Operations Research

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Regenerator Location Problem in Flexible Optical Networks

Barış Yıldız, Oya Ekin Karaşan

informs
http://pubsonline.informs.org/journal/opre/

OPERATIONS RESEARCH
Vol. 65, No. 3, May–June 2017, pp. 595–620
ISSN 0030-364X (print), ISSN 1526-5463 (online)

Regenerator Location Problem in Flexible Optical Networks

Barış Yıldız,[a] Oya Ekin Karaşan[b]

[a]Department of Industrial Engineering, Koç University, Sariyer, 34450 Istanbul, Turkey; [b]Department of Industrial Engineering, Bilkent University, Bilkent, 06800 Ankara, Turkey

Contact: byildiz@ku.edu.tr (BY); karasan@bilkent.edu.tr (OEK)

Received: July 26, 2014
Revised: February 1, 2016
Accepted: October 25, 2016
Published Online in Articles in Advance: April 13, 2017

Subject Classifications: programming: integer: algorithms; facilities/equipment planning: location: discrete; networks/graphs: applications
Area of Review: Games, Information, and Networks

https://doi.org/10.1287/opre.2016.1587

Abstract. In this study, we introduce the regenerator location problem in flexible optical networks. With a given traffic demand, the regenerator location problem in flexible optical networks considers the regenerator location, routing, bandwidth allocation, and modulation selection problems jointly to satisfy data transfer demands with the minimum cost regenerator deployment. We propose a novel branch-and-price algorithm for this challenging problem. Using real-world network topologies, we conduct extensive numerical experiments to both test the performance of the proposed solution methodology and evaluate the practical benefits of flexible optical networks. In particular, our results show that, making routing, bandwidth allocation, modulation selection, and regenerator placement decisions in a joint manner, it is possible to obtain drastic capacity enhancements when only a very modest portion of the nodes is endowed with the signal regeneration capability.

Funding: This work was supported by the Scientific and Technological Research Council of Turkey (TUBITAK) [Grant 2211A].

Keywords: flexible optical networks; regenerator location; relay location; routing; modulation selection; path-segment formulation; branch-and-price

1. Introduction

The wider availability of Internet access, introduction of mobile communication devices (smartphones, tablets, etc.), and booming sector of mobile applications have taken the Internet age to a new stage (Agrawal 2011). In 2011, global mobile data traffic was eight times the size of the whole Internet in 2000, and it is expected to increase 18-fold by 2016 (Index, Cisco Visual Networking 2012). As the growth of the Internet surpasses even the highest estimates, utilized bandwidth of optical fibers rapidly approaches its theoretical limits (Essiambre et al. 2010, Tomkos et al. 2012). Just worsening the problem, the rigid nature of the current optical networks cannot efficiently use available optical bandwidth to support this increasing traffic. The energy consumption of telecommunications networks is also adversely affected by wasteful resource utilization. Such inefficiencies unnecessarily increase the amount of required active network equipment, ultimately increasing the total energy consumption of the network. This is an issue of increasing importance since the power consumption of the Internet is estimated to reach 10% of worldwide energy consumption in a very short time (Global Action Plan 2007). In the United States alone, a 1% saving in the energy consumption of the Internet due to the adoption of energy-efficient network management strategies is estimated to result in savings of \$5 billion per year given that the price of electricity is 17 cents per kWh (Shen and Tucker 2009). The concern over climate change and the heavy carbon footprint of energy generation only increases the importance of the energy efficiency of telecommunications networks.

Motivated by this urgent practical problem, researchers developed the flexible optical network (FON) architecture that can flexibly choose its transmission parameters according to the varying traffic conditions and significantly increase the resource utilization efficiency (Essiambre et al. 2010, Tomkos et al. 2012). The major sources of these inefficiencies, remedies offered by FON architecture, and the algorithmic challenges raised by the adoption of this novel technology can be summarized as follows.

In the current optical network architecture, the available bandwidth is divided into a set of fixed-bandwidth channels, each serving a single transmission demand. However, due to the increasing variability of the offered online services, the capacity demands of connections come from a much broader range with granularities of several gigabits per second to 100 gigabits per second or more. Due to the granularity mismatch between the widths of these channels and demand sizes, the already drained fiber bandwidth cannot be fully utilized (Jinno et al. 2009). On the other hand, with FON, the optical spectrum is divided into fine bandwidths called *slots*, and custom-size bandwidths are created by the contiguous concatenation of those slots. Such custom-size transmission channels can significantly reduce the bandwidth waste and increase the amount of available fiber bandwidth.

The data transfer capacity of backbone networks is not solely dependent on the range of the available bandwidth. Indeed, this capacity is jointly determined by the available bandwidth range of the fiber and the modulation level that induces the amount of data that could be transferred on a fixed bandwidth. Modulation levels with higher bit rates can carry more data on a given bandwidth, but the downside of using higher-bit-rate modulations is the shorter optical reach defined as the maximum distance a signal can traverse before its quality degrades. As a significant source of inefficiency, current optical networks use fixed modulation levels and waste bandwidth by using the same modulation level for both short- and long-distance transmissions (Jinno et al. 2010). FON has been designed to increase the data transmission capacity of optical networks by smartly managing routing and modulation level selection in coordination and, in particular, by utilizing high-bit-rate modulation schemas to increase bandwidth efficiency. However, implementing such an approach is quite challenging due to the optical reach limitations. Optical reach is a decreasing function of the bit rate, and optical reach limitations can significantly restrict the potential gains of flexible modulation selection. One key technology to extend optical reach and overcome this issue is opto-electro-optical (OEO) regeneration. Processed by an OEO regenerator, the optical signal is rejuvenated and after this renewal it can travel up to its optical reach before it arrives to a new regenerator or its final destination. So, with the expense of more capital investment and operating cost (such as energy and maintenance), it is possible to enhance the optical reach of a signal by employing regenerator equipment. Moreover, FON also allows different modulation levels at each segment of a light-path that connects the source of a demand to its destination possibly passing through several regenerators to maintain a certain level of signal quality. So with this new architecture, it is possible to use regenerators to switch modulation formats on a light-path such that the spectrum allocation is minimum while the signal quality is within the predefined limits.

Since OEO regenerators are expensive devices to obtain and operate, there is great motivation to design/operate optical networks with few regeneration points. In short, the better exploitation of the opportunities offered by the FON architecture requires the solution of routing, bandwidth allocation, modulation level selection, and regenerator location problems jointly. The problem of solving all of these problems concurrently is a challenging one. Indeed, researchers indicate that the lack of such an efficient algorithm constitutes a major barrier to the adoption of this novel technology (Tomkos et al. 2012). Despite this urgent need, due to the novelty and the high complexity of the problem, there are not so many studies in the literature that address regenerator location problem in flexible optical networks (RLP-FON). Considering a static demand structure and assuming fixed routes for each transmission demand, Klinkowski (2012) proposes a heuristic algorithm for jointly solving spectrum allocation and regenerator location problems. Similar to our findings, his numerical experiments indicate that a smart placement of regenerators could significantly increase bandwidth efficiency in the network. Relaxing the fixed route assumption, Kahya (2013) presents a sequential solution heuristic approach to solve routing, regenerator location, and spectrum allocation problems in flexible optical networks. In this study, a fixed modulation level is assumed. Motivated by the recent developments in virtualized elastic regenerator (VER) technologies that enable efficient regeneration of various bandwidth super-channels, Jinno et al. (2015) propose a heuristic algorithm for calculating the minimal VER placement, routing, and the least congestion resources assignment in a translucent FON based on Nyquist wavelength-division multiplexing (WDM) super-channels. The authors assume a single modulation level and present computational results which indicate significant efficiency gains in the network due to the strategic VER placements. In a recent study, Wang et al. (2015) investigate the impact of modulation conversion in FONs. The authors present a mixed integer programming (MIP) formulation to solve RLP-FON. Since their formulation cannot solve realistic-size problems, they propose a sequential solution heuristic in which they randomly partition the demand set. Their computational study results show that benefiting from the elastic network structure, proper use of signal regenerators and wavelength converters can significantly decrease the bandwidth requirement depending on the topology of the network. To the best of our knowledge, our study is the first to present an exact algorithm to solve routing, bandwidth allocation, modulation level selection, and regenerator location problems jointly for realistic-size problem instances.

Even in the current networks, regenerators are crucial elements, as regeneration costs make up a significant portion of a network's setup and management costs (Yang and Ramamurthy 2005). Motivated by the practical considerations, the RLP, which tries to find the minimum cost regenerator deployment to facilitate communication between network nodes, has attracted significant research effort in the recent years. Yetginer and Karasan (2003) were the first to introduce the sparse regenerator placement in a static routing environment. Taking the geographical aspect of the RLP into account, Chen et al. (2009) introduce it to the operations research literature, proving its NP-completeness and showing that it can be modeled as a special Steiner arborescence problem. Pointing to the equivalence of the maximum leaf spanning tree problem, the minimum connected dominating set problem and RLP, Sen et al. (2008), Lucena et al. (2010) and, more recently, Gendron et al. (2012) suggest several exact and heuristic algorithms for RLP. Addressing the network survivability concerns, Yildiz and Karasan (2015) introduce two new facets to the problem. They formulate the RLP as a MIP and present an efficient branch-and-cut algorithm which they extend to solve regenerator- and node-reliable versions of the problem. In none of these studies fiber capacity constraints are addressed. Pavon-Mariño et al. (2009) explicitly address the fiber bandwidth capacities and study the RLP in a static demand environment. Different than our work, the authors assume single modulation level and do not consider FON architecture. A MIP formulation that contains a large number of variables is presented. Two heuristic algorithms are proposed to solve large problem instances that cannot be solved by the MIP formulation. In a recent study, considering the FON setting and multiple modulation levels, Castro et al. (2012) investigate the dynamic demand routing and spectrum allocation problem (RSA). The authors present a high-quality heuristic that can solve the dynamic RSA problem, and propose a spectrum reallocation algorithm to deal with the spectrum fragmentation problem which can significantly limit the available fiber bandwidth. Different than this study, the optical reach constraints are not examined.

In this study, we introduce the regenerator location problem in flexible optical networks (RLP-FON). RLP-FON seeks the best routing, modulation level, and regenerator location combination that minimizes the regenerator deployment costs while not using more than a predetermined portion of the fiber bandwidth. In other words, promoting the exclusive capabilities of the new FON architecture, RLP-FON finds the minimum amount of network resources needed to satisfy a given set of transmission demands. Since FON architecture is quite new, despite its immense practical importance, this theoretically challenging problem is not well studied in the literature, and this study is a first attempt to close this gap.

RLP-FON arises both in the design and network management layers. In the design phase, RLP-FON determines the minimum amount of network equipment (regenerators, router chassis, optic line cards, etc.) required to satisfy the targeted demand, whereas in the network operation layer, RLP-FON can help reduce the operating cost of the network by identifying the network elements that could be put into sleep mode when the actual demand is less than the maximum supported demand size. The significance of the latter can be better understood considering the fact that optical backbone networks are designed somewhat to support the worst-case demand scenarios and peak demand is more than two times larger than the minimum observed on the same day (Rizzelli et al. 2012). Indeed, motivated by such an opportunity, hardware developers intensified their research and development efforts on manufacturing network devices with capabilities to go into sleep mode to save energy.

Path-based formulations are very powerful to model problems for which the amount of cost incurred/profit gained or resource depleted depends on the routes chosen. As such, they are widely used in network design and management problems in telecommunications and transportation. Despite their advantages, path-based formulations usually suffer from the exponential number of variables with only a fraction of them actually appearing in a feasible solution. Column generation and branch-and-price methods have been successfully applied to those problems to develop efficient algorithms (Parker and Ryan 1993; Barnhart et al. 1994; Park et al. 1996; Barnhart et al. 1998, 2000; Cohn and Barnhart 2003; Degraeve and Jans 2007; Desaulniers 2010).

Within RLP-FON, for each transmission demand, a routing problem is solved to find a path that connects source and sink nodes and that respects regeneration constraints. However, with a path-based formulation, it is hard to address signal regeneration constraints and consider nonsimple paths while generating new columns in a column generation framework. Because of that, we define path-segments as the simple paths on which the signal does not get into regeneration, and build the routes (both simple and nonsimple) by the proper concatenation of these path-segments.

In this study, we:

- Introduce the RLP-FON problem that adds two new facets to RLP:
  —RLP-FON jointly solves routing, modulation level selection, and regenerator location problems.
  —RLP-FON respects the bandwidth capacity limitations of the fiber links.
- Present a path-segment formulation for RLP-FON and develop a branch-and-price algorithm to solve it. To the best of our knowledge, this is the first study in which path-segments instead of paths are used as the variables in a column generation framework.
- Conduct extensive numerical experiments on realistic reference network topologies to test the computational performance of the proposed algorithm and to offer managerial insights. In particular, results of these experiments show that a strategic deployment of regenerators on a small portion of nodes can achieve capacity enhancements comparable to the case where all of the nodes in the network have regeneration capability.

2. Mathematical Model

In this section, we formally define RLP-FON, present the details of the proposed branch-and-price algorithm, and examine the problem complexity.

2.1. Problem Definition and Notation

For each connection demand, there is a certain amount of data at the origin coded into optical signals to be carried to the destination in a unit time. This coding is done with one of the technologically available modulation levels. For an optical signal, the chosen modulation level determines the number of bandwidth slots required to transfer this signal on a fiber link and sets the optical reach—i.e., the maximum distance to be covered before a regeneration. Higher modulation levels use optical bandwidth more efficiently (require less bandwidth), but they have shorter optical reach. An optical signal is a light-path; that is, a path from the source node to the destination node in the given optical network. When regenerator nodes are visited on this path, the light-path can be viewed as the concatenation of several path-segments where a path-segment is a simple path joining two consecutive regenerators on the light-path or joining a regenerator with the source or the destination node. In other words, except for the one that ends in the destination node, at the end of each path-segment there is a regenerator that restores the signal quality. Regenerators are also capable of recoding and emitting the incoming signal with a different modulation level—i.e., regenerators have the capability to switch the modulation level of an optical signal. Each fiber link in the network has a certain bandwidth capacity which will be consumed by the light-paths passing through it. Considering all of the demands simultaneously, a solution for the RLP-FON needs to respect these bandwidth capacities of fibers. Moreover, the modulation level and the path-segment chosen for a particular demand should be in harmony with respect to optical reach considerations. Thus, the solution of the problem consists of the routing decisions for each demand, location decisions of the regeneration equipment, and the modulation level selections to be used for each demand on each one of its path-segments on its light-path. The objective is to find a solution that minimizes the regenerator deployment costs and obeys the link capacity and optical reach constraints.

We now provide some notation for formalism. Let the undirected weighted graph $G = (N, E)$ represent a flexible optical network instance with node set $N = \{1, 2, 3, \ldots n\}$ and edge set E. Edge lengths and the total number of slots that exist on each fiber link $e \in E$ are denoted by $l(e) \geqslant 0$ and $c(e) \in \mathbb{N}$, respectively. The cost for regenerator placement in node $i \in N$ is denoted by h_i. Induced by the edge set E, we define the arc set A which contains two arcs $\bar{e} = (i, j)$ and $\underline{e} = (j, i)$ for each edge $e = \{i, j\} \in E$ with $l(\bar{e}) = l(\underline{e}) = l(e)$. We define $M = \{1, 2, \ldots, \mu\}$ as the set of modulation levels and assume that the mth component of vector $\Delta = (\Delta^1, \ldots, \Delta^\mu)$ is the threshold of regeneration-free communication (optical reach) for the modulation level m. In other words, two nodes with distance at most Δ^m can communicate without any need for signal regeneration using modulation m. We assume without loss of generality that $\Delta^m \geqslant \Delta^{\bar{m}}$, $\forall\, m < \bar{m}$, and $l(e) \leqslant \Delta^1$ for every $e \in E$ since any edge violating this condition can simply be deleted from G.

Another problem instance parameter is the set of transmission demands $D = \{1, 2, \ldots, \delta\}$. For each $d \in D$, we denote $\mathcal{S}(d)$ as the source node and $\mathcal{T}(d)$ as the destination node, and define $\psi(d)$ as the data rate demanded by d. The number of slots a demand d requires on any fiber link is a function of the modulation level chosen, say m, and the required data transfer rate of the demand $\psi(d)$. For practical purposes, it is important to note that for the same transfer rate $\psi(d)$, higher modulation levels require less bandwidth (less number of slots) but have more limited optical reach. We define $v(d, m)$ as the number of slots a path-segment of demand d occupies on each fiber optical link it traverses for a chosen modulation level m.

A directed path is a sequence of arcs $(a_1, a_2, \ldots, a_\beta)$ with $a_i = (n_{i-1}, n_i) \in A$, $\forall i = 1, \ldots, \beta$ and $n_i \in N$ for $i = 0, \ldots, \beta$. The directed path is nonsimple if it repeats nodes and simple otherwise. Our formulation depends on the notion of *path-segments*. A *path-segment* p is a directed simple path with an associated modulation level $m(p)$. Thus, by associating different modulations to the same simple path, it is possible to generate different path-segments. We denote the source and destination nodes of a path-segment p as $s(p)$ and $t(p)$, respectively. For each *path-segment* p, we denote $\bar{p}$ as the set of edges $e \in E$ such that p passes through $\bar{e}$ or $\underline{e}$, and define the indicator function $I(e, p)$ that returns one if $e \in \bar{p}$ and zero otherwise. The length of a path-segment $l(p) = \sum_{e \in \bar{p}} l(e)$ is the sum of the lengths of the edges contained in $\bar{p}$. In our formulation, we only consider path-segments with total length less than the optical reach of the associated modulation level and call such path-segments *feasible*. More formally, a path-segment p is feasible if $l(p) \leqslant \Delta^{m(p)}$. We define $\mathcal{P}$ as the set of all of those feasible path-segments.

