MULTI-STAGE STOCHASTIC PROGRAMMING FOR DEMAND RESPONSE OPTIMIZATION

A THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF

MASTER OF SCIENCE

IN

INDUSTRIAL ENGINEERING

By

Munise Kübra Şahin

July 2018

Multi-stage Stochastic Programming for Demand Response Optimization
By Munise Kübra Şahin
July 2018

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Hande Yaman Paternotte(Advisor)

Özlem Çavuş İyigün(Co-Advisor)

Çağın Ararat

Haldun Süral

Approved for the Graduate School of Engineering and Science:

Ezhan Karaşan
Director of the Graduate School

ABSTRACT

MULTI-STAGE STOCHASTIC PROGRAMMING FOR DEMAND RESPONSE OPTIMIZATION

Munise Kübra Şahin
M.S. in Industrial Engineering
Advisor: Hande Yaman Paternotte
Co-Advisor: Özlem Çavuş İyigün
July 2018

The increase in the energy consumption puts pressure on natural resources and environment and results in a rise in the price of energy. This motivates residents to schedule their energy consumption through demand response mechanism. We propose a multi-stage stochastic programming model to schedule different kinds of electrical appliances under uncertain weather conditions and availability of renewable energy. We incorporate appliances with internal batteries to better utilize the renewable energy sources. Our aim is to minimize the electricity cost and the residents' dissatisfaction. We use a scenario groupwise decomposition approach to compute lower and upper bounds for instances with a large number of scenarios. The results of our computational experiments show that the approach is very effective in finding high quality solutions in small computation times. We provide insights about how optimization and renewable energy combined with batteries for storage result in peak demand reduction, savings in electricity cost and more pleasant schedules for residents with different levels of price sensitivity.

Keywords: smart grid, demand response, multi-stage stochastic programming, scenario groupwise decomposition.

ÖZET

TALEP TEPKİSİ OPTİMİZASYONU İÇİN ÇOK AŞAMALI RASSAL PROGRAMLAMA

Munise Kübra Şahin
Endüstri Mühendisliği, Yüksek Lisans
Tez Danışmanı: Hande Yaman Paternotte
İkinci Tez Danışmanı: Özlem Çavuş İyigün
Temmuz 2018

Enerji tüketimindeki artış doğal kaynaklar ve çevre üzerinde baskıya neden olup enerji fiyatlarında artışla sonuçlanmaktadır. Bu durum kullanıcıları elektrik tüketimlerini talep tepkisi mekanizmasını kullanarak programlamaya motive etmektedir. Bu çalışmada farklı türlerdeki elektronik aletleri hava koşullarındaki ve yenilenebilir enerji miktarındaki belirsizlikler göz önünde bulundurularak programlayan çok aşamalı rassal bir model önerilmiştir. Yenilenebilir enerjinin daha fazla kullanılabilmesi için batarya bulunduran elektronik aletler de sisteme dahil edilmiştir. Önerilen model kullanıcıların elektrik harcamalarını ve programlamadan kaynaklana-
bilecek memnuniyetsizliklerini en aza indirmeyi amaçlamaktadır. Büyük ölçekli problemlerin çözülebilmesi için senaryo grup bazlı ayrışım metodu kullanılarak alt ve üst sınırlar hesaplanmıştır. Yapılan testler sonucunda kullanılan metodun kısa sürelerde en iyi çözüme yakın çözümler verdiği gösterilmiştir. Farklı fiyat duyarlılığına sahip kullanıcılar göz önüne alınarak optimizasyonun ve yenilenebilir enerjinin batarya ile beraber sisteme dahil edilmesinin her tipte kullanıcı için azami talepte azalma, elektrik harcamalarında tasarruf ve kullanıcının isteklerine daha uygun bir program oluşturulmasıyla sonuçlandığı gözlemlenmiştir.

Anahtar sözcükler: akıllı şebeke, talep tepkisi, çok aşamalı rassal programlama, senaryo grup bazlı ayrışım.

Acknowledgement

First and foremost, I would like to thank my advisors, Prof. Hande Yaman Paternotte and Asst. Prof. Özlem Çavuş İyigün. I am indebted for their understanding and patience especially for the times I believed that writing this thesis would last forever. This thesis would not be possible without their valuable ideas and guidance.

In addition to her outstanding expertise, I am grateful to Prof. Hande Yaman Paternotte to become my role model with her dedication to research. She becomes one of the people I most admire and I am most afraid of disappointing. I could not find motivation required to continue academic research without her confidence in me.

I am grateful to Prof. Haldun Süral and Asst. Prof. Çağın Ararat for their valuable time to read and review this thesis. Their valuable comments improved the thesis significantly.

I am thankful to Duygu İşbilir and Aylin Ünal for always being there for me during my whole education life. Even with all the struggles though, I have had a great time when writing this thesis thanks to their unexpected visits when I could not find time to do anything but study and their ability to cheer me even when my tears came up after manic laughters. My biggest achievement so far is finding these two amazing people.

I am grateful to Basak Erman for the joy she brought into my life and her ability to surprise me everyday even after two years together. I am also grateful to Beyza Çelik for her invaluable company during sleepless nights, her videos that capture the moments I want to forget, and most importantly, to be the person who she is. Looking back over the past two years, if I could do it again, I would do it all again in a heartbeat to be with them for two years.

I am indebted to Yaşar Emre Kara to remind me there is more to my life than research. Without his endless patience in my stressful times and his emotional support when I was terrified of failure, I could not overcome the difficulties in my last year.

I am deeply grateful to my father Şahin Şahin and my mother Yadigar Şahin to always care my happiness more than anything in the world. Without hearing the cheerful voice of my father everyday and rational advices of my mother in every situation, I could not achieve any of the things I have done so far. I am also grateful to my sister Seda Zeynep Şahin and my brother Yahya Kürşat Şahin to know me better than anyone else and never let me be alone. I am truly blessed to have them.

Finally, my special gratitude is for my code to finally decided to do what it was coded for despite all the problems we have had.

To all M.S. students

who spend sleepless nights to figure out why their codes do not work

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The traditional power grid is defined as the electricity system that provides at least one of the following operations: electricity generation, transmission, distribution and control [4]. The primary objective of these operations is to balance supply and demand. With the expected growth in world energy consumption by 48% between 2012 and 2040 [5], keeping demand and supply balanced becomes a challenge for traditional power grids. Although the development of renewable energies has a potential to compensate the increase in the consumption, outdated infrastructure of traditional power grid has difficulties in overcoming increasing volatility in electricity generation [6]. The integration of sustainable sources into the grid aims to satisfy increasing demand with the less possible damage to the environment and calls a need for a more intelligent power grid that can control the increasing volatility in electricity generation and consumption.

The Smart Grid is an improved version of the traditional power grid in a way that can address the problems the traditional power grid is facing. These problems can be classified under three categories. First, traditional grid is designed only for power flow and allows power to flow through one way (from utilities to customers). This one-way communication prevents sources like solar panels to feed the system when needed since the flow from customers to grid is not allowed. In addition, because of the absence of information flow, in traditional power grid, customers do not have any choice for determining their energy usage. In the smart grid, one-way power flow is evolved to two-way communication of power and information. This provides advantages to both utilities and customers. For example, during peak periods, utilities can increase the electricity price and some customers react this increase by reducing their consumption. Besides helping customers to reduce their electricity bills, this reduction helps utilities to smooth the demand profile during peak periods [4]. While the second problem of the traditional power grid is using few sensors on the lines, the third problem is centralized generation structure (only central power stations can generate power) [4]. Since centralized generation leads to long distance transportation of the power, it requires to use long lines to distribute electricity. Placing few sensors on long lines decreases the capability of the system to react rapidly to changes since identifying the location of the problem takes long time. As the system is not capable of tolerating changes rapidly, intermittent nature of renewable sources poses a danger for the traditional power grid [7]. To make the electric energy system suitable to the current development of renewable energies, the transition to the smart grid aims to increase the number of sensors on the lines and apply distributed generation. In distributed generation, in addition to substations, thanks to bidirectional flow of power, customers can act like power stations (power flow can be from customers to the grid when needed). As the number of power stations increases and the power generated by customers is integrated into the grid, distributed generation shortens the transportation distance of power. Also, thanks to multiple sensors on the lines, when a problem occurs, identifying the location and defining a new route to reduce the affected areas become easier in the smart grid. These changes make the electric energy system suitable to the current development of renewable energies as they shorten the reaction time of the system to unexpected changes and integrate customers into the grid through two-way communication.

Integration of customers and renewable energy into the smart grid is promoted by Demand Side Managament (DSM) which is an important component of the smart grid [6]. DSM can be defined as all activities aiming to shift the customers' demand to keep demand and supply balanced and to incorporate renewable energy sources into the grid [8]. There are different activities within the scope of

DSM which aim a reliable and efficient system such as Demand Response (DR). Whereas most of the activities in the DSM scope focus on long-term effects, the effect of DR on the system can be observed in the short run. DR is defined as managing the energy consumption by providing information to customers who can change their consumption patterns with respect to the given information. DR programs aim to smooth the demand profile by motivating customers to reduce their consumption during peak periods or increase during periods with high production and low demand [9]. Since the interaction between utilities and customers are provided throughout DR programs, it becomes essential for the balance between supply and demand. Customers can be classified as large commercial and industrial, small commercial, industrial and residential with respect to the amount of consumption [9]. Since participating in DR programs requires some investments for monitoring and control technologies, in the past, customers with small consumption like residential customers were not encouraged to invest in DR programs, however, this situation changes with the lower-cost equipments provided by the development of technology [9].

Residential energy consumption grows with the population growth accompanied by fewer occupants living in larger houses, the change in people's life style and a higher rate of utilization of electrical appliances. Since some of the electrical appliances, like plug-in hybrid vehicles, have a potential to double residential energy consumption, participation of residential customers into DR programs becomes more crucial. Due to the increase in the energy consumption and the price of electricity, residents are motivated to schedule their energy consumption to reduce their cost of electricity and their negative impact on the environment. With an installation of home energy management system (HEMS) including demand response program, residents can participate in the smart grid system. They can monitor price information through the smart meters and control their own appliances. By the end of 2016, 47% of all residential consumers in U.S. had a smart meter in their homes [10]. Since smart meters provide information about the fluctuation of electricity price between low-demand and high-demand hours, with the help of HEMS, residents can shift their demands to the off-peak hours or hours with high renewable energy levels. Thus, the main objective of HEMS is to reduce the electricity bill of the resident both by scheduling the electrical appliances and increasing the use of renewable energy. To increase the benefits of smart meters, the EU member countries decided to provide smart meters to at least 80% of all residential consumers by 2020 [11]. The increase in the number of residents with smart meters demonstrates the importance of demand response management and the consequent need to develop a method for efficient energy consumption scheduling.

In this study, we propose a multi-stage stochastic nonlinear mixed integer programming model that coordinates different types of appliances in a residential unit in the presence of weather uncertainty. By considering the uncertainty in weather conditions, we aim to model accurately both the energy consumption of the appliances that control the temperature and the availability of the renewable energy. The objective of our model is to minimize the expected electricity cost and the residents' expected dissatisfaction. We use a scenario groupwise decomposition (group subproblem) approach to obtain tight lower and upper bounds for instances with large number of scenarios. In our computational experiments, we analyze the benefits of optimization in demand response and the savings due to renewable energy and battery for residents with different price sensitivity.

The rest of this study is organized as follows. In Chapter 2, we review the relevant studies in the literature. In Chapter 3, we describe the energy consumption characteristics and disutility functions for different appliance types and the electricity pricing scheme. In Chapter 4, we present our model. In Chapter 5, we describe the scenario groupwise decomposition approach and present two different ways to construct group subproblems. We present the results of our computational experiments in Chapter 6. We conclude in Chapter 7.

Chapter 2

Literature Review

In this chapter, we review the literature about demand response programs for home energy management in the smart grid. Since DR programs provide interaction between supply and demand sides, their efficiency has a crucial effect on the capability of the smart grid to balance supply and demand, which results in DR programs becoming an interesting topic for research. Detailed surveys about DR programs are given in [9], [12] and [13]. To increase the efficiency of DR programs, they are generally used with other DSM programs like real-time pricing (RTP). In RTP programs, electricity prices vary at different hours and users are expected to shift their consumption regarding to time-varying prices to reduce their electricity cost. However, since users are not fully knowledgable about how to shift their consumption, they need a system to better utilize the benefits of RTP tariffs. The studies mentioned in this chapter propose scheduling schemes to users to shift their demand where RTP is applied.

The existing studies vary in the types of electrical appliances they consider. Based on the energy consumption characteristics, the appliances can be classified under two major types: appliances with continuous level energy consumption and appliances with discrete level energy consumption. While the energy consumption amount of the appliances with continuous level energy consumption can vary during the operation, this amount is fixed for the appliances with discrete level energy consumption. In [14], energy consumption scheduling scheme for appliances with continuous level energy consumption is proposed for a distributed generation system. They use a game theoretic approach to formulate the problem where the customers are players whose strategy is to schedule their appliances with the aim of minimizing total energy cost. They consider a case where multiple customers use the same utility company. They show that when smart pricing tariffs they propose (based on a game theoretic analysis) are used, the optimal solution is at the Nash equilibrium point of the energy consumption game between the users. They extend this study in [15] by also considering the peak-to-average ratio (PAR) minimization problem. They show that two problems are related to each other: strategies which result with less total energy cost leads to lower PAR values as well. They also provide simulation results which validate that the optimal strategy of the energy consumption game reduces the PAR, the energy cost, and the users' electricity cost.

Residential energy consumption scheduling scheme with an objective of minimizing electricity cost and waiting time is proposed in [3]. In that study, a combination of RTP and inclining block rates (IBR) is used for pricing. In IBR, electricity is charged by a higher price beyond a certain threshold. They consider the scheduling of appliances both with continuous and discrete level energy consumption under two cases. In the first case, all future price parameters within the scheduling horizon are known by the user before determining the schedule. For the second case, users know the future price parameters for the first few slots. To handle the unknown price parameters, they add a price prediction capability to their model. The simulation results demonstrate that the proposed method reduces both users' electricity payment and the PAR.

