MURAT ADİL CAN YILDIZ ESTIMATION OF TERM PREMIA IN TERM STRUCTURE OF TURKISH GOVERNMENT BOND YIELDS Bilkent University 2017

ESTIMATION OF TERM PREMIA IN TERM STRUCTURE OF TURKISH GOVERNMENT BOND YIELDS

A Master's Thesis

by

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September 2017

To my family

ESTIMATION OF TERM PREMIA IN TERM STRUCTURE OF TURKISH GOVERNMENT BOND YIELDS

The Graduate School of Economics and Social Sciences

of

İhsan Doğramacı Bilkent University

by

MURAT ADİL CAN YILDIZ

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Economics.

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ABSTRACT

ESTIMATION OF TERM PREMIA IN TERM STRUCTURE OF TURKISH GOVERNMENT BOND YIELDS

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September 2017

In this thesis, the Turkish Treasury yield curve is estimated by the Nelson-Siegel-Svensson method between January 2010 and December 2016 in a daily frequency. Interest rates taken from estimated yield curves can be used as a benchmark rate to determine the present value of any future cash flow. The main goal of this study is to measure expected future expectations of interest rates and the term premium. After the yield curves are estimated, a multifactor no-arbitrage affine term structure model is used to decompose the yield curve to its term premium and future expected interest rate components.

Keywords: Term Premium, Term Structure of Interest Rates, Yield Curve Fitting

ÖZET

TÜRKİYE DEVLET TAHVİLİ GETİRİLERİNİN VADE YAPISINDAKİ VADE PRİMİ HESAPLANMASI

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Tez Danışmanı: Prof. Dr. Refet S. Gürkaynak

Eylül 2017

Bu çalışmada, devlet iç borçlanma senetlerinin getiri eğrisi Nelson-Siegel-Svensson metodu kullanılarak Ocak 2010 ve Aralık 2016 arasında günlük frekansta tahmin edilmiştir. Tahmin edilen getiri eğrisinden alınan faizler, alındığı gün itibariyle gelecekteki herhangi bir nakit akışının cari değerini belirlemede referans olarak kullanılabilir. Bu çalışmayla geleceğe yönelik faiz beklentilerinin ve vade priminin ölçülmesi amaçlanmıştır. Getiri eğrisi tahmin edildikten sonra, çok faktörlü arbitraja izin vermeyen afin getiri eğrisi modeli kullanılarak vade primi ve geleceğe yönelik faiz beklentisi kısımlarına ayrıştırılmıştır.

Anahtar Kelimeler: Faiz Oranlarının Vade Yapısı, Getiri Eğrisi Hesaplama, Vade Primi

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CHAPTER 1

INTRODUCTION

Term structure of interest rates, also known as the yield curve, can be an important source of information about the future expectations of macroeconomic fundamentals. For example, the yield spread can forecast future inflation (Fama, 1990). Similarly, if investors expect a downturn in the business cycle, they can hedge themselves with a portfolio of bonds with different maturities. When interest rates are set in the market, portfolio holdings of investors affect the shape of the yield curve. By using this logic, Harvey (1993) claimed that the term structure of interest rates could be used to forecast economic growth. Indeed, he successfully predicted an upcoming recession five quarters in advance. The ideas that yield curve carry the information about future expectations of investors and predictive power on some of the future macroeconomic fundamentals make it a key subject for central bank policy and communication decisions (Orphanides & Kim, 2005).

Not all interest rates are set by central banks. They have a policy rate which is usually overnight, or one-week as is the case for Turkey. Hence, central banks make policy on the short end and their policy transmitted through the longer maturities along the yield curve. Understanding this transmission mechanism, the relation between short and long-term rates, is therefore essential for any policy decision. Not only central banks but Treasuries are also interested in the term structure behavior because they must decide how much of the governments' annual borrowing should be financed with long- and short-term debt (Bolder, 2001).

The basic theory of the term structure is the expectations hypothesis. Intuitively, holding a long-term bond until its maturity and rolling over series of short term bonds until the maturity of long term bond gives equal yield. However, since long-term bonds are risky in the short run, investors ask for a compensation for bearing some risk. This compensation is called "term premium". According to the expectations hypothesis, a long maturity bond yield is an average of the expected short-term rates plus a maturity specific "constant" term premium. The yield of *n* period zero coupon bond at *t* denoted by $y_{n,t}$ is then:

$$y_{n,t} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[r_{t+i}] + \phi_n \qquad (1)$$

However, in the literature, there has been evidence of time-varying term premia (e.g., (Campbell & Shiller, 1989) (Bliss & Fama, 1987)). Time variation can be sourced by the fact that price or amount of risk which is demanded for compensation can vary in time. In bad times, investors may ask for higher compensation, which increases the price for risk. Similarly, when there is higher uncertainty in future inflation expectations than usual, the amount of risk may increase (Kim & Orphanides, 2007). Since expectations hypothesis is violated by the evidence on time-varying term premium, we can modify equation (1) as the following:

$$y_{n,t} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[r_{t+i}] + \phi_{n,t} \tag{2}$$

In the literature, there are many studies of modeling yield curve of Turkish Treasury securities. Baki (2006) compared the performance of McCulloch model and Nelson-Siegel model to estimate yield curve. He used only zero-coupon bonds between January 2005 and June 2005 period and found that McCulloch model overperforms Nelson-Siegel model in terms of in-sample goodness of fit. Memiş (2006) include Svensson extension of Nelson-Siegel model to this comparison and found that Svensson's method has superior in sample and out of sample properties. However, these studies used only zero-coupon (discounted) bonds and hence estimated only the short end of the yield curve. Akıncı, Gürcihan, Gürkaynak and Özel (2006), for the first time, included five years fixed coupon bearing bond in the estimation of the yield curve by Svensson extension. They applied event study approach to understand reactions of yield curve to a variety of events such as inflation and monetary policy releases. Alper, Akdemir and Kazimov (2004) used factor analysis to examine the time series properties of yields curves which are estimated by McCulloch and Nelson-Siegel methods. They showed that three factors model is sufficient for both methods.

On the other hand, Artam (2006) included macro variables to model time series properties of the yield curve. He analyzed Ang-Piazessi (AP), Diabold-Rudebusch-Aruoba (DRA) models and his own macro-finance model. In his own model, he used

CPI and Capacity Utilization as macro variables and two latent variables. These macro-finance approaches estimate the joint dynamics of bond yields and macroeconomic variables together with affine term structure models. He found that his own macro-finance model is better at forecasting the yield curve rather than AP and DRA approach as in these models, macroeconomic variables have a smaller impact than latent factors.

In the literature, there is a gap for a study which analyzes the term structure dynamics by decomposing it to expectations and term premia components. This study aims to provide a framework to understand if it is term premium or expectations which captures yield movements of a specific maturity at any day. Firstly, zero coupon yields will be fitted to Turkish Treasury securities by Nelson-Siegel-Svensson method. This model specifies a parametric functional form for forward rates. Then, one can solve for the yields by integrating forward rates over the holding period of the bond. We used Turkish secondary government bond data traded at Borsa İstanbul between the dates of January 2010 to December 2016. Later on, we decomposed these yields to term premia and expectations. For this purpose multi-factor no-arbitrage models have been used in the literature (e.g., (Cochrane & Piazzesi, 2009) (Kim & Wright, 2005)) as they are proposed by Duffee (2002). Although the factors can be related to some macroeconomic variables such as inflation target and output gap (Rudebusch & Wu, 2003), in this proposed model they will be left as latent factors.

