PERIODIC LOCATION ROUTING PROBLEM: AN APPLICATION OF MOBILE HEALTH SERVICES IN RURAL AREAS

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Sinem Savaşer

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Periodic Location Routing Problem: An Application of Mobile Health Services in Rural Areas
By Sinem Savaşer
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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Bahar Yetiş(Advisor)

Hande Yaman Paternotte

Sakine Batun

Approved for the Graduate School of Engineering and Science:

Ezhan Karaşan
Director of the Graduate School

ABSTRACT

PERIODIC LOCATION ROUTING PROBLEM: AN APPLICATION OF MOBILE HEALTH SERVICES IN RURAL AREAS

Sinem Savaşer
M.S. in Industrial Engineering
Advisor: Bahar Yetiş
June 2017

Lack of sufficient healthcare services in rural areas has been a considerable problem throughout the world for a long time. One of the alternative ways to address and solve this problem is providing mobile healthcare services in which the providers are traveling and visiting patients. These services have been obligatory in Turkey since 2010 and there are certain requirements that are enforced by Ministry of Health, such as having multiple routinized visits, having alternative visiting rules and dedicating doctors to specified villages. Based on the characteristics of this problem, it is categorized under Periodic Location Routing Problem (PLRP) literature. The common characteristic of the solution methodologies in the PLRP literature is to predefine a set of alternative schedules and select the best one among those. Unlike the other approaches that have been already studied, the developed integer programming model determines the schedules of the doctors via its constraints, dedicates each doctor to same villages through the planning horizon and satisfies certain visiting rules. The performance of the model is tested by utilizing the data set of Burdur. The proposed model is solved to optimality in reasonable times for the small instances; however, significant optimality gaps remain at the end of predefined time limits of the larger instances. In order to obtain prominent results in shorter durations, a "cluster first, route second" based heuristic algorithm is developed. Based on the computational experiments, it is observed that the solution times are significantly improved and optimal or near-optimal solutions are obtained with the heuristic approach.

Keywords: Mobile healthcare services, periodicity, location routing, integer programming, cluster first route second.

ÖZET

PERİYODİK YER SEÇİMİ VE ROTALAMA PROBLEMİ: KIRSAL KESİMLERDE MOBİL SAĞLIK HİZMETLERİ UYGULAMASI

Sinem Savaşer
Endüstri Mühendisliği, Yüksek Lisans
Tez Danışmanı: Bahar Yetiş
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Kırsal kesimlerde sağlanan sağlık hizmetlerinin yetersizliği bütün dünyada uzun süredir gözlemlenen önemli problemlerden birisidir. Bu problemi çözebilmek için önerilebilecek alternatif yollardan birisi kırsal kesimlere mobil sağlık hizmeti sağlamaktır ve bu kapsamda doktorlar köyleri gezmekte ve hastaları ziyaret etmektedir. Bu hizmet Türkiye'de 2010 yılından itibaren sağlanması zorunlu hale getirilmiştir ve Sağlık Bakanlığı uygulanması gereken bazı şartlar belirlemiştir. Bu gereksinimler köylere gerçekleştirilmesi gereken çoklu ziyaretler, alternatif ziyaret kuralları ve belirli köylere eşleştirilmiş doktorlar olarak sıralanabilir. Karakteristiklerine bakıldığında, problem Periyodik Yer Seçimi ve Rotalama Problemi (PYRP) konusu altında kategorize edilmiştir. PYRP literatüründeki çalışmaların çözüm yöntemlerinin ortak özelliği önceden alternatif çizelge kümesi oluşturmaları ve bunların arasından en iyisini sonuç olarak seçmeleridir. Bu yaklaşımın aksine, geliştirilen tam sayılı programlama modeli, kısıtları ile çizelgeleri belirlemekte, her doktoru planlama süresi boyunca aynı köylere atamakta ve belirlenmiş ziyaret kurallarına uymaktadır. Modelin performansı Burdur şehrine ait bir veri kümesi ile test edilmiştir. Sonuçlara göre önerilen model küçük ölçekli durumlarda kabul edilebilir sürelerde optimum sonucu bulabilmektedir, ancak büyük ölçekli durumlarda belirlenen zaman kısıtlarının sonunda oldukça büyük eniyilik aralıkları kalmaktadır. Bu nedenle daha kısa sürelerde kaliteli sonuçlar bulabilmek için "önce kümele, sonra rotala" temelli bir sezgisel algoritma geliştirilmiştir. Sayısal deneylere göre, sezgisel yöntem ile çözüm sürelerinin önemli ölçüde iyileştirildiği ve optimum ya da optimuma yakın sonuçların elde edildiği görülmüştür.

Anahtar sözcükler: Mobil sağlık hizmetleri, periyodiklik, yer seçimi ve rotalama, tam sayılı programlama, önce kümele sonra rotala.

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Chapter 1

Introduction

Lack of sufficient healthcare services in rural areas has been a considerable problem in the world for a long time. The doctors tend to provide services in urban areas as they prefer to expertise and work under better conditions. Similarly, since the majority of the population is accumulated in large cities, medical centers are mostly located in these areas. On the other hand, the number of medical centers and staff in rural areas are substantially lower. Thus, people living in countryside have to go to the nearest healthcare centers to get medical treatment. In general, these people might not prefer to travel long distances to get routine checkups. More importantly, in an emergency situation, they might not be able to get to a hospital quickly. Therefore, accessing satisfactory healthcare services can be a problem for those living in rural areas.