The study [16] focuses on a setting where electricity is distributed from one utility to different users. They consider the schedule of appliances with continuous level energy consumption with the aim of minimizing the summation of disutility and electricity cost of users in a way that total consumption does not exceed the generated electricity amount.

The study [17] works on the scheduling of household appliances with continuous level energy consumption. As different from other studies in the literature, they consider the case where the load of appliances with continuous level energy consumption is interruptible. They propose both continuous and discrete time formulation minimizing the electricity cost and user's dissatisfaction. They show that although their resulting formulation is non-convex, the continuous time formulation has zero duality gap. Therefore, Lagrangian algorithms can be used to solve it.

The energy consumption of appliances that control the temperature of the environment comprised 40% of total residential energy consumption [18]. This points out the need of scheduling this type of appliances as done by [19] - [20]. However, these studies ignore the randomness in weather conditions and model the problem in a deterministic setting. Since the consumption of these appliances constitutes a big part of total consumption, [21] focuses only on the schedule of air conditioner/heater and water heater with the aim of minimizing the electricity cost and users' discomfort. In addition to appliances that control the temperature of the environment, [19] considers the schedule of appliances both with continuous (including other types of continuous level consumption) and discrete level energy consumption with the aim of minimizing the electricity cost and maximizing users' comfort under the combination of RTP and IBR tariffs. Different from other studies mentioned here, instead of using a weighted combination of different objectives, they minimize the electricity cost while forcing users' comfort to be greater than a threshold value and solve the model by using an iterative algorithm. Algorithm works till the cost value drops below the desired cost value defined by users or when users' comfort equals to zero. In each iteration, they change the threshold value by a small decrement.

Although one of the main purposes of HEMS is to better utilize the renewable energy, to the best of our knowledge, the only study that considers renewable energy in this context is [20]. They consider a single household equipped with appliances both with discrete and continuous level energy consumption. They also incorporate the appliances with internal batteries to the system. Since renewable energy must be consumed at the moment it is produced unless it is stored, using such kind of appliances helps to increase the use of renewable energy. They propose an L_1 regularization technique to deal with the binary variables used to model appliances with discrete level energy consumption. Since they relax binary variables, the reformulated problem is a convex programming problem instead of a mixed integer problem. Incorporation of the appliances with internal batteries is also studied in [22]. In that study, the role of these appliances is to reduce the peak load. They consider the system consisting of one utility company and multiple users. They assume that the utility company is regulated so its main aim is to maximize social welfare rather than its own profit. They propose two different models for utility company and users. They show that if the company uses dynamic pricing, when users schedule their appliances in a way that optimizes their own benefits, social optimality is also achieved.

In Figure 2.1 the classification of the studies which are explained previously is given. In this study, we propose a multi-stage stochastic nonlinear mixed integer programming model that coordinates all types of appliances mentioned above in the presence of weather uncertainty. In particular, our major contributions are the following:

- We propose a novel model to schedule all types of electrical appliances considered in the literature in the presence of renewable energy and batteries.
- We use multi-stage stochastic programming to hedge against the uncertainty in weather conditions, which is critical to model accurately both the energy consumption of the appliances that control the temperature and the availability of the renewable energy.
- Our model is a multi-stage stochastic nonlinear mixed integer programming model. We use a scenario groupwise decomposition (group subproblem) approach to obtain tight lower and upper bounds for instances with large number of scenarios.
- Our computational experiments provide valuable insights on the benefits of optimization in demand response and the savings due to renewable energy and battery for residents with different price sensitivity.

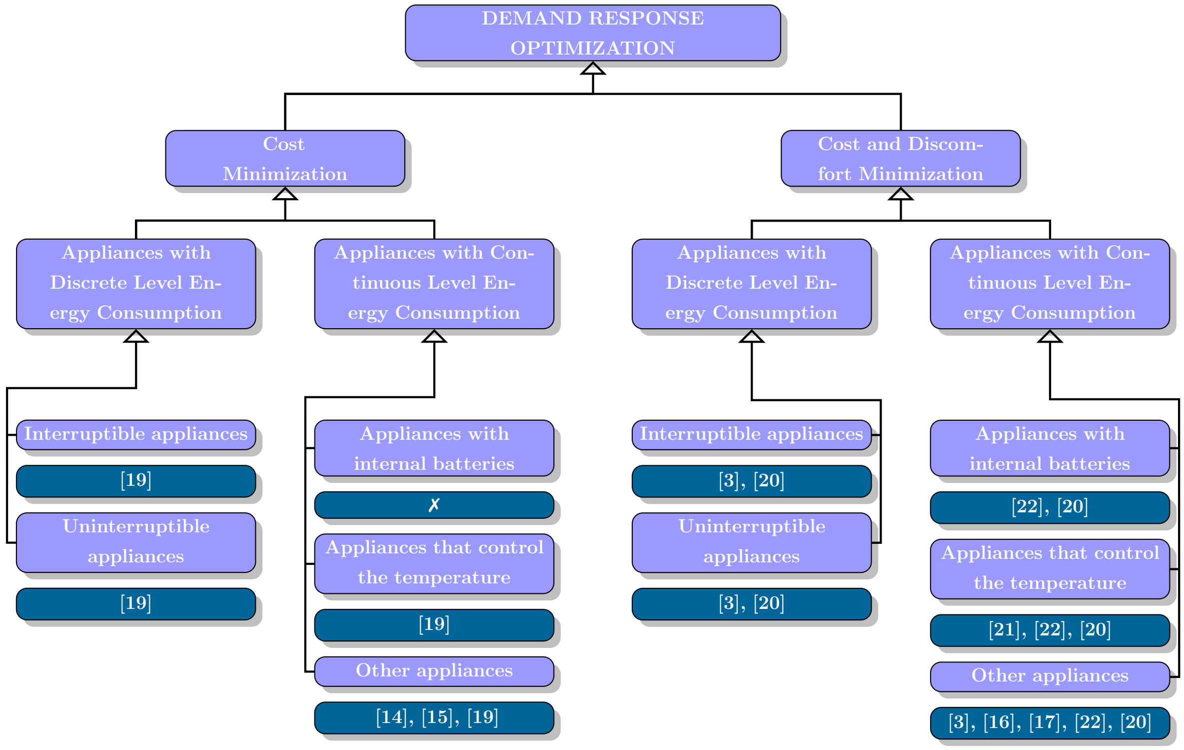


Figure 2.1: Literature comparison

Chapter 3

Problem Setting

We consider a smart home equipped with electrical appliances that are networked together and controlled by Home Energy Management System (HEMS). The communication of the price information between the energy provider and the resident is provided by the smart meter.

We consider a discrete-time model with a finite horizon, where scheduling horizon is divided into time slots of equal duration. The energy consumption scheduling problem aims to achieve a trade-off between minimizing the electricity cost and minimizing the residents' dissatisfaction due to loss of comfort.

3.1 Types of Appliances

Let $\mathcal{A}$ denote the set of appliances networked in this residential unit. This set includes appliances with both continuous and discrete level energy consumption. An appliance a with discrete level energy consumption only operates in on or off status and it consumes a fixed energy level of $\overline{e}_a$ and $\underline{e}_a$ in on and off modes, respectively. Consequently, HEMS decides only when this appliance should be in on mode. On the other hand, for an appliance with continuous level energy consumption, HEMS needs to decide how much energy it consumes in each time period. Each appliance a operates within a user's preferred time interval T_a which includes all integers between $\underline{t}_a - 1$ and $\overline{t}_a + 1$.

We classify the appliances with continuous level energy consumption under three categories as follows:

1) *Type 1* : This type of appliances are equipped with internal batteries. Since they have an ability to store or dispatch energy, they better utilize the renewable energy sources. We denote the set of appliances with battery by $\mathcal{A}_{\mathcal{B}}$.

2) *Type 2* : This type of appliances control the temperature of the environment. An example is air conditioner. We denote the set of such appliances that control the temperature of the environment by $\mathcal{A}_{\mathcal{T}}$.

3) *Type 3* : This type includes the appliances with continuous level energy consumption that are not of *Type 1* and *Type 2*. Let $\mathcal{A}_{\mathcal{C}}$ denote the set of appliances of *Type 3*. An example of this type is refrigerator.

We classify the appliances with discrete level energy consumption as follows:

1) *Type 4* : This type of appliances can be shut down during operation. This means that their loads are interruptible. An example is a hair dryer. We denote the set of appliances with interruptible load by $\mathcal{A}_{\mathcal{I}}$.

2) *Type 5* : For this type of appliances, once the operation starts, it should be run to completion. An example is a washing machine. We denote the set of appliances with uninterruptible load by $\mathcal{A}_{\mathcal{U}}$.

3.2 Disutility Functions

The disutility can have different causes depending on the types of appliances. For an appliance that controls the temperature, the disutility is a result of the discomfort due to low or high inside temperature, whereas for an appliance with discrete consumption, the disutility is a function of earliness of the starting time or the lateness of the finishing time.

The disutility functions for different types of appliances are as follows:

- **Appliances with discrete level energy consumption (types 4 and 5):** For each appliance $a \in \mathcal{A}_{\mathcal{I}} \cup \mathcal{A}_{\mathcal{U}}$ with discrete level energy consumption, the important decisions are starting and finishing times. The disutility for appliance a if it starts operating at time $\hat{\underline{t}}_a$ and finishes at time $\hat{\overline{t}}_a$ is given by $\gamma_a(\hat{\underline{t}}_a, \hat{\overline{t}}_a)$, which is computed as:

$$\gamma_a(\hat{\underline{t}}_a, \hat{\overline{t}}_a) = \underline{\phi}_D\big((\underline{t}_a^{des} - \hat{\underline{t}}_a)_+\big)^2 + \overline{\phi}_D\big((\hat{\overline{t}}_a - \overline{t}_a^{des})_+\big)^2$$

  where $\underline{t}_a^{des}$ and $\overline{t}_a^{des}$, respectively, are the most desirable starting and finishing times and $\underline{\phi}_D$ and $\overline{\phi}_D$, respectively, are the weights of disutility for the earliness of the starting time and the lateness of the finishing time. Note that $(x)_+ := \max\{x, 0\}$. The function $\gamma_a(\hat{\underline{t}}_a, \hat{\overline{t}}_a)$ penalizes starting before and/or ending after the desired times.

- **Appliances that control temperature (type 2):** Let h_a^{comf} be the most comfortable temperature and $h_{a,t}^{in}$ be the inside temperature at the location of appliance $a \in \mathcal{A}_{\mathcal{T}}$ in time period $t \in T_a$ and $\tau_{a,t}(h_{a,t}^{in})$ be the disutility at time t for appliance a. We use the disutility function used in [22] and [20]:

$$\tau_{a,t}(h_{a,t}^{in}) = (h_{a,t}^{in} - h_a^{comf})^2.$$

- **Appliances with battery (type 1):** As in [22] and [20], the disutility $\psi_{a,t}(r_{a,t}, b_{a,t})$ for appliance $a \in \mathcal{A}_{\mathcal{B}}$ with battery depends on $r_{a,t}$, which is the power charged to (when $r_{a,t} \geq 0$) or discharged from (when $r_{a,t} \leq 0$) the appliance in period $t \in T_a$ and $b_{a,t}$, which is the energy level of the appliance a in period t. The disutility is computed as

$$\psi_{a,t}(r_{a,t}, r_{a,t+1}, b_{a,t}) = \eta_1(r_{a,t})^2 - \eta_2 r_{a,t} r_{a,t+1} + \eta_3((\delta \overline{b}_a - b_{a,t})_+)^2,$$

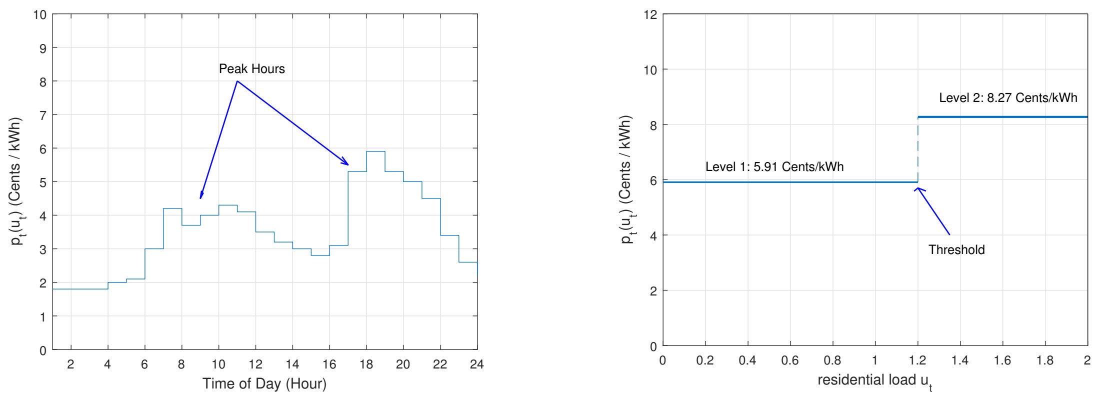
  where η_1, η_2, η_3, and δ are positive constants and $\overline{b}_a$ is the capacity of battery for appliance a. The first term penalizes the damaging effect of charging and discharging, the second term penalizes charging-discharging cycles and the third term penalizes deep discharging, which happens when the energy level of the battery drops below $\delta\bar{b}_a$. The value of δ may vary according to the type of battery.

