicated on the horizontal axis by an arrow, the population in the auxiliary state vanishes and the wavefunction instantaneously collapses onto the qubit-subspace non-demolitionally. Hence, at $t = t_1$ the degenerate levels exchange their population. The bit flip should introduce no relative phase between the qubit states, thus, one needs to know not the probabilities but the amplitudes. These can be directly obtained from the unitary time evolution at $K = 1$ (which corresponds in equation (2) to $V_0^{(1)} = -2\tau$). Evolving the Hamiltonian in (2) at this configuration for $t_1 = \pi/\sqrt{6}$ (in units of $\hbar/\tau$), $t_1 = \pi/\sqrt{6}$ (in units of $\hbar/\tau$) as

$$\exp\left\{-\mathrm{i}t_1\begin{pmatrix}-1 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & -1\end{pmatrix}\right\} = \begin{pmatrix}0 & 0 & -1\\ 0 & 1 & 0\\ -1 & 0 & 0\end{pmatrix} \tag{7}$$

and ignoring the overall phase, we obtain a bit-flip in the qubit subspace.

The second gate manifested by the commensuration condition is the Hadamard gate which is obtained at $K = 3$. In Figure 6 the occupation of the states are plotted as functions of time. The period t_3 at which the instantaneous collapse to the qubit subspace with symmetric occupations occurs is indicated by an arrow. The unitary matrix that performs this operation is

$$\exp\left\{-\mathrm{i}t_3\begin{pmatrix}-1 & V_0^{(3)}/2 & V_0^{(3)}/2\\ V_0^{(3)}/2 & 2 & V_0^{(3)}/2\\ V_0^{(3)}/2 & V_0^{(3)}/2 & -1\end{pmatrix}\right\} = \frac{\mathrm{e}^{\mathrm{i}\alpha}}{\sqrt{2}}\begin{Bmatrix}1 & 0 & -\mathrm{i}\\ 0 & \sqrt{2}\mathrm{e}^{\mathrm{i}\beta} & 0\\ -i & 0 & 1\end{Bmatrix} \tag{8}$$

where $t_3 = \pi/2\,[E_2(V_0^{(3)}) - E_3(V_0^{(3)})] = 0.7043492$ (in units of $\hbar/\tau$) and $V_0^{(3)} = -2\tau(13 + 2\sqrt{22})/9$. The α is an overall phase which is ignored, and β is the phase of the auxiliary level which is irrelevant for the qubit subspace. The gate in (8), after correcting the phase by a relative phase shift becomes a Hadamard gate in the qubit subspace. The phase is corrected by a phase gate which is obtained by turning off V_0 and shifting the levels by H_2 (Eq. (6)). The shifted flux removes the degeneracy with a net effect implied by H_2 and, under the time evolution, phases between the states are induced without changing the populations. The time dependence of the transformation induced by the phase gate is

$$G(\phi) = \mathrm{e}^{-\mathrm{i}\,t\,(H_d+H_2)} = \begin{pmatrix}\mathrm{e}^{\mathrm{i}\phi_1(t)} & 0 & 0\\ 0 & \mathrm{e}^{\mathrm{i}\phi_2(t)} & 0\\ 0 & 0 & \mathrm{e}^{\mathrm{i}\phi_3(t)}\end{pmatrix}\cdot \tag{9}$$

The relative phase $(\phi_1 - \phi_3)/2 = \phi(t)$ applies to the qubit subspace and the phase induced on the auxiliary level can be totally ignored. The relative phase correction needed in equation (8) can be achieved by sandwiching it between the two phase gates $G(\phi = -\pi/4)$. By this demonstration it is also clear how to perform a phase flip which can be obtained by producing $\phi = \pi/2$.

