Comments and Replies

Comments on "The Use of Curl-Conforming Basis Functions for the Magnetic-Field Integral Equation"

Eduard Ubeda and Juan M. Rius

Based on our research experience on low-order curl-conforming discretizations in method of moments (MoM) of the magnetic-field integral equation (MFIE) [1], we would like to comment on the sentence in the third paragraph of [2, subsection B in Section III], where the authors argue that "the diagonal elements of the impedance (MoM-MFIE) matrices are the same for the implementations of the nxRWG and RWG functions." We believe that this cannot be stated in general. Although this statement is true for diagonal impedance elements regarding basis functions expanding coplanar triangles, it cannot be extended to diagonal impedance elements regarding basis functions defined over non-coplanar triangles. From the definition of the Galerkin MoM-MFIE formulation, the self-impedance term due to the mth basis function becomes [1]

$$z_{mm} = \frac{\left\langle \vec{f}_m(\vec{r}), \vec{f}_m(\vec{r}) \right\rangle}{2} - \left\langle \vec{f}_m(\vec{r}), \hat{n}_m \times \int_{\mathrm{CPV}, T_m^+ \cup T_m^-} \vec{f}_m(\vec{r}') \times \nabla' G(\vec{r}, \vec{r}')\, ds' \right\rangle \quad (1)$$

which, depending on the basis function set adopted, RWG or nxRWG, stands for the m-th diagonal impedance elements of, respectively, the RWG MoM-MFIE or nxRWG MoM-MFIE implementation. The two terms in (1) become the diagonal elements of the matrices $[d_{mn}]$ and $[Z_{\mathrm{CPV},mn}]$ following the definition showed in [1], [3] so that $[z_{mn}] = [d_{mn}] + [Z_{\mathrm{CPV},mn}]$. $[Z_{\mathrm{CPV},mn}]$ denotes the Cauchy principal value of the integral expression of the scattered magnetic field and $[d_{mn}]$ includes the evaluation of this integral when $\vec{r} = \vec{r}'$. These two matrices, when implemented for the RWG and nxRWG sets lead, respectively, to $[d_{\mathrm{div},mn}]$, $[Z_{\mathrm{CPV\,div},mn}]$ and $[d_{\mathrm{curl},mn}]$, $[Z_{\mathrm{CPV\,curl},mn}]$, which, as demonstrated in [1], [3], are related as

$$[d_{\mathrm{div},mn}] = [d_{\mathrm{curl},mn}] \qquad [Z_{\mathrm{CPV\,div},mn}] = -[Z_{\mathrm{CPV\,curl},nm}]. \quad (2)$$

For the particular case of the self-impedance terms ($m = n$), we define the values

$$\bar{d} = [d_{\mathrm{div},mm}] = [d_{\mathrm{curl},mm}]$$
$$\bar{c} = [Z_{\mathrm{CPV\,div},mm}] = -[Z_{\mathrm{CPV\,curl},mm}] \quad (3)$$

which, when combined according to (1), lead to the general expression for the diagonal elements of the MoM-MFIE impedance matrices for the RWG and nxRWG sets, respectively, $Z_{\mathrm{div},mm}$ and $Z_{\mathrm{curl},mm}$

$$Z_{\mathrm{div},mm} = \bar{d} + \bar{c} \qquad Z_{\mathrm{curl},mm} = \bar{d} - \bar{c}. \quad (4)$$

For the particular case of basis functions embracing coplanar triangles, $\hat{n}_m$, the normal unitary vector to both triangles T_m^+ and T_m^-, becomes constant and parallel to the cross-product $\vec{f}_m \times \nabla' G$ all over the m-th basis function domain. Hence, for basis functions expanding coplanar triangles, $\bar{c} = 0$ and $z_{\mathrm{div},mm} = z_{\mathrm{curl},mm} = \bar{d}$. However, this is not the case when m corresponds to a sharp-edge arising from the discretization. Indeed, for basis functions defined over adjacent non-coplanar triangles, in general, $\bar{c} \neq 0$ and thus the diagonal elements of the impedance MoM-MFIE matrices for the implementations of the RWG and nxRWG functions, $z_{\mathrm{div},mm}$ and $z_{\mathrm{curl},mm}$, are not the same.