A light-path $P = (p^1 \ldots, p^k)$ is an ordered union of path-segments $p^i, i \in 1, \ldots, k$ where $t(p^i) = s(p^{i+1})$ forall $i = 1, \ldots, k-1$. We call a light-path feasible for a demand $d \in D$ if $s(p^1) = \mathcal{S}(d), t(p^k) = \mathcal{T}(d)$, $l(p^i) \leqslant \Delta^{m(p^i)}, i \in 1, \ldots, k$, and $t(p^i)$ is a regenerator node $\forall i = 1, \ldots, k-1$. The regenerator usage cost for each transmission demand is denoted as η.

A solution for RLP-FON is allowed to use only a portion $\alpha \in (0, 1]$ of the available transmission capacity (bandwidth slots) on a link. That is, the number of *slots* available on a link $e \in E$ is given by $\lfloor c(e) \times \alpha \rfloor$.

Table 1. Outline of Notation

G:	Input graph representing the optical network
N:	Set of nodes in G; $N = \{1, \ldots, n\}$
E:	Set of edges in G
A:	Arc set induced by E
D:	Set of connection demands; $D = \{1, 2, \ldots, \delta\}$
M:	Set of modulation levels; $M = \{1, \ldots \mu\}$
$\mathscr{P}$:	Set of feasible path-segments
$l(e)$:	Length of an edge $e \in E$
$c(e)$:	Number of slots that exist on each fiber link $e \in E$
$\mathscr{S}(d)$:	Source node of a connection demand $d \in D$
$\mathscr{T}(d)$:	Destination node of a connection demand $d \in D$
$s(p)$:	Source node of a path-segment $p \in \mathscr{P}$
$t(p)$:	Destination node of a path-segment $p \in \mathscr{P}$
$m(p)$:	Modulation level used by path-segment $p \in \mathscr{P}$
$\bar{p}$:	Set of edges a path-segment $p \in \mathscr{P}$ passes through
$I(p,e)$:	Indicator function that returns one if $e \in \bar{p}$ and zero otherwise
h_i:	Cost for regenerator placement in node $i \in N$
η:	Regenerator usage cost
α:	Maximum link utilization ratio
Δ^m:	Optical reach for the modulation level $m \in M$
$v(d,m)$:	Number of slots required by demand $d \in D$ transmitted via modulation level $m \in M$

The parameter α actually represents a managerial decision. Due to the quality of service considerations (such as uninterrupted service, accommodating unexpected demands, etc.), network management does not want to use all of the existing bandwidth of a link, and smaller values of α are preferred. However, smaller values for α limit the data transfer capacity of the network and may increase the required number of regenerators. A more detailed discussion about this topic is presented in Section 4. The notation we use throughout this paper is outlined in Table 1.

The formal definition of RLP-FON is as follows.

Definition 1. *Regenerator Location Problem for Flexible Optical Networks* (*RLP-FON*): An RLP-FON instance has associated data $\langle G, D, M, l, c, h, v, \alpha, \eta \rangle$. The aim is to find the minimum cost regenerator deployment and signal routing such that for each $d \in D$ there is a feasible light-path P^d in G such that for all links $e \in E$, $\sum_{d \in D} \sum_{p \in P^d} I(e,p) v(d, m(p)) \leqslant \lfloor c(e) \times \alpha \rfloor$.

2.2. RLP-FON Path-Segment Formulation (PS)

In this subsection, we present the path-segment formulation PS for RLP-FON and provide the details of the proposed branch-and-price algorithm to solve it.

Recall that each demand d is required to follow a union of path-segments from $\mathscr{S}(d)$ to $\mathscr{T}(d)$ with a regenerator at the end of each used path-segment p for which $t(p) \neq \mathscr{T}(d)$. As such, our path-segment formulation admits a very natural representation of signal regeneration constraints. We define the following decision variables.

$$r_i = \begin{cases} 1, & \text{if node } i \in N \text{ is a regeneration point} \\ 0, & \text{otherwise,} \end{cases}$$

$$x_p^d = \begin{cases} 1, & \text{if demand } d \in D \text{ uses path-segment } p \\ 0, & \text{otherwise.} \end{cases}$$

We name $r_i, i \in N$ as the regeneration variables and $x_p^d, d \in D, p \in \mathscr{P}$ as the arc flow variables. With these decision variables, PS can be stated as follows.

$$\min \left\{ \sum_{i \in N} h_i r_i + \sum_{\substack{d \in D \\ p \in \mathscr{P} \\ t(p) \neq \mathscr{T}(d)}} \eta x_p^d \right\} \tag{1}$$

$$\text{s.t.} \quad \sum_{\substack{p \in \mathscr{P}, \\ s(p)=i}} x_p^d - \sum_{\substack{p \in \mathscr{P}, \\ t(p)=i}} x_p^d = \begin{cases} 1, & \text{if } i = \mathscr{S}(d), \\ -1, & \text{if } i = \mathscr{T}(d), \\ 0, & \text{otherwise,} \end{cases} \quad \forall\, i \in N,\, d \in D, \tag{2}$$

$$\sum_{d \in D} \sum_{\substack{p \in \mathscr{P}: \\ e \in \bar{p}}} v(d, m(p)) x_p^d \leqslant \lfloor c(e) \times \alpha \rfloor, \quad \forall\, e \in E, \tag{3}$$

$$\sum_{\substack{p \in \mathscr{P}: \\ t(p)=i}} x_p^d \leqslant r_i, \quad \forall\, d \in D,\, i \in N \backslash \mathscr{T}(d), \tag{4}$$

$$r_i \in \{0,1\}, \quad \forall\, i \in N, \tag{5}$$

$$x_p^d \in \{0,1\}, \quad \forall\, d \in D, p \in \mathscr{P}. \tag{6}$$

The objective function (1) has two components. The first one represents the fixed cost of setting up a regenerator site on a node. Since some of its constituents can be node dependent, regenerator placement cost h_i can take different values for different nodes $i \in N$. The second piece represents the cost of adding a new regenerator device to a regenerator site. Depending on the features of the practical setting, this cost can also represent the signal regeneration cost incurred at the end of a path-segment. Note that, with this flexible construction, the objective function is quite general accommodating the cost structure of a wide range of practical problems. In the next section, we present a more detailed discussion on this topic. Constraints (2) are the flow balance equations that force each demand to be carried from its source to its destination by the concatenation of feasible path-segments. Constraints (3) are the capacity constraints which ensure that the number of slots occupied is not more than the maximum allowed. Constraints (4) enforce regeneration requirements by ensuring regeneration at the end of each feasible path-segment that does not end in the destination node of the associated demand. Constraints (5)–(6) are the domain restrictions. Note that this formulation is equivalent to a flow formulation on a network where different path-segments between pairs of nodes are simply represented by parallel arcs.

Although this formulation considers a static problem where all of the connection demands are known/estimated and regenerator locations are chosen accordingly, it is flexible enough to cover a more realistic case where demands emerge in an incremental manner; that is, when there are a collection of flows already and we need to solve the problem to accommodate additional flows incurring minimum additional cost. In that case, we can simply fix $r_i = 1$ or assume $h_i = 0$ for those regenerators that are in use (already located) to enable existing light-paths. Regarding the edge capacities in the new problem, we have two alternatives. If the initial routes have to remain fixed, we would reduce the number of available bandwidth slots on the edges which have already been used. On the other hand, if we have the rerouting capability, there is no need to update edge capacities and we can solve the problem considering existing and new demands together by simply updating regenerator placement costs as stated above.

2.3. Solution Approach

In this section, we present a novel branch-and-price algorithm to solve PS. Column generation technique is employed to solve the linear relaxation of PS, say PS-LP, and obtain a lower bound for each node of the branch-and-bound tree.

2.3.1. LP Solution (Column Generation).

Pricing Problem: Let RPS be the restricted PS-LP formulation with a fraction of its columns. At every iteration, we determine whether there exists a column with negative reduced cost such that including it in the RPS might improve the objective function. If such columns are detected, we add them to the RPS and repeat the procedure until there is no column left with a negative reduced cost.

Let π_i^d represent the unrestricted dual variables associated with Constraints (2), and κ_e and γ_i^d be the nonnegative dual variables associated with constraints (3) and (4), respectively. For a path-segment p of demand d the reduced cost $\bar{c}_p^d$ for a fixed modulation level m is given as

$$\bar{c}_p^d = \begin{cases} \pi_{t(p)}^d - \pi_{s(p)}^d + \sum_{e \in \bar{p}} v(d,m)\kappa_e, & \text{if } t(p) = \mathcal{T}(d), \\ \pi_{t(p)}^d - \pi_{s(p)}^d + \sum_{e \in \bar{p}} v(d,m)\kappa_e + \gamma_{t(p)}^d + \eta, & \text{if } t(p) \neq \mathcal{T}(d). \end{cases} \tag{7}$$

Definition 2. An ordered node pair $(i,j), \in N \times N$ is called a *plausible-pair* for demand d if the potential difference

$$\pi_{(i,j)}^d = \begin{cases} \pi_j^d - \pi_i^d, & \text{if } j = \mathcal{T}(d), \\ \pi_j^d - \pi_i^d + \gamma_j^d + \eta, & \text{if } j \neq \mathcal{T}(d), \end{cases} \tag{8}$$

is negative. The set of all the *plausible-pairs* for demand d is denoted by Π^d.

To identify columns that price out, it is required to pick out plausible-pairs (i,j) for each demand $d \in D$ and check whether there exists a path p of modulation m from node i to j with length $\sum_{e \in \bar{p}} v(d,m)\kappa_e < -\pi_{(i,j)}^d$. If the signal regeneration was not necessary, such a path could be efficiently identified by solving a shortest path problem for each modulation level $m \in M$, over a graph with arc costs equal to $v(d,m)\kappa_e$ for each arc $\bar{e}, \underline{e} \in A$. However, a path-segment p is feasible only if it satisfies signal regeneration constraint $l(p) = \sum_{e \in \bar{p}} l(e) \leqslant \Delta^{m(p)}$. Thus, the pricing problem actually requires the solution of a number of *constrained shortest path problems* (*CSP*) (Garey and Johnson 1979). Given a directed graph with costs and resources associated with arcs, the CSP problem seeks a minimum cost path from a given source node to a given destination node with a side constraint on the total resource of the path. Our pricing problem can be solved exactly by solving a CSP instance from node i to node j on a directed graph (N,A) where the cost is $v(d,m)\kappa_e$ and the resource is $l(e)$ for each $\bar{e}, \underline{e} \in A$ and the resource limit is Δ^m. We denote this pricing graph as G_m^d.

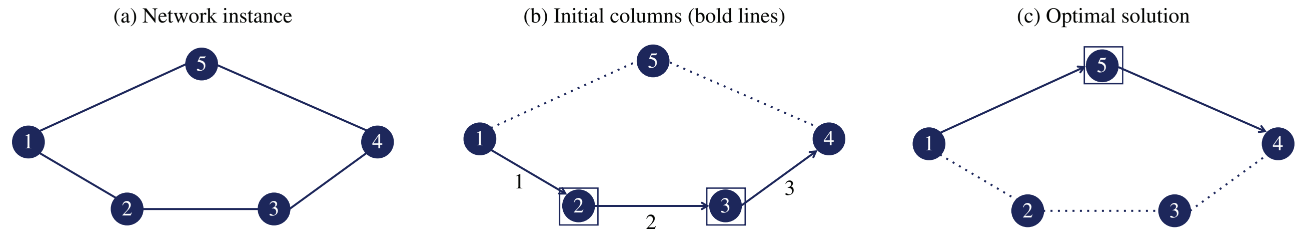
Algorithm 1 (H_k)

Input: $\langle G, D, M, P_{(i,j)}^k, \pi, \kappa, \gamma \rangle$
Output: $\langle \Omega \rangle$

begin
  Set $\Omega = \varnothing$
  for all of the $d \in D$ **do**
    for all of the $(i,j) \in \Pi^d$ **do**
      Set $m = |M|$
      Set *GoToNextPair* = *false*
      while $m > 0$ or *GoToNextPair* = *false* **do**
        Set $\sigma = 1$
        while $\sigma \leqslant k$ or *GoToNextPair* = *false* **do**
          if $\Delta^m \geqslant l(p_{(i,j)}^\sigma)$ and $\pi_{(i,j)}^d + l_m^d(p_{(i,j)}^\sigma) < 0$ **then**
            $\Omega = \Omega \cup \{p_{(i,j)}^\sigma\}$
            GoToNextPair = *true*.
          end
          $\sigma = \sigma + 1$
        end
        $m = m - 1$
      end
    end
  end
end.

Since the number of plausible-pairs is $O(n^2)$, and since CSP is NP-Hard (Garey and Johnson 1979), we propose a heuristic method (H_k) to solve the pricing problem and resort to the exact solution of CSP, which employs the solution approach proposed by Santos et al. (2007), if the heuristic method fails to produce a negative reduced cost column.

Figure 1. A Simple Example Depicting the Fact that Initial Columns Matter



For each node pair $(i, j) \in N \times N$, paths with short lengths are *good* paths in a sense that they can support higher-bit-rate modulations and thus use less network resources to transmit data. Therefore, those paths are more likely to be detected as solutions of the pricing problem. Thus, it is a fruitful idea to store some limited number, say k, of those *good path-segments* and at each pricing step check those paths first before resorting to the costly solution of CSP.

Let $P^k_{(i,j)} = \{p^1_{(i,j)}, p^2_{(i,j)}, \ldots, p^k_{(i,j)}\}$ be the set of k-shortest paths from node i to node j in G with nondecreasing order of lengths. For notational simplicity, we also define the cost of the path-segment $p \in P^k_{(i,j)}$ in the pricing graph G^d_m as $l^d_m(p) = \sum_{e \in \bar{p}} v(d, m)\kappa_e$. Initially, we store k-shortest paths for each node pair (i, j) and call Algorithm 1 (H_k) to detect negative reduced cost path-segments.

If H_k for a chosen k returns $\Omega = \varnothing$, then we continue with the exact solution methodology. When solving the pricing problem after the application of H_k there is no need to consider plausible node pairs (i, j) such that $l(p^k_{(i,j)}) > \Delta^m$. Thus, exact solution of the pricing problem requires significantly less computational effort when we first apply H_k. Note that, once we have the solution of the k-shortest path problem, which we solve just once at the very beginning, H_k requires $O(n^2|D|\mu)$ time when seeking for a negative reduced cost column among all plausible pairs, all demands $d \in D$ and modulation levels $m \in M$. So the time complexity of the heuristic solution of the pricing problem is $O(kn(|A| + n\log n) + n^2|D|\mu)$.

Determining an Initial Set of Columns: Defining variables as the path-segments instead of light-paths diverts from the widely used path-based formulations for which column generation technique has been applied very successfully for a wide range of problems (see Lübbecke and Desrosiers 2005 for a detailed survey). Path-segments as variables necessitate a more careful approach to determine the initial variable pool. In a typical column generation algorithm it is sufficient to have a feasible solution at hand to start the procedure. However, in PS-LP, it is not enough to start with an arbitrary feasible solution. Figure 1 depicts a simple problem instance where there is only one level of modulation with the maximum optical reach of 1 unit. This instance contains a single data transfer demand from 1 to 4 for which just one bandwidth slot is enough to carry the signal with the available modulation. Links contain two slots and have lengths of 1 unit. All nodes have unit regenerator placement costs and $\eta = 0$. Figure 1(b) shows an initial column pool that consists of three path-segments (numerically depicted). With these columns, one can build a feasible solution that requires two regenerators (at nodes 2 and 3). Figure 1(c) depicts the optimal solution with one regenerator at node 5. However, starting with the three path-segments given at (b) there is no way to detect negative reduced cost columns and move to a better feasible solution. Thus, PS-LP is stuck with the initial solution and cannot obtain the optimal solution from here.

We apply Algorithm 2 to obtain the set of initial variables. Note that the data transfer capacity of the network is maximum when all of the nodes have the regeneration capability and each link $e \in E$ uses the most bandwidth efficient (highest bit rate) modulation level m^* such that $l(e) \leqslant \Delta^{m^*}$. Thus, if the restricted problem with the column set Ω_0 is infeasible, PS-LP is infeasible as well. Moreover, for each (d, a) pair $d \in D$, $a \in A$, there exists a variable x^d_p in Ω_0 with $p = a$. Thus, values of all of the dual variables can be properly calculated once a solution is obtained with the variables in Ω_0. Thus, employing Algorithm 2, we can obtain the initial set of columns in $O(\mu|A|)$ time.

Algorithm 2 (Initial variable set generation)

Output: $\langle \Omega_0 \rangle$
begin
  Set $\Omega_0 = \varnothing$
  for all of the $a \in A$ **do**
    —find the highest-bit-rate modulation m^* such that $\Delta^{m^*} \geqslant l(a)$
    —build the single arc path-segment $p = a$ with modulation level $m(p) = m^*$
    —Set $\Omega_0 = \Omega_0 \cup \{x^d_p \mid d \in D\}$
  end
end.

2.3.2. IP Solution.

Branching Rules: One key step toward developing an effective branch-and-price algorithm is the identification of a branching rule which eliminates the fractional solutions but does not disrupt the special structure of the pricing problem. Keeping this in mind, we propose the following two branching rules: one for the regeneration variables r_i and one for the arc flow variables x_p^d.