The realization of the controlled operations with double qubits is an essential requirement of any mechanism of quantum computation. It is possible to obtain a CNOT gate in the quantum system we propose. Both three level systems are initially prepared to be in their qubit subspaces and they are connected by a quantum nondemolitional measurement device which reads the first qubit and depending on its state, induces a static potential $V_0^{(1)}$ in the second qubit to perform the bit flip. The experimental scheme is shown in Figure 7 which employs two mesoscopic rings, a Hall bar in the form of a Corbino disk [25] in the full quantum regime and superconducting loop. The persistent current J_1 in the loop of qubit $\mathbf{Q_1}$ creates a current in the superconducting loop $J_1' = \eta J_1$ where η is the efficiency of current transformation and converts it to voltage

$$V = R_{xy}J_1' = n\frac{h}{e^2}J_1' \tag{10}$$

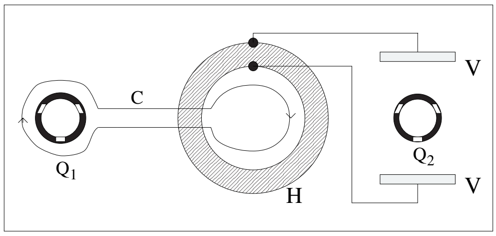

Fig. 7. A sketch of the **CNOT** quantum gate. The loop of the qubit No.1 couples *via* the superconducting loop **C** to quantum Hall bar (**H**) in the form of a Corbino disk. The voltage output $R_{xy}J_1'$ from the disk is supplied (after subtracting a constant value V_0, not shown in the figure) to potential electrodes **V** thus controlling the flip transition in the qubit No. 2.

on the center of nth Hall plateau. (The system is assumed to be initiated such that current in a loop is zero at zero persistent current in a qubit loop; the other possibility could be to include the $-\Phi_0/2$ compensating coil between $\mathbf{Q_1}$ and $\mathbf{H}$ to exclude the large static flux $\Phi_0/2$ in the qubit.) Estimate shows that due to a large value of R_{xy} ($27k\Omega$ on the main Hall plateau), the voltage V is enough to drive the qubit at the efficiency $\eta \sim 0.1$.

The Hall voltage generated in the bar is designed so that either $V_0^{(1)}$ or zero voltage is produced corresponding to the fixed value of the current flowing in one or the other direction. The Hall bar is connected to the V electrodes of qubit $\mathbf{Q_2}$. If the voltage is $V_0^{(1)}$, the bit flip of the second qubit is realized after time t_1 or if the voltage is zero no change is made. The procedure may in principle be executed in a totally reversible way if the Hall bar operates in the manifestly quantum regime. According to measurements [25], longitudinal currents in the contactless realization of the quantum Hall effect (the Corbino geometry) persist for hours, *i.e.* the longitudinal resistance R_{xx} is extremely small, practically a vanishing quantity.

4 Dissipation and decoherence

The main obstacle to many qubit realizations is the decoherence, *i.e.* loss of phase memory in the qubit due to interaction with the environment. Switch operations also, being not fully reversible, result in dissipation and decoherence which effects the evolution of the quantum system. We now examine these main sources of decoherence and estimate the time scale of decoherence for the suggested radiation-free mechanism.

We identify two decoherence regimes depending on the interaction with the external switch or with a continuous background noise field. In the switching phase, the dissipation from the qubit can be calculated semiclassically by using the standard formula for the radiative power

$$W = \frac{2}{3c^3}\langle|\ddot{\mathbf{p}}|^2\rangle \tag{11}$$

in which $\mathbf{p}$ is the dipole moment estimated as $|\mathbf{p}| \sim ed$ ($d = 2R$ is a diameter of the loop) and $|\ddot{\mathbf{p}}|$ estimated as $ed\omega^2$ where characteristic frequency ω is of the order of inverse switching time $t^* \sim \hbar/\tau$ (τ is the hopping energy scale as in (1)). This gives an estimate of the characteristic quality factor ($Q_{diss} = \omega\tau_{diss}$)

$$Q_{diss} \sim \frac{\hbar c\omega_0^2}{e^2\nu_{hopp}^2} \sim \left(\frac{e^2/d}{\tau}\right)^2 \frac{1}{\alpha_0^3} \tag{12}$$

where $\omega_0 = \pi c/d$ is the radiation frequency in a cavity of size d and ν_{hopp} is the hopping frequency of the order of $\tau/\hbar$. α_0 is a fine structure constant $e^2/\hbar c$. For the physical parameters we consider, the typical case of $d \leq 10^{-5}$ cm and $\tau \sim 1$ meV implies a sufficiently high qubit quality factor roughly of the order of $Q_{diss} \sim 10^8$.

