

THEORY OF Q -DEGRADATION AND NONLINEAR EFFECTS IN Nb-COATED SUPERCONDUCTING CAVITIES

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Amplitude-dependent absorption of RF power in superconducting cavity with a wall of normal metal (Cu) covered by a film of superconductor (Nb) is determined by three factors: (1) depairing effect of high-current density on superconducting energy gap; (2) increase of electromagnetic power penetrating to normal metal and Drude-absorbed in it, at higher RF amplitude; (3) thermal heating of both superconducting coating and normal substrate resulting in increase of quasiparticle excitation density at higher RF power (P). Cavity Q -factor is calculated as a function of RF amplitude $A = \sqrt{P}$ and is shown to follow a relationship $\ln Q/Q_0 = -\text{const} \cdot P^\alpha$ with $\alpha = 1$ at low temperature ($T < T^*$) and $\alpha = 1/2$ at $T > T^*$, where T^* is characteristic temperature, $T^* \sim T_c \delta/d$, with δ the London penetration depth and d the thickness of superconducting coating.

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Superconducting cavities for particle accelerators allow receiving values of quality factors $Q \sim 10^{10}$ at RF electric field amplitudes $V_{ac} \sim 10$ MV/m.^{1,2} Similar parameters are realized in cavities made by sputtering of Nb over the surface of Cu.³ Nb-coated cavities can have even higher values of Q at small electric field but show faster Q degradation with increasing amplitude. In the present paper we try to understand factors which limit the ultimate performance of coated cavities. We show that Q -degradation results from the depairing effect of the RF current, from

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the residual amplitude-dependent dissipation of a.c. electromagnetic power in the interface between N and S layers, and from the inhomogeneity and defects in a superconducting coating. By using the local electrodynamics of superconductivity, an expression for the quality factor is derived:

$$Q = \frac{c^3}{8\delta_L^3\omega^2\sigma_2}, \quad (1)$$

showing the $1/\omega^2$ dependence on frequency ω .⁴ δ_L is the London penetration depth of the superconductor and σ_2 the imaginary part of complex conductivity $\sigma = \sigma_1(\omega) + i\sigma_2(\omega)$. In a bilayer system, Q can be expressed through the reflection coefficient R of the RF power from a cavity wall,

$$Q = \frac{\pi}{1 - R}. \quad (2)$$

Dependence of δ_L , σ_2 and R on the amplitude of the RF field determine the amplitude dependence of the quality factor. For the determination of R , we solve the Maxwell equations inside the cavity

$$-\frac{\partial^2 A}{\partial z^2} = \frac{4\pi}{c}j, \quad H = \frac{\partial A}{\partial z}, \quad E = -\frac{1}{c}\frac{\partial A}{\partial t}, \quad (3)$$

with the current density in a wall

$$j = \begin{cases} -(n_s e^2/mc)A + \sigma_2 E, & z < d, \\ \sigma_n E, & z > d, \end{cases} \quad (4)$$

where d is the thickness of superconducting coating. The electromagnetic field inside the cavity at distance z from its surface equals $A = e^{ik} - re^{-ikz}$, where r is the reflection amplitude, and $R = |r|^2$. We get

$$r = \frac{(k_1 - k_2)e^{-k_1 d}(1 - ik/k_1) - (k_1 + k_2)e^{k_1 d}(1 + ik/k_1)}{(k_1 - k_2)e^{-k_1 d}(1 + ik/k_1) - (k_1 + k_2)e^{k_1 d}(1 - ik/k_1)}, \quad (5)$$

with

$$k_1 = \sqrt{\delta_L^{-2} - 2i\delta_{sk}^{-2}}, \quad k_2 = (1 - i)/\delta_{sn}, \quad (6)$$

where δ_{sk} is the skin penetration depth appropriate to normal conductivity of superconductor σ_2 , and δ_{sn} is skin depth of a normal metal:

$$\delta_{sk} = (c^2/2\pi\sigma_2\omega)^{1/2}, \quad \delta_{sn} = (c^2/2\pi\sigma_n\omega)^{1/2}. \quad (7)$$

Penetration of an a.c. field into a superconductor is described with the introduction of a complex penetration depth δ according to

$$\frac{1}{\delta^2} = \frac{1}{\delta_L^2} - \frac{2i}{\delta_{sk}^2}. \quad (8)$$

Two factors determine the amplitude dependence of δ : (1) The depairing effect of finite current; (2) Increase of σ_2 due to the decrease of the energy gap Δ .