3.3 Electricity Price

Since electricity is perishable, the prices set by the electricity retailers fluctuate between low-demand hours and high-demand hours. The most common pricing schemes are *real-time pricing* (RTP), *day-ahead pricing* (DAP), *time-of-use pricing* (TOUP), *critical-peak pricing* (CPP) and *inclining block rates* (IBR). ([3]; see for further references). In our study, as in [3] and [23], we consider a general hourly pricing function that combines RTP and IBR schemes. It also has a DAP structure as we assume that the future price parameters are known by the residents ahead of time. In our price function, prices vary every hour as in RTP, and beyond a certain threshold for total hourly residential load, the electricity price increases to a higher value as in IBR. The cost $\rho_t(u_t)$ of consuming u_t units of electricity in period t depends on the low cost c_t^l, the high cost c_t^h and the threshold $\overline{u}$ as follows:

$$\rho_t(u_t) = \begin{cases} c_t^l u_t & , \text{ if } 0 \leq u_t \leq \overline{u} \\ c_t^h u_t & , \text{ if } u_t > \overline{u}. \end{cases}$$

In numerical studies, price data of Illinois Power Company and British Columbia (BC) Hydro Company are used (Figure 1).



(a) Real-time prices set by Illinois Power Company

(b) Two-level inclining block rates set by BC Hydro

Figure 3.1: Examples of two-non flat pricing models. The real time prices are used by Illinois Power Company on December 15 [1]. The inclining block rates are used by BC Hydro Company in December [2]. (This figure is reproduced from [3].)

Chapter 4

Multi-stage Stochastic Programming Model

In this chapter, we formulate the scheduling problem for smart home appliances as a multi-stage stochastic nonlinear mixed integer program. We assume that the changes in the temperature and in the power of renewable energy are uncertain and this uncertainty is realized gradually at specific time periods called as stages. We also assume that the random temperature and random renewable energy in each stage have discrete distributions with finite number of realizations. Therefore, the uncertainty in the decision process can be represented by a scenario tree. A scenario is defined as a unique path from the root node to a terminal node. Consequently, the number of scenarios is equal to the number of terminal nodes.

We denote the set of all scenarios by $\mathcal{S}$ and the set of scenarios having the same history as scenario s up to stage t by $\mathcal{S}_{s,t}$. The probabiliy of scenario s is represented by $p(s)$ and the scheduling horizon is represented by T. We use the following notation:

Decision Variables

- $u_t^l(s)$: the net energy consumption at low price in period $t \in T$ under scenario $s \in \mathcal{S}$
- $u_t^h(s)$: the net energy consumption at high price in period $t \in T$ under scenario $s \in \mathcal{S}$
- $b_{a,t}(s)$: the energy level of appliance $a \in \mathcal{A}_{\mathcal{B}}$ in period $t \in T_a$ under scenario $s \in \mathcal{S}$
- $r_{a,t}(s)$: the power charged/discharged for appliance $a \in \mathcal{A}_{\mathcal{B}}$ in period $t \in T_a$ under scenario $s \in \mathcal{S}$
- $e_{a,t}(s)$: the energy consumption of appliance $a \in \mathcal{A} \setminus \mathcal{A}_{\mathcal{B}}$ in period $t \in T_a$ under scenario $s \in \mathcal{S}$
- $e^+_{a,t}(s)$: the energy consumption for heating for appliance $a \in \mathcal{A}_{\mathcal{T}}$ in period $t \in T_a$ under scenario $s \in \mathcal{S}$
- $e^-_{a,t}(s)$: the energy consumption for cooling for appliance $a \in \mathcal{A}_{\mathcal{T}}$ in period $t \in T_a$ under scenario $s \in \mathcal{S}$
- $x_{a,t}(s)$: $\begin{cases} 1, & \text{if appliance } a \in \mathcal{A}_{\mathcal{U}} \cup \mathcal{A}_{\mathcal{I}} \text{ starts operationn period } t \in T_a \\ & \text{under scenario } s \in \mathcal{S} \\ 0, & \text{otherwise} \end{cases}$
- $y_{a,t}(s)$: $\begin{cases} 1, & \text{if appliance } a \in \mathcal{A}_{\mathcal{I}} \text{ is in on statue in period } t \in T_a \\ & \text{under scenario } s \in \mathcal{S} \\ 0, & \text{otherwise} \end{cases}$
- $z_{a,t}(s)$: $\begin{cases} 1, & \text{if appliance } a \in \mathcal{A}_{\mathcal{I}} \text{ completes operation in period } t \in T_a \\ & \text{under scenario } s \in \mathcal{S} \\ 0, & \text{otherwise} \end{cases}$
- $w_t(s)$: $\begin{cases} 1, & \text{if energy usage is charged at low price in period } t \in T \\ & \text{under scenario } s \in \mathcal{S} \\ 0, & \text{otherwise} \end{cases}$

- $d_{a,t}(s)$: $(\delta\overline{b}_a - b_{a,t})^+$ deep discharging amount for appliance $a \in \mathcal{A}_{\mathcal{B}}$ in period $t \in T_a$ under scenario $s \in \mathcal{S}$

- $h^{in}_{a,t}(s)$: the temperature inside the place of appliance $a \in \mathcal{A}_{\mathcal{T}}$ in period $t \in T_a$ under scenario $s \in \mathcal{S}$

Parameters

- e^{tot}_a : the total energy required for appliance $a \in \mathcal{A}_{\mathcal{C}}$
- $b_{a,0}$: the initial energy level of the battery for appliance $a \in \mathcal{A}_{\mathcal{B}}$
- $\overline{r}_a$: the maximum charging amount for appliance $a \in \mathcal{A}_{\mathcal{B}}$
- $\underline{r}_a$: the maximum discharging amount for appliance $a \in \mathcal{A}_{\mathcal{B}}$
- $\underline{e}_a$: the minimum energy level required for appliance $a \in \mathcal{A} \setminus \mathcal{A}_{\mathcal{B}}$
- $\overline{e}_a$: the maximum energy level required for appliance $a \in \mathcal{A} \setminus \mathcal{A}_{\mathcal{B}}$
- $\overline{b}_a$: the battery capacity for appliance $a \in \mathcal{A}_{\mathcal{B}}$
- $\underline{h}_a$: the minimum comfortable temperature level for appliance $a \in \mathcal{A}_{\mathcal{T}}$
- $\overline{h}_a$: the maximum comfortable temperature level for appliance $a \in \mathcal{A}_{\mathcal{T}}$
- h^{comf}_a : the most comfortable temperature level for appliance $a \in \mathcal{A}_{\mathcal{T}}$
- $v_t(s)$: the power of renewable energy source in period t $\in T$ under scenario $s \in \mathcal{S}$

- n_a : the total number of time slots required for appliance $a \in \mathcal{A}_{\mathcal{I}} \cup \mathcal{A}_{\mathcal{U}}$ to complete its task

- $h^{out}_{a,t}(s)$: the temperature outside the location of appliance $a \in \mathcal{A}_{\mathcal{T}}$ in period $t \in T_a$ under scenario $s \in \mathcal{S}$

- $\underline{t}^{des}_a$: the most desirable starting time for appliance $a \in \mathcal{A}_{\mathcal{I}} \cup \mathcal{A}_{\mathcal{U}}$
- $\overline{t}^{des}_a$: the most desirable finishing time for appliance $a \in \mathcal{A}_{\mathcal{I}} \cup \mathcal{A}_{\mathcal{U}}$

- $h^{in}_{a,0}(s)$: the initial room temperature for appliance $a \in \mathcal{A}_{\mathcal{T}}$ under scenario $s \in \mathcal{S}$
- α_a, β_a : the thermal characteristics of appliance $a \in \mathcal{A}_{\mathcal{T}}$ and the environment in which appliance operates ($\beta_a > 0$)
- ϕ : the weight of disutility function
- $\underline{\phi}_D$: the weight of disutility for the earliness of the starting time for appliances in $\mathcal{A}_{\mathcal{I}} \cup \mathcal{A}_{\mathcal{U}}$
- $\overline{\phi}_D$: the weight of disutility for the lateness of the finishing time for appliances in $\mathcal{A}_{\mathcal{I}} \cup \mathcal{A}_{\mathcal{U}}$
- ϕ_T : the weight of disutility function for appliances in $\mathcal{A}_{\mathcal{T}}$
- ϕ_B : the weight of disutility function for appliances in $\mathcal{A}_{\mathcal{B}}$

An efficient energy consumption schedule can be obtained by solving the following model:

$$\begin{aligned}\mathbf{min} \sum_{s\in\mathcal{S}} p(s)\Bigg(& \sum_{t\in T}\Big(c^l_t u^l_t(s) + c^h_t u^h_t(s)\Big) + \phi\Bigg(\sum_{a\in\mathcal{A}_{\mathcal{I}}\cup\mathcal{A}_{\mathcal{U}}}\Bigg(\underline{\phi}_D \sum_{t=\underline{t}_a}^{\underline{t}^{des}_a}(t-\underline{t}^{des}_a)^2 x_{a,t}(s) \\ & + \overline{\phi}_D \sum_{t=\overline{t}^{des}_a}^{\overline{t}_a}(t-\overline{t}^{des}_a)^2 z_{a,t}(s)\Bigg) + \phi_T \sum_{a\in\mathcal{A}_{\mathcal{T}}}\sum_{t\in T_a}\Big(h^{in}_{a,t}(s) - h^{comf}_a\Big)^2 \\ & + \phi_B \sum_{a\in\mathcal{A}_{\mathcal{B}}}\sum_{t\in T_a}\Big(\eta_1(r_{a,t}(s))^2 - \eta_2 r_{a,t}(s) r_{a,t+1}(s) + \eta_3(d_{a,t}(s))^2\Big)\Bigg)\Bigg)\end{aligned} \quad (4.1)$$

s.t.

$$\underline{e}_a \le e_{a,t}(s) \le \overline{e}_a, \qquad \forall a \in \mathcal{A}_{\mathcal{C}}, t \in T_a, s \in \mathcal{S} \quad (4.2)$$

$$\sum_{t\in T_a} e_{a,t}(s) = e^{tot}_a, \qquad \forall a \in \mathcal{A}_{\mathcal{C}}, s \in \mathcal{S} \quad (4.3)$$

$$\underline{e}_a \le e^+_{a,t}(s) \le \overline{e}_a, \qquad \forall a \in \mathcal{A}_{\mathcal{T}}, t \in T_a, s \in \mathcal{S} \quad (4.4)$$

$$\underline{e}_a \le e^-_{a,t}(s) \le \overline{e}_a, \qquad \forall a \in \mathcal{A}_\mathcal{T}, t \in T_a, s \in \mathcal{S} \tag{4.5}$$

$$e_{a,t}(s) = e^-_{a,t}(s) + e^+_{a,t}(s), \qquad \forall a \in \mathcal{A}_\mathcal{T}, t \in T_a, s \in \mathcal{S} \tag{4.6}$$

$$\begin{aligned} h^{in}_{a,t}(s) = h^{in}_{a,t-1}(s) + \alpha_a\big(h^{out}_{a,t}(s) - h^{in}_{a,t-1}(s)\big) \\ + \beta_a\big(e^+_{a,t}(s) - e^-_{a,t}(s)\big), \end{aligned} \qquad \forall a \in \mathcal{A}_\mathcal{T}, t \in T_a, s \in \mathcal{S} \tag{4.7}$$

$$\underline{h}_a \le h^{in}_{a,t}(s) \le \overline{h}_a, \qquad \forall a \in \mathcal{A}_\mathcal{T}, t \in T_a, s \in \mathcal{S} \tag{4.8}$$

$$b_{a,t}(s) = \sum_{u=1}^{t} r_{a,u}(s) + b_{a,0}, \qquad \forall a \in \mathcal{A}_\mathcal{B}, t \in T_a, s \in \mathcal{S} \tag{4.9}$$

$$0 \le b_{a,t}(s) \le \overline{b}_a, \qquad \forall a \in \mathcal{A}_\mathcal{B}, t \in T_a, s \in \mathcal{S} \tag{4.10}$$

$$\underline{r}_a \le r_{a,t}(s) \le \overline{r}_a, \qquad \forall a \in \mathcal{A}_\mathcal{B}, t \in T_a, s \in \mathcal{S} \tag{4.11}$$

$$d_{a,t}(s) \ge \delta\,\overline{b}_a - b_{a,t}(s), \qquad \forall a \in \mathcal{A}_\mathcal{B}, t \in T_a, s \in \mathcal{S} \tag{4.12}$$

$$d_{a,t}(s) \ge 0, \qquad \forall a \in \mathcal{A}_\mathcal{B}, t \in T_a, s \in \mathcal{S} \tag{4.13}$$

$$r_{a,\bar{t}_a+1}(s) = 0, \qquad \forall a \in \mathcal{A}_\mathcal{B}, s \in \mathcal{S} \tag{4.14}$$

$$e_{a,t}(s) = \underline{e}_a + (\overline{e}_a - \underline{e}_a)y_{a,t}(s), \qquad \forall a \in \mathcal{A}_\mathcal{I}, t \in T_a, s \in \mathcal{S} \tag{4.15}$$

$$\sum_{t \in T_a} y_{a,t}(s) = n_a, \qquad \forall a \in \mathcal{A}_\mathcal{I}, s \in \mathcal{S} \tag{4.16}$$

$$\sum_{u=t+1}^{\bar{t}_a - n_a + 1} x_{a,u}(s) + y_{a,t}(s) \le 1, \qquad \forall a \in \mathcal{A}_\mathcal{I}, t \in [\underline{t}_a, \bar{t}_a - n_a], \; s \in \mathcal{S} \tag{4.17}$$

$$\sum_{u=\underline{t}_a + n_a - 1}^{t-1} z_{a,u}(s) + y_{a,t}(s) \le 1, \qquad \forall a \in \mathcal{A}_\mathcal{I}, t \in [\underline{t}_a + n_a, \bar{t}_a],$$

$$s \in \mathcal{S} \quad (4.18)$$

$$\sum_{t=\underline{t}_a+n_a-1}^{\bar{t}_a} z_{a,t}(s) = 1, \qquad \forall a \in \mathcal{A}_{\mathcal{I}}, s \in \mathcal{S} \qquad (4.19)$$