We proceed as follows. Chapter 2 is adapted to basics of yield curve fitting. In this section, we describe Nelson-Siegel-Svensson method and estimation procedure.

Chapter 3 includes the term structure model. Finally, chapter 4 presents the conclusion of this study.

CHAPTER 2

YIELD CURVE FITTING

Turkish Treasury issues debt only for a couple of days in a month and unfortunately, it does not issue a full spectrum of zero coupon bonds at each tender. Instead, under its issuance strategy, Treasury issues bonds and securities with different maturities and coupon rates. Hence, it is not possible to observe yield curve each trade date. We can, however, trace the prices of treasury bonds in the secondary market and try to come with an estimated yield curve. This procedure is called the yield curve estimation (Gurkaynak, Sack, & Wright, 2006).

Another issue with the yield curve estimation is the coupon effect. Inconveniently, almost all bonds with maturity higher than one year are coupon bearing bond. Often, yield to maturity of a coupon bond is used to represent the yield of the corresponding maturity of the bond. Yield to maturity is the internal rate of return for the coupon bond, that is, the constant interest rate that makes the present value of the coupon payments and the face value equal to the price of the bond. However, it is unfavorable. To see this, consider the following bond valuation formula for a coupon bearing bond where *c* is the annual coupon rate, *m* is time to maturity and *d(t, t+k)* is the discount function between *t* and *t+k*.

$$P(t,t+m) = \sum_{k=1}^{m} cd(t,t+k) + 100d(t,t+m) \quad (3)$$

In this formula, each cash flow is discounted with the corresponding spot rate between time t and *t+k*, where *k* = 1,..,*m*. Now, by using the definition of yield to maturity, we can write the bond valuation formula in the following way:

$$P(t,t+m) = \sum_{k=1}^{m} c\ exp\left(-\frac{y(t,t+m)}{100}k\right) + 100exp\left(-\frac{y(t,t+m)}{100}m\right) \quad (4)$$

Notice that, coupon payments and face value is discounted with a constant yield to maturity. We can think of yield to maturity as a weighted average of spot rates. However, with this formulation when c is larger, spot rates for corresponding coupon payments gain more weight. Different coupon rates for a given maturity gives different yield to maturities. Hence, yield to maturity cannot be trusted to be a substitute of zero coupon rate (Svensson, 1994).

Yield curve estimation literature focuses on two classes of models. McCulloch model, use piecewise cubic polynomials to estimate yield curve. The method suggests that dividing maturity range into subintervals and define a cubic polynomial for each subinterval such that observed prices of securities captured by the spline. Each polynomial must be connected at knot points carefully to ensure continuity and differentiability. Although this method is flexible enough to capture idiosyncratic variation in existing bond prices, it has a disadvantage of poor estimates of forward rates especially at long maturities (S.Shea, 1984)

The other widely used model is Nelson-Siegel model. This is a parametric class of model in a sense that only a few parameters govern the movements of the yield curve. Next section is committed for a detailed explanation of this model.

2.1 Nelson-Siegel-Svensson Model

Nelson and Siegel assume that the instantaneous forward rate is a solution with equal roots[1] to a second order differential equation (Nelson & Siegel, 1987).

$$f(m, b) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1}) \tag{5}$$

Where $b = (\beta_0, \beta_1, \beta_2, \tau_1)$.

In this form of forward rates, first term β_0 is a constant and second term is decreasing with τ_1. $\beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1})$ term generates the hump shape. When maturity goes to infinity, only β_0 governs the forward rate and when maturity goes to zero, $\beta_0 + \beta_1$ governs the forward rate. With these components, this specification can generate a variety of shapes of yield and forward curves.

However, convexity effect[2] tends to pull down the yields on longer maturities and may generate a second hump. To incorporate this effect in the yield curve, Svensson added a fourth component with two new parameters to Nelson and Siegel's forward specification.

[1] Model is over-parametrized if the solution is unequal roots.
[2] For a more general discussion, see (Gurkaynak *et al.*, 2006)

$$f(m,b) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right) \qquad (6)$$

Where $b = (\beta_0, \beta_1, \beta_2, \beta_3 \tau_1, \tau_2)$.

These parameters can be interpreted as the factors (historical principle components) and their loadings. In Diebold & Li (2006), they interpret β_0 as the level factor since loading on β_0 is 1 and at the limit does not decay to zero. Loading on β_1 stars from 1 but decays to 0 with a rate τ_1. Since it affects short maturities more than long ones, β_1 is the slope. β_2 starts from zero, increases and decays to zero. It generates a hump shape hence curvature. If we apply the same logic to β_3, it is similar to β_2 in the sense that it affects the medium ranged maturities but with a different decay component. It generates the second hump which is to span convexity effect.

If instantaneous forward rate *f(s)* is expected short rate *s* years ahead, then yield of an *m-t* years maturing bond must be average forward rate during this horizon. That is:

$$y(t,m) = \frac{1}{m-t} \int_0^{m-t} f(s)ds \qquad (7)$$

This integration results in the following equation for spot rates:

$$y(m,b) = \beta_0 + \beta_1 \frac{1 - \exp(-\frac{m}{\tau_1})}{\frac{m}{\tau_1}} + \beta_2 (\frac{1 - \exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} - \exp\left(-\frac{m}{\tau_1}\right)) + \beta_3 (\frac{1 - \exp\left(-\frac{m}{\tau_2}\right)}{\frac{m}{\tau_2}} - \exp\left(-\frac{m}{\tau_2}\right)) \qquad (8)$$

This is the zero-coupon yield curve. We can obtain discount function by:

$$d(m,b) = \exp(-\frac{y(m,b)}{100}m) \tag{9}$$

2.2 Data and Estimation

Since Nelson-Siegel-Svensson method is used to determine zero coupon yield curve, we have six parameters to estimate. This can be achieved either by minimizing the sum of squared yield or price errors. Minimizing yield errors necessitates running a Newton-Raphson type of an algorithm to obtain yield to maturity of a given coupon bond. This procedure demands high computational power and hence it is slower than minimizing price errors. For that reason, parameters are estimated by minimizing price errors.

One issue arises from minimizing price errors. Derivative of the log price of a bond with respect to its yield is $-D_{MOD}$ which is the modified duration. Modified duration can be interpreted as the sensitivity of bond price to changes in yield (Gurkaynak *et al.*, 2006). Short maturities are insensitive to changes in its yield. Because of this, if price errors are minimized, there may be large yield error for short maturities. To get rid of this effect, we can weight the square of each bond's price error by its inverse duration before minimizing the sum of squared errors. This provides a better fit of the yield curve by giving more weight to price errors of short term bonds.