This problem can be observed in Turkey very clearly. According to a research done by Ministry of Health in 2008, the average death rate among new born babies throughout the country is 1.6%, whereas this number increases to 3.9% in rural areas. The vaccination rate of the children up to the age of 2 is 60% in rural areas which is 14% less than the average of Turkey. While the average of the country goes up to 91%, 74% of the mothers are assisted and guided by a medical staff during birth in the underdeveloped regions and only 33% of these staff consists of doctors as the remaining ones are nurses and midwifes [1].

Several solutions could be utilized in order to solve this issue. Doctors could be encouraged to practice in rural areas with some privileges and promotions. More investments could be utilized in rural areas in order to increase the number of medical centers. Another approach could be utilizing mobile healthcare services in these areas. With this system, medical staff could travel to the villages to visit patients and provide primary healthcare services and perform basic checkups.

This thesis addresses the option of improving the healthcare system in rural areas by focusing on mobile healthcare services. The aim of the study is to determine the weekly schedules of the healthcare providers so that they spend the minimum amount of time while they are traveling between villages. In order to achieve this, a mathematical formulation is generated according to the problem specific requirements which makes use of the well-known vehicle routing problem (VRP). Additionally, in order to obtain results in shorter times without compromising the solution qualities, a two-stage heuristic algorithm is introduced in which clusters are generated first and routes are determined second.

The remainder of this study is organized as follows: in Chapter 2, an overview of the world's and Turkey's healthcare systems is presented and they are compared with each other in terms of both primary and mobile healthcare services. Chapter 3 introduces the problem specific requirements in detail and defines the problem considered in this thesis. Chapter 4 reviews the most relevant vehicle routing literature related with this work. Moreover, the distinctive characteristics of the problem studied are pointed out and the contributions of this work are emphasized.

In Chapter 5, an integer programming model developed for the problem is presented. In this model, each healthcare provider's schedules are determined while satisfying the necessary requirements. Technical details of the formulation are explained explicitly in this chapter. In the following chapter, the results of the computational studies are presented and performance of the formulation is evaluated. Various elements' effects on computational times are also investigated and discussed.

In Chapter 7 the details of the *Cluster First, Route Second* based heuristic algorithm are discussed. Two variations of this approach are developed in order to observe the effects of various elements in solution qualities. Chapter 8 is dedicated to the computational studies of the heuristic algorithm. The results of both variants are compared with each other and mathematical formulation in terms of solution times and qualities.

The thesis ends with an overview of the work done and also with some guidelines for future research with Chapter 9.

Chapter 2

Healthcare Systems in the World

Healthcare is defined as diagnosis, treatment and prevention of disease, injury, illness and other physical and mental impairments in human beings. The healthcare services can be mainly categorized into three as primary, secondary and preventive healthcare services.

Primary healthcare addresses the basic health problems that arise in the community and provides curative services to those who are in need. The services provided as primary healthcare are grouped in several categories in Declaration of Alma Ata in 1978 by World Health Organization (WHO). These services can be listed as follows [2]:

- Promotion of food supply and appropriate nutrition,
- Adequate amount of clean water and basic sanitation,
- Education on preventing and controlling health problems,
- Maternal and child healthcare,
- Immunization against the major infectious diseases,
- Treatment of common and locally endemic diseases and injuries,
- Supply of necessary drugs.

Secondary healthcare includes the services provided by medical specialists to patients who have a disease/illness in a certain organ system rather than a general problem. Preventive healthcare includes the services provided prior to a disease. In other words, these services are given to the community to be able to prevent diseases from occurrence. There are a lot of services for adults, women and children. Some of the main services provided are as follows: blood pressure, cholesterol, depression, diabetes, Hepatitis B and C, HIV and lung cancer screenings for adults, breast and cervical cancer and osteoporosis screenings for women, autism and obesity screening and immunization vaccines for children [3].

Besides all these, there is another type of service provided to the people in need, called mobile healthcare services. In this system, medical staff travels and visits the patients and gives the necessary diagnosis and treatment to them, instead of patients visiting medical centers to get service. Depending on the situation, mobile healthcare may include primary, secondary and/or preventive healthcare services.

In general, governments are obliged to provide healthcare services to their citizens. It is observed that almost every country is capable of providing primary, secondary and preventive healthcare services. The service providers are mostly the physicians, nurses, therapists and pharmacists. Besides having some common characteristics, these services show variations based on the policies of countries in which they are practiced. In order to understand how these services differ, several countries' policies across the world are discussed and compared with each other next. While selecting the countries to examine, different development levels are considered as well as various geographical locations.

The United States of America (USA)'s healthcare system is largely provided by private funds. It could be said that around 70% of the capital is funded by private organizations whereas the remaining 30% is funded publicly. 16% of the citizens do not have any health insurance, which corresponds to approximately 50 million people [4]. The government spent 17.1% of its gross domestic product (GDP) for healthcare in 2013 [5]. Besides the healthcare services provided at hospitals and medical clinics, mobile healthcare services are also utilized in the system. These mobile services include medical examinations, surgical operations and laboratory test services. Physicians are giving primary healthcare services, whereas specialists provide secondary healthcare to the cases with continuous care and chronic treatments [6].