$$\sum_{t=\underline{t}_a}^{\bar{t}_a-n_a+1} x_{a,t}(s) = 1, \qquad \forall a \in \mathcal{A}_{\mathcal{I}} \cup \mathcal{A}_{\mathcal{U}}, s \in \mathcal{S} \qquad (4.20)$$

$$e_{a,t}(s) = \underline{e}_a + (\overline{e}_a - \underline{e}_a)\Big(\sum_{u=\max\{\underline{t}_a, t-n_a+1\}}^{t} x_{a,u}(s)\Big), \qquad \forall a \in \mathcal{A}_{\mathcal{U}}, t \in T_a, s \in \mathcal{S} \qquad (4.21)$$

$$u_t^l(s) + u_t^h(s) \geq \sum_{a \in \mathcal{A} \setminus \mathcal{A}_{\mathcal{B}}} e_{a,t}(s) + \sum_{a \in \mathcal{A}_{\mathcal{B}}} r_{a,t}(s) - v_t(s), \quad \forall t \in T, s \in \mathcal{S} \qquad (4.22)$$

$$0 \leq u_t^l(s) \leq \overline{u}\, w_t(s), \qquad \forall t \in T, s \in \mathcal{S} \qquad (4.23)$$

$$0 \leq u_t^h(s) \leq M(1 - w_t(s))\,, \qquad \forall t \in T, s \in \mathcal{S} \qquad (4.24)$$

$$e_{a,t}(s') = e_{a,t}(s), \qquad \forall a \in \mathcal{A} \setminus \mathcal{A}_{\mathcal{B}}, s \in \mathcal{S}, s' \in \mathcal{S}_{s,t}, t \in T_a \quad (4.25)$$

$$e_{a,t}^+(s') = e_{a,t}^+(s), \qquad \forall a \in \mathcal{A}_{\mathcal{T}}, s \in \mathcal{S}, s' \in \mathcal{S}_{s,t}, t \in T_a \quad (4.26)$$

$$e_{a,t}^-(s') = e_{a,t}^-(s), \qquad \forall a \in \mathcal{A}_{\mathcal{T}}, s \in \mathcal{S}, s' \in \mathcal{S}_{s,t}, t \in T_a \quad (4.27)$$

$$r_{a,t}(s') = r_{a,t}(s), \qquad \forall a \in \mathcal{A}_{\mathcal{B}}, s \in \mathcal{S}, s' \in \mathcal{S}_{s,t}, t \in T_a \quad (4.28)$$

$$x_{a,t}(s) \in \{0,1\}, \qquad \forall a \in \mathcal{A}_{\mathcal{I}} \cup \mathcal{A}_{\mathcal{U}}, t \in T_a, s \in \mathcal{S} \qquad (4.29)$$

$$y_{a,t}(s) \in \{0,1\}, \qquad \forall a \in \mathcal{A}_{\mathcal{I}}, t \in T_a, s \in \mathcal{S} \qquad (4.30)$$

$$z_{a,t}(s) \in \{0,1\}, \qquad \forall a \in \mathcal{A}_{\mathcal{I}}, t \in T_a, s \in \mathcal{S} \qquad (4.31)$$

$$w_t(s) \in \{0,1\} \qquad \forall t \in T, s \in \mathcal{S} \qquad (4.32)$$

The objective function (4.1) is the sum of expected electricity cost and expected disutility.

For appliances of *Type 3* (appliances with continuous consumption), constraints (4.2) and (4.3) ensure that energy consumption is between minimum standby power level and maximum power level and that the total energy requirement is provided within the specified time interval, respectively.

Constraints (4.4) - (4.8) are given for appliances of *Type 2* (appliances that control temperature). As above, constraints (4.4) - (4.6) ensure that energy consumption is between minimum and maximum levels. Constraints (4.7) are the balance equations for inside temperature and energy consumption. Constraints (4.8) ensure that the temperature is in the range that user defines as comfortable.

Constraints (4.9) - (4.14) are for the appliances of *Type 1* (appliances with battery). Constraints (4.9) compute the battery energy level at each period. Constraints (4.10) ensure that the total charge stored in the battery does not exceed its capacity. Constraints (4.11) bound the battery charging/discharging amount. Constraints (4.12) and (4.13) compute the amount of deep discharging. Constraints (4.14) ensure that disutility does not occur outside the specified time interval.

Constraints (4.15) - (4.20) are for appliances of *Type 4* (interruptible appliances with discrete consumption). Constraints (4.15) compute the energy consumption: if the appliance is off, the energy consumption is equal to the minimum standby power level and if it is on, the energy consumption amount equals the maximum power level. Constraints (4.16) ensure that the appliance is on for the number of periods required to fully complete its task. Constraints (4.17) and (4.18) allow operation of an appliance only between its starting and finishing time. Constraints (4.19) and (4.20) ensure that there is one starting and one finishing time for each appliance.

Constraints (4.20) and (4.21) are for appliances of *Type 5* (uninterruptible appliances with discrete consumption). One starting time is chosen for each appliance due to constraints (4.20). Once an appliance starts, it operates consecutively for the required number of periods. Constraints (4.21) compute the energy consumption amount.

Constraints (4.22) - (4.24) compute the energy consumption amount at low and high prices. Constraints (4.22) give the relation between the net energy request, energy consumption amount, and renewable energy amount. Constraints (4.23) and (4.24) decide whether the energy usage is charged at low price with regard to threshold value $\overline{u}$. The value of big M is taken to be equal to the sum of the maximum energy level of all appliances that are not of *Type 1* (appliances with battery) and the maximum charging amount of appliances with battery.

Constraints (4.25) - (4.28) are non-anticipativity constraints to ensure that for any stage t, decision variables that have a common history of uncertainty till stage t must have the same value at that stage.

Finally, constraints (4.29) - (4.32) are variable restrictions.

Chapter 5

Solution Method

Multi-stage stochastic programming problems are in the framework of extremely challenging problems since their size grows exponentially with the number of stages. Our problem has additional difficulties due to nonlinear objective function and integer variables. In the literature, some exact solution techniques exist for the multi-stage stochastic programming problems such as stochastic dual dynamic programming (SDDP) [24] and Lagrangian relaxation of non-anticipativity constraints [25]. Since both techniques require convexity to obtain an exact solution, they are not applicable for the problems with integer variables. However, a lower bound can be obtained for multi-stage stochastic integer problems using these methods. An extension of SDDP method proposed by [26] makes the method applicable for problems with integer variables by enforcing the integrality requirements only in the forward steps and relaxing in the backward steps. If the state variables are binary and the cuts satisfy some sufficient conditions, [27] proves that SDDP can provide an exact solution for the mixed integer multi-stage stochastic problems. However, even so, SDDP cannot be used in our problem since it also requires the assumption of stagewise independency in the stochastic process. [28] demonstrates that Lagrangian relaxation of non-anticipativity constraints within a branch and bound procedure can be used to obtain an exact solution for multi-stage stochastic integer problems. However, since complete recourse assumption is a requirement for the branch and bound procedure, this method is also not applicable for our problem. Besides these techniques, scenario groupwise decomposition approach which is recently proposed by [29] can be used to obtain bounds for multi-stage stochastic integer problems. In this approach, the problem is divided into smaller problems called as group subproblems. Group subproblems are defined over a reduced number of scenarios and each of them has the same number of stages as the original problem. The problem is solved for each group subproblem separately rather than being solved for all scenarios simultaneously. Besides easy implementation, since it does not rely on restrictive assumptions such as stagewise independency, complete recourse and convexity, scenario groupwise decomposition is applicable for a wide range of problems including our problem. Group subproblem approach is used in the literature to calculate lower and upper bounds for multistage stochastic programs [30] and [31].

5.1 A Lower Bound

In this section, we provide a lower bound on the optimal value of our problem using the scenario groupwise decomposition approach of [29]. In this method, a lower bound is obtained by solving the problem separately for subsets of the scenarios called as groups. We define the set of groups as a partition of $\mathcal{S}$. Note that $\mathcal{S}$ denotes the finite set of all scenarios. Let $\hat{\mathcal{S}}$ be the set of groups and $I_{\hat{\mathcal{S}}}$ be the set of indices of groups in $\hat{\mathcal{S}}$. A collection $\hat{\mathcal{S}} = (\hat{\mathcal{S}}_i)_{i \in I_{\hat{\mathcal{S}}}}$ of subsets of $\mathcal{S}$ is called scenario partition if it satisfies $\cup_{i \in I_{\hat{\mathcal{S}}}} \hat{\mathcal{S}}_i = \mathcal{S}$ and $\hat{\mathcal{S}}_i \cap \hat{\mathcal{S}}_j = \emptyset$ for all $\{i, j\} \subseteq I_{\hat{\mathcal{S}}}$ such that $i \neq j$. In this case, for each $i \in I_{\hat{\mathcal{S}}}$, $\hat{\mathcal{S}}_i$ is called a group. Note that groups do not have to be disjoint to obtain valid bounds, as mentioned in [29]. However, for ease of representation, we consider disjoint groups. The problem defined over a group is called as group subproblem.

After a partition is determined, the probability of each scenario $s \in \mathcal{S}$ is adjusted to achieve a lower bound. The probability of each group $\hat{\mathcal{S}}_i$ is the summation of the probabilities of scenarios in that group, that is, $p(\hat{\mathcal{S}}_i) = \sum_{s \in \hat{\mathcal{S}}_i} p(s)$ for all $i \in I_{\hat{\mathcal{S}}}$. The conditional probability of each scenario s is $\hat{p}_i(s) = p(s)/p(\hat{\mathcal{S}}_i)$

for all $s \in \hat{\mathcal{S}}_i$ and $i \in I_{\hat{\mathcal{S}}}$, which is the ratio of the probability of the scenario to the probability of its group. Once the conditional probabilities are calculated, the weighted summation of the optimal values of the group subproblems provides a lower bound for our minimization problem as proved in [29]. Let $z^*(\mathcal{S})$ be the optimal value of our problem with scenario set $\mathcal{S}$ and $\underline{z}(\hat{\mathcal{S}})$ be a lower bound on $z^*(\mathcal{S})$ obtained using scenario partition $\hat{\mathcal{S}}$. The following inequality is satisfied by all $\hat{\mathcal{S}}$ selections as proved in Proposition 4.4 in [29]:

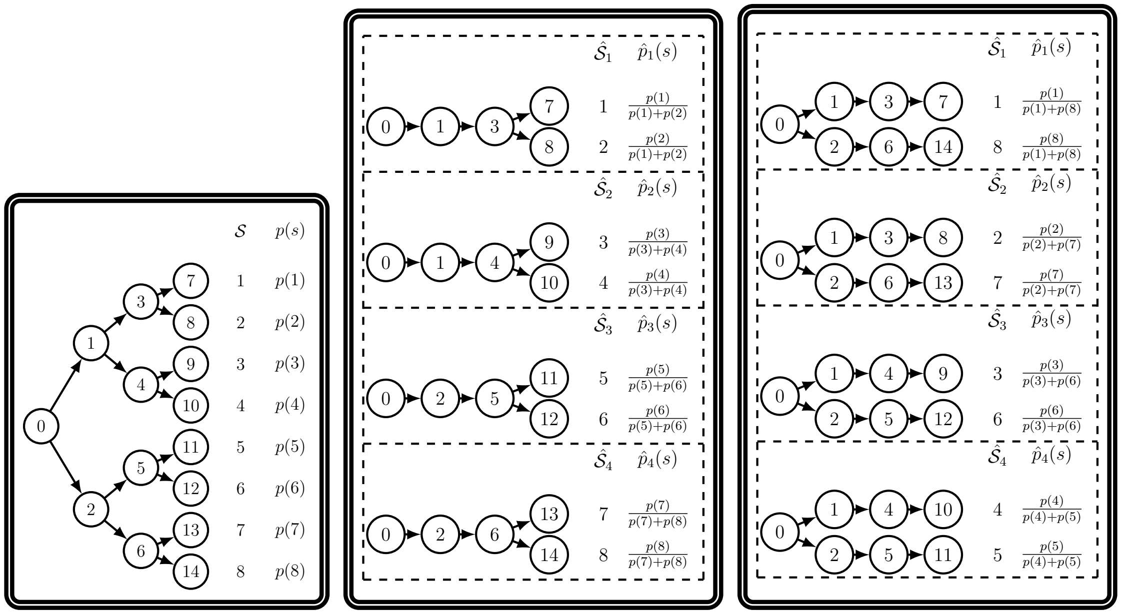
$$\underline{z}(\hat{\mathcal{S}}) = \sum_{i \in I_{\hat{\mathcal{S}}}} p(\hat{\mathcal{S}}_i) \cdot z^*(\hat{\mathcal{S}}_i) \leq z^*(\mathcal{S}). \tag{5.1}$$

The quality of the lower bound depends on the selection and size of groups as demonstrated in [29]. To see the effect of different partition selections, we construct the group subproblems with size $|\hat{\mathcal{S}}_i| = {}^{|\mathcal{S}|}/_{|I_{\hat{\mathcal{S}}}|}$ for all $i \in I_{\hat{\mathcal{S}}}$ with the following two partitioning methods:

1) *Consecutive:* We select ${}^{|\mathcal{S}|}/_{|I_{\hat{\mathcal{S}}}|}$ consecutive scenarios using the order that they appear in the scenario tree. This collection constructs the first group subproblem. We repeat this procedure for the rest of the scenarios till all of them are selected.

2) *Half-and-half:* We select ${}^{|\mathcal{S}|}/_{|I_{\hat{\mathcal{S}}}|}$ scenarios half of which is taken from the leaf nodes with the smallest indices and half is taken from the largest indices. This collection constructs the first group subproblem. We repeat this procedure for the rest of the scenarios till all of them are selected.