One disadvantage of the weighted sum of squares emerges if one includes estimation of the yields of maturities shorter than three months. Bonds with less than three months to maturity, since errors are weighted by inverse duration, gain very high weight and behave oddly (Gurkaynak *et al.*, 2006).

Objective function to be minimized is then:

$$\min_{b \in B} \sum_{i=1}^{n} \frac{(P_i^o - P_i^e(b))^2}{D_{MOD,i}} \tag{10}$$

Where n is the number of observed bonds in corresponding trade date. P_i^o is the observed (market) price of bond i. $P_i^e(b)$ is bond price estimated by yield curve generated by parameter set b.

Bond prices are obtained from Borsa Istanbul. However, for each trade date, a basket of bonds which are used in estimation should be determined. For that reason, we selected the bonds according to following criteria:

1- We excluded all corporate bonds.
2- If value date is different than trade date, the bond is excluded.
3- All the bonds with floating coupon rate (varying duration) are excluded such as Revenue or Inflation Indexed bonds.
4- Because of the technical reasons mentioned above, any maturity with less than 91 days to maturity is excluded.

After elimination, there were bond baskets which the number of securities with fixed or no coupon payment vary 12 to 24 for each trade date. Estimation is made in Matlab according to the procedure explained above. Zero coupon rates of 3 month, 2-

5-10 year maturities for the period between 2010-2016 are shown in Figure 1 and in Figure 2 in appendices.

2.3 Fitting Error

Root Mean Square Price Errors (RMSPE) is used for the performance of fits. Figure 3 shows the RMSPE of each trade date and also maturity specific RMSPE in the given period. RMSPE in the given period is 17 basis points. The ability to fit the yields for specific maturity ranges are also calculated. RMSPE of the short end (3 months to 2 years) is 11 basis points where medium range (2 to 5 years) and long range (5 to 10 years) are both 20 basis points. Hence, the model made a better job fitting of the short end.

CHAPTER 3

TERM STRUCTURE MODEL

Main objective and contribution of this study is to understand how risk preference of investors, together with their future expectations of short rates, affected yield curve movements.

Within a long-time period, it is more likely to see a hike in interest rates which decreases the bond price than within a shorter period. Moreover, it is easier for short bonds to wait until maturity. This makes long maturity bonds more vulnerable than short maturity bonds to interest rate risk if they are to be held over a short horizon. Unless long bond yields are adjusted for that risk, arbitrage opportunities exist. Therefore, movements in cross section of yields are tied together (Piazzesi, 2010).

To compute risk-adjusted path of future short rates, a no-arbitrage affine term structure model will be used. Since bonds with different maturities are traded at the same time in liquid markets, any arbitrage opportunity is cleared out. The model is affine in the sense that, bond yields are affine function of some state variables.

First three principal components of the yields, which are taken from the fitted yield curve for maturities 3-6 month, 1-2-3-5-10 years, are used as state variables. According to principle components analysis, these three components explain 99 percent of the variation of bond yields and can generate good fit of the data.

Three-factor model specifies X_t as an n dimensional vector of latent variables follow the Gaussian VAR (1) process under the physical or data generating measure P:

$$X_{t+1} = \mu^P + \phi^P X_t + \Sigma\varepsilon_{t+1} \tag{11}$$

Where $\varepsilon_{t+1}|\mathcal{F}_t \backsim N(0,I)$ and $\{\mathcal{F}_t : t \geq 0\}$ is a filtration under probability space (Ω, F, P). For identification purposes, we used Cholesky factorized lower triangular part of the covariance matrix Σ as it is proposed by Dai and Singleton (2000). The instantaneous rate is an affine function of state variables such that:

$$r_t = \delta_0 + \delta_1 X_t \tag{12}$$

The strong result of Harrison & Kreps (1979) implies that under no arbitrage, there exists an equivalent martingale measure under which asset prices are martingales. This means that, under risk-neutral measure Q, asset prices depend only on expected future short rates. Under Q measure, state variables follow:

$$X_{t+1} = \mu^Q + \phi^Q X_t + \Sigma\varepsilon_{t+1} \tag{13}$$

We can express the density of any random variable under Q w.r.t P by L_t such that expectation of a random variable Z_t under Q is equal to expectation $Z_t L_t$ under P. Assume L_t follows log-normal process:

$$L_{t+1} = L_t \exp(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}) \tag{14}$$

Where the price of risk vector is also affine in terms of state vector:

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{15}$$

Then under P, we can price any asset by the Stochastic Discount Factor (SDF) with the following form after substitution of r:

$$m_{t+1} = \exp(-r_t)\frac{L_{t+1}}{L_t} = \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1} - \delta_0 - \delta_1 X_t\right) \tag{16}$$

Definition of measure Q implies that holding period return on *n* period bond is equal to the short rate for all *n*. By using this fact, we can calculate bond prices recursively by:

$$p_t^{n+1} = E_t[m_{t+1} p_{t+1}^n] \tag{17}$$

Calculation provides the following formula for log bond price:

$$\log(p_t^n) = \overline{A}_n + \overline{B}_n X_t \tag{18}$$

Where $\overline{A}_n$ and $\overline{B}_n$ follows:

$$\overline{B}_{n+1} = \overline{B}_n(\phi^P - \Sigma\lambda_1) - \delta_1 \tag{19}$$

$$\overline{A}_{n+1} = \overline{A}_n + \overline{B}_n(\mu^P - \Sigma\lambda_0) + \frac{1}{2}\overline{B}_n'\Sigma\Sigma'\overline{B}_n - \delta_0 \tag{20}$$

with $\overline{A}_1 = -\delta_0$ and $\overline{B}_1 = -\delta_1$ [3]. These iterative equations are the cross-sectional restrictions. Starting from $\overline{A}_1$ and $\overline{B}_1$, by solving for $\overline{A}_n$ and $\overline{B}_n$, these equations

[3] For complete derivation of recursive relation check (Ang & Piazzesi, 2003).

determine the price of the n period bond while imposing no-arbitrage along the yield curve.

We can also show the relation between P and Q related parameters. By using equation (15) for the price of risk, low of motion for the state can be expressed as:

$$\begin{aligned} X_{t+1} &= \mu^Q + \phi^Q X_t + \Sigma\lambda_t + \Sigma\varepsilon_{t+1}{}^{4} \\ &= (\mu^Q + \Sigma\lambda_0) + (\phi^Q + \Sigma\lambda_1)X_t + \Sigma\varepsilon_{t+1} \end{aligned} \tag{21}$$

If we match these parameters with their counterpart P measure parameters, we obtain:

$$\mu^Q = \mu^P - \Sigma\lambda_0 \tag{22}$$

$$\phi^Q = \phi^P - \Sigma\lambda_1 \tag{23}$$

Yield on *n* period zero coupon bond is:

$$y_t^n = -\frac{log p_t^n}{n} = A_n + B_n' X_t \tag{24}$$

Where $A_n = -\frac{\overline{A}_n}{n}$ and $B_n = -\frac{\overline{B}_n}{n}$.