In Canada, healthcare is delivered through a publicly funded healthcare system with some minor exceptions. All of the citizens of the country are insured by the national health insurance program [7]. Total budget utilized for healthcare constitutes 10.9% of the GDP of Canada in 2013 [5]. On the other hand, mobile services are considered under enlarged healthcare services, which includes nursery home visiting and mobile clinics. The physicians mainly provide secondary healthcare services in the mobile system in order to follow up the patients who have already gone under surgical treatment. As this is the case, mobile services are provided whenever necessary, therefore there are not any regulations on this system [8].

Healthcare system in United Kingdom (UK) is funded by the government and national health insurance covers every citizen living in the country [9]. In 2013, 9.1% of the GDP of UK is reserved for healthcare [5]. In this country mobile healthcare services are not provided.

France is another country which funds its healthcare system publicly. They are also utilizing the national health insurance program in order to provide insurance to their citizens. With 11.7% of GDP provided for healthcare services, France is one of the countries that invests most to this area [5]. Besides the common healthcare providers, general practitioners are also working in healthcare [10]. In terms of mobile services, elderly people are the main beneficiaries. General practitioners who are giving home care services are providing primary services to the elderly as it is harder for them to go to hospitals and minor complaints such as pain, infections, etc. can be cured at home easily. Despite the fact that the country makes use of mobile healthcare services, it is indicated that this system is not substantially developed [11].

Healthcare in Norway is funded mostly by public sources, while a very small portion of it is funded by private organizations. This country is another one that provides every citizen a health insurance by national health insurance program. The government of Norway spent 9.3% of its GDP for healthcare in 2013 [5]. In addition to the regular providers, general practitioners are also providing service and almost every Norwegian have chosen their practitioners and registered to their choices [12]. In terms of mobile services, they only provide limited primary healthcare services to the children [13].

Egypt is one of the countries that its healthcare system is funded publicly and privately together. Around half of the citizens already have an health insurance while another 30% of them enrolled to have one. The government aims to decrease the percentage of uninsured population in the following years [14]. In terms of the investments on healthcare, 5.1% of 2013's GDP is utilized for these services in Egypt [5]. Even though the percentage of the reserved budget to healthcare is relatively lower than afore mentioned countries, Egypt benefits from mobile services more than most of them. There are more than 500 mobile health clinics and they only provide primary healthcare. It is found out that urban areas are taking advantage of mobile services twice more than rural areas [15].

Thailand's healthcare system is mostly funded by public sources. According to the data obtained, 99.5% of the people have a health insurance [16]. In the last years, around 4.6% of the GDP is used for healthcare services [5]. Thailand is another country that implements mobile services commonly. These services can be categorized into two according to the beneficiaries as elderly and poor people. For the elderly, nurses travel and provide information on health and social services in addition to providing primary healthcare. These visits are controlled by the government and they have to be done at least once in a month. For the poor areas, physicians are providing primary healthcare but they work with voluntary groups [17].

India's healthcare system is mostly funded privately; it could be said that only 30% of the capital is funded by public where the remaining 70% depends on private organizations. India is one of the countries that has very low insurance rate. According to recent surveys, it is found out that at most 25% of the citizens have a health insurance [18]. The budget reserved for healthcare is 4.0% of the total GDP of 2013 [5]. Besides the regular care givers, traditional healers also have an effective role in this area [19]. In terms of mobile healthcare, mostly rural areas benefit from the services. Mobile healthcare clinics are providing primary and after birth care centers are providing secondary healthcare services [20]. There is also a program called Rural Unit for Health and Social Affairs (RUHSA) which is established in 1977. According to their policy, there are 5 family care volunteers in a team, where each of them is responsible from 200 families. There exists 16 groups of 1000 families and 4 mobile teams. These families are visited and provided primary healthcare by a mobile medical team once in a week [21].

Healthcare in Brazil is funded by public and private organizations jointly. It is recorded that at least 25% of the people have a health insurance [22]. Although insurance rate is low, it is observed that Brazil spent 9.7% of its GDP for healthcare services in 2013 [5]. Mobile healthcare services are not provided by the government.

In Turkey, healthcare depends on both private and public funds. It is observed that the insurance rates are increased in the last years and reached to 99.5% of the population. Total budget reserved for healthcare constitutes 5.6% of the GDP in year 2013 [5]. In 2010, a new type of healthcare providers is introduced to the system, that are family practitioners. Every citizen is registered to the closest family practice center and the practitioners are responsible for providing primary healthcare services to them [23]. Moreover, with this new system, mobile healthcare services became more effective. These family practitioners are also responsible from providing primary healthcare in rural areas which do not have any medical centers. The visiting frequencies of the mobile teams depend on the population of the villages which require this type of service [24].

As it could be seen, basic and mobile healthcare policies show variations from one country to another. The summary of these policies for each country can be found in Table 2.1. One of the most important observations made is that even though there are some exceptions, mobile services are commonly used throughout the world for different beneficiaries and purposes such as serving elderly, children, poor or rural areas. However, these services have started to be provided in the recent years in most of the countries and are not completely developed. The widespread but rudimentary utilization of mobile services indicates that this area could be studied to improve the quality and effectiveness of the system.