On the other hand, in the idling phase, the qubit may experience transitions between the qubit eigenstates due to the interaction with the environment field. The primary source being the dipole radiation, these secondary effects result in the fluctuations in the flux from its desired value. Considering that all electromagnetic fluctuation effects (including the low temperature as well as zero point oscillations in an electromagnetic cavity) will one way or another couple to the flux in the ring, a general decoherence model can be devised by assuming that the flux phase α in equation (1) is allowed to fluctuate as a result of the electromagnetic background noise. Hence we consider in equation (1) $\alpha \rightarrow \alpha + \hat{\Delta\Phi}/\Phi_0$ where α is the desired flux phase required for the implementation of the qu-gates and $\hat{\Delta\Phi}$ some flux fluctuation associated with the background noise field where we will assume that $|\hat{\Delta\Phi}|/(\Phi_0\alpha) \ll 1$. By using equation (1) we formulate the coupling of the noise field to the ring as

$$H_{dec} = \tau \sum_{n=0}^{2} a_n^\dagger\, a_{n+1} \left(\mathrm{e}^{2\pi \mathrm{i}\hat{\Delta\Phi}/\Phi_0} - 1\right) + \text{h.c.} \tag{13}$$

where $\hat{\Delta\Phi}$ is the operator associated with the environmental background field. The fluctuations in the flux induced by the background field deforms the transitions between the energy configuration and hence upsets the time evolution of the qubit states.

A general background noise field can be formulated as

$$\mathbf{A} = \sum_{\mathbf{k}} \mathbf{e_k} \left(\frac{4\pi\hbar c^2}{V\omega_{\mathbf{k}}}\right)^{1/2} \left(c_{\mathbf{k}} + c_{-\mathbf{k}}^\dagger\right) \mathrm{e}^{\mathrm{i}\mathbf{kr}} \tag{14}$$

where $c_{\mathbf{k}}^\dagger$ ($c_{\mathbf{k}}$) are the photon creation and annihilation operators. The coupling of the environment to the qubit arises from the Hamiltonian in equation (1) with an additional radiative field as

$$\begin{aligned} H &= -\tau \sum_{n=1}^{3} a_n^\dagger a_{n+1} \mathrm{e}^{\mathrm{i}\alpha} \exp\left(\frac{\mathrm{i}e}{\hbar c}\int_n^{n+1} \mathbf{A} d\mathbf{l}\right) + \text{h.c.} \\ &= H_0 + H_{int} \end{aligned} \tag{15}$$

where H_0 is the Hamiltonian in (1), and H_{int} is given in the diagonal basis of H_0 by the matrix

$$\begin{aligned} (H_{int})_{m,m'} = -\mathrm{i}\frac{\tau}{3}\mathrm{e}^{\mathrm{i}\alpha} \sum_{\mathbf{k}} \frac{|\Delta\Phi_{\mathbf{k}}|}{\Phi_0} \mathrm{e}^{2\pi \mathrm{i} n(m-m')/3} \\ \times \mathrm{e}^{-2\pi \mathrm{i} m'/3} \Lambda(\mathbf{k}) \left(c_{\mathbf{k}} + c_{-\mathbf{k}}^\dagger\right) \end{aligned} \tag{16}$$

where

$$\Lambda(\mathbf{k}) = \oint \mathrm{d}\theta\, \mathbf{e_k} \cdot \mathbf{n}_\theta\, \mathrm{e}^{\mathrm{i}Rk\cos\theta}, \tag{17}$$

with $\mathbf{n}_\theta$ as the angular unit vector, arising from the line integral of the background environment field in equation (16). We also assumed that the fluctuations are weak and expanded the $\mathrm{e}^{\frac{\mathrm{i}e}{\hbar c}\int \mathbf{A}\cdot d\mathbf{l}}$ to first order in the background field. In (16) we defined $|\Delta\Phi_{\mathbf{k}}| = R(\frac{4\pi\hbar c^2}{V\omega_{\mathbf{k}}})^{1/2}$ as a typical order of magnitude in the background flux noise.