Because it affects the constant term A_n in yield equation, λ_0 affects the long-run mean of yields. A non-zero λ_1 enters the yield equation via B_n hence affects the time variation in term premium. When both λ_0 and λ_1 are zero, risk neutral measure and

[4] Intuitively, this is applying the Girsanov Theorem reversely. Instead of hiding the term premium in the error term under Q measure, this time we extract it under P measure.

data generating measure coincide. A non-zero λ_0 and a zero λ_1 implies constant term premium whereas a non-zero λ_1 implies time-varying term premium.

3.1 Data and Estimation

Data comes from two sources. Yields between 3 months to 10 years maturities are taken from Svensson curve fitted to Treasury securities as explained in Chapter 2. Weighted Average Cost of the CBRT[5] Funding (WACF) is used as the short rate. CBRT has been releasing WACF data since 3 January 2011. Accordingly, yield decomposition also starts from this date. Data is in daily frequency. Between 3 January 2011 and 23 December 2016, it makes 1499 observations. For each trade date, observations consist of overnight WACF as the short rate and yields from 3 and 6 month and 1,2,3,5 and 10 year maturities.

If y_t^o is 8x1 vector of daily observations of yields and X_t is an 3x1 vector of unobserved state variables, the model can be represented in the following way:

$$y_{t+1}^o = A + BX_{t+1} \tag{25}$$

$$X_{t+1} = \mu^P + \phi^P X_t + \Sigma\varepsilon_{t+1} \qquad \varepsilon_t \sim N(0, I) \tag{26}$$

Where A is an 8x1 vector and B is an 8x3 matrix. ε_t is mean zero and has a 3x3 covariance matrix.

[5] Central Bank of the Republic of Turkey

Then, parameter vector to be estimated is:

P = ($\mu^P, \phi^P, \mu^Q, \phi^Q, \Sigma, \delta_0, \delta_1$) where μ is 3x1, ϕ is 3x3, μ^Q is 3x1, ϕ^Q is 3x3, Σ is 3x3, δ_0 is 1x1 and δ_1 are 3x1 matrices. It makes total 37 parameters to be estimated.

Alternatively, one can estimate the λ_0 and λ_1 parameters instead of μ^Q and ϕ^Q. This makes no difference because if any of two parameters set between P, Q and price of risk related parameters are known, then the third one can be calculated by equations (22) and (23).

If we make state space estimation, equation (25) would be measurement equation and equation (26) would be state transition equation, together with the diagonal 8x8 covariance matrix of measurement equation Gaussian error, it makes total of 45 parameters. Then model can be estimated by a Kalman Filter based maximum likelihood in a single step. When we tried to estimate all the parameters in a single step, results were unable to mimic the data unless we provide initial conditions which are extremely close to their estimated values. For that reason, the model is estimated in two-step as Ang and Piazzesi (2003) offered. In the first step, P measure related parameters (μ, ϕ, Σ) and short rate parameters (δ_0, δ_1) are estimated. To estimate (μ, ϕ) parameters, state variables, first three principle components of bond yields, are regressed onto their one period lags with simple OLS. Then, residuals from VAR(1) are used to calculate covariance matrix Σ. However, for identification purposes explained in previous section, we Cholesky factorized this covariance matrix and use the resulting lower triangular part for the rest of the estimation. Similarly, short rates are regressed onto state variables with OLS to determine (δ_0, δ_1).

In the second step, Q related parameters are estimated. Since there is a linear relation between yields and state variables in the measurement equation, minimizing the sum of squared deviations is identical to MLE. We estimated (μ^Q, ϕ^Q) by minimizing the sum of squared deviations. This way we can reduce the number of parameters to be estimated in a single step to 12 which also boosts the speed of estimation. Estimation results of parameters are indicated in Table 1. Overall RMSE of the model is 18 basis points where it is only 1 basis point in overnight rates and 26 basis points in 10 year yields.

After parameter estimates are obtained, decomposition can be done. To calculate the expected return of an n period bond on a trade date, starting from corresponding state variables, the future evolution of state variables can be predicted from transition equation. Since the short rate is an affine function of factors, expected future instantaneous short-term interest rates can be estimated. Hence, term premia on bond which matures n-period after can be estimated by subtracting the average of expected future short rates from yield on an n-period zero coupon bond. Figure 4 indicates the result of yield decomposition for 3 months and 1, 3 and 10 year maturities.

According to model decomposition, movements in 3 month bond yields are mostly due to the change in future expectations. Term premium is less responsive to yield movements. As maturity increases, especially after 3 years, expectations almost do not change and almost all of the variations in bond yields are attributed to term premium. This is not surprising since this decomposition based on VAR forecasts. Because parameters related to drift are not time variant in our model, after some point, state variables and consequently future short rates converge to their long run

Table 1: Model Estimates

P related parameters

	μ	ϕ matrix x1	x2	x3	Σ matrix ($x10^{-6}$) x1	x2	x3
x_1	0.0011 (0.0006)	0.9963 (0.0021)	-0.017 (0.0067)	-0.0201 (0.0118)	8.148	-0.7557	-0.5292
x_2	-0.0005479 (0.0004)	0.002 (0.0013)	0.9883 (0.0041)	-0.0067 (0.0071)	-0.7557	2.9808	-0.5979
x_3	0.00032925 (0.0003)	0.0015 (0.0011)	-0.0046 (0.0035)	0.9746 (0.0061)	-0.5292	-0.5979	2.1878

Q related parameters

	μ^Q	ϕ^Q matrix x1	x2	x3
x_1	0.0012 (0.0441)	1.0014 (0.0052)	-0.0156 (0.1006)	-0.0151 (0.3594)
x_2	-0.0019 (0.0045)	0.0021 (1.2552)	0.9806 (0.0035)	-0.0213 (0.1488)
x_3	0.0020 (0.0205)	0.0011 (0.0017)	-0.0128 (0.1023)	0.9854 (0.01)

Short rate parameters

δ_0	δ_1
-0.00072489 (0.0000263)	0.3233 (0.0000869)
	0.7521 (0.0002741)
	0.5711 (0.0004814)

Price of risk parameters λ_t

	λ_0	λ_1 matrix x1	x2	x3
x_1	-0.0083 (0.0023)	-1.757 (0.0032)	-0.4782 (0.0035)	-1.7535 (0.0015)
x_2	0.8162 (0.0024)	-0.3195 (0.0019)	4.4363 (0.0012)	8.2735 (0.0017)
x_3	-1.3303 (0.0003)	0.1025 (0.0005)	4.6875 (0.0004)	-9.6362 (0.0005)

The table reports parameter estimates for the model with 3 latent factors with state transition equation (26), short rate equation (12) and the market price of risk equation (15). Standard errors are provided under each parameter within parenthesis. The sample period is 3 Jan 2011 to 23 Dec 2016 with 1499 observations in daily frequency.

mean. This implies almost the same expectations of long term yields at any time over the sample period. We know that long run expectations are adjusted to macroeconomic news (Gürkaynak, Sack, & Swanson, 2005). However, it is not desirable to capture long yield movements by expectations or term premium only. In this regard, it is better not to overtrust this decomposition but to consider it as a demonstration of how to use this model.