Branching on Regeneration Variables: Encountering a fractional solution, we first detect fractional regeneration variables and branch on the variable $0 < r_i < 1$ where $i = \arg\min_{j \in N}\{|r_j - 0.5|\}$.

Note that in formulation PS, arc flow variables x_p^d are tied to the regeneration variables r_i by the constraints (4). Thus, branching decisions on regeneration variables affect a significant number of arc flow variables. Let r_i have a fractional value:

- *Branching-cut*-1 $r_i = 0$: In this case the set of arc flow variables $\underline{X}_i = \{x_p^d \mid d \in D, \mathcal{T}(d) \neq i \text{ and } p \in \mathcal{P}, t(p) = i\}$ are implicitly set to zero. Thus, we must make sure that in the pricing problem any path-segment $x_p^d \in \underline{X}_i$ should not appear as a negative reduced cost column. This can be easily done by setting $\gamma_i^d = \infty$ $\forall\, d \in D, \mathcal{T}(d) \neq i$. Note that with this modification, only the lengths of the arcs in the pricing graph are changed and the structure of the pricing problem is not affected.
- *Branching-cut*-2 $r_i = 1$: Implementation of this branching cut is straightforward and does not require any change in the pricing problem.

Branching on Arc Flow Variables: Our branching rule on the arc flow variables is closely related with the one proposed by Barnhart et al. (2000). For a branching rule which is based on the arc flow variables, it is very likely that branching cuts destroy the special structure of the pricing problem. One remedy is to consider original links and base the branching decisions on the usage of an arc in A by a demand $d \in D$.

We derive our branching rule by observing that if an arc flow variable has a fractional value, then there must exist a node $i \in N$ such that there are at least two variables $x_{p^1}^d > 0, x_{p^2}^d > 0$ where $s(p^1) = s(p^2) = i$. We call node i the *root node*. There are two possibilities to consider. Either fractional path segments follow the same route but stop at different regenerators or there exists a node $\bar{\imath}$ after which these path segments diverge. The first case is easy to handle since it can only occur when $x_{p^1}^d$ and $x_{p^2}^d$ use different modulations (note that if they use the same modulation, this fractional solution cannot be an extreme point) and we can find a partition M^1, M^2 of the modulation levels set such that $m(p^1) \in M^1$ and $m(p^2) \in M^2$. Then we can generate a branching by restricting the modulation levels for path segments that are utilized by demand d and originate from root node i. In one branch, we don't allow path segments to use modulation levels $m \in M^1$; in the other branch, M^2 modulations are not allowed. Such a branching would not disturb the structure of the pricing problem since we simply do not solve it for those modulation levels banned for the current branch and bound node. The second case is more complicated, and we explain our branching policy in more detail. For the distinct path-segments p^1 and p^2, starting with the *root node* and inspecting one arc at a time, we can find two different arcs a^1 and a^2 where $s(a^1) = s(a^2) = \bar{\imath}$. The node $\bar{\imath}$ is called the *divergence node*. We denote the set of arcs originating from $\bar{\imath}$ as $A(\bar{\imath})$ and let $A(\bar{\imath}, a^1)$ and $A(\bar{\imath}, a^2)$ represent a partition of $A(\bar{\imath})$ where $A(\bar{\imath}, a^1)$ contains a^1 and $A(\bar{\imath}, a^2)$ contains a^2. Let $\mathcal{P}(a)$ denote the set of path-segments containing arc $a \in A$. Now consider the following two sets of arc flow variables.

- $X^1 = \{x_p^d \mid s(p) = i, p \in \mathcal{P}(a) \text{ for some } a \in A(\bar{\imath}, a^1)\}$
- $X^2 = \{x_p^d \mid s(p) = i, p \in \mathcal{P}(a) \text{ for some } a \in A(\bar{\imath}, a^2)\}$

The main idea for the branching rule for the flow variables follows from the observation that in the optimal solution either arc flow variables in X^1 or those in X^2 are all set to zero.

- *Branching-cut*-1 $\sum_{x_p^d \in X^1} x_p^d = 0$: In this case, the set of arc flow variables in X^1 are set to zero. Let i be the root node and $\bar{\imath}$ be the divergence node. To force this constraint in the following pricing problems, we simply remove arcs $A(\bar{\imath})$ from the arc set of the constrained shortest path instances $\langle G_m^d, i, j, l_m^d, l, \Delta^m \rangle$ $\forall\, m \in M$, $d \in D$ and $j \in N$, where G_m^d is the input graph, i is the origin node, j is the destination node, l_m^d is the cost vector, and l is the resource vector. Similarly for H_k, we can simply update $l_m^d(\bar{e}) = \infty, \forall \bar{e} \in A(\bar{\imath})$ when calculating the lengths of the paths $p \in P_{(i,j)}^k$ $\forall\, j \in N$.
- *Branching-cut*-2 $\sum_{x_p^d \in X^2} x_p^d = 0$: The implementation of this branching cut is analogous to the previous one.

2.4. Heuristic Solutions

The bulk of the columns generated in the branch-and-price algorithm is actually obtained during the column generation in the root node. Thus, solving the problem with only those columns can provide a good heuristic for RLP-FON. We call this heuristic H-Root and apply it to obtain an initial feasible solution to reduce the overall size of the branch-and-bound tree. Obviously, a similar procedure can be applied at any given branch-and-bound node (other than the root node). Indeed, during our implementation phase, at the end of some definite intervals, we pause the branch-and-price algorithm and try to find an integer solution with the columns generated so far.

3. Insights to Problem Complexity

In this section, we investigate theoretical results about the complexity of RLP-FON and its special cases in an attempt to understand the challenges involved. We start by presenting the computational complexity of RLP-FON.

Theorem 1. *RLP-FON is NP-Hard.*

Proof. Chen et al. (2009) prove that the RLP is NP-Hard. The result follows from the observation that the RLP is a special case of RLP-FON where

- only a single modulation level m is considered,
- regenerator usage cost η is zero,
- between any node pair $(i,j) \in N \times N$ such that $i < j$, the set D contains a demand $d_{i,j}$ with a fixed transmission rate (i.e., we have $\sigma = v(d,m)$ for each demand $d \in D$),
- all of the links $e \in E$ have capacities $c(e) = (n \cdot (n-1)/2)\sigma$. □

Establishing the computational complexity of RLP-FON, the next question is to explore what makes this problem hard. For most network problems, properties of the input graph are important dimensions in an answer to this question. For various well-known NP-hard problems, there are polynomial time algorithms to solve them if the input graph has some special structure. However, as we show by the next theorem, this is not the case for RLP-FON, which retains its computational challenge even when we consider a tree as an input graph.

Theorem 2. *RLP-FON is NP-hard even if the input graph is a tree.*

Proof. We provide a polynomial time reduction that transforms an arbitrary 0-1 knapsack instance into an RLP-FON instance on a tree.

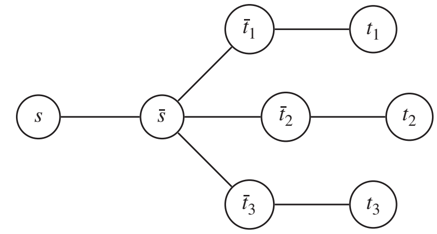
Consider a knapsack instance with n items of capacity W with z_i corresponding to the value and w_i corresponding to the weight of item $i \in \{1,\dots,n\}$ respectively. We construct an RLP-FON instance as follows.

- $N = \{s, \bar{s}, \bigcup_{i=1}^{n} t_i, \bigcup_{i=1}^{n} \bar{t}_i\}$,
- $E = \{\{s,\bar{s}\}, \bigcup_{i=1}^{n}\{\bar{s},\bar{t}_i\}, \bigcup_{i=1}^{n}\{t_i,\bar{t}_i\}\}$,
- $l(e) = 1$ and $c(e) = W \ \forall e \in E$,
- $h_s = h_{\bar{s}} = \infty$, $h_{t_i} = 0$ and $h_{\bar{t}_i} = z_i$ for $i \in \{1,2,\dots,n\}$, and $\eta = 0$,
- $M = \{1,2\}$ with $\Delta^1 = 2$ and $\Delta^2 = 3$,
- $D = \{1,\dots,n\}$ where $\mathcal{S}(i) = s, \mathcal{T}(i) = t_i$ for $i \in \{1,\dots,n\}$,
- $v(i,1) = 0$ and $v(i,2) = w_i, \forall i \in \{1,2,\dots,n\}$.

Figure 2 depicts a small example of building the input graph G for a given knapsack instance.

Note that, for the given RLP-FON instance, we have two choices for each demand $i \in D$. We can either use modulation level 2 and reach to the destination without any regeneration but occupying w_i number of slots on the edge $\{s,\bar{s}\}$ or we can use modulation level 1 and reach to the destination by visiting a regenerator at $\bar{t}_i$ and without consuming any bandwidth on the edge $\{s,\bar{s}\}$. Observing that it is always advantageous to use modulation 2 and our freedom of using this

Figure 2. Depiction of a Small Transformation Example with Three Knapsack Items 1, 2, and 3



modulation is limited with the W capacity of the bottleneck edge $\{s,\bar{s}\}$, it is easy to see the relation between the given knapsack problem and its RLP-FON transformation. Item i for $i \in \{1,\dots,n\}$ will be chosen in an optimal solution of the knapsack problem if and only if demand i in RLP-FON uses modulation level 2 in an optimal solution for RLP-FON, and the RLP-FON instance will have a solution of cost at most Z if and only if the knapsack instance has a solution with value at least $\sum_{i=1}^{n} z_i - Z$. □

The number of transmission demands is another significant dimension of the problem complexity. As we see with the following two theorems, while there is a polynomial time algorithm to solve RLP-FON when we consider a single transmission demand, RLP-FON is still a challenging problem even if we have only two transmission demands.

Theorem 3. *There is a polynomial time algorithm to solve an RLP-FON instance with $|D| = 1$.*

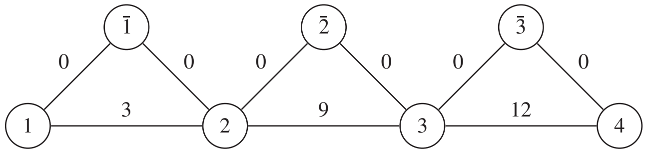
Proof. Let $\langle G, D, M, l, c, h, v, \alpha, \eta \rangle$ be an RLP-FON instance with $D = \{d\}$. For each modulation level $i \in M$, we denote its *feasible graph* $G_i = (N, E_i)$ as the subgraph of $G = (N,E)$ where $E_i = \{e \in E \mid v(d,i) \leqslant \lfloor c(e)\alpha \rfloor\}$. For each feasible graph G_i, consider the closure graph $G_i^c = (N, E_i^c)$. The notion of closure graph is introduced by Chen et al. (2009) and used in Yildiz and Karasan (2015). Namely, $\{i,j\} \in E_i^c$ if and only if the length of the shortest path from i to j in G_i is at most Δ^i. Now we define the united closure graph $G^c = (N, E^c)$ where $E^c = \bigcup_{i=1}^{\mu} E_i^c$.

Note that we can generate feasibility graphs and solve all pairs shortest path problem on these graphs to obtain their closure graphs in polynomial time. So the generation of the united closure graph can be accomplished in polynomial time. Let each edge in E^c have cost η and node $i \in N$ have cost h_i. Solving a node weighted shortest path problem on G^c from $\mathcal{S}(d)$ to $\mathcal{T}(d)$ gives the optimal solution to the RLP-FON instance. □

Theorem 4. *RLP-FON is NP-hard even if $|D| = 2$.*

Proof. We prove this theorem by showing that a set partitioning problem can be reduced to an RLP-FON instance with two transmission demands. Let

Figure 3. Depiction of a Small Transformation Example for $S = \{3, 9, 12\}$



$S = \{a_1, \ldots, a_{|S|}\}$ where $\sum_{i=1}^{|S|} a_i$ is even be an arbitrary instance of a set partitioning problem. We construct an RLP-FON instance as follows.

- $N = \{\bigcup_{i=1}^{|S|} \{i, \bar{\imath}\}, |S| + 1\}$,
- $E = \{\bigcup_{i=1}^{|S|} \{i, \bar{\imath}\}, \bigcup_{i=1}^{|S|} \{\bar{\imath}, i+1\}, \bigcup_{i=1}^{|S|} \{i, i+1\}\}$,
- $l(\{i, \bar{\imath}\}) = l(\{\bar{\imath}, i+1\} = 0$ for $i \in \{1, \ldots, |S|\}$,
- $l(\{i, i+1\}) = a_i$ for $i \in \{1, \ldots, |S|\}$,
- $c(e) = 1$ for $e \in E$,
- $h_i = 1$ for $i \in N$, $\eta = 0$, $\alpha = 1$,
- $D = \{1, 2\}$, $\mathcal{S}(1) = \mathcal{S}(2) = 1$ and $\mathcal{T}(1) = \mathcal{T}(2) = |S| + 1$,
- $M = \{1\}$ and $\Delta^1 = \sum_{i=1}^{|S|} a_i / 2$,
- $v(1, 1) = v(2, 1) = 1$.

In Figure 3, we present a small example of transforming a set partitioning problem into an RLP-FON instance with the desired properties.

Now observe that any solution to the given RLP-FON instance will construct two edge disjoint light-paths, say π_1 and π_2, for each connection demand in D. Let $S_1 = \{a_i \in S \colon (i, \bar{\imath}) \in \pi_1\}$ and $S_2 = S \backslash S_1$. Then the given set partitioning instance has a solution if and only if RLP-FON has a zero cost solution. □

3.1. Uncapacitated Edges

Obviously, having capacity limits for the edges increases the difficulty of RLP-FON. Now we investigate the case in which these constraints are relaxed. In many studies motivated by different practical applications, uncapacitated edges are considered for regenerator/relay/refueling station placement (Yetginer and Karasan 2003, Kuby and Lim 2005, Cabral et al. 2007, Pachnicke et al. 2008, Chen et al. 2009, Üster and Kewcharoenwong 2011, Flammini et al. 2011, Kewcharoenwong and Üster 2014, Yildiz and Karasan 2015, Chen et al. 2015, Yildiz et al. 2016). We denote this problem as RLP-FON-U. Note that, once the edge capacities are relaxed, we can find the optimal routing by using only the lowest modulation level that can transmit signals furthest since occupying more bandwidth is not an issue when the capacity limits are not considered for the edges. However, such a simplification does not make the problem an easy one. Indeed, RLP is a special case of RLP-FON-U in which all nodes are required to communicate with each other and the regenerator usage cost η is assumed to be zero. Thus, RLP-FON-U is also NP-hard.

Depending on the application area, one of the two costs—regenerator placement or usage costs—may dominate the overall cost. Now we first consider the case where the regenerator placement costs are negligible and show that there exists a polynomial time algorithm to solve this special case.

Theorem 5. *There exists a polynomial time algorithm to solve an instance $\langle G, D, l, h, \eta \rangle$ of RLP-FON-U where $h_i = 0$ for each $i \in N$.*

Proof. By solving an all pairs shortest path problem on G and considering the reach limit Δ^1, we can easily generate a closure graph with edge lengths all equal to regeneration cost η. Then solving the RLP-FON-U instance entails finding the shortest path from $\mathcal{S}(d)$ to $\mathcal{T}(d)$ on this closure graph for each demand $d \in D$. □

The second case we investigate is the one for which the regenerator placement costs dominate the total cost. Unlike the previous case, this time RLP-FON-U remains an NP-Hard problem. So we consider special network topologies (i.e., path and ring networks) for which we can present interesting characterizations for the optimal solutions and attain polynomial time algorithms.

Considering a special case of RLP-FON-U by assuming unit costs for the regenerator placement, unit edge lengths, and fixed routes for OD pairs, Flammini et al. (2011) show that there exist polynomial time algorithms to solve path (line) and ring network topologies. Now we show that these results can be extended even when unit cost and fixed route assumptions are relaxed.

Although path networks are seldom used in real-world applications, investigating properties of the optimal solutions for these networks can be quite useful in developing solution algorithms for general networks. Consider an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$ where the input graph $G = (N, E)$ is a path (line). Assume without loss of generality that the nodes are labelled from 1 to n such that $\{i, j\} \in E$ if and only if $|i - j| = 1$. For $i \leqslant j$, let $[i, j] = \{k \in N \mid k \leqslant j$ and $i \leqslant k\}$ be the set of nodes lying in the interval from i through j in this ordering. Since G is undirected, without loss of generality, we may assume that $\mathcal{S}(d) < \mathcal{T}(d)$ with respect to this ordering for each $d \in D$.