Since the *consecutive* method constructs the groups with scenarios having more common nodes, it is expected that *consecutive* method has an advantage in computation time over the *half-and-half* method per group. On the other hand, grouping scenarios with less common nodes, as in the *half-and-half* method, shortens the computation time to compute an upper bound as will be explained in detail in Section 5.2.



(a) The original problem (b) Group subproblems of the *consequtive* method (c) Group subproblems of the *half-and-half* method

Figure 5.1: Group subproblems of the *consecutive* and *half-and-half* methods for groups of two scenarios for a four stage problem

These partition strategies are illustrated in Figure 5.1 over a four stage scenario tree with two branches in each stage. The scenario tree is provided in (a). The construction of groups of size two and the adjusted probabilities of each scenario under this construction are demonstrated in (b) and (c) for the *consecutive* and *half-and-half* strategies, respectively.

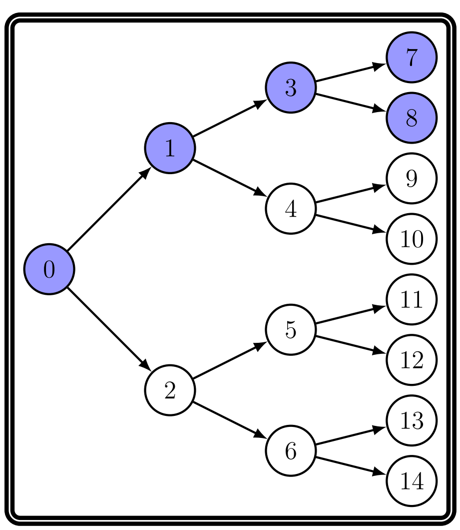
5.2 An Upper Bound

We use an optimal solution of each group subproblem to generate a feasible solution to the original problem as in [29]. Let $x^*(S,t)$ be the restriction of an optimal solution of our problem with scenario set S to the first t stages. For a given scenario partition $\hat{\mathcal{S}}$, to obtain an upper bound for our problem, the optimal solution $x^*(\hat{\mathcal{S}}_i, t)$ is substituted to the original problem one by one for each group of scenarios $\hat{\mathcal{S}}_i$, $i \in I_{\hat{\mathcal{S}}}$. In other words, we set the values of variables corresponding to $\hat{\mathcal{S}}_i$ in the first t stages to their optimal values in the subproblem and solve the resulting problem for the remaining variables. Each resulting problem is called as residual problem and denoted by $\mathcal{P}(\hat{\mathcal{S}}_i, t)$. Then, the residual problems are solved for the remaining decision variables. This procedure is illustrated in Figure 5.2 for the four stage problem provided in Figure 5.1. Residual problems obtained after the substitution of $x^*(\hat{\mathcal{S}}_1, 4)$ into the original problem are demonstrated in (a) and (b) for *consecutive* and *half-and-half* methods, respectively. Note that, residual problems consist only of decision variables and constraints associated with the non-colored nodes.

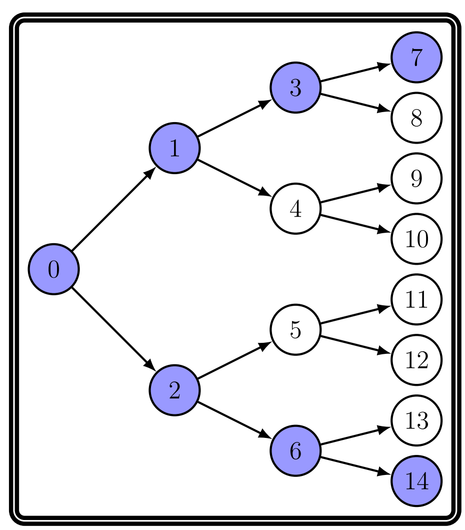
After substitution, if $\mathcal{P}(\hat{\mathcal{S}}_i, t)$ is feasible, its optimal value represented by $z^*(\hat{\mathcal{S}}_i, t)$ is a valid upper bound on $z^*(\mathcal{S})$. If $\mathcal{P}(\hat{\mathcal{S}}_i, t)$ is infeasible, then we set $z^*(\hat{\mathcal{S}}_i, t)$ to ∞. Note that, to reduce the possibility of infeasibility, small t values can be used. Additionally, since the number of fixed variables in $\mathcal{P}(\hat{\mathcal{S}}_i, t)$ reduces as t decreases, the resulting bound may get stronger. After solving $\mathcal{P}(\hat{\mathcal{S}}_i, t)$ for all $\hat{\mathcal{S}}_i$, $i \in I_{\hat{\mathcal{S}}}$, the minimum of the obtained upper bounds is selected:

$$z^*(\hat{\mathcal{S}}, t) = \min_{i \in I_{\hat{\mathcal{S}}}} z^*(\hat{\mathcal{S}}_i, t)$$

where $z^*(\hat{\mathcal{S}}, t)$ is the upper bound provided by scenario partition $\hat{\mathcal{S}}$.



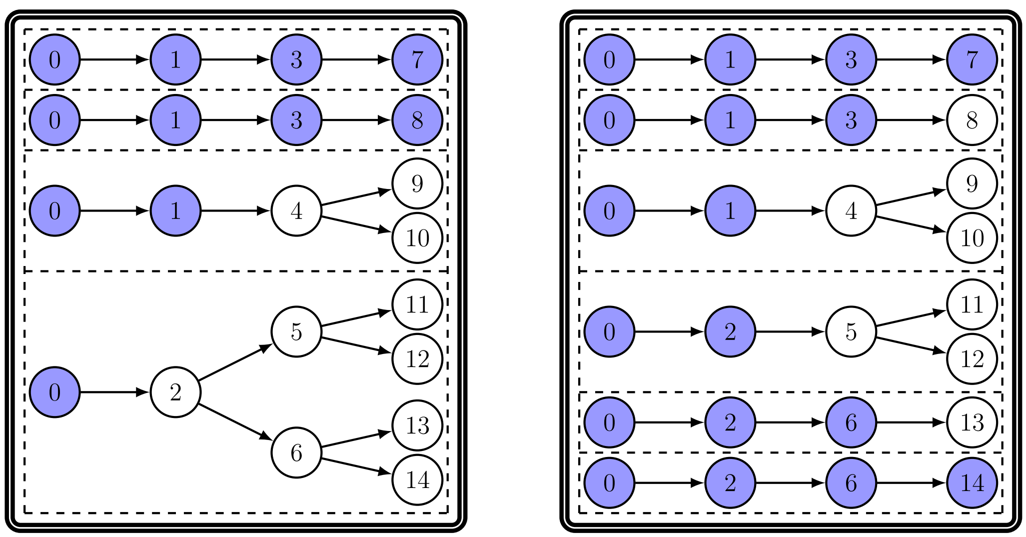
(a) $\mathcal{P}(\hat{\mathcal{S}}_1, 4)$ in *consecutive* method



(b) $\mathcal{P}(\hat{\mathcal{S}}_1, 4)$ in *half-and-half* method

Figure 5.2: Problems solved to compute an upper bound in different selection strategies

The residual problems have less decision variables and constraints compared to the original problem, furthermore, they are decomposable into smaller problems. Therefore, each residual problem may require far less computation time than the original problem. Figure 5.3 provides the scenario decompositions of the residual problems given in Figure 5.2. As seen in Figure 5.3, the *consecutive* and *half-and-half* methods differ with regard to the number and sizes of the problems obtained after decomposition. In the *consecutive* method, since the scenarios in the group subproblems have more common nodes, the sizes of the problems after decomposition are greater. Consequently, we expect that solving the residual problem in the *half-and-half* method takes less time compared to the *consecutive* method.



(a) *Consecutive* method

(b) *Half-and-half* method

Figure 5.3: Scenario decomposition of $\mathcal{P}(\hat{\mathcal{S}}_1, 4)$ for different selection strategies

Chapter 6

Computational Results

In this chapter, we present the results of our computational experiments. We first investigate the sizes of the instances that are solved to optimality using a general purpose solver and the quality of bounds obtained with the scenario groupwise decomposition for large instances. Second, we investigate the gains of demand response optimization for residents with different levels of price sensitivity. Third, we analyze the savings due to the use of renewable energy and batteries.

6.1 Scenario generation

We consider a time period of 24 hours and use equal number of periods in each stage. For instance, if we have eight stages, each stage consists of three hours. The random parameters are the temperature and the power of renewable energy. In our study, we consider the solar energy as our renewable energy source. The power of renewable energy can be calculated using global horizontal irradiance (GHI) data (total amount of solar irradiance on the horizontal surface of the ground [32]) and the features of the solar panel such as area, tilt angle and the conversion efficiency.

To generate the data of temperature and the power of renewable energy randomly, we use the hourly temperature and GHI data obtained from the dataset of National Renewable Energy Laboratory [33]. The correlation between GHI and temperature is calculated as 40% when considering the daytime data of the year 2005. This correlation can also be observed in Figure 6.1. Despite the positive correlation, high temperature and low amount of solar radiation can be observed at the same time because of the solar absorption by clouds, as mentioned in [34] and [35]. We use the real temperature and GHI data for each hour to take into account both the solar absorption and the positive correlation. We provide two data sets generated by the following two methods:

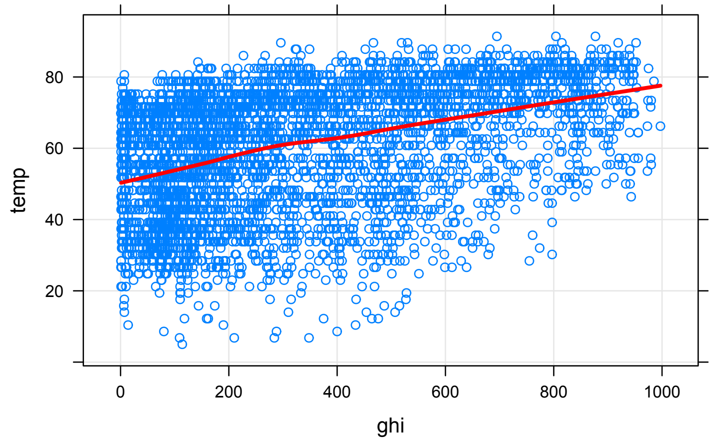


Figure 6.1: The relation between temperature and GHI

- *Data set-1*: This data set represents the cases with a good forecast for temperature and GHI. During the data generation process, the data of one month of a specific year are used. Firstly, a random day in the given month is chosen. For each hour of the first stage, corresponding temperature and GHI values of this day are used. When the stage is over, two branches emanate: one branch continues to use the data of this day and for the other branch, the temperature of the last hour of previous stage is compared with the temperature of the same hour of each day in the given month and the data of the day having closest temperature are used. The same procedure is applied for the rest of the stages.

- *Data set-2*: This data set represents the cases with a relatively bad forecast for temperature and GHI. Different from *Data set-1*, the data of a given month for different years are used. Data generation process is almost the same as *Data set-1*. As explained above, two branches emanate from each node, one branch continues to use the same year data of previous stage and for the other branch, the data of a randomly selected day of a randomly selected year are assigned.

For computational experiments, we consider a smart home equipped with five electrical appliances, each from different types, where HEMS is used to schedule their operations in a day in July. We set $\phi_T = 0.5$, $\phi_B, \underline{\phi}_D, \overline{\phi}_D = 1$ (the weights of disutility functions) and $\eta_1 = 5e^{-7}, \eta_2 = 4e^{-7}, \eta_3 = 1, \delta = 0.2$ (the parameters in the disutility function of appliances with battery). Note that the disutility function of appliances with battery is a convex function for the used parameters.

Our data is available at `https://github.com/DemandResponseOptimization/datasets`.

6.2 Computation times and quality of bounds

All experiments are carried out on a 64-bit machine with Intel Xeon E5-2630 v2 processor at 2.60 GHz and 96 GB of RAM using Java and CPLEX 12.6.

To measure the limits of a general purpose solver and to see the quality of bounds obtained by the scenario groupwise decomposition approach, we first consider small instances. First, we analyze the impact of group size and the number of stages for which the variables are fixed on the gap between lower and upper bounds. In Table 6.1, we report the averages of results for six instances from dataset 1 with six stages (32 scenarios) and a more detailed version is given in Appendix A. The first column gives the number of scenarios in each group and the second column gives the number of stages for which we fix the variables to compute an upper bound. We give the computation times and the gaps obtained by the *half-and-half* and *consecutive* methods. We set time limit of one hour to compute the lower bound and one hour to compute the upper bound. Since each bound computation requires solution of several problems, we distribute the hour equally for each problem. For instance, when each group subproblem contains two scenarios, we need to solve 16 subproblems to compute a lower bound. We set a time limit of 3600/16 seconds to each subproblem. To compute an upper bound, we set different time limits to each subproblem directly proportional to their sizes. If subproblem is not solved to optimality within the time limit, the best bounds are used.

Table 6.1: Comparion of gaps and solution times for instances with six stages

group size	number of stages fixed	*half-and-half* method time (sec)	*half-and-half* method opt gap (%)	*consecutive* method time (sec)	*consecutive* method opt gap (%)
2	1	2537.47	0.57	2269.42	0.68
	2	63.41	0.61	1063.85	0.63
	3	53.39	0.62	1057.05	0.63
	4	50.97	0.62	1054.83	0.63
	5	49.27	0.62		
4	1	2204.91	0.49	1711.04	0.60
	2	122.69	0.56	842.57	0.60
	3	115.6	0.58	793.80	0.60
	4	115.39	0.58		
8	1	2860.53	0.63	1722.61	0.41
	2	1274.68	0.71	880.56	0.45
	3	1270.76	0.73		
16	1	4259.95	1.92	4096.77	2.23
	2	3244.75	2.01		

If the *half-and-half* method is used, the smallest gap is observed when we solve subproblems of four scenarios and fix the variables in the first stage. However, it takes a long time to compute the bounds. The next best case arises again with subproblems of four scenarios but this time we fix the variables in the first two stages and this saves a significant amount of computation time. A better gap is obtained by the *consecutive* method when the group size is eight and the variables in the first two stages are fixed. However, this again requires a longer computation time. Based on this analysis, we decided to look closely on different instances when we take subproblems of size four and fix the first two stages. To compute upper bounds, we use the solutions from two randomly selected group subproblems. We report the results in Table 6.2 for 15 instances. For each instance, we report the lower and upper bounds computed using both selection methods as well as the time required to compute both bounds and the gaps. We also report the optimality gaps given by the solver in the same amount of time. Additionally, in the last column, we report the gaps for the solver with a time limit of one hour.