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Reply to "Comments on 'The Use of Curl-Conforming Basis Functions for the Magnetic-Field Integral Equation'"

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We thank E. Úbeda and J. M. Rius for their interest in our work. In [1, p. 1920], the statement "the diagonal elements of the impedance matrices are the same for the implementations of the $\hat{\boldsymbol{n}} \times \mathrm{RWG}$ and the RWG functions" and (14) are not correct, in general. However, we emphasize that the rest of the paper and the results are not contaminated with this error.

Consider the general expression (15) in [1] for the interactions of the half $\hat{\boldsymbol{n}} \times \mathrm{RWG}$ functions, i.e.,

$$Z_{mn,ij}^{\mathrm{nRWG}} = \int_{S_{m,i}} d\boldsymbol{r}\, \boldsymbol{t}_{m,i}^R(\boldsymbol{r}) \cdot \boldsymbol{b}_{n,j}^R(\boldsymbol{r}) + \int_{S_{n,j}} d\boldsymbol{r}'\, \boldsymbol{b}_{n,j}^R(\boldsymbol{r}') \cdot \hat{\boldsymbol{n}}' \times \int_{S_{m,i}} d\boldsymbol{r}\, \boldsymbol{t}_{m,i}^R(\boldsymbol{r}) \times \nabla g(\boldsymbol{r}, \boldsymbol{r}') \quad (1)$$

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where $\boldsymbol{t}^R_{m,i}$ and $\boldsymbol{b}^R_{n,j}$ are the half RWG testing and basis functions located at the edges m and n, respectively. The right-hand side of the equation above is composed of two terms. The limit term

$$L^{\mathrm{nRWG}}_{mn,ij} = \int_{S_{m,i}} d\boldsymbol{r}\boldsymbol{t}^R_{m,i}(\boldsymbol{r}) \cdot \boldsymbol{b}^R_{n,j}(\boldsymbol{r}) \tag{2}$$

exists only when the half functions are located on the same triangle. However, the principal-value term

$$P^{\mathrm{nRWG}}_{mn,ij} = \int_{S_{n,j}} d\boldsymbol{r}'\boldsymbol{b}^R_{n,j}(\boldsymbol{r}') \cdot \hat{\boldsymbol{n}}' \times \int_{S_{m,i}} d\boldsymbol{r}\boldsymbol{t}^R_{m,i}(\boldsymbol{r}) \times \nabla g(\boldsymbol{r},\boldsymbol{r}') \tag{3}$$

is nonzero, if the triangles of the half functions are not in the same plane. Therefore, when the half functions are located on the same triangle, the principal-value term vanishes. We also note that

$$L^{\mathrm{nRWG}}_{mn,ij} = L^{\mathrm{RWG}}_{mn,ij} \tag{4}$$

$$P^{\mathrm{nRWG}}_{mn,ij} = -P^{\mathrm{RWG}}_{nm,ij} \tag{5}$$

and the interactions calculated for the $\hat{\boldsymbol{n}} \times \mathrm{RWG}$ and the RWG functions are closely related.

The three special cases for the matrix elements of the $\hat{\boldsymbol{n}} \times \mathrm{RWG}$ implementations can be derived as follows.