Lemma 1. *Consider an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$ where $\eta = 0$ and $D = \{1, 2\}$. If $[\mathcal{S}(2), \mathcal{T}(2)] \subset [\mathcal{S}(1), \mathcal{T}(1)]$, then solving the problem for only $D = \{1\}$ provides the optimal solution for $D = \{1, 2\}$.*

Proof. It is clear that a subset of the regenerator locations enabling a feasible light-path from $\mathcal{S}(1)$ to $\mathcal{T}(1)$ with Δ^1 reach limitation will enable a feasible light-path from $\mathcal{S}(2)$ to $\mathcal{T}(2)$ as well. □

Lemma 2. *Consider an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$ where $\eta = 0$ and $D = \{1, 2\}$. If $[\mathcal{S}(1), \mathcal{T}(1)] \cap$*

$[\mathcal{S}(2), \mathcal{T}(2)] = \emptyset$, *then it is possible to attain an optimal solution by solving the single demand subproblems individually.*

Proof. Let R_1 and R_2 be the regenerator locations when solving the given RLP-FON-U instance with $D = \{1\}$ and $D = \{2\}$, respectively. It is clear that $R_1 \cup R_2$ will be an optimal solution for $D = \{1, 2\}$. □

Lemma 3. *Consider an RLP-FON-U instance* $\langle G, D, l, h, \eta\rangle$ *where* $\eta = 0$ *and* $D = \{1, 2\}$. *Assume* $[\mathcal{S}(1), \mathcal{T}(1)] \cap [\mathcal{S}(2), \mathcal{T}(2)] \neq \emptyset$, $[\mathcal{S}(1), \mathcal{T}(1)] \not\subset [\mathcal{S}(2), \mathcal{T}(2)]$, $[\mathcal{S}(2), \mathcal{T}(2)] \not\subset [\mathcal{S}(1), \mathcal{T}(1)]$, *and* $\mathcal{S}(1) < \mathcal{S}(2)$. *Then one of the following must hold.*

- Solving the single demand subproblems individually will provide an optimal solution for the original problem,
- Solving the single demand subproblem from $\mathcal{S}(1)$ to $\mathcal{T}(2)$ will provide an optimal solution for the original problem.

Proof. Let R be an optimal solution of the RLP-FON-U instance. There are two cases to consider.

- *Case*-1: There are two feasible light-paths connecting $\mathcal{S}(1)$ to $\mathcal{T}(1)$ and $\mathcal{S}(2)$ to $\mathcal{T}(2)$ visiting nonintersecting sets of regenerators. In this case, it is clear that the first claim holds.
- *Case*-2: Any two light-paths connecting $\mathcal{S}(1)$ to $\mathcal{T}(1)$ and $\mathcal{S}(2)$ to $\mathcal{T}(2)$ in an optimal solution share at least one regenerator. Let r be such a shared regenerator. It is clear that nodes $\mathcal{S}(1), \mathcal{S}(2), \mathcal{T}(1)$ and $\mathcal{T}(2)$ all have feasible light-paths to connect to r. Let π_1 and π_2 be the light-paths through which nodes $\mathcal{S}(1)$ and $\mathcal{T}(2)$ connect to r. Then the light-path $\pi = (\pi_1, \pi_2)$ is a feasible one that connects $\mathcal{S}(1)$ to $\mathcal{T}(2)$ as desired. By Lemma 1, the regenerator locations on π provide feasible light-paths for both demands. □

Theorem 6. *There is a polynomial time algorithm to solve an RLP-FON-U instance* $\langle G, D, l, h, \eta\rangle$, *if* $\eta = 0$ *and the input graph G is a path.*

Proof. Without loss of generality, assume that the demand set cannot be partitioned into nonoverlapping intervals; otherwise, one can solve the problem by solving the disjoint intervals independently as suggested by Lemma 2. In a similar fashion, assume that $[\mathcal{S}(d), \mathcal{T}(d)] \not\subset [\mathcal{S}(d'), \mathcal{T}(d')]$ for distinct $d, d' \in D$ since otherwise demand d can be ignored without loss of generality using Lemma 1. Assume the demands are ordered such that $\mathcal{S}(1) < \mathcal{S}(2) < \cdots < \mathcal{S}(\delta)$.

Let $Z(i), i = 1, 2, \ldots, \delta$ be the optimal solution value of the RLP-FON-U instance $\langle G_i, D_i, l, h, \eta\rangle$ where $G_i = [\mathcal{S}(\delta - i + 1), \mathcal{T}(\delta)]$ and $D_i = D \setminus \bigcup_{j=1}^{\delta-i}\{j\}$, i.e., $Z(i)$ is the optimal solution of the problem considering only the last i demands in D. In particular, $Z(\delta)$ is the optimal solution value for $\langle G, D, l, h, \eta\rangle$. We also define $\bar{Z}(i)$ as the optimal solution value of the RLP-FON-U instance considering the single OD pair $(\mathcal{S}(1), \mathcal{T}(i))$. Finally, let $\mathcal{T}(i^*) \in \{\mathcal{T}(1), \mathcal{T}(2), \ldots, \mathcal{T}(n)\}$ be the largest index node that $\mathcal{S}(1)$ would have a feasible light-path in an optimal solution of $\langle G, D, l, h, \eta\rangle$. Now observe that for $\langle G, D, l, h, \eta\rangle$, adding an artificial demand $d^* = (\mathcal{S}(1), \mathcal{T}(i^*))$ to D does not change the optimal solution and at the presence of d^* we can disregard demands $i \leqslant i^*$ since they are all dominated by it. Moreover, the fact that $\mathcal{S}(1)$ cannot reach nodes $\mathcal{T}(i), i > i^*$ in an optimal solution implies that none of the demands $i > i^*$ uses a regenerator that could be reached by $\mathcal{S}(1)$ via a feasible light-path since otherwise $\mathcal{S}(1)$ could reach $\mathcal{T}(i)$ first reaching the shared regenerator. So we have

$$Z(\delta) = \begin{cases} \bar{Z}(i^*) + Z(\delta - i^*), & \text{if } i^* < \delta, \\ \bar{Z}(\delta), & \text{if } i^* = \delta. \end{cases} \tag{9}$$

Since we have only δ possible values for i^*, and $\bar{Z}(1), Z(1)$ can both be calculated in polynomial time by Theorem 3, a dynamic programming algorithm can find $Z(\delta)$ in polynomial time by recursively calculating $Z(i)$ for each $i < \delta$. □

In telecommunications, ring networks are pervasive as cost-effective and easy-to-implement solutions to protect network traffic against edge and node failures (Vachani et al. 1996). As minimal cycles, they have interesting properties from a theoretical perspective as well. So we pay a special attention to these networks and present two important results regarding the RLP-FON-U.

Let $G = (N, E)$ represent a ring. For a pair of nodes a and b on this ring, let the interval $[a, b]$ depict the path starting from a and traversing the ring in a clockwise manner to reach node b. Let $\langle G, D, l, h, \eta\rangle$ be an RLP-FON-U instance where $\eta = 0$, the input graph G is a ring, and R is the set of regenerator locations in an optimal solution. Consider the graph $T = (R, \bar{E})$ where $\bar{E} = \{\{i, j\}$: i and j are two consecutive nodes of R on G with clockwise distance at most $\Delta^1\}$.

Proposition 1. *The graph T is a forest.*

Proof. Assume to the contrary that $\bar{E}$ includes a cycle. Let $r \in R$ be a regenerator node. Let r_1 and r_2 be the neighbors of r in the clockwise and counter-clockwise directions on the ring. Any node i in N which communicates through r will communicate through either r_1 or r_2; thus, removal of r will not spoil feasibility which contradicts the optimality of R. □

Theorem 7. *There is a polynomial time algorithm to solve an RLP-FON-U instance* $\langle G, D, l, h, \eta\rangle$ *when the input graph G is a ring and* η *is zero.*

Proof. For the sake of simplicity, and without loss of generality, we assume that there are at least four regenerator locations in any optimal solution. Note that if this is not the case, one can find an optimal solution by a simple enumeration since the feasibility of a given regenerator set can be checked in polynomial time. We also assume that there is no OD pair in D with shortest path distance less than or equal to Δ^1 since it could simply be omitted from D without loss of generality. Let Y^* be the value of the optimal solution for the problem instance $\langle G, D, l, h, \eta\rangle$ and $\mathcal{R}$ the collection of sets of feasible regenerator locations for this instance.

Let $i, j \in N$ be such that the length of the interval $[i, j]$ denoted as $l_{[i,j]}$ is larger than Δ^1. We name nodes i and j as detachment and attachment nodes, respectively. Considering the demand set $D_{[i,j]} = \{(i, j)\}$ with the single OD pair (i, j) and the path graph $G_{[i,j]}$ derived from G by removing all of the edges on $[i, j]$, we obtain the reduced problem instance $\langle G_{[i,j]}, D_{[i,j]}, l, h, \eta\rangle$ for which we denote the optimal regenerator locations as $R_{[i,j]}$. We define

$$\bar{Y}_{[i,j]} = \begin{cases} \sum_{i\in R_{[i,j]}} h_i, & \text{if } R_{[i,j]} \in \mathcal{R}, \\ \infty & \text{otherwise.} \end{cases} \tag{10}$$

Note that if T is a tree, then $Y^* = \min_{i,j\in N} \bar{Y}_{[i,j]}$.

If T is a forest with more than one disconnected trees, then there exist two sets of detachment–attachment nodes $(i, j), (l, m)$ that we can visit i, j, l, m when traversing G in the clockwise direction. Let $\langle G_{[i,j]}, D^1_{[i,j][l,m]}, l, h, \eta\rangle$ and $\langle G_{[i,j]}, D^2_{[i,j][l,m]}, l, h, \eta\rangle$ be two RLP-FON-U instances where the demand set $D^k_{[i,j][l,m]}$, $k = 1, 2$ initially set to D is updated as follows. For an OD pair $(s, t), s \in [i, j]$, if (s, t) can reach each other using i, j, l or m as a regenerator, we remove it from $D^k_{[i,j][l,m]}$, $k = 1, 2$. If this is not the case, we proceed as follows.

- If s can reach only one of the regenerators, say i, we replace (s, t) with (i, t) in $D^k_{[i,j][l,m]}, k = 1, 2$
- If s can reach both i and j,
  - —if $t \in [m, i]$, we replace the OD pair (s, t) with (i, t) in $D^k_{[i,j][l,m]}, k = 1, 2$ (similarly we replace it with (j, t) if $t \in [j, l]$)
  - —if $t \notin [m, i]$ and:
    - ∗ t can reach only one of the regenerators, say m, we replace (s, t) with (i, m) in $D^k_{[i,j][l,m]}, k = 1, 2$
    - ∗ t can reach both regenerators l and m, we replace (s, t) with (i, m) and (j, l) in $D^1_{[i,j][l,m]}$ and $D^2_{[i,j][l,m]}$, respectively.

Now let $R^k_{[i,j][l,m]}$ be the optimal regenerator locations for the problem instances $\langle G_{[i,j]}, D^k_{[i,j][l,m]}, l, h, \eta\rangle, k = 1, 2$, respectively. We define

$$\bar{Y}^k_{[i,j][l,m]} = \begin{cases} \sum_{i\in R^k_{[i,j][l,m]}} h_i, & \text{if } R^k_{[i,j][l,m]} \in \mathcal{R}, \\ \infty & \text{otherwise.} \end{cases} \tag{11}$$

Let $\bar{Y}_{[i,j][l,m]} = \min_{k=1,2} \bar{Y}^k_{[i,j][l,m]}$. Then it is clear that if T is a forest with more than one tree, we have $Y^* = \min\{\bar{Y}_{[i,j][l,m]} \mid i, j, l, m \in N, l_{[i,j]} > \Delta^1 \text{ and } l_{[l,m]} > \Delta^1\}$.

By Proposition 1, it must be the case that T is a single tree or a forest with more than one disconnected trees. We have shown that for each case, we can enumerate a polynomially bounded number of RLP-FON-U instances on path networks, $O(|N|^2)$ for the single tree case and $O(|N|^4)$ for the forest case, and find the optimal solution. Hence, the result follows by Theorem 6. □

3.2. Computational Performance of the Branch-and-Price Algorithm for Special Problem Instances

In this subsection, we investigate the computational performance of our branch-and-price algorithm on some special problem instances. In the first part, we present some problem instances for which we can put a theoretical performance guarantee for our pricing algorithm; in the second part, we propose a useful optimality cut to improve the PS formulation when regenerator usage costs are assumed to be negligible.

3.2.1. Ring Networks. In many practical settings, ring networks are quite common in telecommunications. For these networks our H_k heuristic can solve the pricing problem exactly when $k = 2$. This is simply due to the fact that in these networks, there could be at most two different simple paths that connect any two nodes. Since the computational complexity of our heuristic solution approach is $O(kn(|A| + n\log n) + n^2|D|\mu)$, we can solve the pricing problem in polynomial time. Obviously, for any network where the number of feasible path-segments between any two nodes is bounded by some positive integer K (such as path and tree networks), H_K can solve the pricing problem exactly in polynomial time. Moreover, since we solve the K-shortest path problem just once in the beginning, such a computational efficiency can boost the performance of the branch-and-price algorithm significantly.

3.2.2. Networks with Equal Edge Lengths. Optical or electrical signals do not lose their quality just traveling long distances, they also deteriorate when they pass through a switch, router, or any other network device represented as a node. So especially when the distances between the network nodes are not large, the main concern becomes the number of nodes a signal visits instead of the total distance it travels. Considering this situation, there is a wide stream of RLP literature which considers the number of hops as the reach constraint instead of the distance travelled (Ramamurthy et al. 1999, Huang et al. 2005, Cardillo et al. 2006, Zsigmond et al. 2007, He et al. 2007, Pachnicke et al. 2008, Manousakis et al. 2009). Note that considering such hop constraints is equivalent to having equal edge lengths in the input graph and using the reach limit as usual. For this case, solving the pricing problem can be accomplished in polynomial time by iterating the Bellman–Ford shortest path algorithm on the pricing graph as many times as the allowed number of hops.

3.2.3. Strengthening the PS Formulation. In this part, we present a cut to tighten the LP lower bound and improve the strength of the branching cuts specifically for r variables when the regenerator usage cost η is assumed to be zero.

Definition 3. A node $i \in N$ is called an internal node of *path-segment* p if it is visited by p and it is neither the source nor the destination node of p. The set of path-segments that contain a node i as an internal node is denoted by $\mathscr{P}(i)$.

Proposition 2. *Let (x, r) be a feasible solution of an RLP-FON instance. If there exists a node $i \in N$ for which $r_i = 1$ and i is an internal node of a path-segment p such that $x_p^d = 1$ for some $d \in D$, then there exists an alternative feasible solution $(\bar{x}, r)$ which satisfies the following conditions.*

- i is not an internal node of any path-segment p that satisfies $\bar{x}_p^d = 1$ for some $d \in D$,
- for each arc $\bar{e} \in A$ the number of slots occupied by the solution $(\bar{x}, r)$ is less than or equal to that of (x, r).

Proof. Let (x, r) be a feasible solution of an RLP-FON instance and assume $r_i = 1$ and i is an internal node of a path-segment $p = (p_1, p_2)$ where $t(p_1) = s(p_2) = i$. Let $\bar{D} = \{d \in D \mid x_p^d = 1\}$. Assume $\bar{D}$ is not empty.

Since $l(p) > l(p^1)$ and $l(p) > l(p^2)$, we can choose $m(p^1) \geqslant m(p)$ and $m(p^2) \geqslant m(p)$. Now we modify x by setting $x_p^d = 0$ and $x_{p^1}^d = x_{p^2}^d = 1\ \forall\, d \in \bar{D}$ and obtain the vector $\bar{x}$. By our assumption, we have $r_i = 1$ and $r_{t(p)} = 1$. Thus, replacing x with $\bar{x}$ does not necessitate a change in the number and location of regenerators. The same will be true if $t(p) = \mathscr{T}(d)$ holds as well. Moreover, $m(p^1) \geqslant m(p)$ and $m(p^2) \geqslant m(p)$ implies that p^1 and p^2 utilize optical bandwidth more efficiently and the amount of bandwidth slots used by $(\bar{x}, r)$ is less than or equal to the one used by (x, r). Since i was arbitrary and this procedure can be repeated as many times as needed, the result follows. □

By Proposition 2, the following is an optimality cut for PS.

$$\sum_{\substack{d \in D,\\ p \in \mathscr{P}(i)}} x_p^d \leqslant K(1 - r_i), \quad \forall\, i \in N, \tag{12}$$

where K is a large number.

The proposed cut forces that if a node $i \in N$ is chosen as a regeneration node (i.e., $r_i = 1$), none of the path-segments utilized by a positive x variable should contain i as an internal node. The modified formulation containing (12) is denoted as $\overline{PS}$. Note that choosing $K = \lfloor c(e^*) \times \alpha \rfloor$, where e^* is the highest capacity edge, is sufficient to assure the validity of the cut, since each path-segment x_p^d occupies at least 1 bandwidth slot.

In a branch-and-price framework, adding cuts to the model requires special attention since maintaining the structure of the pricing problem is crucial to retaining tractability of the solution approach. In our case, we can easily modify the reduced cost calculations and preserve the special structure of the pricing problem as follows.

Let θ_i, $i \in N$ be the dual variables associated with the constraints (12) in $\overline{PS}$ and p^o denote the set of internal nodes of a path-segment p. With the following modifications, solution of the pricing problem for $\overline{PS}$ follows exactly the same steps explained in Subsection 2.3.1.

- The reduced cost calculation (7) is changed as:

$$\bar{c}_p^d = \begin{cases} \pi_{t(p)}^d - \pi_{s(p)}^d + \sum_{e \in \bar{p}} v(d, m)\kappa_e + \sum_{i \in p^o} \theta_i, & \\ \qquad \text{if } t(p) = \mathscr{T}(d), & \\ \pi_{t(p)}^d - \pi_{s(p)}^d + \sum_{e \in \bar{p}} v(d, m)\kappa_e + \sum_{i \in p^o} \theta_i + \gamma_{t(p)}^d, & \\ \qquad \text{if } t(p) \neq \mathscr{T}(d). & \end{cases} \tag{13}$$

- The length function for the pricing graph $l_m^d(e)$ is modified as

$$l_m^d(\bar{e}) = \begin{cases} v(d, m)\kappa_e + \theta_i, & \text{if } t(\bar{e}) \in p^o, \\ v(d, m)\kappa_e, & \text{o.w.} \end{cases} \tag{14}$$

The main function of the cut (12) is not to raise the LP bound in the root node; it helps to speed up the solution procedure by reinforcing the strength of the branching cuts that remove the fractional solutions for the regeneration variables r. This is because when a variable r_i is set to its upper bound one, variables x_d^p, $\forall\, d \in D$ and $p \in \mathscr{P}(i)$ are all forced to zero with the presence of the constraints (12). As such, we need to modify our branching rule for the Branching-cut-2 as follows.