Table 6.2: Comparison of bounds, gaps and solution times for instances with six stages, when groups have four scenarios and the first two stages are fixed in computing upper bounds

	half-and-half **method**					***consecutive*** **method**					**CPLEX**
	lower bound	**upper bound**	**sol time (sec)**	**opt gap (%)**	**CPLEX opt gap (%)**	**lower bound**	**upper bound**	**sol time (sec)**	**opt gap (%)**	**CPLEX opt gap (%)**	**opt gap in 3600 sec (%)**
	131.65	131.82	27.16	0.13	6.66	131.71	131.78	355.3	0.05	6.56	4.87
	158.61	159.69	53.37	0.68	9.64	158.36	159.37	165.64	0.64	9.64	7.64
	142.39	143.9	303.93	1.05	8.49	142.3	143.11	1855.11	0.57	6.85	6.74
	133.58	133.81	27.43	0.17	7.24	133.65	133.8	379.24	0.12	5.81	4.69
	153.84	154.77	26.36	0.60	7.82	153.85	154.34	296.09	0.32	7.82	6.64
	161.45	161.95	16.02	0.31	18.28	161.42	161.78	56.32	0.22	13.42	4.26
	150.17	150.39	17.01	0.15	7.36	150.09	150.83	18.07	0.49	7.36	4.96
	158.84	160.15	268.08	0.82	12.60	158.51	159.96	214.33	0.90	12.6	8.65
	148.00	148.83	88.27	0.56	19.41	147.98	148.82	486.02	0.56	9.25	5.87
	143.69	144.41	58.23	0.50	9.24	143.68	144.24	254.6	0.39	9.1	5.54
	211.03	211.94	25.35	0.43	21.52	210.45	211.92	281.38	0.69	19.73	6.99
	189.36	190.15	12.78	0.41	22.45	189.47	190.2	71.79	0.39	22.45	1.85
	151.37	151.96	17.66	0.39	7.97	151.32	151.9	59.16	0.39	7.94	6.08
	178.42	178.89	15.53	0.26	16.69	178.53	178.82	38.17	0.16	16.69	5.83
	157.82	159.74	273.73	1.20	10.22	157.47	159.49	1169.08	1.27	8.52	8.39
avg			82.06	0.51	12.37			380.02	0.48	10.92	5.93

We see that the quality of bounds computed using both selection methods are very similar. However, the *half-and-half* selection requires a smaller amount of time to compute the bounds. The average gap for the *half-and-half* method is 0.51% and the average computation time is 82 seconds. The solver stops with an average gap of 12.37% in the same amount of time. When given the computation times of the *consecutive* method, the gap of the solver decreases to 10.92%. After one hour, this gap drops to 5.93%, which is significantly larger than the gap of the scenario groupwise decomposition approaches.

As the *half-and-half* method gives smaller computation times, we use it to compute bounds for larger instances. In Table 6.3, we report the results for instances from datasets 1 and 2, respectively, with eight stages (128 scenarios) under different threshold values when each group contains four scenarios, the decisions in the first three stages are fixed and optimal solutions of four randomly selected group subproblems are used in computing upper bounds.

Table 6.3: Results for instances with eight stages

$\overline{u}$	dataset 1 sol time (sec)	dataset 1 opt gap (%)	dataset 1 CPLEX opt gap (%)	dataset 1 CPLEX opt gap in 3600 sec (%)	dataset 2 sol time (sec)	dataset 2 opt gap (%)	dataset 2 CPLEX opt gap (%)	dataset 2 CPLEX opt gap in 3600 sec (%)
1.8	1962.65	0.58	9.87	8.39	2056.24	1.80	12.53	8.6
	2128.24	0.34	7.95	7.80	2491.96	1.43	7.8	7.62
	2129.39	0.72	8.96	8.41	2126.88	0.60	9.47	9.13
	2210.99	0.96	10.68	9.77	2206.33	0.85	8.08	7.78
	2012.08	0.33	9.22	9.16	3594.08	0.98	7.14	7.14
	1092.17	0.51	11.56	9.63	2247.10	0.92	7.62	5.55
avg	1922.59	0.57	9.71	8.86	2453.76	1.10	8.77	7.64
2.2	618.54	0.40	6.38	4.53	807.89	1.71	8.91	7.24
	53.95	0.10	1.63	0.96	721.89	0.78	16.07	7.30
	51.32	0.23	1.59	1.08	1468.1	1.11	15.02	10.85
	86.14	0.59	2.34	1.32	1156.85	0.85	8.83	6.6
	44.23	0.10	1.67	1.02	120.59	0.39	2.19	1.05
	1163.25	0.63	10.52	7.81	1009.09	0.89	15.13	12.11
avg	336.24	0.34	4.02	2.79	880.73	0.95	11.02	7.52

The results show that the decomposition approach is able to compute high quality solutions in reasonable computation times whereas the solver stops with significantly larger gaps after one hour of computation. We also observe that instances from dataset 2 with a smaller threshold are more challenging.

Finally, we present results for 12 stage problems (2048 scenarios) in Table 6.4. Here, we use groups of size eight. To compute upper bounds, optimal solutions of four random group subproblems are used and the decisions in the first four stages are fixed. As the problem is very large, we increase the time limit to two hours to compute the lower bound and two hours to compute the upper bound.

With the increase in the number of stages, we observe that the quality of bounds degrade. However, these instances are very challenging and the bounds of the scenario groupwise decomposition approach are significantly better than those of the solver in the same amount of time. The observation on the difficulty of instances from dataset 2 with a smaller threshold remains valid with 12 stages.

Table 6.4: Results for instances with 12 stages

$\overline{u}$	dataset 1			dataset 2		
	sol time (sec)	**opt gap (%)**	**CPLEX opt gap (%)**	**sol time (sec)**	**opt gap (%)**	**CPLEX opt gap (%)**
1.8	14431.62	12.70	28.65	14343.22	10.77	25.59
	14156.04	11.99	25.57	12533.56	12.8	23.49
	14353.66	8.92	26.93	14142.96	12.03	22.99
	10332.87	10.95	22.7	14099.57	14.6	25.06
	14239.2	16.38	28.74	13749.75	13.2	23.18
	14430.57	10.75	29.76	13520.34	17.41	25.82
avg	13657.30	11.95	27.06	13731.57	13.50	24.36
2.2	9380.30	1.98	8.26	12602.52	13.60	30.03
	13660.67	4.31	29.57	14046.66	8.92	28.25
	12851.99	0.71	30.45	12397.66	5.16	27.71
	13800.13	7.23	27.53	13448.18	10.13	19.47
	6933.12	0.75	2.05	11017.74	6.11	27.92
	6639.77	0.42	1.42	8368.98	3.6	29.74
avg	10544.33	2.57	16.55	11980.29	7.92	27.19

Based on the results of this experiment, we can conclude that the stochastic demand response optimization problem is very difficult to solve exactly using a general purpose solver. Even the instances with six stages are not solved to optimality in an hour. The decomposition approach, on the other hand, gives very good quality bounds for instances with up to eight stages. However, the gap increases as the number of stages increases. Finally, instances with higher variability are more difficult to solve.

6.3 Gains due to demand response optimization

In our second experiment, we investigate the gains of demand response optimization for residents with different price sensitivity. To this end, we use instances with four stages (eight scenarios), which are all solved to optimality. We set the threshold value to 1.8 for the rest of the experiments. To model different levels of price sensitivity, we vary the disutility weight ϕ as $0.1, 0.5, 1, 5$. The value 0.1 models a resident who is very sensitive to price whereas the value 5 models a resident who is not sensitive to price and values his/her comfort.

In Table 6.5, we report the improvements in the electricity cost and the weighted sum of electricity cost and disutility (our objective function) when we use optimization for four types of residents. For the case without optimization, we schedule the appliances in two ways. First we minimize the discomfort: The continuous appliances use electricity uniformly over the day. The temperature is set to its comfortable value and interruptible and uninterruptible appliances are used in the middle of their desired time intervals. No battery is used, hence the renewable energy is utilized when available. In the case of optimization, we solve our model. We report the average results over ten instances.

We observe that there is significant gain for all types of residents. There is clearly a big difference between the improvements in the electricity cost for residents that are very sensitive and insensitive to price. In the case of price insensitive residents, optimization still reduces the electricity cost by 17% with a little increase in the disutility.

Table 6.5: The average gains of optimization compared to a schedule that minimizes disutility

	very price sensitive resident	**price sensitive resident**	**less price sensitive resident**	**price insensitive resident**
gain in cost (%)	41.25	26.58	22.42	17.05
loss in disutility	18.65	9.49	6.72	1.73
total gain (%)	32.45	22.15	19.24	16.22

In the second approach, we schedule the appliances in a greedy way to minimize cost. The continuous appliances operate at maximum energy level during the required number of periods with the lowest electricity price and operate at the remaining energy level at the period with the next lowest electricity price to provide total energy requirement. They operate at minimum energy level for the rest of the day. If the inside temperature is in the allowed range without heating or cooling, the temperature appliance does not operate. Otherwise, the temperature is set to its minimum or maximum comfortable value depending on the outside temperature. Interruptible and uninterruptible appliances are used in time periods that provide the lowest total electricity cost to complete their tasks in their desired interval. No battery is used. We report the average results over ten instances in Table 6.6.

Table 6.6: The average gains of optimization compared to a greedy schedule with cost objective

	very price sensitive resident	**price sensitive resident**	**less price sensitive resident**	**price insensitive resident**
loss in cost (%)	3.08	22.38	26.64	32.89
gain in disutility(%)	87.91	98.60	99.43	99.98
total gain (%)	49.17	81.88	89.85	97.74

We observe that even if there is an increase in the electricity cost, the overall gain is significant for all types of residents as in the previous case. While improvement in disutility is more than 85% for all types of residents, it is above 99% for less price sensitive and price insensitive residents.

These results show that important gains are possible using optimization in demand response for all types of residents.

Next, we are interested in the tradeoff between the electricity cost and the distutility as well as the amount of electricity consumption exceeding threshold value for different types of residents. In Table 6.7, we report the average results over ten instances. Since keeping the temperature close to the comfortable value requires more energy than keeping it in the comfortable interval, total electricity consumption increases as price sensitivity of residents decreases. When appliances are used in their most desirable time intervals, the threshold value is exceeded more often and the electricity is charged at the higher price. Consequently, residents who are insensitive to price pay 43.37% more than the ones who are very sensitive to price while decreasing their disutilities by 86.78%. Almost half of the energy consumption of the price insensitive residents is charged at the higher price whereas the percentage is less than 18% for residents who are very sensitive to price.

Table 6.7: Results on electricity consumption above the threshold, cost and disutility

	very price sensitive resident	price sensitive resident	less price sensitive resident	price insensitive resident
electricity consumption exceeding the threshold value	6.81	14.63	16.80	21.10
total net electricity consumption	37.87	43.79	44.77	45.67
total cost	117.71	146.77	155.61	168.76
total disutility	17.77	10.34	8.12	2.35

6.4 Gains due to the incorporation of appliances that control the temperature

In the next experiment, we investigate the gain of the incorporation of appliances that control the temperature for residents with different price sensitivity. For the case without their incorporation into the system, we set the values of decision variables corresponding to the appliances that control the temperature to the required energy consumptions of these appliances to set the temperature to its most comfortable value and solve the remaining problem to schedule other appliances. We report the average results over ten instances in Table 6.8.

Table 6.8: The average gains of the incorporation of appliances that control the temperature

	very price sensitive resident	price sensitive resident	less price sensitive resident	price insensitive resident
gain in cost (%)	25.54	8.49	5.79	1.4
loss in disutility(%)	92.73	56.06	61.71	72.6
total gain (%)	15.26	5.16	3.2	0.8

We observe that the improvement in the electricity cost decreases as residents care more about their comfort. Since the desire to keep the temperature close to the its most comfortable value increases as the price sensitivity of residents increases, the effect of the incorporation of appliances that control the temperature on the schedule of other appliances decreases. Consequently, the overall gain of their incorporation reduces as price sensitivity of residents increases.

6.5 Gains due to battery and renewable energy

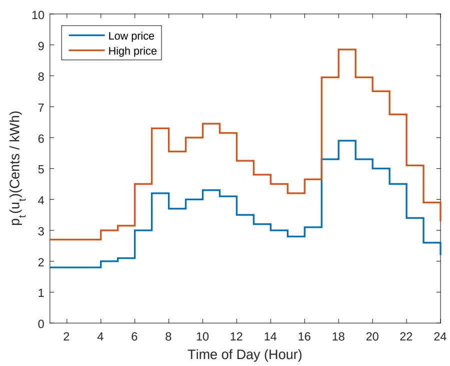
In our final experiment, we investigate the impact of battery and renewable energy on the electricity costs and disutilities of residents.

We present average results over ten instances in Table 6.9. Without the renewable energy, the use of batteries already results in significant improvements in the objective function for all types of residents. The gain due to renewable energy is similar for all resident types and it is a remarkable amount. But even further improvements are possible when the renewable energy is combined with batteries.