To affect the long run expectations, Kim and Orphanides (2007) included survey forecasts of short term interest rates to the Kalman Filter based estimation of the term structure model. By providing expected future path of the short rates as an additional information, they can produce the variation in the long-term expectations of short rate. When they compared the survey forecasts of long-term yields with those based on expectations hypothesis and the model implied expectations, they found that the model estimated with survey data can generate implied forecast of long-term interest rates that captures some of the deviations of survey forecasts of long-term interest rates from the expectations hypothesis.

When we look at the estimation results of the price of risk λ_t, for any date, we see that price of risk correspondent to level and curvature factors are negative whereas price of risk of the slope factor is positive. This factor determines the effect of shocks to SDf. Since -λ_t is positive for level factor, any positive shock decreases the SDF resulting an increase in yields. In Dewachter and Lyrio (2006), they provide macroeconomic interpretation for latent factors. Their findings indicate that, level factor is higly correlated to long run inflation expectations. With this regard, higher future expectations for inflation lowers marginal utility hence increases bond yields.

However, in our model, we have fixed endpoints for VAR estimation. When yields are increasing in long maturities, then we say its the term premium that is increasing.

Although the model with only latent variables has low forecasting power on long horizons, it does a better job on the short horizon. Starting from 2013, CBRT conducts and releases expectations surveys in every month. To compare model implied expectations to survey expectations we used the survey for the average WACF expectation of prevailing month. To form expectations over 3 months horizon, we calculated the average of expectations of the prevailing and the forthcoming two months. Figure 5 indicates this comparison. Model implied expectations move parallel to survey results.

3.2 Results

So far, we have estimated zero coupon yields and introduced a model to help us understand what was driving yield movements. This section is dedicated to demonstrating model capabilities. We will examine a subperiod in 2013 summer.

3.2.1 2013 Summer Crisis

After 2008 crisis, Fed had been conducting large scale of a Quantitative Easing (QE) program. After FOMC meeting on 22 May 2013, Fed chairman Ben Bernanke addressed Joint Economic Committee saying first that it is not efficient to do it immediately but considering incoming information Fed may reduce the scale of its Asset Purchasing Program in next two FOMC meetings (Bernanke, 2013). After this declaration, markets reacted immediately by bonds were being sold off and equities showed higher volatility especially in developing countries. This event named as Taper Tantrum. In Turkey, however, a civil unrest began six days after Bernanke's talk on 27 May 2013 and remained over a month on the agenda. Although we had seen similar behavior in other developing countries' bonds market, claiming which event was dominant along the crisis period requires further study. In this example, without asserting a claim about the source of the crisis, we will name this period as 2013 summer crisis and use our model to understand what was happening.

Figure presents the decomposition results beginning from 26 May 2013 to 28 August 2013. In 10-year yields, we see that expectations form a straight line. Although a possible permanent change in Fed's QE program would affect expectations over longer than 10 years horizon, we see no change in expectations. As it is mentioned before, this is because there is no mechanism in the model to affect long horizon expectations. Therefore, term premium is responsible for all the movement on the long end of the yield curve. On the short end, however, we can trust our measurement of future expectations. Throughout the summer period, we observe over 400 basis points hike in 3-month yields. The contribution of expectations over this period is up to 273 basis points whereas term premium contributes up to 183 basis points. That term premium increases significantly on each maturity glitters at the first glance. Although it is hard to identify the effect of each event, we can expect that civil unrest may be responsible for the increase in term premium since investors might ask higher risk premium as they are afraid that civil disobedience in the country can affect the overall stability of the country. On the other hand, an increase in expectations may be more related to Fed's policy changes since such an important policy change in Fed's policy will be transmitted into developing economies' markets especially it is expected to be more long-termed. Hence it is very likely that the changes in expectations are more likely to be affected by Bernanke's talk. If there were a mechanism in the model to move long-term expectations, it would be very likely to observe how Taper Tantrum would affect long-run expectations (e.g. 10-year expectations). Unfortunately, this issue will be illuminated in the future study.

CHAPTER 4

CONCLUSION

In this paper, we fitted the Turkish Treasury yield curves for maturities between three months to ten years within a sample period of 2010 to 2016. We used a parsimonious model which smooths out the idiosyncratic variation of individual securities. After obtaining zero coupon rates, we used a multifactor no-arbitrage affine term structure model to measure real expectations of future spot interest rates by decomposing yields of a range of maturities.

Our model implied that future expectations are dominant in capturing yield movements at the short end, whereas term premium was dominant at the long end. We compared model implied expectations for short maturities to CBRT survey expectations and found that it is successful in capturing the trend. Our model attributes “all” the variation in longer maturity yield movements to term premium. To be able to move future expectations in the longer horizon, we wish to include survey forecasts of interest rates as Kim and Orphanides (2007) in a future study.

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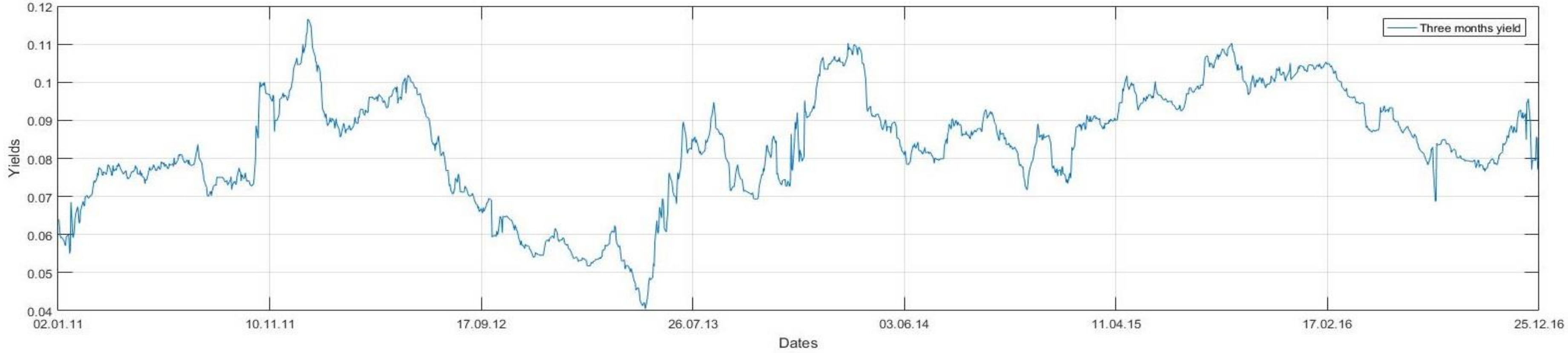
APPENDICES