- *Branching-cut*-2 $r_i = 1$: In this case, the set of arc flow variables $\bar{X}_i = \{x_p^d \mid d \in D, p \in \mathscr{P} \text{ and } i \in p^o\}$ are implicitly set to zero. To make sure that any path-segment $x_p^d \in \bar{X}_i$ would not appear as a solution of the pricing problem, we can simply remove the node i from all of the pricing graphs where i is neither the source nor the sink node. Similarly for H_k, we can update $l_m^d(\bar{e}) = \infty$, $\forall\, \bar{e} \in \{a \in A \mid s(a) = i, \mathscr{T}(d) \neq i \text{ and } \mathscr{S}(d) \neq i\}$ and implement the algorithm without any change.

4. Numerical Experiments

Extensive numerical experiments are conducted to both test the performance of the proposed solution methodology and derive insights from the instances closely representing the real-world problems. We implemented the branch-and-price algorithm using Java under Linux and CPLEX 12.4. All experiments are done on an AMD Opteron Processor 6282 SE machine with 2 GB RAM.

Table 2. Topological Parameters

			Node degree			Edge length (km)		
Network	#nodes	#edges	Min	Max	Mean	Min	Max	Mean
NSF	14	21	2	4	3	312	3,408	1,299.1
COST-266	28	41	2	5	2.9	218	1,500	625.4

Table 3. Modulation Level Parameters

Modulation format (m)	Δ^m (km)	DTR (Gbps) 10	40	100	400
BPSK (1)	2,880	2	4	8	32
QPSK (2)	1,080	2	2	4	16
8-QAM (3)	630	2	2	4	12
16-QAM (4)	270	2	2	2	8

4.1. Network and Traffic

For our experiments, we studied two well-known network topologies from the literature: NSF-US network (Figure 4(a)) and COST-266 Pan European network (Figure 4(b)) (Hulsermann et al. 2004). Table 2 presents the topological parameters of these networks. In this table, for each optical network, we denote the minimum (min), maximum (max), and average (mean) values for the node degrees and physical edge lengths in kilometers.

For both network topologies, we studied problem instances with 75, 100, 125, and 150 number of transmission demands. For each connection, the demanded data transfer rate (DTR) is assumed to be a uniform random variable that attains values of 10, 40, 100, and 400 Gbps. For each demand, we randomly choose an origin–destination (O-D) pair among all possible node pairings. For the O-D pair selection, we investigate two cases: *uniform distribution* (U) and *traffic density distribution* (TD). In the first case, all pairs have equal probability, whereas in the second case, the probability of a pair is assumed to be proportional to its IP traffic volume as reported in Hulsermann et al. (2004).

Abiding by the general approach in the RLP literature (Kim and Seo 2001, Yetginer and Karasan 2003, Pachnicke et al. 2008, Chen et al. 2009, Kewcharoenwong and Üster 2014, Duarte et al. 2014, Yildiz and Karasan 2015, Chen et al. 2015), we assumed $\eta = 0$ and considered unit cost for the regenerator placement. There are mainly two practical reasons for such an approach. The first one is the high set up and maintenance costs of regeneration sites for OEO regenerators (Yang and Ramamurthy 2005, Wang et al. 2015). Thus, placement and operation of such a regenerator site is much more costly when compared with the addition of an extra regenerator slot in an existing site. Secondly, once a regenerator site is deployed on a node, it is very costly to relocate it. Thus, as a strategic level problem, RLP is more concerned about the locations of the regenerator sites rather than the number of regenerator devices in each site. Moreover, as we have shown in the previous section, the problem gets harder under the cost structure when the regenerator placement costs are dominant.

4.2. Fiber and Modulation Level Parameters

Cables are assumed to be nonzero dispersion-shifted fiber (NZDSF) and four modulation formats are considered: BPSK, QPSK, 8-QAM, and 16-QAM. The number of frequency slots per fiber is 360 (Klinkowski 2012), and Table 3 shows the optical reaches (Δ) (Bosco et al. 2011) and the number of slots required by each modulation format (Klinkowski 2012).

We generated problem instances with six different α values from the set $C = \{\alpha_{\min}, 0.2, 0.4, 0.6, 0.8, 1\}$ where the value $\alpha_{\min}$ represents the smallest α value for which the problem is feasible with the given set of parameters. Since the problem has no solution for $\alpha < \alpha_{\min}$, for each problem setting, we studied $\alpha \in C$ such that $\alpha \geqslant \alpha_{\min}$.

There are two main motivations to generate problem instances with the minimum bandwidth allocation (i.e., $\alpha = \alpha_{\min}$). From the algorithmic point of view, as our numerical results will attest, problem instances with the limited arc capacities are harder to solve, and those hard problem instances are required for a comprehensive performance test of algorithmic efficiency. On the other hand, from the managerial perspective, $\alpha_{\min}$ constitutes an upper bound on the spectrum efficiency in a network. Thus, it is interesting to solve problem instances with the minimum α values to see the trade-off between the number of regenerators and bandwidth utilization efficiency. Indeed, finding the $\alpha_{\min}$ value for each problem setting is an optimization problem in its own right. Thus, we solve the following MIP for finding the minimum α value for which the problem stays feasible when deploying regenerators at each node of the network.

$$\min \ \alpha_{\min} \tag{15}$$

$$\text{s.t.} \ \sum_{\substack{\bar{e}\in A:\\ s(\bar{e})=i}} x^d_{\bar{e}} - \sum_{\substack{\bar{e}\in A:\\ t(\bar{e})=i}} x^d_{\bar{e}} = \begin{cases} 1, & \text{if } i=\mathcal{S}(d)\\ -1, & \text{if } i=\mathcal{T}(d)\\ 0, & \text{otherwise,}\end{cases} \quad \forall i\in N, d\in D, \tag{16}$$

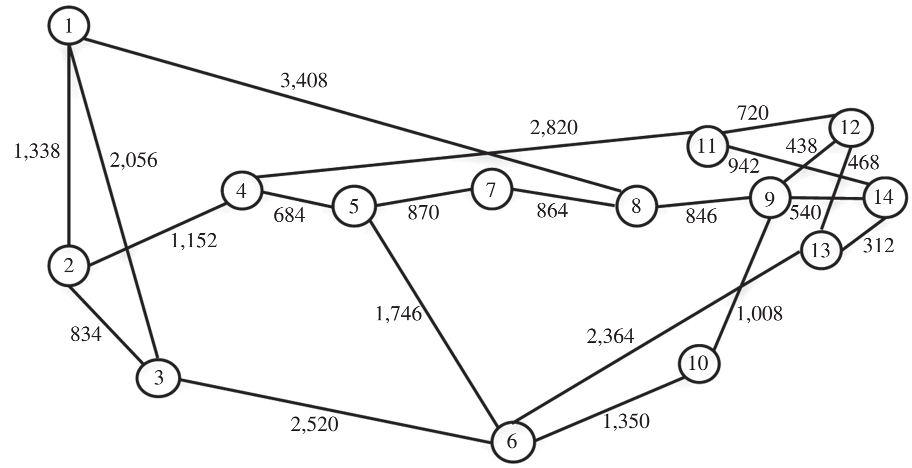
$$\sum_{d\in D} v(d, m^*_{\bar{e}})(x^d_{\bar{e}} + x^d_{\underline{e}}) \leqslant c(e)\times\alpha_{\min}, \quad \forall e\in E, \tag{17}$$

$$x^d_{\bar{e}} \in \{0,1\}, \quad \forall d\in D, \bar{e}\in A. \tag{18}$$

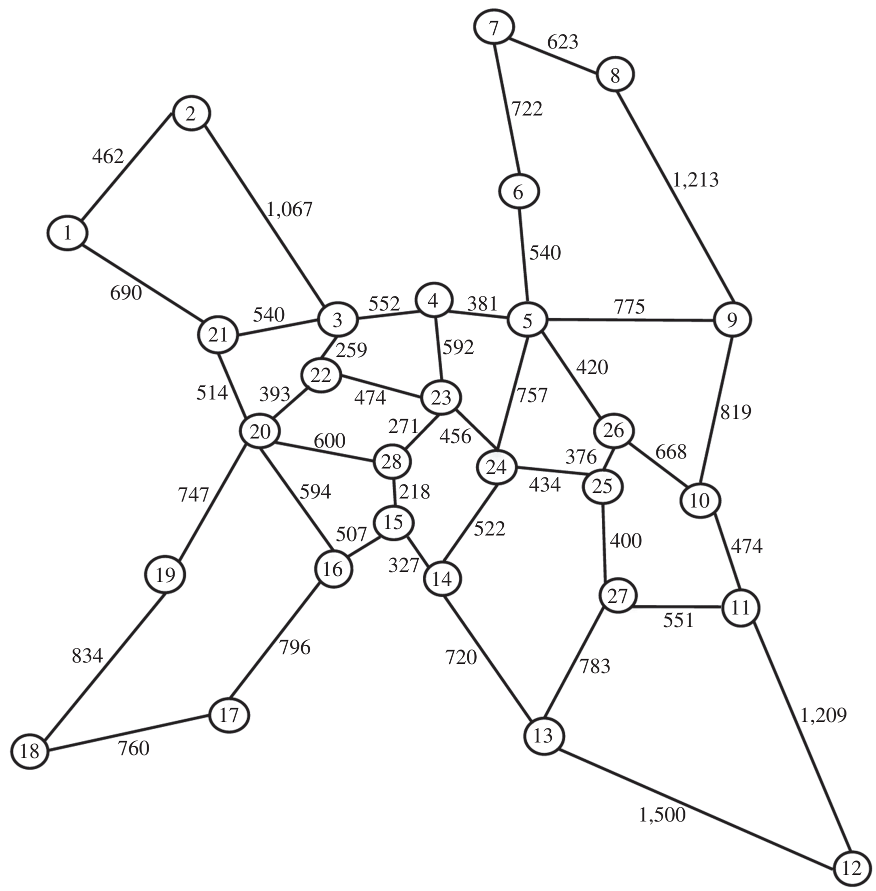
The above formulation assumes that each node in the network has the regeneration capacity. The objective is to minimize the $\alpha_{\min}$ value for the given problem instance. Constraints (16) are the flow balance equations that force each demand to be carried from its source to its destination. The decision variable $x^d_{\bar{e}}$ attains the value one if the route for the demand $d \in D$ includes the arc

Figure 4. Network Topologies; Link Lengths in km

(a) NSF US network



(b) COST-266 Pan European network



$\bar{e} \in A$ and zero otherwise. Constraints (17) are the capacity constraints which ensure that the total number of slots occupied is not more than the maximum allowed. For each arc $\bar{e} \in A$, the best modulation $m^*_{\bar{e}}$ is defined as the highest modulation level which has an optical reach larger than or equal to the length of the arc. We have $m^*_{\bar{e}} = m^*_{\underline{e}}$ since $l(\bar{e}) = l(\underline{e}) = l(e)$. Also note that $m^*_{\bar{e}}$ is indeed the most spectrum-efficient feasible modulation since higher modulation levels can transmit more data with less number of bandwidth slots. Constraints (18) are the variable restrictions.

4.3. Implementation Details

Before presenting the results of the numerical experiments, we briefly state the implementation details of the proposed algorithm.

Table 4. Results for COST-266 Network with Uniform Demand Distribution

$\|D\|$	*Seed*	α	*R.LP*	*HS*	*H.RT*	*NR*	*NR.LB*	*RT*	*#BB*	*#Col.G.*	*% SW.*	*Reg. loc.*
75	1	0.23	3.00	8	37	8	7	**3,600**	410	21,765	53.3	5 6 9 14 15 20 23 24
		0.40	1.50	2	14	2	2	19	0	9,819	21.3	4 14
		0.60	1.50	2	4	2	2	7	0	4,102	24.0	5 17
		0.80	1.50	2	3	2	2	6	0	4,242	26.7	5 17
		1.00	1.50	5	4	2	2	12	6	5,253	21.3	4 27
	2	0.27	2.52	5	38	5	5	223	29	18,845	49.3	5 6 20 23 25
		0.40	1.50	2	35	2	2	43	0	14,167	33.3	5 23
		0.60	1.50	2	6	2	2	9	0	4,674	40.0	4 5
		0.80	1.50	2	4	2	2	7	0	4,385	52.0	4 5
		1.00	1.50	2	3	2	2	6	0	4,070	21.3	5 17
	3	0.13	2.62	6	33	6	6	353	73	18,049	49.3	3 5 14 15 20 24
		0.20	1.50	3	21	2	2	168	75	15,224	21.3	3 24
		0.40	1.50	2	3	2	2	6	0	3,846	21.3	10 20
		0.60	1.50	4	4	2	2	12	7	6,028	10.7	3 13
		0.80	1.50	4	4	2	2	12	7	5,983	32.0	4 5
		1.00	1.50	2	1	2	2	5	0	5,334	4.0	9 16
	4	0.16	6.04	12	52	12	11	**3,600**	3,777	9,754	57.3	3 4 5 6 10 13 14 15 16 20 24 27
		0.20	2.70	6	48	6	6	868	113	22,633	52.0	5 14 16 22 24 26
		0.40	1.50	2	5	2	2	8	0	5,265	41.3	4 5
		0.60	1.50	2	3	2	2	6	0	4,617	8.0	4 13
		0.80	1.50	2	2	2	2	5	0	4,027	37.3	5 14
		1.00	1.50	2	2	2	2	4	0	4,363	1.3	9 14
	5	0.29	1.50	2	23	2	2	34	0	14,597	20.0	23 24
		0.40	1.50	2	10	2	2	13	0	6,153	9.3	14 26
		0.60	1.50	2	3	2	2	6	0	4,857	6.7	9 20
		0.80	1.50	4	4	2	2	13	6	6,258	37.3	23 28
		1.00	1.50	2	4	2	2	7	0	4,341	30.7	5 17
100	1	0.26	5.73	11	204	10	10	414	75	14,752	53.0	3 4 5 6 9 14 15 20 23 24
		0.40	1.50	3	55	3	2	168	10	22,279	32.0	5 22 24
		0.60	1.50	2	4	2	2	10	0	6,127	16.0	13 23
		0.80	1.50	3	6	2	2	30	14	7,591	15.0	10 14
		1.00	1.50	2	9	2	2	14	0	6,410	26.0	5 19
	2	0.33	3.66	6	66	6	6	363	26	22,108	40.0	5 6 9 20 23 24
		0.40	1.63	3	134	3	3	224	4	22,460	34.0	5 23 25
		0.60	1.50	7	14	2	2	99	75	13,323	23.0	5 20
		0.80	1.50	7	11	2	2	28	5	8,586	24.0	4 25
		1.00	1.50	8	9	2	2	29	9	8,213	20.0	11 20
	3	0.19	4.20	8	54	8	7	**3,600**	1,590	21,501	55.0	4 5 6 14 15 20 23 24
		0.20	3.01	7	44	6	6	482	75	25,693	46.0	4 5 6 20 23 25
		0.40	1.50	2	17	2	2	27	0	13,523	15.0	10 23
		0.60	1.50	4	8	2	2	25	6	8,278	14.0	3 9
		0.80	1.50	7	7	2	2	22	5	9,245	24.0	14 24
		1.00	1.50	4	7	2	2	20	5	8,211	18.0	3 14
	4	0.21	6.76	12	69	12	11	**3,600**	1,701	20,007	56.0	4 5 6 9 13 14 15 16 20 22 24 25
		0.40	1.50	2	29	2	2	44	0	17,230	14.0	4 15
		0.60	1.50	2	7	2	2	12	0	6,200	22.0	14 24
		0.80	1.50	3	5	2	2	22	6	7,544	14.0	6 15
		1.00	1.50	3	8	2	2	21	4	7,423	33.0	23 24
	5	0.32	2.07	5	58	4	4	956	75	32,616	48.0	5 20 23 24
		0.40	1.50	3	60	2	2	356	75	23,494	26.0	23 24
		0.60	1.50	2	5	2	2	11	0	6,182	13.0	9 23
		0.80	1.50	2	5	2	2	10	0	6,455	11.0	1 24
		1.00	1.50	5	7	2	2	29	11	8,414	25.0	10 23

For each problem instance, we run H-Root to find an initial feasible solution and repeat this procedure at every 75 branch-and-bound nodes to improve the current solution at hand. As we present in the following section, our numerical experiments show that H-Root can produce very high-quality solutions and boosts the performance of the branch-and-price algorithm drastically.