In the current-carrying state, BCS density of states $N(\epsilon)$ changes from the value $N(\epsilon) = |\epsilon|/\sqrt{\epsilon^2 - \Delta^2} \theta(|\epsilon| - \Delta)N(0)$ (Figure 1, curve 1) to the dependence

$$\frac{N_s(\epsilon)}{N(0)} = \begin{cases} \left[\frac{\sqrt{(\epsilon + p_F v_s)^2 - \Delta^2} - \sqrt{(\epsilon - p_F v_s)^2 - \Delta^2}}{2p_F v_s}, \right. \\ \quad \left. \epsilon > \Delta + p_F v_s, \right. \\ \left. \frac{\sqrt{(\epsilon + p_F v_s)^2 - \Delta^2}}{2p_F v_s}, \quad \Delta - p_F v_s < \epsilon < \Delta + p_F v_s. \right. \\ \left. 0, \quad \epsilon < \Delta - p_F v_s, \right. \end{cases}$$

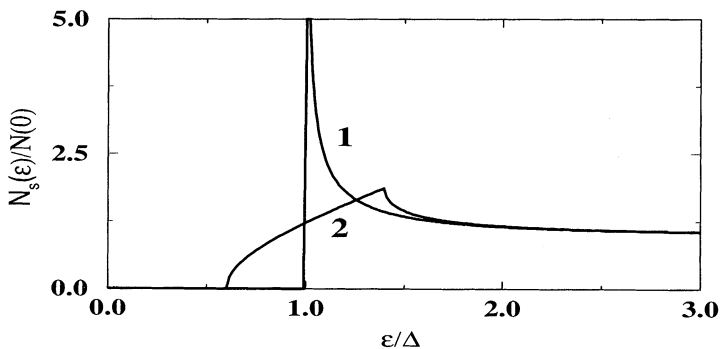


FIGURE 1 Density of states in the BCS superconductor at $j=0$ (curve 1) and in a current-carrying state corresponding to $p_F v_s/\Delta = 0.4$ (curve 2).

shown in curve 2. At the same time, the energy gap in a current-carrying state decreases according to the relation

$$\Delta = \Delta_0 - p_F v_s, \quad (9)$$

where the supervelocity v_s is related to the magnetic field at the surface of the superconductor $v_s = eH\delta_L/mc$.

Since the concentration of quasiparticles decreases exponentially with temperature

$$n_{qp} = \frac{2\Delta}{T} n e^{-\Delta/T} \ln \frac{T}{\omega}, \quad (10)$$

this results in a decrease of the quality factor

$$Q_1 = Q_1^0 \exp(-ev_F H\delta_L/cT). \quad (11)$$

Another source of RF dissipation in the cavity results from the electric field penetration to the normal substrate. Nevertheless the electric field in the substrate is very small in comparison to the electric field at surface, it sees much greater amount of excitations responding to the field thus making a comparable contribution to losses. For a quality factor Q_2 corresponding to this effect, we get

$$Q_2 = Q_2^0 \exp(-2d/\delta_L). \quad (12)$$

The amplitude dependence of δ_L is determined through the relations

$$\delta_L = \left(\frac{mc^2}{4\pi n_s e^2} \right)^{1/2}, \quad n_s = n_s^0 (1 - v_s^2/v_c^2). \quad (13)$$

In the above equations, $Q_{1,2}^0$ are the factors

$$Q_1^0 \simeq \frac{\lambda \delta_{sn}^2}{8\delta_L^3} e^{\Delta/T}, \quad Q_2^0 \simeq \frac{\lambda \delta_{sn}}{32\delta_L^2} e^{2d/\delta_L}. \quad (14)$$

Since losses related to both mechanisms add, we receive for the overall quality factor

$$Q^{-1} = Q_1^{-1} + Q_2^{-1}, \quad (15)$$

which results in the expression

$$Q = \left[\frac{1}{Q_1^0} e^{H_{ac}/H_1} + \frac{1}{Q_2^0} e^{(H_{ac}/H_2)^2} \right]^{-1}, \quad (16)$$

where H_1 and H_2 are characteristic amplitudes

$$H_1 = \frac{cT}{ev_F \delta_L}, \quad H_2 = 1.76 \frac{cT_c}{ev_F \sqrt{d\delta_L}}. \quad (17)$$

The second term in Eq. (16) is specific to the Nb-coated cavity and does not appear in a cavity of pure Nb. At low temperatures, when Q_1^0 is much larger than Q_2^0 , $\ln Q$ is expected to decrease linearly with RF power whereas at low temperatures it scales with the RF amplitude. The crossover temperature between these asymptotic regimes, T^* , is determined from the condition $H_1 = H_2$ giving

$$T^* = \Delta \frac{\delta_L}{2d} = 0.88 T_c \frac{\delta_L}{d}. \quad (18)$$

The dependence of Q on the RF power is presented in Figure 2. (It can be trusted only qualitatively at large P since vortex nucleation generally starts at currents much smaller than the depairing currents.)

At large RF amplitude, the current becomes amplitude dependent owing to the depairing effect of v_s on n_s , Eq. (13). As a result, a.c. current acquires odd harmonics 3ω , 5ω , etc. Since nonlinearity is small, only third harmonic may effectively contribute. The harmonic $A_{3\omega}$ is not in resonance in a cavity, therefore the power reflected in this harmonic is excluded from further amplification and will contribute effectively to losses in Eq. (16).