1) When the basis and testing functions related to edges m and n do not overlap in space, we obtain

$$\begin{aligned} Z^{\mathrm{nRWG}}_{mn} &= Z^{\mathrm{nRWG}}_{mn,11} + Z^{\mathrm{nRWG}}_{mn,12} + Z^{\mathrm{nRWG}}_{mn,21} + Z^{\mathrm{nRWG}}_{mn,22} \\ &= P^{\mathrm{nRWG}}_{mn,11} + P^{\mathrm{nRWG}}_{mn,12} + P^{\mathrm{nRWG}}_{mn,21} + P^{\mathrm{nRWG}}_{mn,22} \\ &= -P^{\mathrm{RWG}}_{nm,11} - P^{\mathrm{RWG}}_{nm,12} - P^{\mathrm{RWG}}_{nm,21} - P^{\mathrm{RWG}}_{nm,22} \\ &= -Z^{\mathrm{RWG}}_{nm,11} - Z^{\mathrm{RWG}}_{nm,12} - Z^{\mathrm{RWG}}_{nm,21} - Z^{\mathrm{RWG}}_{nm,22} \\ &= -Z^{\mathrm{RWG}}_{nm}. \end{aligned} \tag{6}$$

2) When $m = n$, i.e., when the basis and testing functions are the same

$$\begin{aligned} Z^{\mathrm{nRWG}}_{mm} &= Z^{\mathrm{nRWG}}_{mm,11} + Z^{\mathrm{nRWG}}_{mm,12} + Z^{\mathrm{nRWG}}_{mm,21} + Z^{\mathrm{nRWG}}_{mm,22} \\ &= L^{\mathrm{nRWG}}_{mm,11} + P^{\mathrm{nRWG}}_{mm,12} + P^{\mathrm{nRWG}}_{mm,21} + L^{\mathrm{nRWG}}_{mm,22} \\ &= L^{\mathrm{RWG}}_{mm,11} - P^{\mathrm{RWG}}_{mm,12} - P^{\mathrm{RWG}}_{mm,21} + L^{\mathrm{RWG}}_{mm,22} \\ &= Z^{\mathrm{RWG}}_{mm,11} - Z^{\mathrm{RWG}}_{mm,12} - Z^{\mathrm{RWG}}_{mm,21} + Z^{\mathrm{RWG}}_{mm,22}, \end{aligned} \tag{7}$$

which is different from the self interactions of the RWG functions, i.e.,

$$Z^{\mathrm{RWG}}_{mm} = Z^{\mathrm{RWG}}_{mm,11} + Z^{\mathrm{RWG}}_{mm,12} + Z^{\mathrm{RWG}}_{mm,21} + Z^{\mathrm{RWG}}_{mm,22}. \tag{8}$$

3) Let $m \neq n$, but the basis and testing functions overlap on a triangle. Assume that the first triangle of m is the same as the second triangle of n. Then

$$\begin{aligned} Z^{\mathrm{nRWG}}_{mn} &= Z^{\mathrm{nRWG}}_{mn,11} + Z^{\mathrm{nRWG}}_{mn,12} + Z^{\mathrm{nRWG}}_{mn,21} + Z^{\mathrm{nRWG}}_{mn,22} \\ &= P^{\mathrm{nRWG}}_{mn,11} + L^{\mathrm{nRWG}}_{mn,12} + P^{\mathrm{nRWG}}_{mn,21} + P^{\mathrm{nRWG}}_{mn,22} \\ &= -P^{\mathrm{RWG}}_{nm,11} + L^{\mathrm{RWG}}_{mn,12} - P^{\mathrm{RWG}}_{nm,21} - P^{\mathrm{RWG}}_{nm,22} \\ &= -Z^{\mathrm{RWG}}_{nm,11} + Z^{\mathrm{RWG}}_{mn,12} - Z^{\mathrm{RWG}}_{nm,21} - Z^{\mathrm{RWG}}_{nm,22}. \end{aligned} \tag{9}$$

As a consequence, implementations of MFIE with the $\hat{\boldsymbol{n}} \times \mathrm{RWG}$ functions can easily be obtained from the existing implementations of MFIE with the RWG functions by simple modifications, such as changing the signs of some terms.

REFERENCES

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