	General Healthcare Services				Mobile Healthcare Services		
	Fund Provider	**Insurance Rate**	**GDP % in 2013**	**Different Service Provider**	**Service Type**	**Service Provider**	**Beneficiary**
USA	70% Private, 30% Public	84%	17.1	-	Primary & Secondary Healthcare	Medical Staff and Physicians	-
CANADA	Public with Minor Exceptions	National Health Insurance	10.9	-	Secondary Healthcare	Physicians	Follow-up Patients
UK	Public	National Health Insurance	9.1	-	-	-	-
FRANCE	Public	National Health Insurance	11.7	General Practitioners	Primary Healthcare	General Practitioners	Elderly
NORWAY	Public with Minor Exceptions	National Health Insurance	9.6	General Practitioners	Primary Healthcare	-	Children
EGYPT	Public and Private	50% Has, 30% Enrolled	5.1	-	Primary Healthcare	-	Every citizen
THAILAND	Mostly Public	99.5%	4.6	-	Primary Healthcare	Physicians and Nurses	Elderly & Poor Areas
INDIA	70% Private, 30% Public	At Most 25%	4.0	Traditional Healers	Primary Healthcare	Family Care Volunteers	Rural Areas
BRAZIL	Public and Private	At Least 25%	9.7	-	-	-	-
TURKEY	Public and Private	99.5%	5.6	Family Practitioners	Primary Healthcare	Family Practitioners	Rural Areas

Table 2.1: Comparison of the Healthcare Systems of Some Countries

Besides all these characteristics discussed, lack of healthcare in rural areas is a joint problem of the world despite the huge differences between developing and developed countries. There are many examples of this problem that can be observed in various countries. For instance in USA, 20% of people, corresponding to more than 60 million, live in rural areas but only 9% of the doctors are serving in these areas, which creates lack of access to healthcare providers in rural areas [25]. Canada indicates that there is shortage of hospital beds and medical practitioners in rural areas [4]. Throughout the history, the main problem for

Thailand's healthcare system is the inadequate number of physicians in rural areas [26]. France is trying to encourage practitioners to work specifically in these areas [27].

It is clear that providing healthcare in rural areas is a necessity and it is believed that mobile healthcare services could be the solution to solve this problem. However, if the system is not structured well and services are not provided effectively, healthcare providers may cover excessive distances as well as spending a lot of non-value added time. This will result in substantial increases in the costs, which will be reflected as excessive expenditure and budget violations to the governments. While providing mobile services, Operations Research (OR) tools can be utilized to develop more efficient methodologies. With the help of these, the operational costs can be decreased and the workforce can be utilized in a more efficient manner. The quality of the service provided will indirectly increase as the practitioners will spend the time for treatments and medical procedures instead of spending it for their travels between villages. Therefore, in this thesis, mobile healthcare services is aimed to be improved via OR tools and methodologies.

Chapter 3

Problem Definition

Mobile healthcare service is the transportation of care providers to the patients as opposed to the general practice where patients visit the medical centers. In the previous chapter it is pointed out that this application area is quite new; however, it is also widely used across many villages of Turkey. Family practitioners are obliged to travel for certain time periods to those villages which do not have any medical center since 2010. Monthly schedules for these practitioners are generated according to certain requirements. With the help of OR approaches the time spent on roads and travel expenses can be reduced while determining these schedules.

The aim of this study is to generate cost efficient monthly service schedules for each family practitioner. In addition to this, each practitioner should be assigned to a medical center that they leave and return (origin and end points) at the beginning and end of each week for paperwork and reporting purposes. These points are referred as base hospitals in the remainder of this study and another target is to determine the base hospital of each practitioner in the least costly manner. While determining schedules and base hospitals, there are some problem specific requirements that have to be satisfied and these are enforced by the Ministry of Health and published in the Official Journal [28].

First of all, according to the population size, each village has a minimum visiting hour limit per month. When a working day is accepted as 8 hours (from 8.00 am to 5.00 pm with 1 hour break), it can be said that a period of 4 hours corresponds to a half of a day. In order to simplify the parameters, frequencies are determined in terms of number of half days a village has to be visited in a month. There are also alternative visiting rules for each frequency level that have to be satisfied. For each population level, the corresponding visiting hours, frequencies and alternative visiting rules are given in Table 3.1.

Population Size	**Minimum Visiting Hours (per month)**	**Frequencies (half-day/month)**	**Visiting Rule Alternatives**
≤ 100	4	1	1 half-day in a month
≤ 300	8	2	1 day in a month 1 half-day in each two weeks
≤ 750	16	4	1 day in each two weeks 1 half-day in each week
≤ 1000	32	8	1 day in each week 1 half-day in each 2.5 days
> 1000	48	12	1.5 days in each week

Table 3.1: Frequencies and visiting rules of the villages according to the population size

As it can be seen from the table, a small village with less than 100 people living has to be visited one half day per month. A population size between 100 and 300 people is equal to visiting a village 8 hours in a month, which is actually 2 half days/month. A medium sized village with 300-750 residents has to be visited at least 4 half days in a 4 week period. Practitioners have to visit villages 8 half days in their monthly schedule if the population size changes between 750 and 1000. When the number of residents exceeds 1000, the frequency rate can go up to 12 half days in a month which corresponds to 48 visiting hours.

There are certain visiting rule alternatives for each frequency level. The villages with frequency 1 can be visited for a half day anywhere available on the schedule. The ones with frequency 2 can be visited two half days consecutively, which corresponds to a single day, or one half day in every two week periods. Similarly, the villages with frequency of 4 can get service two half days in a row in every two weeks, or a half day in each week. The villages with frequency 8 can be visited two half days consecutively in each week, or single half day in every 5 half day periods. There is a single alternative for the villages with 12 frequency, which is visiting those villages 3 half days in a row, i.e. 1.5 days, in each week. The schedules have to be constructed according to these visiting rules. As it could be observed, the aim here is to stabilize the time periods between two consecutive visits so that the practitioners have more balanced schedules and patients can access to healthcare services in an equally distributed manner.