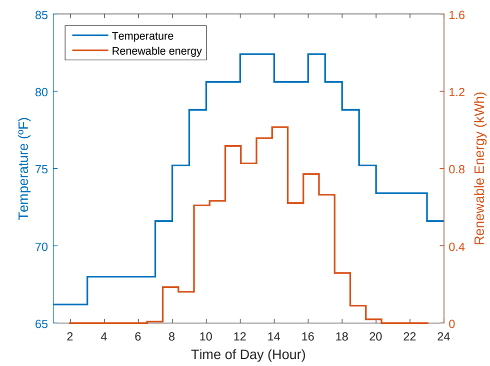
Table 6.9: Results of gains due to battery and renewable energy

	very price sensitive resident	price sensitive resident	less price sensitive resident	price insensitive resident
gain due to battery (%)	3.16	3.48	3.67	4.21
gain due to renewable energy (%)	25.31	25.10	24.82	24.84
gain due to battery and renewable energy (%)	27.11	28.34	28.66	28.74

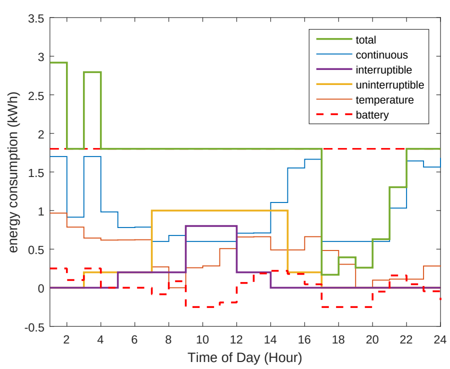
Finally, to see the impact of the renewable energy and the battery on electricity consumption during a day, we plot the consumption due to different types of appliances for an instance over a scenario in Figure 6.2. Low and high price values and the temperature and available renewable energy over a day are depicted in (a) and (b), respectively. From (c) and (d), it can be deduced that using a battery decreases the electricity consumption in time periods with higher prices (between 5 and 10 P.M.). In (c), battery stores electricity when the price is low. While the battery feeds the system when interruptible appliance operates, it stores electricity when it does not operate or when the amount of renewable energy is high. This flexibility enables the appliances to work during their desirable time intervals while keeping total consumption under the threshold value. When the battery is not used, in (d), although the uninterruptible appliance moves away from its desirable time interval, total consumption exceeds the threshold value in a time period with a relatively higher price. The comparison of (c) and (e) shows that when renewable energy is not available, consumption shifts to hours with cheaper price. In addition to the increase in the number of time periods in which electricity is charged at the higher price, the amount charged at this price also increases without renewable energy. Figures (e) and (f) show that the battery provides savings even when renewable energy is not available. While the periods in which the threshold is exceeded are generally those with lower electricity price in (e), when the battery is removed, we encounter also periods with high consumption at relatively higher prices in (f).



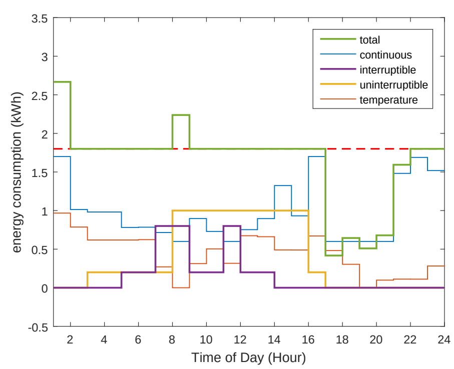
(a) Low and high price values



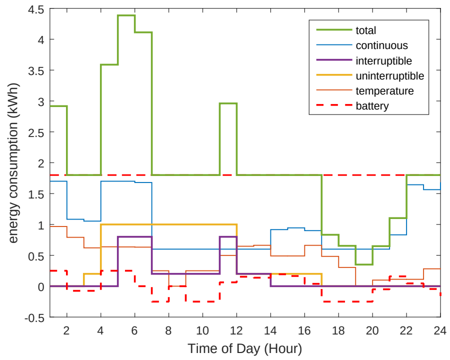
(b) Temperature and renewable energy



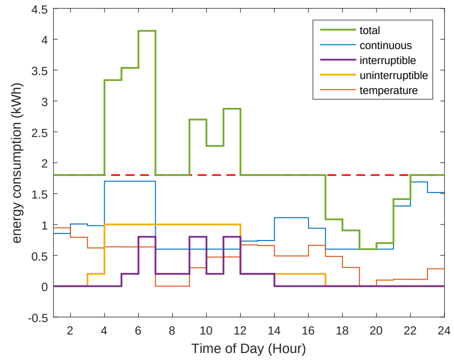
(c) Energy consumption



(d) Energy consumption without battery



(e) Energy consumption without renewable energy



(f) Energy consumption without battery and renewable energy

Figure 6.2: Effect of battery and renewable energy on the daily consumption

Chapter 7

Conclusion

In this study, we presented a model for demand response optimization in which we considered all types of appliances already mentioned in the literature as well as renewable energy and batteries. We also incorporated the uncertainty in temperature and the availability of renewable energy using multi-stage stochastic programming. We provided a decomposition scheme that produced good quality bounds for large instances. We also conducted a detailed experiment that provided insights about the gains in cost and utility for residents with different price sensitivity. We observed that optimization can provide significant improvements both in cost and utility for all types of residents. The same is also true for renewable energy and batteries. With these three tools combined, the peak demand can be reduced significantly, providing savings for residents, flexibility for the electricity providers and less damage to the environment.

Our model is a multi-stage stochastic mixed integer nonlinear program and is quite challenging to solve exactly. The 0-1 variables are used to model the nonlinear structure of the price function and the schedules of interruptible and uninterruptible appliances. The uninterruptible appliances are modelled easily as the only decision is about their start times. However the interruptible appliances have a more interesting structure; we need to keep track of their start and finish times as well as the periods in which they function. A future research is to work on a strong formulation for the schedule of a single interruptible appliance. The inequalities for this structure can be used to strengthen the overall model and shorten the computation times. We find it also interesting to investigate how such strong formulations can be used in other decomposition approaches such as SDDP. Since SDDP is a stagewise decomposition approach, it would perform better for problems with large number of stages. However, since it depends on convexity and stagewise independency assumptions, it cannot be applied to our problem directly, therefore, solving our problem with SDDP is also another future research direction.

Bibliography

[1] A. Illinois, "Real-time pricing for residential customers." `http://www.ameren.com/Residential/ADC_RTP_Res.asp`, 2009.

[2] B. Hydro, *BC Hydro Annual Report 2009.* BC Hydro, 2009.

[3] A.-H. Mohsenian-Rad and A. Leon-Gracia, "Optimal residential load control with price prediction in real-time electricity pricing environments," *IEEE Transactions on Smart Grid*, vol. 1, pp. 120 – 133, 2010.

[4] X. Fang, S. Misra, G. Xue, and D. Yang, "Smart grid?the new and improved power grid: A survey," *IEEE communications surveys & tutorials*, vol. 14, no. 4, pp. 944–980, 2012.

[5] J. Conti, P. Holtberg, J. Diefenderfer, A. LaRose, J. T. Turnure, and L. Westfall, "International energy outlook 2016 with projections to 2040," tech. rep., USDOE Energy Information Administration (EIA), Washington, DC (United States). Office of Energy Analysis, 2016.

[6] P. Palensky and D. Dietrich, "Demand side management: Demand response, intelligent energy systems, and smart loads," *IEEE transactions on industrial informatics*, vol. 7, no. 3, pp. 381–388, 2011.

[7] F. Li, W. Qiao, H. Sun, H. Wan, J. Wang, Y. Xia, Z. Xu, and P. Zhang, "Smart transmission grid: Vision and framework," *IEEE transactions on Smart Grid*, vol. 1, no. 2, pp. 168–177, 2010.

[8] M. Alizadeh, X. Li, Z. Wang, A. Scaglione, and R. Melton, "Demand-side management in the smart grid: Information processing for the power switch," *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 55–67, 2012.

[9] P. Siano, "Demand response and smart grids-a survey," *Renewable and sustainable energy reviews*, vol. 30, pp. 461–478, 2014.

[10] U. EIA, "Electric power sales, revenue, and energy efficiency form eia-861 detailed data files," 2017.

[11] C. F. Covrig, M. Ardelean, J. Vasiljevska, A. Mengolini, G. Fulli, E. Amoiralis, M. Jiménez, and C. Filiou, "Smart grid projects outlook 2014," *Joint Research Centre of the European Commission: Petten, The Netherlands*, 2014.

[12] J. S. Vardakas, N. Zorba, and C. V. Verikoukis, "A survey on demand response programs in smart grids: Pricing methods and optimization algorithms," *IEEE Communications Surveys & Tutorials*, vol. 17, no. 1, pp. 152–178, 2015.

[13] R. Deng, Z. Yang, M.-Y. Chow, and J. Chen, "A survey on demand response in smart grids: Mathematical models and approaches," *IEEE Transactions on Industrial Informatics*, vol. 11, no. 3, pp. 570–582, 2015.

[14] A.-H. Mohsenian-Rad, V. W. Wong, J. Jatskevich, and R. Schober, "Optimal and autonomous incentive-based energy consumption scheduling algorithm for smart grid," in *Innovative Smart Grid Technologies (ISGT), 2010*, pp. 1–6, IEEE, 2010.

[15] A.-H. Mohsenian-Rad, V. W. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE transactions on Smart Grid*, vol. 1, no. 3, pp. 320–331, 2010.

[16] P. Samadi, A.-H. Mohsenian-Rad, R. Schober, V. W. Wong, and J. Jatskevich, "Optimal real-time pricing algorithm based on utility maximization for smart grid," in *Smart Grid Communications (SmartGridComm), 2010 First IEEE International Conference on*, pp. 415–420, IEEE, 2010.

[17] N. Gatsis and G. B. Giannakis, "Residential demand response with interruptible tasks: Duality and algorithms," in *Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on*, pp. 1–6, IEEE, 2011.

[18] R. Energy, "Energy efficiency trends in residential and commercial buildings," 2010.

[19] S. Althaher, P. Mancarella, and J. Mutale, "Automated demand response from home energy management system under dynamic pricing and power and comfort constraints," *IEEE Transactions on Smart Grid*, vol. 6, no. 4, pp. 1874–1883, 2015.

[20] K. M. Tsui and S.-C. Chan, "Demand response optimization for smart home scheduling under real-time pricing," *IEEE Transactions on Smart Grid*, vol. 3, no. 4, pp. 1812–1821, 2012.

[21] S. Tiptipakorn and W.-J. Lee, "A residential consumer-centered load control strategy in real-time electricity pricing environment," in *Power Symposium, 2007. NAPS'07. 39th North American*, pp. 505–510, IEEE, 2007.

[22] N. Li, L. Chen, and S. H. Low, "Optimal demand response based on utility maximization in power networks," in *Power and Energy Society General Meeting, 2011 IEEE*, pp. 1–8, IEEE, 2011.

[23] Z. Zhao, W. C. Lee, Y. Shin, and K.-B. Song, "An optimal power scheduling method for demand response in home energy management system," *IEEE Transactions on Smart Grid*, vol. 4, no. 3, pp. 1391–1400, 2013.

[24] M. V. Pereira and L. M. Pinto, "Multi-stage stochastic optimization applied to energy planning," *Mathematical programming*, vol. 52, no. 1-3, pp. 359–375, 1991.

[25] R. A. Collado, D. Papp, and A. Ruszczyński, "Scenario decomposition of risk-averse multistage stochastic programming problems," *Annals of Operations Research*, vol. 200, no. 1, pp. 147–170, 2012.

[26] J. F. Bonnans, Z. Cen, and T. Christel, "Energy contracts management by stochastic programming techniques," *Annals of Operations Research*, vol. 200, no. 1, pp. 199–222, 2012.

[27] J. Zou, S. Ahmed, and X. A. Sun, "Nested decomposition of multistage stochastic integer programs with binary state variables," *Optimization Online*, vol. 5436, 2016.

[28] R. Schultz, "Stochastic programming with integer variables," *Mathematical Programming*, vol. 97, no. 1-2, pp. 285–309, 2003.

[29] B. Sandikçi and O. Y. Özaltin, "A scalable bounding method for multistage stochastic programs," *SIAM Journal on Optimization*, vol. 27, no. 3, pp. 1772–1800, 2017.

[30] F. Maggioni, E. Allevi, and M. Bertocchi, "Monotonic bounds in multistage mixed-integer stochastic programming," *Computational Management Science*, vol. 13, no. 3, pp. 423–457, 2016.

[31] F. Maggioni and G. C. Pflug, "Bounds and approximations for multistage stochastic programs," *SIAM Journal on Optimization*, vol. 26, no. 1, pp. 831–855, 2016.

[32] M. J. Reno, C. W. Hansen, and J. S. Stein, "Global horizontal irradiance clear sky models: Implementation and analysis," *SANDIA report SAND2012-2389*, 2012.

[33] NREL, "Nsrdb data viewer," 2018. Retrieved February 8, 2018, `https://maps.nrel.gov/nsrdb-viewer`.

[34] R. D. Cess, M. Zhang, P. Minnis, L. Corsetti, E. Dutton, B. Forgan, D. Garber, W. Gates, J. Hack, E. Harrison, *et al.*, "Absorption of solar radiation by clouds: Observations versus models," *Science*, vol. 267, no. 5197, pp. 496–499, 1995.

[35] S.-C. Tsay, M. D. King, R. F. Cahalan, and W. K.-M. Lau, "Absorption of solar radiation by clouds: A second look at irradiance measurements," 2001.