For the exact solution of the pricing problem, we employed the state-of-the art algorithm proposed by Santos et al. (2007) (Alg 3 by their notation). As a subroutine, Alg 3 requires the solution of several k-shortest path problems, and the authors use the algorithm presented in de Azevedo et al. (1994) for this task. Although very efficient, this algorithm can produce nonsimple paths and performs rather poorly in our

Table 4. (Continued)

$\|D\|$	*Seed*	α	*R.LP*	*HS*	*H.RT*	*NR*	*NR.LB*	*RT*	*#BB*	*#Col.G.*	*% SW.*	*Reg. loc.*
125	1	0.32	4.63	9	66	9	9	191	26	18,262	57.6	4 5 6 9 14 15 20 22 24
		0.40	1.78	4	87	4	4	687	28	37,169	44.0	5 15 23 24
		0.60	1.50	3	36	2	2	193	75	22,814	16.0	3 24
		0.80	1.50	4	11	2	2	88	75	12,872	18.4	23 24
		1.00	1.50	6	11	2	2	49	15	13,058	34.4	5 20
	2	0.40	2.98	6	216	6	5	**3,600**	551	35,617	48.0	3 5 6 15 23 24
		0.60	1.50	3	151	2	2	435	75	29,420	28.0	3 5
		0.80	1.50	8	22	2	2	146	75	16,685	9.6	11 14
		1.00	1.50	2	7	2	2	15	0	8,626	26.4	5 15
	3	0.23	4.25	9	44	8	8	589	75	32,706	54.4	4 5 6 7 14 20 23 25
		0.40	1.50	2	43	2	2	65	0	21,391	17.6	14 24
		0.60	1.50	3	15	2	2	54	19	11,774	16.8	4 14
		0.80	1.50	7	10	2	2	36	7	11,457	23.2	23 24
		1.00	1.50	8	11	2	2	27	4	9,321	17.6	4 27
	4	0.29	5.68	11	58	10	10	352	75	20,767	55.2	4 5 6 13 14 15 16 22 24 26
		0.40	1.79	4	107	4	4	814	33	35,450	44.0	3 5 14 24
		0.60	1.50	2	34	2	2	45	0	15,497	19.2	14 23
		0.80	1.50	5	23	2	2	60	19	10,327	18.4	24 28
		1.00	1.50	5	8	2	2	29	9	9,626	26.4	5 28
	5	0.37	2.65	6	86	6	6	1,809	146	43,467	52.0	5 14 19 20 23 25
		0.40	2.13	4	80	4	4	362	11	30,128	47.2	5 20 23 24
		0.60	1.50	3	63	2	2	329	75	26,272	28.8	23 24
		0.80	1.50	2	5	2	2	13	0	7,728	24.0	5 23
		1.00	1.50	4	15	2	2	45	7	10,982	44.0	4 5
150	1	0.36	4.50	8	62	8	8	205	21	16,932	54.0	4 5 6 14 15 20 23 24
		0.40	2.35	5	69	5	5	254	18	30,980	45.3	3 5 14 15 24
		0.60	1.50	2	63	2	2	90	0	24,709	32.7	4 5
		0.80	1.50	3	42	2	2	185	75	17,660	12.0	6 14
		1.00	1.50	4	11	2	2	136	75	14,450	6.0	17 24
	2	0.49	2.92	6	156	5	5	1,800	75	48,989	32.7	5 6 9 23 24
		0.60	1.61	3	163	3	3	361	5	40,331	22.7	4 6 24
		0.80	1.50	2	40	2	2	58	0	16,952	22.7	3 5
		1.00	1.50	2	10	2	2	23	0	10,800	6.7	4 27
	3	0.31	3.58	6	257	6	6	572	33	32,811	34.7	3 5 6 9 15 24
		0.40	1.54	3	95	2	2	542	75	34,671	28.0	5 23
		0.60	1.50	2	49	2	2	66	0	20,486	19.3	20 24
		0.80	1.50	8	20	2	2	156	75	16,714	6.0	3 13
		1.00	1.50	6	24	2	2	105	36	16,419	20.0	3 10
	4	0.36	5.41	9	199	9	9	283	13	15,145	48.0	3 4 5 6 13 14 15 20 24
		0.40	2.94	6	303	6	6	706	46	36,044	50.7	4 5 13 15 20 24
		0.60	1.50	3	112	2	2	338	75	32,311	26.7	5 23
		0.80	1.50	3	23	2	2	115	62	17,032	35.3	5 23
		1.00	1.50	2	24	2	2	35	0	12,711	26.7	3 5
	5	0.38	3.47	7	56	7	7	601	64	42,469	54.0	4 5 13 14 16 20 23
		0.40	2.99	6	80	6	6	849	62	44,329	42.0	5 14 16 20 23 24
		0.60	1.50	3	157	2	2	564	75	37,924	25.3	24 28
		0.80	1.50	5	28	2	2	167	75	18,406	16.0	3 28
		1.00	1.50	6	25	2	2	162	75	16,697	25.3	23 28

pricing graph instances that can include arcs with zero length. Therefore, different than Santos et al. (2007), we implemented Yen's loopless k-shortest path algorithm (Yen 1971) as a subroutine in Alg 3.

Our preliminary results have shown that for all of the problem instances we have studied, employing heuristic H_k has been very useful to reduce solution times. For some problem instances (mostly for those with $\alpha_{\min}$ in COST-266 network), we were not able to find the optimal solution within the time limit of one hour unless we apply H_k. Since the 4th and 15th shortest loopless paths between any two nodes of the NSF and COST-266 networks, respectively, have lengths more than the optical reach of the BPSK modulation (2,880 km), the pricing problem is solved exactly by the heuristics H_4 and H_{15}. Hence, as the solution of the pricing problem, we use H_4 and H_{15} for NSF and COST-266 networks, respectively.

We conduct a best bound search to explore the branch-and-bound tree. Although this strategy cannot explore as many nodes as the depth first search strategy, our computational studies showed that it can

Table 5. Results for COST-266 Network with Traffic Density Demand Distribution

$\lvert D\rvert$	*Seed*	α	*R.LP*	*HS*	*H.RT*	*NR*	*NR.LB*	*RT*	*#BB*	*#Col.G.*	*% SW.*	*Reg. loc.*
75	1	0.18	5.17	10	33	10	10	218	42	14,669	50.7	1 3 5 6 7 14 15 20 21 22 24
		0.20	3.26	6	41	6	6	189	18	17,946	52.0	3 5 6 20 23 24
		0.40	1.50	2	10	2	2	16	0	11,138	16.0	9 28
		0.60	1.50	2	3	2	2	7	0	6,072	28.0	23 26
		0.80	1.50	3	4	2	2	18	8	6,596	20.0	16 24
		1.00	1.50	3	2	2	2	12	5	6,711	18.7	4 14
	2	0.14	5.64	12	21	11	10	**3,600**	4,528	9,385	49.3	3 4 5 14 15 16 20 21 23 24 26
		0.20	1.76	4	47	4	3	**3,600**	1,088	22,945	42.7	3 5 22 24
		0.40	1.50	2	3	2	2	6	0	4,202	16.0	17 24
		0.60	1.50	2	2	2	2	6	0	4,725	0.0	9 21
		0.80	1.50	2	2	2	2	5	0	4,087	21.3	24 28
		1.00	1.50	2	2	2	2	6	0	4,192	17.3	4 14
	3	0.15	3.57	9	40	8	8	262	75	14,120	53.3	3 4 5 14 15 20 22 24
		0.20	1.70	4	33	4	4	237	33	20,547	38.7	3 5 22 24
		0.40	1.00	1	2	1	1	6	0	5,976	12.0	3
		0.60	1.00	1	2	1	1	5	0	4,601	22.7	3
		0.80	1.00	1	2	1	1	5	0	4,811	20.0	3
		1.00	1.00	1	2	1	1	5	0	4,023	21.3	3
	4	0.28	3.07	6	36	6	6	143	20	13,670	38.7	3 5 14 16 20 23
		0.40	1.54	3	45	2	2	184	75	17,244	30.7	22 24
		0.60	1.50	3	8	2	2	69	75	6,583	9.3	10 14
		0.80	1.50	2	3	2	2	6	0	3,628	16.0	1 24
		1.00	1.50	3	4	2	2	11	5	5,574	20.0	4 25
	5	0.16	8.49	15	23	14	13	**3,600**	3,490	7,400	58.7	3 4 5 6 13 14 15 20 21 22 23 24 26 27
		0.20	3.33	7	35	6	6	450	75	15,863	38.7	3 5 9 20 23 24
		0.40	1.50	2	7	2	2	11	0	8,213	16.0	3 9
		0.60	1.50	2	2	2	2	5	0	3,331	21.3	3 13
		0.80	1.50	2	2	2	2	5	0	3,437	21.3	19 24
		1.00	1.50	2	2	2	2	5	0	3,895	13.3	19 26
100	1	0.21	6.11	12	39	12	12	105	20	9,965	57.0	3 4 5 6 7 14 15 16 20 21 22 24
		0.40	1.50	2	33	2	2	52	0	19,702	16.0	22 24
		0.60	1.50	4	10	2	2	34	10	10,007	42.0	5 23
		0.80	1.50	3	4	2	2	23	8	10,600	44.0	4 5
		1.00	1.50	3	7	2	2	24	6	9,637	29.0	5 15
	2	0.22	5.16	10	25	10	10	174	40	13,698	44.0	3 4 5 14 15 16 17 20 23 24
		0.40	1.50	3	31	2	2	236	75	19,887	23.0	23 24
		0.60	1.50	5	8	2	2	67	75	8,456	21.0	23 24
		0.80	1.50	2	3	2	2	8	0	4,645	11.0	9 14
		1.00	1.50	3	4	2	2	17	6	6,538	21.0	14 23
	3	0.20	4.39	9	31	9	8	**3,600**	1,483	23,023	53.0	3 4 14 15 16 20 22 24 27
		0.40	1.50	2	15	2	2	23	0	12,525	23.0	23 24
		0.60	1.50	3	9	2	2	24	7	7,847	23.0	14 24
		0.80	1.50	2	3	2	2	7	0	4,612	25.0	3 13
		1.00	1.50	3	6	2	2	15	3	7,442	42.0	23 24
	4	0.33	3.37	7	45	6	6	390	75	27,118	41.0	3 5 14 16 20 23
		0.40	2.13	4	50	4	4	328	21	28,533	39.0	3 5 20 23
		0.60	1.50	2	19	2	2	25	0	10,958	24.0	14 24
		0.80	1.50	2	5	2	2	11	0	5,317	20.0	3 13
		1.00	1.50	2	2	2	2	7	0	5,044	25.0	3 9
	5	0.22	6.86	12	31	12	11	**3,600**	1,125	16,181	61.0	3 4 5 6 13 14 15 20 21 22 24 27
		0.40	1.50	2	34	2	2	48	0	15,599	20.0	3 24
		0.60	1.50	2	8	2	2	14	0	9,783	11.0	16 24
		0.80	1.50	6	5	2	2	16	5	6,097	11.0	4 27
		1.00	1.50	2	2	2	2	7	0	4,747	26.0	23 28

converge much faster by exploiting the high-quality heuristic solution of H_k that can prune a significant part of the search tree.

4.4. Performance of the Branch-and-Price Algorithm

In this subsection, we investigate the performance of the proposed solution methodology and discuss the effects of the various problem parameters on the difficulty of the resulting instances.

Our experimental design has 480 problem instances in total (two networks, four demand sizes, two demand distributions, six α choices, and five random seeds). However, for some instances, $\alpha = 0.2, 0.4$, and even $\alpha = 0.6$ are larger than $\alpha_{\min}$ and hence there is no feasible solution. Tables 4–7 report the solutions of the

Table 5. (Continued)

$\|D\|$	*Seed*	α	*R.LP*	*HS*	*H.RT*	*NR*	*NR.LB*	*RT*	*#BB*	*#Col.G.*	*% SW.*	*Reg. loc.*
125	1	0.26	6.26	13	55	12	12	187	75	13,689	55.2	1 3 4 5 6 7 14 15 20 21 22 24
		0.40	1.74	3	70	3	3	115	1	25,145	37.6	3 22 24
		0.60	1.50	2	37	2	2	48	0	15,076	6.4	9 16
		0.80	1.50	2	9	2	2	18	0	8,843	20.0	3 9
		1.00	1.50	4	7	2	2	49	23	12,239	12.8	9 23
	2	0.26	5.28	9	33	9	9	334	54	18,718	43.2	3 5 14 15 16 19 20 23 24
		0.40	1.76	4	107	3	3	1,056	75	37,491	27.2	3 5 16
		0.60	1.50	3	17	2	2	138	75	13,172	21.6	14 24
		0.80	1.50	2	10	2	2	19	0	6,969	30.4	14 24
		1.00	1.50	5	8	2	2	26	6	8,032	20.8	5 23
	3	0.26	4.10	8	51	8	8	913	110	32,527	44.0	3 4 15 16 20 21 22 24
		0.40	1.50	3	87	2	2	502	75	27,471	21.6	3 5
		0.60	1.50	2	12	2	2	18	0	7,837	25.6	3 24
		0.80	1.50	4	8	2	2	32	8	11,077	35.2	3 5
		1.00	1.50	4	7	2	2	30	8	10,870	22.4	5 23
	4	0.41	3.05	6	59	6	6	253	26	29,247	38.4	3 5 14 16 20 23
		0.60	1.50	2	76	2	2	113	0	20,730	24.8	22 24
		0.80	1.50	2	25	2	2	36	0	16,851	22.4	3 15
		1.00	1.50	2	9	2	2	18	0	7,270	21.6	20 24
	5	0.28	6.98	13	41	12	11	**3,600**	75	18,344	54.4	3 4 5 13 14 15 20 21 24 26 27 28
		0.40	1.61	3	72	3	3	155	3	22,477	32.0	3 5 24
		0.60	1.50	2	15	2	2	34	0	20,330	4.8	9 14
		0.80	1.50	6	9	2	2	117	75	15,128	13.6	3 13
		1.00	1.50	2	6	2	2	12	0	6,320	17.6	23 24
150	1	0.31	4.95	9	70	9	9	284	27	25,216	52.0	1 3 5 6 15 20 21 22 24
		0.40	2.21	4	332	4	3	**3,600**	779	44,496	44.7	3 5 20 24
		0.60	1.50	3	53	2	2	284	75	30,497	26.7	24 28
		0.80	1.50	2	12	2	2	26	0	9,671	3.3	9 14
		1.00	1.50	3	12	2	2	46	9	15,357	10.0	4 11
	2	0.33	5.27	10	53	10	10	378	53	26,104	46.7	3 5 14 15 16 17 19 20 21 23
		0.40	3.20	6	92	6	6	804	35	45,207	36.7	3 15 16 17 20 24
		0.60	1.50	3	110	2	2	323	75	26,560	24.7	3 15
		0.80	1.50	3	22	2	2	113	75	15,019	14.0	4 15
		1.00	1.50	5	17	2	2	79	35	12,884	40.0	4 5
	3	0.33	4.66	9	198	9	9	1,233	139	40,488	32.0	3 4 14 16 17 20 21 23 24
		0.40	2.83	5	121	4	4	934	75	44,488	32.7	3 20 24 28
		0.60	1.50	3	44	2	2	190	75	25,936	8.7	10 15
		0.80	1.50	3	18	2	2	140	75	15,966	5.3	11 28
		1.00	1.50	3	13	2	2	80	31	15,993	30.7	3 28
	4	0.49	3.23	7	106	7	7	480	50	35,523	46.0	3 4 5 14 16 20 23
		0.60	1.88	4	128	4	4	859	24	45,629	32.7	15 20 23 24
		0.80	1.50	3	95	2	2	444	75	35,013	28.0	22 24
		1.00	1.50	2	33	2	2	47	0	21,249	14.7	24 28
	5	0.34	6.60	11	244	11	10	**3,600**	75	11,607	52.0	3 4 5 6 14 15 16 20 21 22 24
		0.40	3.22	5	222	5	5	295	6	20,825	38.7	3 5 20 23 24
		0.60	1.50	2	77	2	2	98	0	23,414	18.7	24 28
		0.80	1.50	2	19	2	2	31	0	15,521	6.7	9 28
		1.00	1.50	4	23	2	2	147	75	13,260	11.3	1 24

branch-and-price algorithm for the total of 316 problem instances for which feasible solutions exist. In these tables, $|D|$ is the number of connection demands, seed is the key used to generate random numbers for the relevant problem instances, $R.LP$ is the solution of the linear relaxation, HS is the number regenerators found by the heuristic solution H-Root, $H.RT$ is the run time (in seconds) for H-Root, NR is the solution found by the branch-and-price algorithm, $NR.LP$ is the proven lower bound for the number of regenerators, RT is the solution time (in seconds) for the branch-and-price algorithm, $\#BB$ is the number of branch-and-bound nodes explored by the branch-and-price algorithm, $\#Col.G.$ is the total number of columns generated during the branch-and-price algorithm, $\%SW.$ shows the percentage of demands that have gone through at least one modulation swap (at a regenerator node) on its path, and finally *Reg. Loc.* reports the nodes that are chosen to be a regenerator point by the branch-and-price algorithm.