There is another requirement which is related to the alternative visiting rules of different frequency levels. This requirement is that the services have to be provided at the same slot in each week to the villages that are visited in multiple weeks. For instance, if a village with frequency 4 is visited on only Wednesday morning in the first week, then it cannot be visited on Thursday afternoon in the next week but has to be visited Wednesday morning in the following three weeks. Similarly, a village with frequency 2 can be visited on Monday afternoon in third week, if and only if it is visited on Monday afternoon also in the first week. By this way, it will be easier for the patients to follow the arrival plan of the doctor and the possible confusions can be avoided.

One of the other problem specific requirements besides the visiting frequencies and rules is that the practitioners are dedicated to the villages. In other words, when a practitioner is assigned to certain villages, s/he will be responsible from the same ones in the remaining part of his/her schedule. Since the practitioner who started the treatment and who knows the patient history could give better decisions in monitoring the patient in the following weeks, this statement is also included in the regulatory of the Ministry of Health [28].

Finally, the base hospitals for the practitioners are not known. In other words, the start and end points of the tours of each practitioner need to be selected among the existing medical centers in this problem in such a way that the total travel distance is kept at minimum.

Eventually, the unique characteristics of the problem studied in this thesis can be summarized as follows:

- Visiting frequencies depend on the population sizes.
- There are alternative visiting rules for each frequency level.
- Services must be provided at the same slot each week.
- Practitioners are dedicated to their villages.
- Base hospitals should be selected for each practitioner.

In addition to the requirements that are explained, there are also several assumptions made while determining the schedules of the practitioners. First, it is known that the staff leaves the base hospital at the beginning of each week and returns to it when the week is over. On the other hand, it is assumed that the practitioners are staying at the villages they visit during the weekdays. Another assumption is that the practitioners are able to travel between villages in the middle of the day without arriving late to the next destination.

Considering these requirements and assumptions, the aim of this study is to determine the monthly schedules of the family practitioners and their base hospitals so that they spend the minimum amount of time during traveling, which will be both cost and manpower efficient. As the routes of the practitioners are going to be generated, VRP will be the basis of this study. In addition to the routing decisions, location decisions are also given, which indicates that location routing problems (LRP) are going to be considered. However, there are villages where multiple visits in the planning horizon are required so that periodicity has to be taken into account. Consequently, Periodic VRP (PVRP) plays a crucial role in the process of defining schedules. If the two major research areas are combined, i.e., LRP and PVRP, the problem considered in this study can be categorized as Periodic Location Routing Problem (PLRP).

Chapter 4

Literature Review

As it is explained, the problem considered in this thesis can be classified as a PLRP. However, the literature on this research area is not rich yet and it is required to investigate the dynamics of PVRP and LRP separately. In this chapter, first a brief introduction to VRP and its extensions is going to be presented. Then, PVRP and LRP literatures are going to be reviewed in detail and finally the limited number of articles on PLRP are going to be discussed. At the end, the unique characteristics of the solution methodology utilized in this study are going to be explained and contributions of this thesis are going to be emphasized.

4.1 Vehicle Routing Problem and Extensions

VRP is the distribution of goods or services from depots to the customers while optimizing the routes between these points in a such a way that the customer demands are satisfied without violating any problem specific constraints. There are quite a lot of variations of this problem: Capacitated VRP (CVRP), VRP with Time Windows (VRPTW), Green VRP (GVRP), Multiple Depots VRP (MDVRP), Split-delivery VRP (SDVRP), Periodic VRP (PVRP) are being the most known and studied ones [29]. There is also another related area that is called Location Routing Problem (LRP), in which locations of the depots are considered as decisions in addition to the routing decisions.

There are quite a lot review studies conducted on VRP in the recent years by various researchers; such as Cacares-Cruz et al. [29], Golden et al. [30] and Laporte [31]. The first problem addressed in VRP literature was a CVRP studied by Dantzig and Ramser [32] under the name of "The truck dispatching problem" in 1959. In the following years, all afore mentioned VRP extensions are widely studied by many researchers and numerous algorithms are developed which include both exact and heuristic approaches. Among the large number of studies conducted in the last 60 years, PVRP is going to be addressed in detail next as periodic routing problems have an essential role for this thesis.

4.2 Periodic Vehicle Routing Problem

PVRP is a variation of the classical VRP in which the vehicle routes are constructed over a multiple day horizon. In the standard PVRP, customers require to be visited one or more times within the planning period and the visit combinations are selected from an available set of alternatives. The aim of PVRP is to find the allocation of customers to the predefined schedules such that each node is visited required number of times while minimizing the total cost. There are some surveys published on this specific topic by Francis et al. [33] and Campbell and Wilson [34].

This problem has a broad application area, which may be divided into three main categories as pickup, delivery and on-site services [34]. The majority of the pickup problems consider waste and recyclable collection problems [35, 36, 37, 38]. A research on distribution of fast-moving consumer goods to its stores is the first delivery application of the PVRP in the literature [39]. Grocery store distributions [40], replenishment of vending machine stocks [41] and routing for blood banks [42] are also considered as periodic inventory routing problems. Finally, periodic maintenance of elevators [43], home healthcare services [44] and assigning teaching assistant to the disabled people [45] are in the category of on-site service applications of the PVRP. The summary of the application areas of the PVRP can be found in Table 4.1.