The decoherence can be estimated from the first order correction to the state vector of qubit Ψ. Assume that the qubit is originally in a state $\Psi_0 = |0\rangle$ which is one of the eigenstates of H_0. At a time t the change in the state due to the coupling to the background field will be

$$|\delta\Psi\rangle = \sum_{m=0}^{3} C_{0m}(t)|m\rangle \quad (18)$$

where

$$C_{0m}(t) = \frac{\mathrm{i}e\tau}{\hbar c}\sum_{\mathbf{k}} \mathrm{e}^{\mathrm{i}\alpha} \Lambda(\mathbf{k}) \frac{\mathrm{e}^{\mathrm{i}(\varepsilon_0-\varepsilon_m-\omega_k)t}-1}{\varepsilon_0-\varepsilon_m-\omega_{\mathbf{k}}} \cdot \quad (19)$$

Square of modulus of $\delta\Psi$ has linear dependence on t at large t, consistent with the "Golden rule", from which we estimate the characteristic quality factor due to the environmental decoherence

$$Q_{dec} \sim \frac{\hbar^2 c\omega_0^2}{e^2 \nu_{hopp} \Delta\varepsilon} \quad (20)$$

where $\Delta\varepsilon$ is a characteristic energy of transition or the environment temperature. We now compare the radiative quality factor Q_{diss} given by equation (12) with the environmental one in equation (20). If $\Delta\varepsilon \simeq \hbar\nu_{hopp}$ then $Q_{diss} = Q_{dec}$ and hence $Q_{dec} \sim 10^8$ for the parameters used in equation (12). In case of transition between the non-degenerate states ($0 \rightarrow 1$ and $1 \rightarrow 2$) $\Delta\varepsilon$ is of order of ν_{hopp} and for the $0 \rightarrow 2$ we assume $\Delta\varepsilon \sim k_B T$ considering $k_B T \ll \hbar\nu_{hopp}$.

The above estimates show that the decoherence effects are relatively small for small loops with the highly transparent hopping channels allowing in principle up to 10^8 coherent operations with the qubit. Our estimate shows that, with the radiation free coupling between the qubits, the transitions between the degenerate states are more protected against decoherence compared to non-degenerate ones. Another problem is the "instrumental decoherence" related to the Ohmic bath spectrum (the Nyquist noise) which is known to be much stronger at low frequencies than the vacuum fluctuations of the field. Indeed, manipulation of qubits while transforming quantum information with quantum gates is assumed by applying static voltages for precisely defined (short) intervals. During this process the low frequency tail of the voltage fluctuations is naturally cut out and the strong low frequency sector of the noise is eliminated. This mechanism can be provided either by resistive sources or by the electrometric devices which reduce the noise. Mention that our qugates do not bear any quantum restrictions (those including the Planck's constant) regarding the precision of the voltage amplitudes and durations.

Similarly to system noise, the multilevel structure (the higher energy levels above the Λ-shaped structure) will cause a deviation in the time evolution of qubit from the one calculated on basis of the idealized three-level model of equations (1, 2). This effect is similar to transverse relaxation in the NMR experiments and vanishes at small enough $\tau/\Delta E$ ratio, *i.e.* at deep enough potential wells creating qubit sites. In principle, a proper error correction scheme can be added to (at least partly) compensate for the effect of extra levels by renormalizing the parameters (V^*, t^* - the voltage amplitudes and durations) of the qugate transformations ("tuning of the qugate"). This problem will be discussed elsewhere.

5 Physical implementation

Among the crucial points in the computation are a reproducible initialization, storing the information until the final readout, and an accurate readout which we examine respectively. For the initialization, the magnetic flux is shifted adiabatically from half flux quantum and the system is allowed to relax to the nondegenerate lowest energy state $|0\rangle$ by spontaneous emission. We either leave the state there or, by applying a Hadamard gate, a Schrödinger Cat state is obtained which is conventionally the initial state in some quantum computing algorithms, in particular Shor's algorithm for factorizing large integers [3].

Concerning the *read-out* and *read-in*, for parallel read-out in a large scale computation some of the produced data needs to be read in parallel and simultaneously in a number of qubits. This implies that the information in some qubits must be stored until all necessary operations are performed and during this time the qubit subspace should be free of dissipation. The coherence can be maintained by adopting the $H_1 = 0, H_2 = 0$ case as the idling configuration in the doubly degenerate eigenbasis of H_0. The degenerate configuration helps the states to maintain relative phase coherence.

In conclusion, we suggested a radiation free mechanism whose physical Hamiltonian allows for coupled qubit/qugate storage and processing of information in a reversible, scalable way with reduced decoherence effects. The system implementation remains to be a future task which may become less demanding due to high degree of flexibility in setup organization regarding in particular the use of multi-loop qubits and quantum mesoscopic effects other than the original Aharonov-Bohm one, including the Berry phase and spin-orbit interaction induced persistent currents.

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