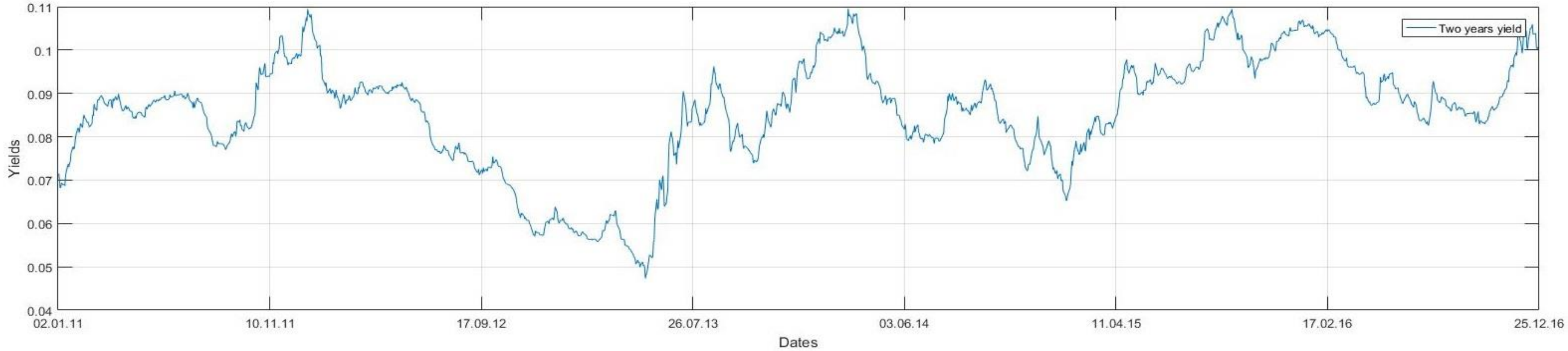


Figure 1: HISTORICAL 3 MONTH AND 2 YEAR YIELDS IN SAMPLE PERIOD

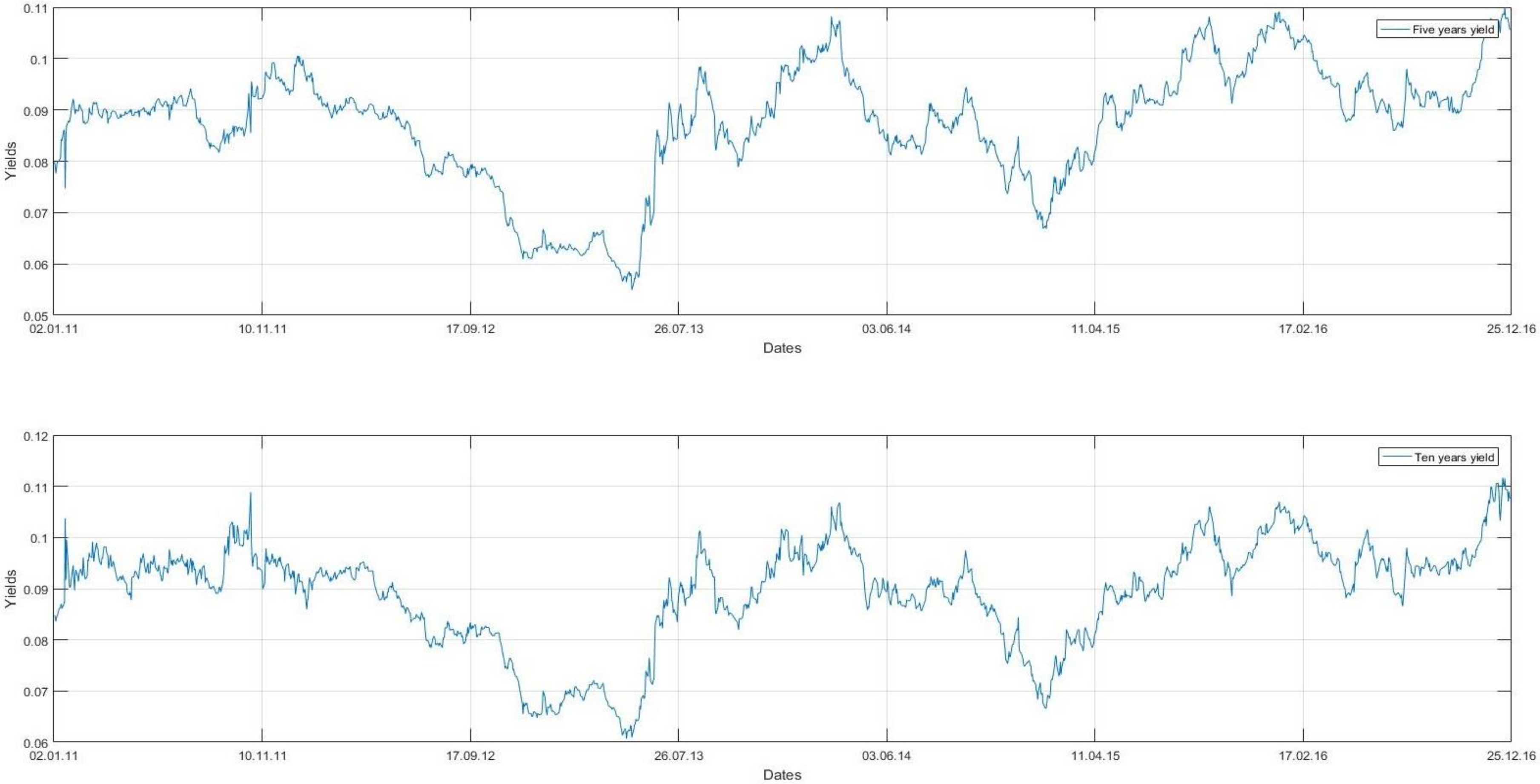


Figure 2: HISTORICAL 5 YEAR AND 10 YEAR YIELDS IN SAMPLE PERIOD