Table 6. Results for NSF Network with Uniform Demand Distribution

$\|D\|$	*Seed*	α	*R.LP*	*HS*	*H.RT*	*NR*	*NR.LB*	*RT*	*#BB*	*#Col.G.*	*% SW.*	*Reg. loc.*
75	1	0.48	2.11	3	1	3	3	3	0	901	5.3	3 4 6
		0.60	2.00	3	1	3	3	3	1	718	8.0	3 4 6
		0.80	2.00	2	1	2	2	2	0	673	2.7	4 6
		1.00	2.00	2	0	2	2	1	0	619	18.7	4 11
	2	0.44	2.19	4	1	4	4	10	9	1,635	13.3	3 4 6 7
		0.60	2.00	3	1	3	3	3	2	675	14.7	3 4 6
		0.80	2.00	3	1	3	3	4	3	998	21.3	3 4 11
		1.00	2.00	2	0	2	2	1	0	487	21.3	4 11
	3	0.42	2.29	4	2	4	4	6	3	1,323	12.0	3 4 6 11
		0.60	2.00	3	1	3	3	3	1	844	5.3	3 4 6
		0.80	2.00	2	1	2	2	2	0	724	2.7	4 6
		1.00	2.00	2	0	2	2	2	0	614	22.7	4 11
	4	0.41	2.17	3	1	3	3	3	0	824	10.7	3 4 6
		0.60	2.00	2	1	2	2	2	0	601	2.7	4 6
		0.80	2.00	2	0	2	2	1	0	536	18.7	4 11
		1.00	2.00	2	0	2	2	2	0	574	20.0	4 11
	5	0.51	2.19	4	1	4	4	5	3	1,054	9.3	3 4 6 11
		0.60	2.01	4	1	3	3	7	22	948	14.7	3 6 7
		0.80	2.00	3	1	3	3	2	1	813	2.7	3 4 6
		1.00	2.00	2	1	2	2	2	0	631	1.3	4 6
100	1	0.68	2.20	4	4	4	4	10	2	1,541	12.0	3 4 6 11
		0.80	2.02	3	2	3	3	5	0	1,310	9.0	3 4 6
		1.00	2.00	3	2	3	3	5	1	1,066	15.0	4 6 11
	2	0.62	2.17	3	2	3	3	5	0	1,340	10.0	3 4 6
		0.80	2.00	4	2	3	3	9	12	1,235	17.0	3 4 6
		1.00	2.00	3	1	3	3	4	2	917	16.0	3 4 6
	3	0.67	2.15	3	2	3	3	5	0	1,313	7.0	3 4 6
		0.80	2.00	3	1	3	3	4	1	1,057	5.0	3 4 6
		1.00	2.00	3	1	3	3	4	1	1,041	4.0	4 6 11
	4	0.44	2.37	4	3	4	4	9	3	1,603	17.0	3 4 6 11
		0.60	2.00	3	1	3	3	6	2	1,213	9.0	4 6 11
		0.80	2.00	2	1	2	2	3	0	845	3.0	4 6
		1.00	2.00	2	1	2	2	2	0	801	20.0	4 11
	5	0.62	2.17	3	2	3	3	4	0	1,243	8.0	3 4 6
		0.80	2.00	4	1	3	3	11	10	1,326	7.0	3 4 6
		1.00	2.00	3	1	3	3	3	1	922	6.0	3 4 6
125	1	0.84	2.22	4	4	4	4	14	4	2,106	11.2	3 4 6 11
		1.00	2.04	3	2	3	3	7	0	1,804	5.6	3 4 6
	2	0.86	2.11	3	2	3	3	6	0	1,544	11.2	3 4 6
		1.00	2.00	3	2	3	3	7	1	1,195	10.4	3 4 6
	3	0.77	2.12	3	3	3	3	8	0	1,843	8.8	3 4 6
		0.80	2.08	3	2	3	3	6	0	1,737	11.2	3 4 6
		1.00	2.00	3	2	3	3	5	1	1,340	10.4	3 4 6
	4	0.65	2.41	4	6	4	4	14	2	1,933	12.8	3 4 6 11
		0.80	2.13	3	3	3	3	8	0	1,855	9.6	3 4 6
		1.00	2.00	3	1	3	3	6	2	1,176	10.4	3 4 6
	5	0.76	2.18	4	4	4	4	10	3	1,877	18.4	3 4 6 11
		0.80	2.13	3	2	3	3	6	0	1,318	8.0	3 4 6
		1.00	2.00	3	2	3	3	6	1	1,183	5.6	3 4 6
150	1	0.96	2.25	4	6	4	4	14	4	2,262	12.7	3 4 6 11
		1.00	2.20	4	11	4	4	16	2	1,954	14.0	3 4 6 11
	2	1.04	2.12	4	4	4	4	12	4	2,140	14.7	3 4 6 11
	3	0.94	2.14	3	3	3	3	8	0	1,980	6.7	3 4 6
		1.00	2.07	3	2	3	3	7	0	1,656	11.3	3 4 6
	4	0.79	2.45	4	5	4	4	14	2	2,056	14.7	3 4 6 11
		0.80	2.42	4	5	4	4	13	2	1,914	12.7	3 4 6 11
		1.00	2.13	3	4	3	3	9	0	2,209	9.3	3 4 6
	5	0.93	2.15	3	4	3	3	8	0	1,888	6.7	3 4 6
		1.00	2.07	3	3	3	3	7	0	1,524	8.0	3 4 6

Table 7. Results for NSF Network with Traffic Density Demand Distribution

|D|	*Seed*	*α*	*R.LP*	*HS*	*H.RT*	*NR*	*NR.LB*	*RT*	*#BB*	*#Col.G.*	*% SW.*	*Reg. loc.*
75	1	0.43	2.25	3	1	3	3	2	0	661	2.7	3 4 6
		0.60	2.00	3	1	3	3	2	1	616	1.3	3 4 6
		0.80	2.00	2	1	2	2	2	0	546	0.0	4 6
		1.00	2.00	2	1	2	2	2	0	555	13.3	2 7
	2	0.38	2.34	5	2	5	5	7	5	1,227	32.0	3 4 6 9 11
		0.60	2.00	3	1	3	3	2	1	899	2.7	3 4 6
		0.80	2.00	3	1	3	3	3	3	972	6.7	3 4 6
		1.00	2.00	2	0	2	2	2	0	650	30.7	4 11
	3	0.48	2.09	3	1	3	3	2	0	713	6.7	3 4 6
		0.60	2.00	3	1	3	3	2	1	739	10.7	3 4 6
		0.80	2.00	3	1	3	3	2	2	687	8.0	3 6 7
		1.00	2.00	2	0	2	2	2	0	571	22.7	4 11
	4	0.71	2.08	3	1	3	3	2	0	749	9.3	3 4 6
		0.80	2.00	3	1	3	3	2	1	759	6.7	3 4 6
		1.00	2.00	3	1	3	3	2	1	719	9.3	3 4 6
	5	0.67	2.19	4	2	4	4	6	4	1,138	9.3	3 4 6 11
		0.80	2.00	3	1	3	3	3	1	846	8.0	3 4 6
		1.00	2.00	3	1	3	3	3	1	785	4.0	3 4 6
100	1	0.52	2.19	4	2	3	3	18	53	1,524	14.0	3 5 6
		0.60	2.04	3	1	3	3	3	0	904	4.0	3 4 6
		0.80	2.00	2	1	2	2	2	0	697	0.0	4 6
		1.00	2.00	2	1	2	2	2	0	722	2.0	4 6
	2	0.63	2.25	4	3	4	4	9	3	1,298	18.0	3 4 6 11
		0.80	2.00	3	1	3	3	4	0	1,251	5.0	3 4 6
		1.00	2.00	3	1	3	3	4	1	1,045	18.0	4 6 11
	3	0.72	2.00	3	1	3	3	4	1	951	9.0	3 4 6
		0.80	2.00	3	1	3	3	3	1	916	10.0	3 4 6
		1.00	2.00	3	1	3	3	3	1	925	7.0	3 6 7
	4	0.79	2.10	4	1	4	3	5	2	1,029	7.0	3 4 6 7
		0.80	2.08	3	1	3	3	4	0	1,001	7.0	3 4 6
		1.00	2.00	3	1	3	3	4	1	953	7.0	3 4 6
	5	0.75	2.22	3	2	3	3	6	0	1,199	7.0	3 4 6
		0.80	2.14	3	2	3	3	4	0	1,112	8.0	3 4 6
		1.00	2.00	3	1	3	3	4	1	1,117	5.0	3 4 6
125	1	0.62	2.28	4	3	4	4	10	4	1,527	7.2	3 4 6 11
		0.80	2.00	3	2	3	3	7	1	1,546	16.8	3 5 6
		1.00	2.00	3	1	3	3	4	1	948	1.6	4 6 11
	2	0.77	2.22	4	3	4	4	10	3	1,711	16.0	3 4 6 11
		0.80	2.18	4	4	4	4	11	3	1,798	4.8	3 4 6 11
		1.00	2.00	3	2	3	3	5	1	1,390	3.2	3 4 6
	3	0.89	2.04	3	2	3	3	5	0	1,392	8.0	3 4 6
		1.00	2.00	3	2	3	3	5	1	1,237	9.6	3 4 6
	4	0.98	2.15	4	2	4	4	9	3	1,494	8.0	3 4 6 7
		1.00	2.13	4	2	4	4	11	4	1,786	8.8	3 4 6 7
	5	0.93	2.17	4	2	3	3	26	75	2,282	8.0	3 4 6
		1.00	2.08	3	3	3	3	8	0	1,397	5.6	3 4 6
150	1	0.73	2.28	4	4	4	4	9	2	1,695	7.3	3 4 6 11
		0.80	2.19	4	3	3	3	16	50	1,974	14.0	3 5 6
		1.00	2.00	3	2	3	3	7	1	1,428	4.7	3 4 6
	2	0.88	2.24	4	4	4	4	9	3	1,774	17.3	3 4 6 11
		1.00	2.11	4	6	4	4	11	3	1,957	5.3	3 4 6 11
	3	1.01	2.07	3	3	3	3	7	0	1,569	6.0	3 4 6
	4	1.25	2.14	4	6	4	4	12	3	1,888	13.3	3 4 6 7
	5	1.09	2.21	3	4	3	3	8	0	1,816	5.3	3 4 6

As we can see from the results, 303 out of 316 problems were solved optimally, and for the remaining 13 the optimality gap was reduced to just one regenerator within the given time limit of 3,600 seconds. For 12 out of the 13 unsolved problem instances, the bandwidth utilization level is equal to the minimum (i.e., $\alpha = \alpha_{\min}$).

Table 8 depicts a summary of the results in Tables 4–7. Cells occupied with "—" are the cases where the related problem has no feasible solution. It is easy to read from Table 8 that the smaller α values (limited link capacities) make the RLP-FON instances harder to solve. This is due to the fact that the short supply of bandwidth slots entails that a high percentage of the flow variables assume fractional values in the LP solution of the problem and the branch-and-bound tree grows significantly.

The results also show that problem instances with the NSF network are much easier to solve than those of COST-266. This is in part due to the higher number of nodes and edges in the latter. However, the substantial difference in the solution times points to a more significant effect being in play. In the NSF network, the mean edge length is more than two times larger than that of the COST-266 network, and consequently the optical reach constraints are more binding. As a result, for each demand, the number of alternative paths is rather limited for the NSF network compared to the COST-266 network. Moreover, the topology is quite different between the two networks. In the NSF network, most of the nodes are along the periphery. On the other hand, the COST-266 network has a much more crowded core which contains more than half of the nodes. Such a composition significantly increases the number of alternative routings for each demand and makes problem instances challenging.

Not surprisingly, problem instances with a higher number of demands are harder to solve. What is interesting is the higher solution times for the problem instances with the *traffic density* demand distribution. A possible explanation for this result could be the higher concentration of connection demands on some specific node pairs which exacerbates the problem of bandwidth capacity limitations.

Our numerical experiments show that H-Root can produce high-quality solutions. Among the 316 problem instances, H-Root could find the optimal solution in 214 (67.72%). For the 65 instances out of the remaining 102, H-Root could find a solution with an optimality gap of just one regenerator. Interestingly, for some instances, H-Root would return a higher number of regenerators when α increases (e.g., Table 4; $|D| = 75$, *Seed* = 1). The reason for such a result is the lack of some critical path-segments that are not generated in the root node when α is larger and that LP relaxation of the problem has to be solved with a more restricted set of columns.

Table 8. Exact Solution Results for COST-266 and NSF Networks

			Average regenerator number						Average run time (seconds)						Average #*BB* nodes						Average #*Col* generated					
Network	Demand	$\|D\|$	$\alpha_{\min}$	0.2	0.4	0.6	0.8	1.0	$\alpha_{\min}$	0.2	0.4	0.6	0.8	1.0	$\alpha_{\min}$	0.2	0.4	0.6	0.8	1.0	$\alpha_{\min}$	0.2	0.4	0.6	0.8	1.0
COST-266	TD	75	9.8	5.0	1.8	1.8	1.8	1.8	1,564.4	1,119.1	44.6	18.3	7.7	7.8	1,631.0	303.5	15.0	15.0	1.6	2.0	11,848.8	19,325.3	9,354.6	5,062.4	4,511.8	4,879.0
		100	9.8	—	2.4	2.0	2.0	2.0	1,573.9	—	137.3	33.0	13.0	14.0	548.6	—	19.2	18.4	2.6	3.0	17,997.0	—	19,249.2	9,410.2	6,254.2	6,681.6
		125	9.4	—	2.8	2.0	2.0	2.0	1,057.4	—	456.8	70.2	44.3	27.1	68.0	—	38.5	15.0	16.6	7.4	22,505.0	—	28,146.0	15,429.0	11,773.6	8,946.2
		150	9.2	—	4.8	2.4	2.0	2.0	1,195.3	—	1,408.2	350.7	150.9	80.1	68.8	—	223.8	49.8	45.0	30.0	27,787.6	—	38,754.0	30,407.2	18,238.0	15,748.6
	U	75	6.6	4.0	2.0	2.0	2.0	2.0	1,561.9	518.2	17.7	8.1	8.5	6.8	857.8	94.0	0.0	1.4	2.6	1.2	16,602.0	18,928.5	7,850.0	4,855.6	4,979.0	4,672.2
		100	8.0	6.0	2.4	2.0	2.0	2.0	1,786.7	482.1	163.6	31.3	22.3	22.6	693.4	75.0	17.8	16.2	6.0	5.8	22,196.8	25,693.0	19,797.2	8,022.0	7,884.2	7,734.2
		125	7.8	—	3.5	2.0	2.0	2.0	1,308.3	—	482.2	211.3	68.7	32.9	174.6	—	18.0	48.8	35.2	7.0	30,163.8	—	31,034.5	21,155.4	11,813.8	10,322.6
		150	7.0	—	4.8	2.2	2.0	2.0	692.1	—	587.9	283.8	136.3	92.0	41.2	—	50.3	31.0	57.4	37.2	31,269.2	—	36,506.0	31,152.2	17,352.8	14,215.4
NSF	TD	75	3.6	—	—	3.0	2.8	2.4	4.1	—	—	2.1	2.5	2.1	1.8	—	—	1.0	1.4	0.4	897.6	—	—	751.3	762.0	656.0
		100	3.4	—	—	3.0	2.8	2.8	8.1	—	—	3.4	3.6	3.4	11.8	—	—	0.0	0.2	0.8	1,200.2	—	—	904.0	995.4	952.4
		125	3.6	—	—	—	3.5	3.2	11.9	—	—	—	8.7	6.7	17.0	—	—	—	2.0	1.4	1,681.2	—	—	—	1,672.0	1,351.6
		150	3.6	—	—	—	3.0	3.5	8.9	—	—	—	15.8	8.9	1.6	—	—	—	50.0	2.0	1,748.4	—	—	—	1,974.0	1,692.5
	U	75	3.6	—	—	2.8	2.4	2.0	5.4	—	—	3.6	2.3	1.6	3.0	—	—	5.2	0.8	0.0	1,147.4	—	—	757.2	748.8	585.0
		100	3.4	—	—	3.0	2.8	2.8	6.6	—	—	6.2	6.3	3.7	1.0	—	—	2.0	4.6	1.0	1,408.0	—	—	1,213.0	1,154.6	949.4
		125	3.6	—	—	—	3.0	3.0	10.6	—	—	—	6.7	6.3	1.8	—	—	—	0.0	1.0	1,860.6	—	—	—	1,636.7	1,339.6
		150	3.6	—	—	—	4.0	3.3	11.2	—	—	—	13.1	9.8	2.0	—	—	—	2.0	0.5	2,065.2	—	—	—	1,914.0	1,835.8

Table 9. Number of Required Regenerators for COST-266 Network with Traffic Density Demand Distribution

	Percentage of bandwidth allocated (α)								
$\|D\|$	0.14	0.20	0.22	0.26	0.33	0.40	0.60	0.80	1.00
75	10	3	3	2	2	2	2	2	2
100	—	—	10	6	3	2	2	2	2
125	—	—	—	9	5	3	2	2	2
150	—	—	—	—	10	6	2	2	2

4.5. Managerial Insights

The relation between the number of regenerators and the spectral efficiency is an interesting one for network managers who want to deploy/operate a minimum number of regenerators to lower capital investment and operational costs (such as energy and maintenance costs), but who also want to achieve higher bandwidth utilization efficiency to be able to satisfy more demand and build resilience against failures in network components. As detailed below, our results show that with FON architecture, a smart deployment of a rather limited number of regenerators can achieve high levels of bandwidth efficiency. These promising results indicate that FON architecture can simultaneously provide significant cost reductions and capacity enhancements.

Table 9 shows the lower bounds for the optimal solution values of the 27 problem instances with COST-266 network and traffic density distribution of 75, 100, 125, and 150 connection demands generated by random number seed 2. Cells depicted with "—" are the cases where the problem has no feasible solution, and for each row, the first cell with a numeric value is the case where the percentage of allocated bandwidth is the minimum (i.e., $\alpha = \alpha_{\min}$). To leave no dents in Table 9, we generate and solve additional problem instances, using different α values than we have in Table 5.