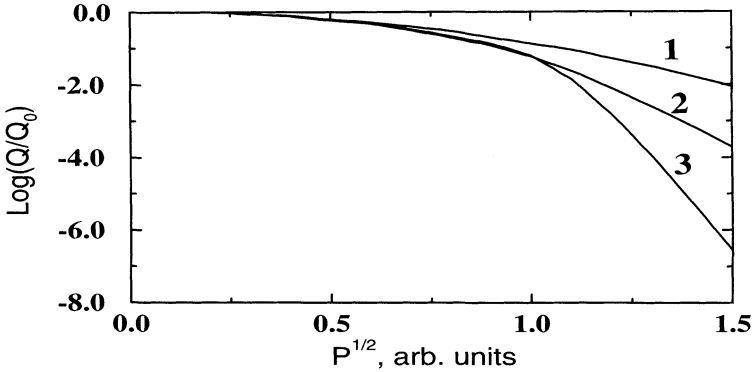


FIGURE 2 Q vs RF power P at $H_1=1.33H_2$ at various values of temperature. 1 - $T/T^*=0.05$, 2 - $T/T^*=0.1$ and 3 - $T/T^*=1.0$.

The equation for the vector potential inside the cavity,

$$\frac{\partial^2 A}{\partial z^2} - \frac{mc^2}{4\pi n_s^0 e^2} (1 - A^2/A_c^2) - \frac{4\pi\sigma_2}{c^2} \frac{\partial A_c}{\partial t} = 0, \tag{19}$$

is first solved neglecting the dissipation. This gives the spatial dependence of the vector potential

$$A(z) = \frac{A(0)}{\cosh(z/\delta_L) + \sqrt{1 - A^2(0)/2A_c^2} \sinh(z/\delta_L)}. \tag{20}$$

By differentiating $A(z)$ with respect to z and putting $z = 0$ we receive the boundary condition at the surface

$$(dA/dz)_{z=0} + \frac{A(0)}{\delta_L} \sqrt{1 - A^2(0)/2A_c^2} = 0. \tag{21}$$

By including the dissipative term and solving perturbatively for A at $A \ll A_c$, we get a contribution to formula (15)

$$Q_3^{-1} \simeq 2 \left(\frac{A(0)}{A_c} \right)^2 \left(\frac{c\delta_L}{\omega} \right)^3. \tag{22}$$

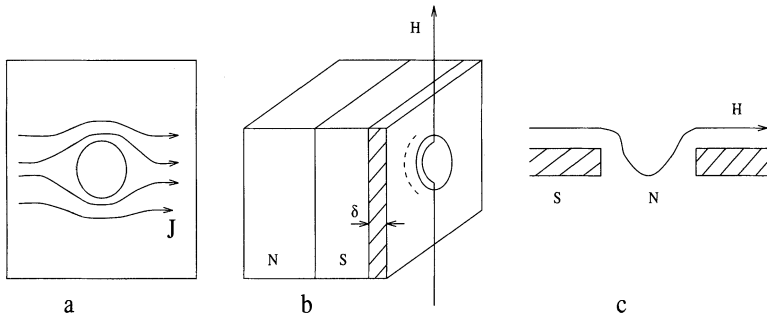


FIGURE 3 Magnetic fields and currents near the opening in the superconducting coating. (a) Streamlines of the supercurrents, (b, c) lines of force of magnetic field near the orifice.

The last factor is of the order of the ratio of penetration depth to the wavelength of electromagnetic field in vacuum, and is a very small quantity. So, practically, we can neglect the harmonic contribution to losses, except at very high frequencies.

The above mechanisms of Q -degradation are applicable below the critical field which is of the order of the threshold field for vortex nucleation. In nonuniformly-coated mixed superconductor-normal metal cavities, another mechanism may be responsible for the Q -degradation. Assume a model in which superconducting film has a small opening (Figure 3(a)). Near the opening, at small RF amplitude, the supercurrent bypasses the normal spot and therefore may not have a significant effect on the cavity's Q . What is important, however, is that the geometry of lines of force of magnetic field near the opening changes from parallel to inclined with respect to surface orientation (Figure 3(b) and (c)). Therefore the critical magnetic field near the opening is lower than that at the rest of the surface. At increasing RF amplitude, this "weak spot" will first respond to vortex penetration. The a.c. current will start flowing through the spot rather than bypassing it resulting in Joule heating and in the expansion of normal layer to the adjacent part of the opening. Since characteristic Q value of normal metal cavity $Q_n \sim 10^5$ is much smaller than superconducting Q value $Q \sim 10^{10}$, a very small fraction of spotted surface ($\Delta S/S \sim 10^{-5}$) may significantly decrease the overall quality factor at increased RF amplitude thus creating the main source of Q degradation.

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