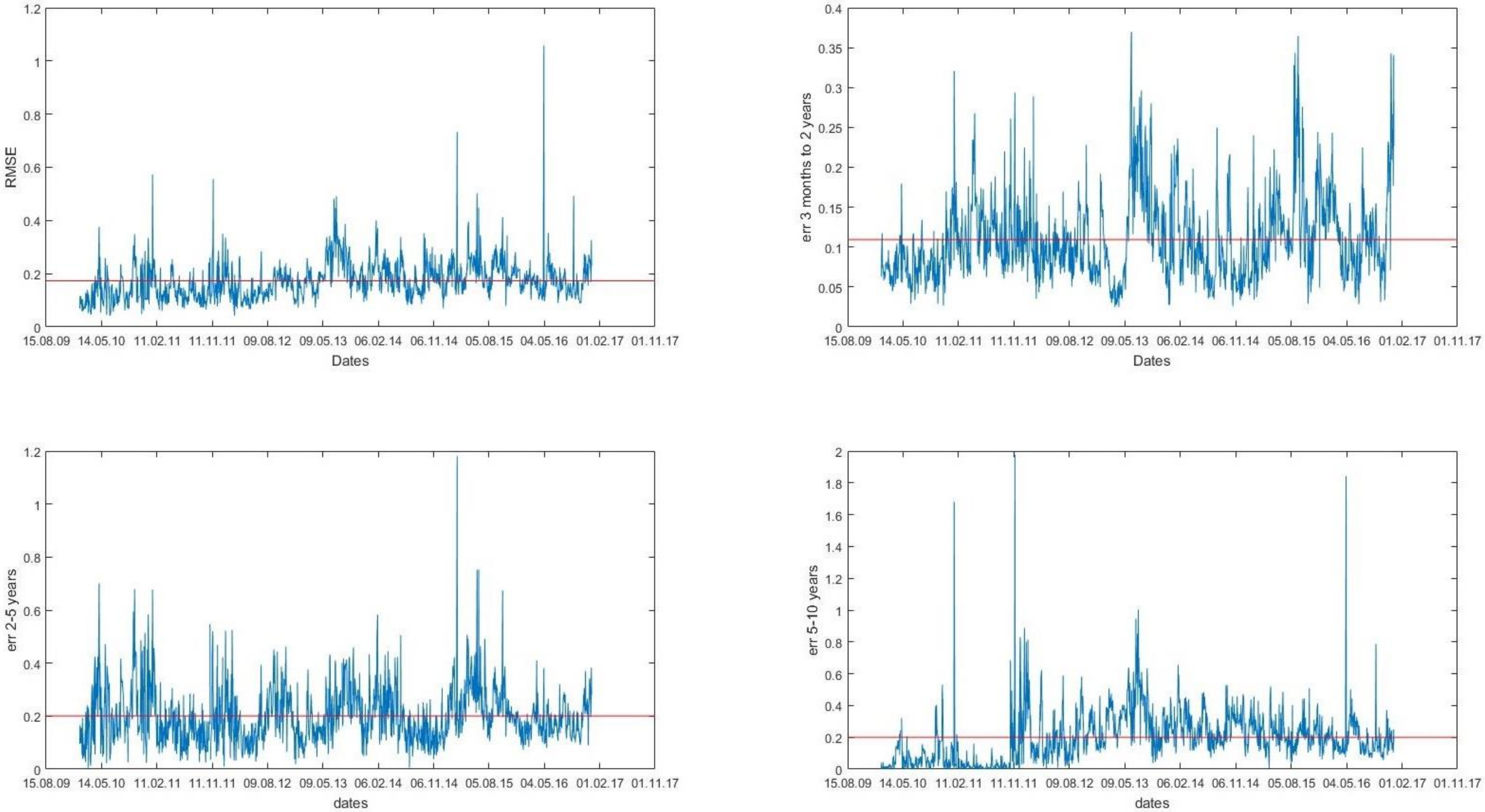


Figure 3: RMSPE BY INDICATED MATURITY BIN

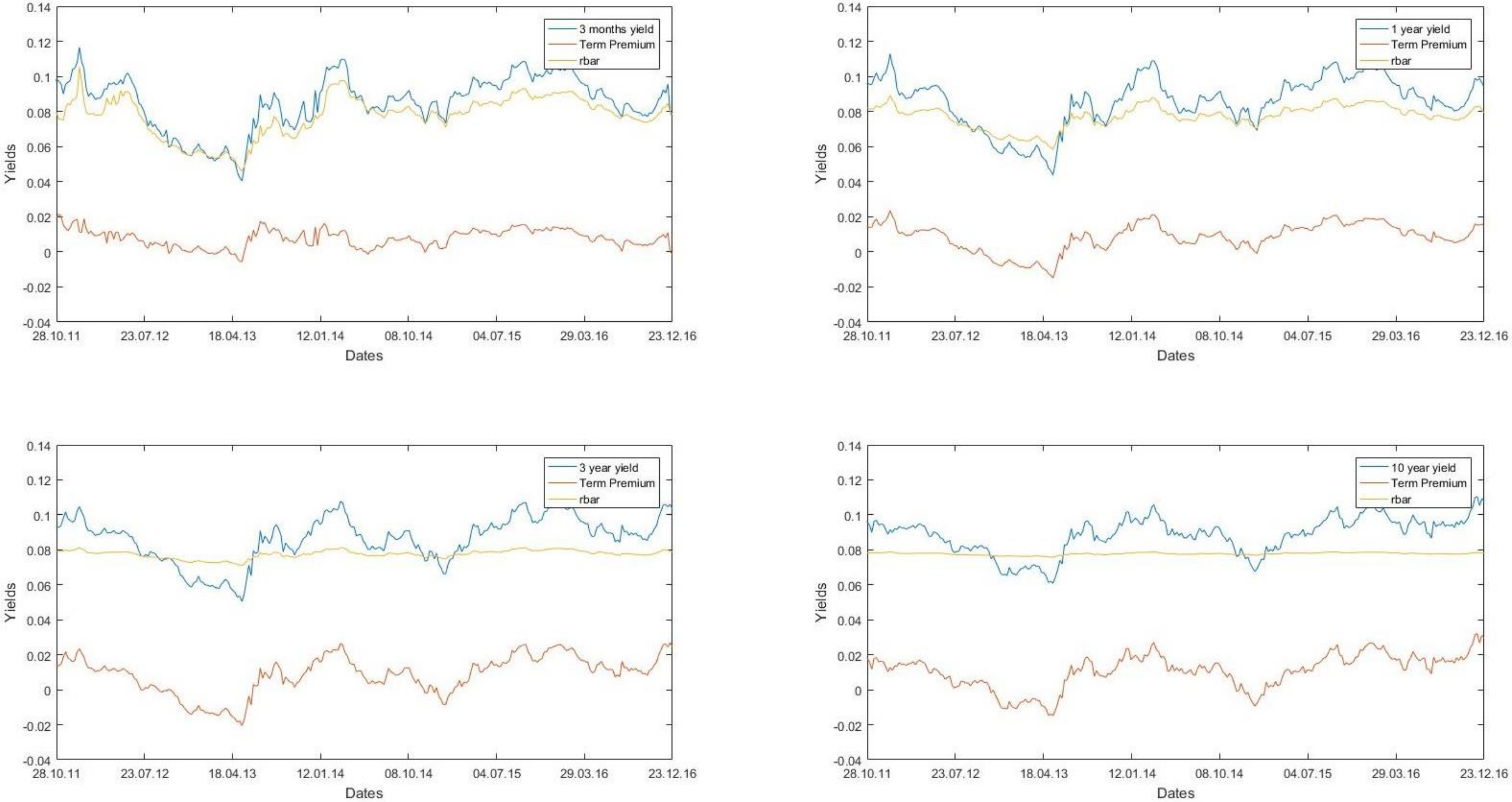


Figure 4: YIELD DECOMPOSITION FOR INDICATED MATURITIES