Appendix A

Results

Table A.1: Comparison of bounds, gaps and solution times of *half-and-half* method for instances with six stages

$\lvert\hat{\mathcal{S}}_i\rvert$	$\underline{z}(\hat{\mathcal{S}})$	sol time (sec)	j	$\overline{z}(\hat{\mathcal{S}})$ #	avg	min	max	sol time (sec) avg	min	max	total	the best opt gap (%)
1	210.28	8.17	1	32	224.95	214.85	228.63	86.25	34.86	105.27	8170.18	0.75
			2	32	221.04	211.87	228.92	42.65	2.82	58.02		
			3	32	221.07	211.88	228.97	42.22	2.54	57.97		
			4	32	221.07	211.88	228.97	42.02	2.44	57.62		
			5	32	221.07	211.88	228.97	41.93	2.14	57.8		
2	210.97	9.61	1	16	212.63	212.36	213.22	160.39	159.66	160.98	2801.52	0.46
			2	16	212.15	211.95	212.62	4.45	3.26	5.72		
			3	16	212.23	211.96	212.7	3.22	2.16	5.2		
			4	16	212.23	211.96	212.7	3.2	2.01	4.73		
			5	16	212.24	211.96	212.72	3.24	2.33	4.75		
4	211.03	18.66	1	8	211.84	211.84	211.84	273.34	272.86	273.85	2290.56	0.38
			2	8	212.2	211.92	212.62	4.28	3.15	5.58		
			3	8	212.25	211.92	212.67	3.21	2.11	4.24		
			4	8	212.26	211.92	212.67	3.16	2.1	3.79		
8	211.12	49.01	1	4	211.84	211.84	211.84	323.92	306.05	343.46	1376.6	0.34
			2	4	212.28	211.92	212.62	4.53	3.29	5.2		
			3	4	212.33	211.92	212.67	3.44	2.41	3.97		
16	210.82	3225.01	1	2	211.84	211.84	211.84	351.91	349.35	354.47	3937.25	0.48
			2	2	212.08	212.02	212.14	4.2	3.82	4.59		
1	157.42	9.85	1	32	161.07	159.02	164.76	88	31.46	112.2	8098.97	1.01
			2	32	160.5	159.03	162.86	42.49	3.22	59.49		
			3	32	160.51	159.04	162.87	42	2.47	58.76		
			4	32	160.51	159.04	162.87	42.04	2.63	58.87		
			5	32	160.58	159.03	162.87	38.26	2.68	58.97		
2	157.79	31.32	1	16	159.13	158.91	159.89	179.31	69.41	224.24	3036.81	0.7
			2	15	159.59	159.03	160.19	2.83	2.1	4.46		
			3	15	159.62	159.05	160.23	1.93	1.33	3.47		
			4	15	159.62	159.05	160.23	1.98	0.91	3.19		
			5	15	159.62	159.05	160.23	2.06	1.13	3.37		
4	157.82	269.38	1	8	159.02	158.91	159.15	301.58	57.72	450.29	2736.25	0.69
			2	8	159.56	159.06	159.95	2.84	2.05	3.76		
			3	8	159.6	159.08	159.97	1.96	1.36	2.95		
			4	8	159.6	159.08	159.97	1.98	1.04	3.45		

8	156.79	2893.99	1	4	158.98	158.91	159.03	388.73	96.06	717.29	4466.84	1.33
			2	4	159.48	159.07	159.95	2.87	2.11	3.58		
			3	3	159.64	159.15	159.97	2.06	1.23	2.92		
16	152.55	3589.85	1	2	158.97	158.92	159.02	408.23	70.43	746.04	4414.69	4.01
			2	2	159.35	159.07	159.62	4.18	2.19	6.18		

Table A.2: Comparison of bounds, gaps and solution times of *consecutive* method for instances with six stages

$\lvert\hat{\mathcal{S}}_i\rvert$	$\underline{z}(\hat{\mathcal{S}})$	**sol time (sec)**	j	$\overline{z}(\hat{\mathcal{S}})$				**sol time (sec)**				**the best opt gap (%)**
				#	**avg**	**min**	**max**	**avg**	**min**	**max**	**total**	
2	210.35	7.41	1	16	213.77	212.36	217.91	127.55	35.39	161.15	5425.67	0.7
			2	16	213.61	211.84	218.01	70.77	2.94	114.05		
			3	16	213.84	211.84	218.03	70.3	2.54	113.62		
			4	16	213.69	211.84	218.15	70.02	2.88	113.59		
4	210.45	13.45	1	8	212.59	211.84	214.85	214.15	35.53	274.07	3759.46	0.65
			2	8	212.87	211.84	215.86	127.37	3.19	226.58		
			3	8	212.88	211.84	215.9	126.73	2.77	226.68		
8	210.83	47.04	1	4	212.59	211.84	214.85	272.3	34.97	355.3	1776.05	0.48
			2	4	212.75	211.87	215.12	159.96	3.64	281.12		
16	211.84	2726.38	1	0	-	-	-	307.38	307.09	307.67	3341.14	-
2	157.43	12.77	1	16	159.6	158.91	160.62	156.4	34.41	224.21	5413.60	0.93
			2	14	159.79	158.91	161.28	69.33	5	115.02		
			3	14	159.8	158.91	161.29	68.9	4.14	114.31		
			4	14	159.81	158.92	161.29	68.79	4.06	114.37		
4	157.47	43.8	1	8	159.33	158.91	160.6	227.05	29.35	450.24	3848.415	0.91
			2	7	159.48	158.91	160.98	109.96	2.92	226.13		
			3	7	159.48	158.91	160.99	109.75	2.22	225.29		
8	157.91	745.66	1	4	159.4	158.91	160.59	455.73	76.5	900.19	3375.78	0.63
			2	4	159.56	158.93	160.68	201.8	5.08	451.42		
16	152.92	3590.21	1	2	159	158.91	159.1	648.46	66.34	1230.58	4887.13	3.77

Table A.3: Comparison of bounds, gaps and solution times of *half-and-half* method for instances with eight stages

$\lvert\hat{\mathcal{S}}_i\rvert$	$\underline{z}(\hat{\mathcal{S}})$	**sol time (sec)**	j	$\overline{z}(\hat{\mathcal{S}})$				**sol time (sec)**				**the best opt gap (%)**
				#	**avg**	**min**	**max**	**avg**	**min**	**max**	**total**	
1	187.38	40.02	1	128	223.06	223.06	223.06	28.54	28.27	29.03	22160	8.88
			2	128	207.3	205.87	210.99	28.28	23.85	29.25		
			3	128	206.73	205.65	210.69	23.79	18.81	27.81		
			4	128	206.73	205.65	210.7	23.11	18.06	27.24		
			5	128	206.73	205.65	210.72	23.05	17.69	26.76		
			6	128	206.74	205.65	210.72	23.01	17.77	26.7		
			7	128	206.74	205.65	210.73	23.03	17.92	26.88		
2	187.38	34.64	1	64	223.06	223.06	223.06	56.52	56.37	56.78	16705.42	0.58
			2	64	189.89	188.8	192.83	52.54	44.67	56.85		
			3	64	189.4	188.48	192.58	31.83	22.19	38.47		
			4	64	189.41	188.51	192.58	30	18.96	36.08		
			5	64	189.42	188.51	192.59	29.8	19.05	36.33		
			6	64	189.42	188.51	192.59	29.84	18.75	35.85		
			7	64	189.43	188.51	192.59	29.95	18.74	36.33		
4	187.45	49.17	1	32	223.06	223.06	223.06	112.72	112.62	112.83	13569.82	0.51
			2	32	188.99	188.59	189.69	97.52	80.37	112.79		
			3	32	188.98	188.41	190.56	54.97	30.02	60.12		
			4	32	189	188.44	190.6	52.5	26.09	60.01		
			5	32	189.01	188.44	190.62	52.39	27.04	59.81		
			6	32	189.01	188.44	190.62	52.43	27.39	59.56		

8	187.54	151.16	1	16	220.46	220.29	223.06	225.22	225.16	225.3	11132.47	0.46
			2	16	188.87	188.45	189.67	175.83	136.3	194.97		
			3	16	188.95	188.41	190.39	97.58	31.44	116.34		
			4	16	188.98	188.44	190.43	93.93	27.26	115.69		
			5	16	188.98	188.44	190.44	93.76	26.81	115.54		
16	183.93	2850.7	1	8	211.18	205.72	220.29	450.21	450.16	450.27	11918.03	2.35
			2	8	188.54	188.36	188.79	332.82	257.41	354.82		
			3	8	188.91	188.55	189.9	179.44	48.09	229.15		
			4	8	188.96	188.55	189.98	170.95	35.98	228.1		
1	159.79	35.65	1	128	188.13	180.9	202.21	28.54	28.25	29.34	22631.97	4.66
			2	128	180.77	168.12	191.4	28.42	24.93	29.53		
			3	118	178.69	167.66	191.29	25.97	20.46	28.95		
			4	118	178.68	167.6	191.36	24.51	20.18	27.81		
			5	118	178.69	167.6	191.36	24.4	19.68	27.16		
			6	118	178.69	167.6	191.36	24.36	19.64	27.49		
			7	118	178.68	167.6	191.36	24.43	19.9	27.71		
2	159.82	62.52	1	64	187.04	175.58	200.72	56.55	56.38	56.79	17397.14	1.12
			2	64	170.48	163.38	179.71	52.56	45.83	56.84		
			3	57	166.69	161.64	172	38.72	25.33	54.17		
			4	57	166.73	161.9	171.38	33.65	23.36	44.59		
			5	57	166.73	161.63	172.32	33.16	23.05	44.06		
			6	57	166.59	161.64	172.32	33.19	22.97	44.03		
			7	57	166.64	161.66	171.52	33.16	22.63	44.01		
4	159.93	393.02	1	32	186.28	175.78	200.72	112.73	112.62	112.91	15056.42	0.79
			2	32	168.22	162.7	175.35	102.83	89.15	112.99		
			3	31	165.05	161.2	170.72	68.48	42.01	98.78		
			4	31	165.26	161.21	171.02	59.81	40.13	78.81		
			5	31	165.21	161.21	171.07	59.22	39.49	78.55		
			6	31	165.22	161.21	171.02	59.18	39.7	78.84		
8	158.43	3284.84	1	16	185.2	175.83	199.43	225.22	225.14	225.32	15458.92	1.7
			2	16	165.9	161.73	173.69	198.13	174.75	225.35		
			3	16	164.25	161.17	169.3	120.76	76.35	181.75		
			4	16	164.27	161.21	169.3	108.68	72.25	148.36		
			5	16	164.26	161.21	169.3	108.1	71.59	147.85		
16	154.44	3600.7	1	8	185.59	176.3	197.7	450.22	450.16	450.33	12864.48	4.23
			2	8	162.59	161.35	164.78	330.99	280.65	450.26		
			3	8	162.09	161.26	163.9	197.92	127.11	297.71		
			4	8	162.12	161.27	163.95	178.85	125.2	285.58		

Table A.4: Comparison of bounds, gaps and solution times of *conseuctive* method for instances with eight stages

$\|\hat{\mathcal{S}}_i\|$	$\underline{z}(\hat{\mathcal{S}})$	**sol time (sec)**	j	$\overline{z}(\hat{\mathcal{S}})$				**sol time (sec)**				**the best opt gap (%)**
				#	**avg**	**min**	**max**	**avg**	**min**	**max**	**total**	
2	187.45	24.46	1	64	223.06	223.06	223.06	56.59	56.42	56.98	18319.40	8.85
			2	64	206.46	205.75	209.63	54.91	44.83	56.87		
			3	64	206.2	205.64	209.73	44.27	32.74	52.25		
			4	64	206.2	205.64	209.74	43.36	31.63	51.3		
			5	64	206.21	205.64	209.74	43.36	31.54	51.59		
			6	64	206.21	205.64	209.74	43.37	31.83	51.39		
4	187.54	31.2	1	32	223.06	223.06	223.06	112.74	112.64	113.28	14961.68	8.8
			2	32	206.06	205.79	207.06	105.2	87.04	112.77		
			3	32	206.06	205.64	207.13	83.73	61.23	101.25		
			4	32	206.06	205.64	207.17	82.49	59.87	100.27		
			5	32	206.07	205.64	207.18	82.41	60.17	100.11		
8	187.58	77.55	1	16	218.81	205.72	223.06	225.24	225.16	225.41	12082.86	0.39
			2	16	203.09	188.31	206.25	203.18	171.34	225.31		
			3	16	203.08	188.48	206.23	161.48	117.53	199.71		
			4	16	203.09	188.52	206.23	160.43	116.45	198.55		
16	182.8	2277.42	1	8	214.82	205.72	220.29	450.2	450.16	450.25	11434.17	2.91
			2	8	200.74	188.28	205.9	388.55	347.54	450.22		
			3	8	200.82	188.33	205.9	305.84	237.82	346.73		

2	159.89	33.47	1	64	185.99	175.84	200.71	56.55	56.39	56.96	18721.03	4.57
			2	64	179.36	168.12	191.37	55.07	47.83	57.05		
			3	60	177.76	167.61	191.29	48.18	37.3	56.72		
			4	60	177.75	167.55	190.32	45.62	36.82	52.16		
			5	60	177.78	167.55	191.34	45.52	36.66	52.43		
			6	60	177.77	167.55	191.34	45.52	36.56	52.11		
4	159.96	106.24	1	32	186.92	175.83	199.81	112.74	112.62	112.9	15289.76	5.82
			2	32	179.49	169.85	191.11	107.59	89.93	112.79		
			3	30	178.01	169.86	191.1	89.03	69.83	112.8		
			4	30	178.01	169.87	191.12	85.49	65.82	101.08		
			5	30	178.01	169.87	191.12	85.31	66.42	100.9		
8	159.87	1192.01	1	16	184.26	175.83	197.82	225.21	225.1	225.28	13488.03	4.53
			2	16	175.14	167.7	185.8	210.1	174.55	225.3		
			3	16	174.59	167.46	185.78	169.69	126.35	201.53		
			4	16	174.52	167.46	185.78	163.51	122.93	198.84		
16	155.28	3600.74	1	8	181.32	175.58	185.6	450.21	450.15	450.32	13121.17	7.54
			2	8	172.2	167.94	179.27	417.19	345.65	450.27		
			3	8	171.49	167.94	175.72	322.65	239.81	398.12		