Pick-Up Services		Delivery Services		On-Site Services	
Articles	**Application Area**	**Articles**	**Application Area**	**Articles**	**Application Area**
Beltrami and Bodin (1974) [35]	Waste Collection	Golden and Wasil (1987) [39]	Coca-Cola Distribution	Blakeley et al. (2003) [43]	Elevator Maintanance
Angelelli and Speranza (2002) [36]	Waste Collection	Gaur and Fisher (2004) [40]	Grocery Store Distribution	An et al. (2012) [44]	Home Healthcare
Bommisetty et al. (1998) [37]	Recyclable Collection	Rusdiansyah and Tsao (2005) [41]	Vending Machine Replenishment	Maya et al. (2012) [45]	Teaching Assistance Planning
Teixeria et al. (2004) [38]	Recyclable Collection	Hemmelmayr et al. (2009) [42]	Blood Bank		

Table 4.1: PVRP Application Areas

This problem emerged fifteen years after the introduction of classical VRP, by Beltrami and Bodin [35]. The authors were motivated by a municipal waste collection problem in New York City, in which garbage sites need to be visited with different frequencies with only two alternative schedules. They did not formulate or define the problem formally, but developed a heuristic and proved that the problem is more difficult and complex than classical VRP. In 1979, Russell and Igo [46] called the problem "Assignment Routing Problem" and solved the routing problem after selecting a schedule for each node. Christofides and Beasley [47] provided the first mathematical formulation and named the problem as "The Period Routing Problem". The problem could not be solved optimally due to its complexity. In the same year, Tan and Beasley [48] proposed an integer relaxation of the formulation and improved the computational time of the previous heuristics for larger instances. In 1991, Russell and Gribbin [49] introduced local search algorithm on top of Tan and Beasley's work [48]. Chao et al. [50] aimed to escape from the local optima with their heuristic method in 1995. After this study, a 32 instance sized data set is constructed, which is referred as the "old data set" in the literature. First 10 instances were obtained from Christofides and Beasley [47], next three were taken from Russell and Gribin's work [49], and the last 19 of them were introduced by Chao et al. [50]. Most of the following studies compared their solution qualities and computational times using this data set.

Cordeau et al. [51] solved the PVRP in 1997 with a tabu search heuristic, in which it is aimed to go to a better solution in the neighborhood at each iteration. It is observed that this method outperforms all of the previous studies when the results are compared with the old data set. Alegre et al. [52] used an evolutionary method to solve the PVRP, which is an adaptation of scatter search in 2007. However, neither the solution quality, nor the computational times of this study show a significant improvement. In the same year, Mourgaya and Vanderbeck [53] utilized a heuristic based on column generation method. In the first step, customers are assigned to schedules in the decreasing order of demands. Column generation is applied as the second step in the methodology. In 2009, Hemmelmayr et al. [54] applied a variable neighborhood search (VNS) heuristic to the PVRP by moving to a better solution in the neighborhood. The routes are constructed by using Clarke and Wright algorithm. Gulczynski et al. [55] proposed a new heuristic method in 2011 for solving the PVRP. Customers are assigned to schedules with a mixed-integer programming model which minimizes the maximum amount of demand served on one day. Clarke and Wright algorithm is used for determining the routes. After obtaining the initial solution, improvements such as 2-opt, local search, etc. are applied. Since at every iteration a mathematical model is run, computational times of the heuristic is higher than the previous methods; however, in terms of solution quality, this study outperformed most of the methodologies. Cordeau and Maischberger [56] in 2012 offered a parallel iterated tabu search heuristic. This method is able to find the best solutions for most of the cases of the old data set; however, all of them are a tie with one of the previous works. Therefore, it could not improve any of the solutions found until 2012. Cacchiani et al. [57] proposed a heuristic algorithm based on the linear programming (LP) relaxation of a set-covering problem in 2014. The LP relaxation is solved by column generation, where columns are generated by an iterated local search algorithm.

In the studies that are examined above, a set of alternative visiting combinations for each frequency level is defined as a parameter and the solution methodology, whether it is an exact or heuristic algorithm, chooses the best among the elements of this set. For instance, let's assume that a demand point has to be visited 3 times/week. Then, the possible visiting combinations for this point could be *“Monday, Wednesday, Friday”* or *“Monday, Tuesday, Thursday”*. In this manner, a set of alternatives are generated in a logical way and the solution methodology picks the most convenient combination depending on the objective function. However, it is both time consuming and inefficient to define all visiting combinations in the algorithms in most of the cases, especially when the number of possible schedules increases exponentially. Naturally, unless all of the combinations are defined, the solution may be suboptimal. There are two main studies found in the literature which do not utilize predefined set of schedules. Instead, An et al. [44] and Maya et al. [45] develop mathematical models which generate the service schedules of nurses that are providing home healthcare in Korea and teaching assistants for disabled students in Netherlands, respectively, without taking any visiting combinations as an input.

In addition, dedicating vehicles to certain customers or enforcing visiting rules are not common in this area. There are a few studies in the literature that consider these characteristics. For instance, Smilowitz et al. [58] ensures that the drivers are responsible for the same customers since the familiarity with them can be beneficial for customer satisfaction. Maya et al. [45] is another study that assigns teaching assistants to same students as it will be easier for disabled children to get used to a single person. An et al. [44] does not consider assigning one nurse to same patients. On the other hand, the patients have visiting frequencies with certain rules; such as some has to be visited once in every two days, while others have to be visited once in every 6-day periods. This study is the only one in the literature which implements alternative visiting rules.