A quick look at the table reveals that certain combinations of regenerator deployment and bandwidth utilization (α) levels could be very attractive for the network management. For example, the first row ($|D| = 75$) of the table shows that with just three regenerators (deploying regenerators at 11% of nodes), it is possible to satisfy all of the demand with only 20% of the available bandwidth in each link. Note that for this case, the lowest possible utilization level is 0.14, which requires 10 regenerators. Moreover, Tables 4–7 together show that regardless of the network and demand distribution differences, the same conclusion stays valid. Our results also show that some nodes are more likely to appear as regeneration nodes (in the optimal solutions, regenerators are placed/activated on those nodes). In Table 10, each row depicts the results of five different RLP-FON instances with the COST-266 network and traffic density demand distribution. For each node, the percentage of solutions for which that node appears in the set of regenerator nodes is given in the table. For example, looking at the first row, we can see that nodes 3, 5, 14, and 20 have been chosen as the regenerator nodes at all of the five problem instances solved with the minimum bandwidth allocation ($\alpha = \alpha_{\min}$), whereas nodes 1, 13, and 26 were selected just once. Looking at the table, one can see that the optimal locations of the regenerators do not change drastically with the fluctuations in the demand. From Tables 6 and 7, we can see that the same conclusion is also true for the NSF network for which the various combinations of the nodes 3, 4, 6, and 11 constitute approximately 89.3% of the optimal solutions. Thus, the solutions obtained by the proposed algorithm can be considered as somewhat robust solutions. This is a desired property for network management, especially when it is hard to accurately estimate the demand at the time of planning.

Another interesting data point in the given tables is the percentage of modulation swaps. We call it a modulation swap if a demand uses more than one modulation level (by going through modulation conversion in a regenerator node) on its light-path. Our results show that, in general, more strict bandwidth limitations necessitate more modulation swaps to satisfy connection demand with less network resources. For example, in the first line of Table 4, 53.3% of the demands have gone through at least one modulation swap on their determined light-paths when the allowed bandwidth is at its minimum ($\alpha = \alpha_{\min}$). This number reduces to 21.3% when all of the bandwidth is allowed to be used. A very similar trend is clearly visible in other tables as well. Thus, we can see that, at least according to the results of our numerical experiments, FON architecture's novel capability of multimodulation transmission appears to be very useful in increasing resource utilization efficiency for the optical networks.

5. Conclusions

This study revisits the regenerator location problem from the flexible optical network architecture perspective and introduces RLP-FON to the operations research (OR) literature. Since the concept of FON architecture is quite new (due to the recent maturation of the enabling hardware technologies), this practically significant and theoretically interesting problem is not well studied in the literature. One of the purposes of this study is to draw the attention of OR researchers to this gap in the literature and promote new studies in this promising area of research.

For the considered problem, we developed a path-segment–based formulation. Our solution methodology introduces a new perspective to general path-based formulations. In particular, our novel path-segment formulation makes it easy to consider

Operations Research, 2017, vol. 65, no. 3, pp. 595–620, © 2017 INFORMS

Table 10. Percentage of Nodes Appearing as Regeneration Points in the Optimal Solutions for the Problem Instances with COST-266 Network and Traffic Density Demand Distribution

		Node number																											
$\|D\|$	α	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
75	$\alpha_{\min}$	20	0	100	60	100	40	20	0	0	0	0	0	20	100	80	40	0	0	0	100	60	60	60	60	0	20	0	0
	0.20	0	0	100	0	100	25	0	0	25	0	0	0	0	0	0	0	0	0	0	50	0	50	50	100	0	0	0	0
	0.40	0	0	40	0	0	0	0	0	40	0	0	0	0	0	0	0	20	0	0	0	0	20	0	40	0	0	0	20
	0.60	0	0	40	0	0	0	0	0	20	20	0	0	20	20	0	0	0	0	0	0	20	0	20	0	0	20	0	0
	0.80	20	0	20	0	0	0	0	0	0	0	0	0	0	0	0	20	0	0	20	0	0	0	0	80	0	0	0	20
	1.00	0	0	20	60	0	0	0	0	0	0	0	0	0	40	0	0	0	0	20	0	0	0	0	0	20	20	0	0
100	$\alpha_{\min}$	0	0	100	80	80	40	20	0	0	0	0	0	20	100	80	80	20	0	0	100	40	60	40	60	0	0	20	0
	0.40	0	0	40	0	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20	0	20	60	80	0	0	0	0
	0.60	0	0	0	0	20	0	0	0	0	0	0	0	0	40	0	20	0	0	0	0	0	0	40	80	0	0	0	0
	0.80	0	0	40	40	20	0	0	0	20	0	0	0	40	20	0	0	0	0	0	0	0	0	0	0	0	0	20	0
	1.00	0	0	20	0	20	0	0	0	20	0	0	0	0	20	20	0	0	0	0	0	0	0	60	20	0	0	0	20
125	$\alpha_{\min}$	20	0	100	60	80	20	20	0	0	0	0	0	20	80	80	60	0	0	20	100	60	40	40	60	0	20	20	0
	0.40	0	0	100	0	75	0	0	0	0	0	0	0	0	0	0	25	0	0	0	0	0	25	0	50	0	0	0	0
	0.60	0	0	20	0	0	0	0	0	40	0	0	0	0	40	0	20	0	0	0	0	0	20	0	60	0	0	0	0
	0.80	0	0	80	0	20	0	0	0	20	0	0	0	20	20	20	0	0	0	0	0	0	0	0	20	0	0	0	0
	1.00	0	0	0	0	40	0	0	0	20	0	0	0	0	0	0	0	0	0	0	20	0	0	80	40	0	0	0	0
150	$\alpha_{\min}$	20	0	100	60	80	40	0	0	0	0	0	0	0	80	60	80	40	0	20	100	80	40	60	60	0	0	0	0
	0.40	0	0	100	0	50	0	0	0	0	0	0	0	0	0	25	25	25	0	0	100	0	0	25	100	0	0	0	25
	0.60	0	0	20	0	0	0	0	0	0	20	0	0	0	0	60	0	0	0	0	20	0	0	20	60	0	0	0	40
	0.80	0	0	0	20	0	0	0	0	40	0	20	0	0	20	20	0	0	0	0	0	0	20	0	20	0	0	0	40
	1.00	20	0	20	40	20	0	0	0	0	0	20	0	0	0	0	0	0	0	0	0	0	0	0	40	0	0	0	40

nonsimple path solutions and include some special constraints on the paths which are otherwise harder to incorporate in a plain path-based formulation.

We propose an efficient branch-and-price algorithm to solve the problem. We conducted extensive numerical experiments to test the performance of the proposed algorithm. As explained above, RLP-FON requires the solution of regenerator location, routing, spectrum allocation, and modulation selection problems jointly. The performance of the proposed algorithm is comparable to the state-of-the-art heuristic algorithms that solve these problems sequentially. From the practical point of view, these numerical studies provide significant managerial insights about this urgent problem. In particular, our findings show that FON architecture makes it possible to enhance network capacity and reduce the capital and operational costs of the optical network.

Although the practical motivation for RLP-FON comes from the telecommunications applications, this theoretically interesting problem and its simple extensions can actually appear in various application settings. In its current form, arc costs are not considered in the RLP-FON formulation (they are simply assumed to be zero). But this is only because of the practical setting of the problem where arc costs are either negligible or do not scale up to the regenerator deployment costs. From the mathematical point of view, adding arc costs does not disturb the main structure of the proposed algorithm at all. Note that adding arc costs to the objective function or considering new constraints on some resource usage at arcs can be included in the pricing problem by just adding a new term in the calculation of arc lengths of the pricing graph. Thus, our formulation and solution methodology can be easily adapted to the rather general multicommodity, multimodal flow problems; as such, we leave it for a future study to investigate the application of the proposed solution methodology in different contexts than the optical networks.

References

Agrawal GP (2011) *Optical Fibers* (John Wiley & Sons, Hoboken, NJ), 24–78.

Barnhart C, Hane C, Vance P (2000) Using branch-and-price-and-cut to solve origin-destination integer multicommodity flow problems. *Oper. Res.* 48(2):318–326.

Barnhart C, Hane CA, Johnson EL, Sigismondi G (1994) A column generation and partitioning approach for multi-commodity flow problems. *Telecommunication Systems* 3(3):239–258.

Barnhart C, Johnson EL, Nemhauser GL, Savelsbergh MW, Vance P (1998) Branch-and-price: Column generation for solving huge integer programs. *Oper. Res.* 46(3):316–329.

Bosco G, Curri V, Carena A, Poggiolini P, Forghieri F (2011) On the performance of Nyquist-WDM terabit superchannels based on PM-BPSK, PM-QPSK, PM-8QAM or PM-16QAM subcarriers. *J. Lightwave Tech.* 29(1):53–61.

Cabral EA, Erkut E, Laporte G, Patterson RA (2007) The network design problem with relays. *Eur. J. Oper. Res.* 180(2):834–844.

Cardillo R, Curri V, Mellia M (2006) Considering transmission impairments in configuring wavelength routed optical networks. *Optical Fiber Comm. Conf.* (Optical Society of America), OFG6.

Castro A, Velasco L, Ruiz M, Klinkowski M, Fernández-Palacios J, Careglio D (2012) Dynamic routing and spectrum (re)allocation in future flexgrid optical networks. *Comput. Networks* 56(12):2869–2883.

Chen S, Ljubić I, Raghavan S (2009) The regenerator location problem. *Networks* 55(3):205–220.

Chen S, Ljubić I, Raghavan S (2015) The generalized regenerator location problem. *INFORMS J. Comput.* 27(2):204–220.

Cohn A, Barnhart C (2003) Improving crew scheduling by incorporating key maintenance routing decisions. *Oper. Res.* 51(3): 387–396.

de Azevedo J, Silvestre Madeira J, Vieira Martins E, Pires F (1994) A computational improvement for a shortest paths ranking algorithm. *Eur. J. Oper. Res.* 73(1):188–191.

Degraeve Z, Jans R (2007) A new Dantzig-Wolfe reformulation and branch-and-price algorithm for the capacitated lot-sizing problem with setup times. *Oper. Res.* 55(5):909–920.

Desaulniers G (2010) Branch-and-price-and-cut for the split-delivery vehicle routing problem with time windows. *Oper. Res.* 58(1): 179–192.

Duarte A, Martí R, Resende M, Silva R (2014) Improved heuristics for the regenerator location problem. *Internat. Trans. Oper. Res.* 21(4):541–558.

Essiambre R, Kramer G, Winzer P, Foschini G, Goebel B (2010) Capacity limits of optical fiber networks. *J. Lightwave Tech.* 28(4): 662–701.

Flammini M, Marchetti-Spaccamela A, Monaco G, Moscardelli L, Zaks S (2011) On the complexity of the regenerator placement problem in optical networks. *IEEE/ACM Trans. Networking* 19(2):498–511.

Garey M, Johnson D (1979) *Computers and Intractability: A Guide to the Theory of NP-Completeness* (W.H. Freeman, New York).

Gendron B, Lucena A, Cunha A, Simonetti L (2012) Benders decomposition, branch-and-cut and hybrid algorithms for the minimum connected dominating set problem. Technical report, Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT).

Global Action Plan (2007) An inefficient truth. Technical report.

He J, Brandt-Pearce M, Pointurier Y, Brown CL, Subramaniam S (2007) Adaptive wavelength assignment using wavelength spectrum separation for distributed optical networks. *Proc. IEEE Internat. Conf. Communications, ICC '07* (IEEE, Piscataway, NJ), 2406–2411.

Huang Y, Heritage JP, Mukherjee B (2005) Connection provisioning with transmission impairment consideration in optical WDM networks with high-speed channels. *J. Lightwave Tech.* 23(3): 982–993.

Hulsermann R, Betker A, Jager M, Bodamer S, Barry M, Spath J, Gauger C, Kohn M (2004) A set of typical transport network scenarios for network modeling. *Proc. 5th ITG Workshop Photonic Networks*, 65–72.

Index, Cisco Visual Networking (2012) Global mobile data traffic forecast update, 2011–2016. *Cisco white paper*.

Jinno M, Takagi T, Kiyokawa K (2015) Minimal virtualized-elastic-regenerator placement and least congestion resources assignment for translucent elastic optical networks. *Optical Fiber Communication Conference* (Optical Society of America, Washington, DC), paper Th3J.2.

Jinno M, Kozicki B, Takara H, Watanabe A, Sone Y, Tanaka T, Hirano A (2010) Distance-adaptive spectrum resource allocation in spectrum-sliced elastic optical path network [topics in optical communications]. *IEEE Comm. Magazine* 48(8):138–145.

Jinno M, Takara H, Kozicki B, Tsukishima Y, Sone Y, Matsuoka S (2009) Spectrum-efficient and scalable elastic optical path network: Architecture, benefits, and enabling technologies. *IEEE Comm. Magazine* 47(11):66–73.

Kahya A (2013) Routing spectrum allocation and regenerator placement in flexible-grid optical networks. Ph.D. thesis, Bilkent University, Bilkent, Turkey.

Kewcharoenwong P, Üster H (2014) Benders decomposition algorithms for the fixed-charge relay network design in telecommunications. *Telecomm. Systems* 56(4):441–453.

Kim S, Seo S (2001) Regenerator placement algorithms for connection establishment in all-optical networks. *IEEE Proc. Comm.* 148(1):25–30.

Klinkowski M (2012) On the effect of regenerator placement on spectrum usage in translucent elastic optical networks. *14th Internat. Conf. Transparent Optical Networks, ICTON* (IEEE, Piscataway, NJ).

Kuby M, Lim S (2005) The flow-refueling location problem for alternative-fuel vehicles. *Socio-Economic Planning Sci.* 39(2): 125–145.

Lübbecke M, Desrosiers J (2005) Selected topics in column generation. *Oper. Res.* 53(6):1007–1023.

Lucena A, Maculan N, Simonetti L (2010) Reformulations and solution algorithms for the maximum leaf spanning tree problem. *Comput. Management Sci.* 7(3):289–311.

Manousakis K, Christodoulopoulos K, Kamitsas E, Tomkos I, Varvarigos EA (2009) Offline impairment-aware routing and wavelength assignment algorithms in translucent WDM optical networks. *J. Lightwave Tech.* 27(12):1866–1877.

Pachnicke S, Paschenda T, Krummrich PM (2008) Physical impairment based regenerator placement and routing in translucent optical networks. *Optical Fiber Communication Conference* (Optical Society of America, Washington, DC), paper OWA2.

Park K, Kang S, Park S (1996) An integer programming approach to the bandwidth packing problem. *Management Sci.* 42(9): 1277–1291.

Parker M, Ryan J (1993) A column generation algorithm for bandwidth packing. *Telecomm. Systems* 2(1):185–195.

Pavon-Mariño P, Azodolmolky S, Aparicio-Pardo R, Garcia-Manrubia B, Pointurier Y, Angelou M, Sole-Pareta J, Garcia-Haro J, Tomkos I (2009) Offline impairment aware RWA algorithms for cross-layer planning of optical networks. *J. Lightwave Tech.* 27(12):1763–1775.

Ramamurthy B, Datta D, Feng H, Heritage JP, Mukherjee B (1999) Impact of transmission impairments on the teletraffic performance of wavelength-routed optical networks. *J. Lightwave Tech.* 17(10):1713–1723.

Rizzelli G, Morea A, Tornatore M, Rival O (2012) Energy efficient traffic-aware design of on–off multi-layer translucent optical networks. *Comput. Networks* 56(10):2443–2455.

Santos L, Coutinho-Rodrigues J, Current J (2007) An improved solution algorithm for the constrained shortest path problem. *Transportation Res. Part B Methodological* 41(7):756–771.

Sen A, Murthy S, Bandyopadhyay S (2008) On sparse placement of regenerator nodes in translucent optical network. *2008 IEEE Global Telecomm. Conf., GLOBECOM '08* (IEEE, Piscataway, NJ).

Shen G, Tucker R (2009) Energy-minimized design for IP over WDM networks. *J. Optical Comm. Networking* 1(1):176–186.

Tomkos I, Palkopoulou E, Angelou M (2012) A survey of recent developments on flexible/elastic optical networking. *14th Internat. Conf. Transparent Optical Networks, ICTON* (IEEE, Piscataway, NJ).

Üster H, Kewcharoenwong P (2011) Strategic design and analysis of a relay network in truckload transportation. *Transportation Sci.* 45(4):505–523.

Vachani R, Shulman A, Kubat P (1996) Multicommodity flows in ring networks. *INFORMS J. Comput.* 8(3):235–242.

Wang X, Brandt-Pearce M, Subramaniam S (2015) Impact of wavelength and modulation conversion on translucent elastic optical networks using MILP. *J. Optical Comm. Networking* 7(7):644–655.

Yang X, Ramamurthy B (2005) Sparse regeneration in translucent wavelength-routed optical networks: Architecture, network design and wavelength routing. *Photonic Network Comm.* 10(1): 39–53.

Yen J (1971) Finding the k shortest loopless paths in a network. *Management Sci.* 17(11):712–716.

Yetginer E, Karasan E (2003) Regenerator placement and traffic engineering with restoration in GMPLS networks. *Photonic Network Comm.* 6(2):139–149.

Yildiz B, Karasan OE (2015) Regenerator location problem and survivable extensions: A hub covering location perspective. *Transportation Res. Part B: Methodological* 71:32–55.
Yildiz B, Arslan O, Karasan OE (2016) A branch and price approach for routing and refueling station location model. *Eur. J. Oper. Res.* 248(3):815–826.
Zsigmond S, Németh G, Cinkler T (2007) Mutual impact of physical impairments and grooming in multilayer networks.Tomkos I, Neri F, Pareta JS, Bruin XM, Lopez SS, eds. *Optical Network Design and Modeling* (Springer, Berlin), 38–47.

Baris Yildiz is an assistant professor in the College of Engineering at Koc University. His research focuses on network design problems and their applications in telecommunication and transportation networks.

Oya Ekin Karasan is a professor of industrial engineering at Bilkent University. Her research interests are in discrete optimization with applications in scheduling, logistics, and network design.