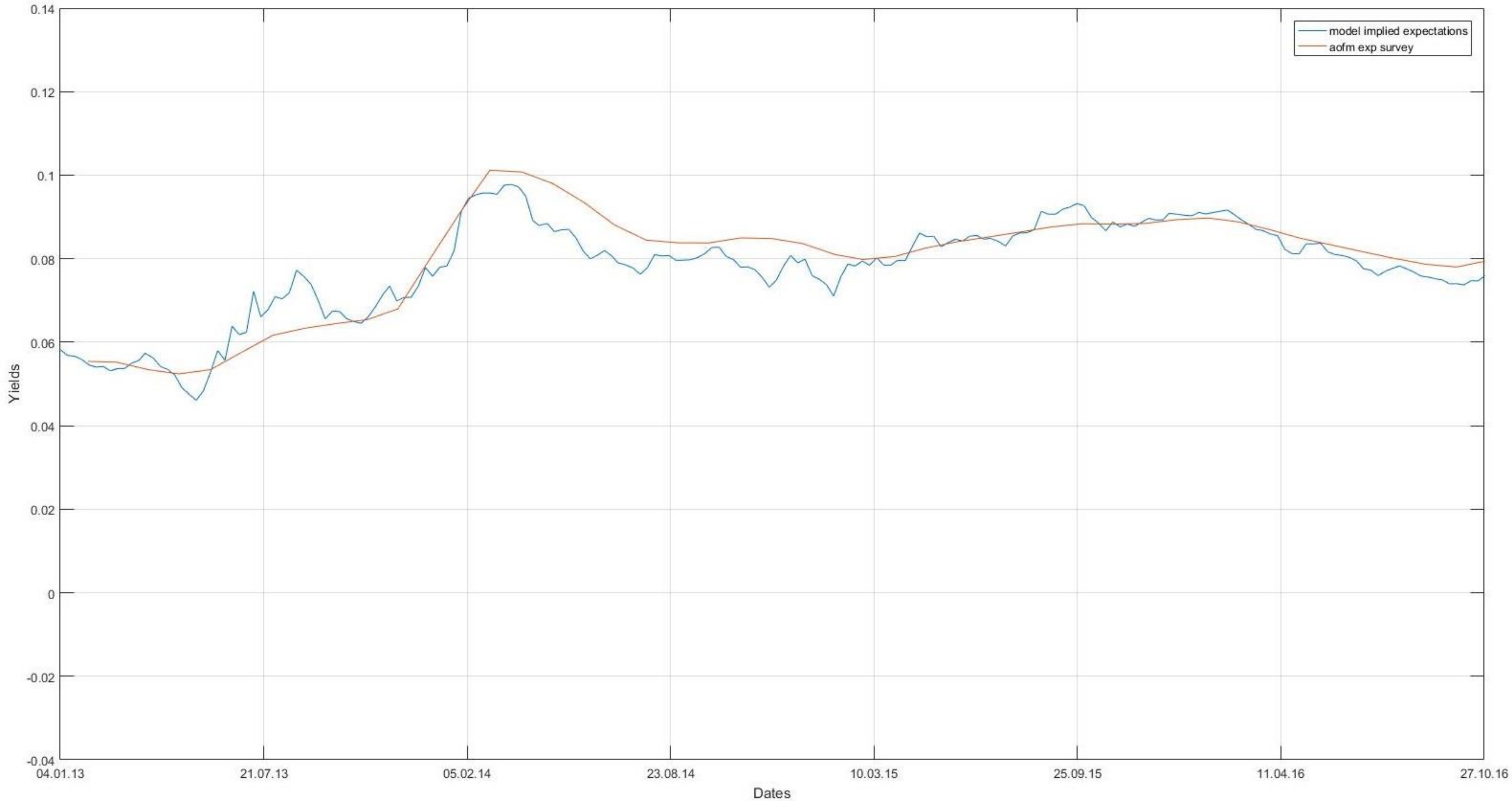


Figure 5: COMPARISON OF MODEL IMPLIED EXPECTATIONS AND SURVEY EXPECTATION

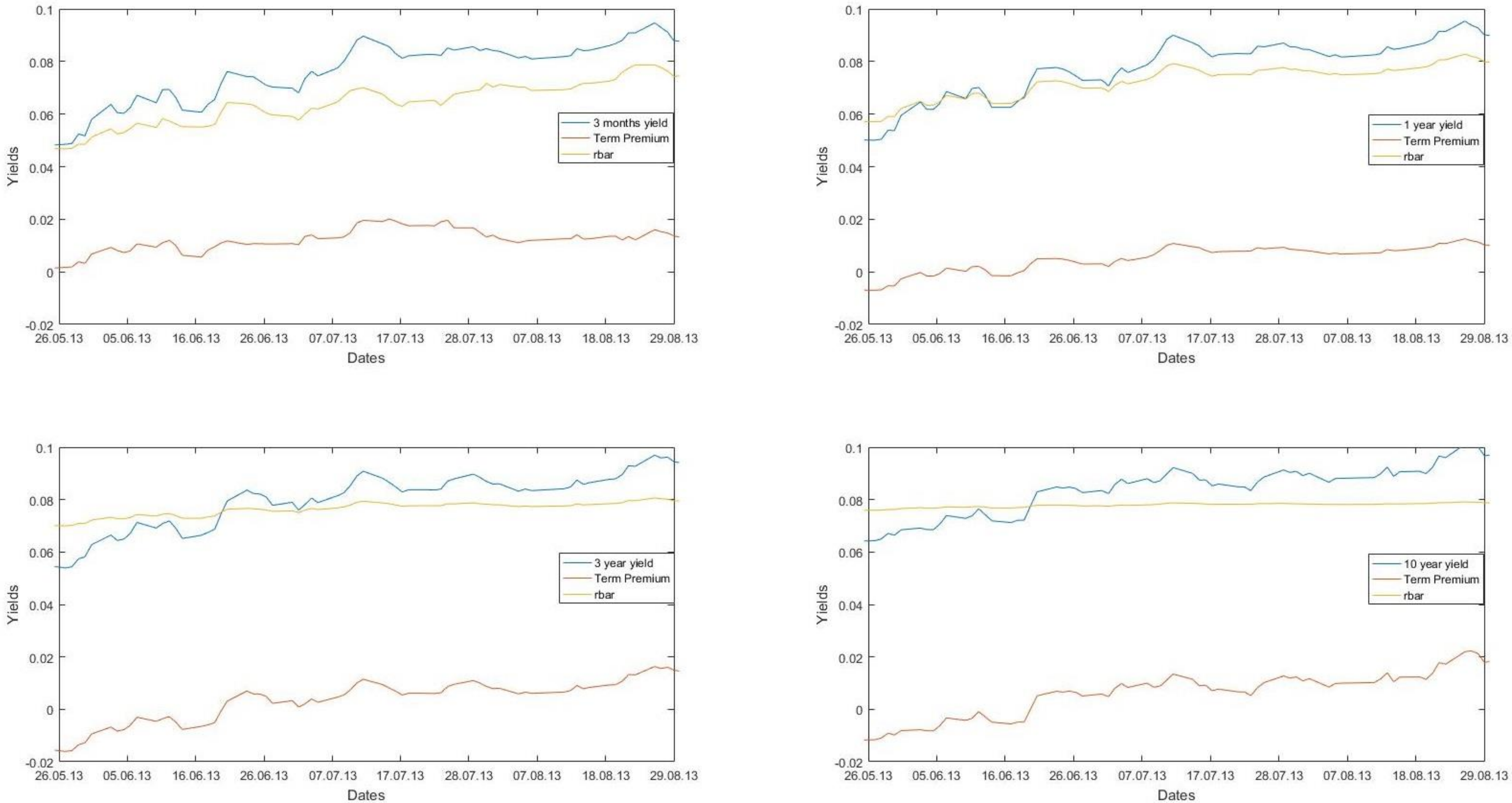


Figure 6: 2013 SUMMER CRISIS YIELD DECOMPOSITION