The summary of the studies explained above can be found in Table 4.2. As it could be seen, there have been a lot of effort put on the PVRP in the last 40 years. On the other hand, majority of the studies define the schedules in advance and do not consider the requirements that this problem have, i.e., having alternative visiting rules and dedicated vehicles. Therefore, it can be said that none of the studies in the PVRP literature completely overlap with the problem that is considered in this thesis.

Authors	Schedules	Dedicated Vehicles	Visiting Rules	Solution Methodology
Tan and Beasley (1984) [48]	Predefined	✘	✘	LP Relaxation
Russell and Gribbin (1991) [49]	Predefined	✘	✘	IP and Savings Algorithm
Chao et al. (1995) [50]	Predefined	✘	✘	LP Relaxation
Cordeau et al. (1997) [51]	Predefined	✘	✘	Tabu Search Heuristic
Alegre et al. (2007) [52]	Predefined	✘	✘	Adaptation of Scatter Search
Mourgaya and Vanderbeck (2007) [53]	Predefined	✘	✘	Column Generation Based Heuristic
Hemmelmayr et al. (2009) [54]	Predefined	✘	✘	Variable Neighborhood Search
Gulczynski et al. (2011) [55]	Predefined	✘	✘	IP Based Heuristic
Cordeau and Maischberger (2012) [56]	Predefined	✘	✘	Parallel Iterated Tabu Search Heuristic
An et al. (2012) [44]	Decided by the model	✘	✔	Two-Phase Heuristic Algorithm
Maya et al. (2012) [45]	Decided by the model	✔	✘	Auction Algorithm
Cacchiani et al. (2014) [57]	Predefined	✘	✘	Column Generation Based Heuristic

Table 4.2: PVRP Summary

It should be noted that, this problem also has extensions like classical VRP such as; PVRP with Time Windows, Multi-Depot PVRP, PVRP with Service Choice, PVRP with Intermediate Facilities and Multi-Objective PVRP [34]. However, since these extensions are not in the scope of this thesis, they are not examined in detail.

4.3 Location Routing Problem

Location Routing Problem (LRP) integrates two decision levels simultaneously; a strategic level decision that is locating the depots and a tactical level decision that is determining vehicle routes from each depot to the customers. The objective of this problem is to minimize the overall fixed and operational costs. These two decisions are generally addressed separately in order to reduce the complexity of the overall problem; however, it has been proven that this strategy only leads to suboptimal solutions [59]. Therefore, in the last 30 years, LRP gained more and more significance and is one of the active areas in the OR literature. The most recent review articles on this topic are conducted by Nagy and Salhi [60] in 2007 and Prodhon and Prins [61] in 2014 and in both studies, the development of LRP and existing solution approaches are discussed.

As OR is an application-oriented discipline, there are many application areas of LRP as it is the case for PVRP. For instance in terms of healthcare services, blood banking [62] and medical evacuation [63] are studied under LRP literature. Certain studies consider newspaper [64], grocery store [65], bill [66] or parcel distribution [67] via determining the locations of the distribution centers and routes to operate from them to deliver the goods to the customers. Similar studies to locating rubber plants [68] or biofuel refineries [69] and routing the vehicles to transfer the goods to their destinations are also studied in LRP literature. In humanitarian relief operations LRP is commonly applied to inventory prepositioning [70] and relief item distribution [71] problems. Waste collection [72] and hazardous material transportation [73] could be given as other application areas of this OR problem. The summary of the application areas of LRP can be found in Table 4.3.

Articles	Application Area	Articles	Application Area	Articles	Application Area
Or and Pierskalla (1979) [62]	Blood Banking	Kulcar (1996) [72]	Waste Collection	Alumur and Kara (2007) [73]	Hazardous Material Transportation
Jacobsen and Madsen (1980) [64]	Newspaper Distribution	Chan et al. (2001) [63]	Medical Evacuation	Bai et al. (2011) [69]	Biofuel Refinery Location
Nambiar et al. (1981) [68]	Rubber Plant Location	Lin et al. (2002) [66]	Bill Delivery	Rath and Gutjahr (2014) [70]	Inventory Prepositioning
Semet and Taillard (1993) [65]	Grocery Store Distribution	Wasner and Zapfel (2004) [67]	Parcel Pick-up and Delivery	Moreno et al. (2016) [71]	Relief Item Distribution

Table 4.3: LRP Application Areas

Watson-Gandy and Dohrn [74] is one of the first studies that considers routing of vehicles which distribute food and drinks while locating depots. However, since the computers and optimization tools were not developed 50 years ago, it was not possible to obtain results. The benefits of addressing location and routing decisions at the same time are quantified by Salhi and Rand [59] for the first time in 1989, a decade after the definition of the problem. Since then, there have been many attempts to solve the LRP in various contexts. As the problem is reduced to classical VRP when the number of alternative locations is set to 1, it could be said that the LRP is an NP-Hard problem. Therefore, the majority of the solution approaches consist of heuristics along with a few exact algorithms.

On of the earliest studies developing an exact method for LRP is by Laporte et al. [75]. The authors consider a cutting plane algorithm to address the problem. Another study of Laporte et al. [76] reformulates the problem into a traveling salesman problem via graph transformation and applies branch-and-bound algorithm to be able to solve large instances. Labbé et al. [77] formulate plant-cycle location problem and develop a branch-and-cut algorithm to find the optimal solution. Belenguer et al. [78] also utilize a branch-and-cut algorithm on the capacitated LRP and strengthen their model via a new set of valid inequalities. Finally, Baldacci et al [79] use a variant of set partitioning formulation and various bounding procedures that are based on dynamic programming. In their methodology, they decompose the LRP into multi-capacitated depot VRPs.

There are quite a lot of heuristic algorithms developed for the LRP where the ones that have major contributions are going to be examined here. Albareda-Sambola et al. [80] determined an initial solution via the LP relaxation of their mathematical formulation and improved this solution by utilizing tabu search heuristic. Melechovský et al. [81] also followed a similar pattern while developing an algorithm for the LRP. They find an initial solution with the p-median approach and utilize VNS meta-heuristic to obtain better results. Prins et al. proposed two different methodologies to solve the problem in 2006 [82] and 2007 [83]. In their earlier study [82], they developed a two-stage meta-heuristic in which they construct an initial solution with a greedy randomized heuristic (GRASP) first and improve this solution by local search later. In the following study [83], they utilized Lagrangean relaxation with tabu search algorithm. At the beginning they aggregate the customers and routes into super-customers and solve a facility location problem via Lagrangean relaxation on the assignment constraints. Later, they improve the routing results they obtained from MDVRP using tabu search.

During the same year, Barreto et al. [84] presented several hierarchical and non-hierarchical clustering techniques to solve the capacitated LRP. In 2010, Duhamel et al. [85] determined depot locations and so called giant tours with GRASP and split those tours according to vehicle and depot capacities while minimizing the total cost. Derbel et al. [86] propose a genetic algorithm combined with an iterative local search in order to make location and routing decisions simultaneously in an effective way. The idea of Prins et al. [83], which is addressing location and routing decisions in an order, is later adapted by Escobar et al. [87] and Ting and Chen [88] in 2013. Escobar et al. [87] utilized a hybrid heuristic algorithm where Ting and Chen [88] preferred an ant colony optimization algorithm. Most recently, Rath and Gutjahr [70] presented a MIP and VNS based approach to solve the LRP.

There are extensions of LRP such as multi-echelon LRP, multi-objective LRP or multi-period LRP. In addition to the deterministic studies that are discussed here, there also exists quite a lot of stochastic approaches in the literature. As the extent of this study does not cover those versions of the LRP, they are not examined in detail.

4.4 Periodic Location Routing Problem

PLRP can be defined as the combination of PVRP and LRP and it covers all of the decision levels observed in both problem types. The aim of the problem is finding the locations of the depots, assigning schedules to customers and determining the vehicle routes through the planning horizon while minimizing the total cost. The PLRP literature is not broad as PVRP or LRP; it has been only studied since 2007 and 7 studies have been published in this period of time. The problem is reduced to the VRP when the depot location decision is excluded and planning horizon length is set to 1. Therefore, it could be said that the PLRP is NP-hard and the literature consists of heuristic approaches rather than exact solution algorithms, as in PVRP and LRP.

PLRP is mostly studied by Prodhon in the literature. In her first study [89], a mathematical model is not provided but instead a metaheuristic approach is introduced to solve such a problem. The developed heuristic consists of three steps that are location, allocation and routing and includes local search for improvements. The algorithm is applied to three different set of instances; one day horizon (LRP), single depot (PVRP) and PLRP instances. By this way, the method is compared with the existing heuristics and its solution performances are evaluated. Prodhon and Prins [90] provides a memetic algorithm with population management. It is observed that this methodology outperforms the previous iterative heuristic approach both in terms of solution quality and computational times of PLRP instances. The next two studies by Prodhon, [91, 92] operate through an Evolutionary Local Search (ELS) algorithm. These two studies focus on the periodic decisions by improving the assignments of the customers to the visiting combinations unlike the previous two studies. Both algorithms provided better solutions than the former studies. The latter article also has a mathematical model with predefined schedules. However, it is indicated that the linear program is capable of solving small instances so that it is required to provide heuristics to handle the large PLRP instances.

Pirkweiser and Raidl [93] develop an integer linear programming (ILP) based very large neighborhood search (VLNS) algorithm for this problem. Three different procedures are applied to the solutions which are changing depot locations and visit combinations iteratively and modifying the daily routes by removing and reinserting the customers. Hemmelmayr [94] also utilizes a variant of LNS algorithm; sequential and parallel LNS. The algorithm basically destroys the solution by removing customers and repairs it by adding them to another route or visit combination. It is observed that this algorithm improves the PLRP instances significantly in terms of solution quality. The most recent study on this topic is by Koç [95]. He introduces Heterogeneous PLRP (HP), HP with Time Windows (TW) and Homogeneous PLRP with TW and provides a formulation in which he benefits from the predefined set of visit combinations. For the large scale instances, the solutions are obtained via utilizing Unified-Adaptive LNS (U-ALNS) meta-heuristic. The classification of the PLRP can be found in Table 4.4.

Authors	Schedules	Dedicated Vehicles	Visiting Rules	Solution Methodology
Prodhon (2007) [89]	Predefined	✘	✘	Iterative Metaheuristic
Prodhon and Prins (2008) [90]	Predefined	✘	✘	Memetic Algorithm
Prodhon (2009) [91]	Predefined	✘	✘	ELS
Pirkweiser and Raidl (2010) [93]	Predefined	✘	✘	VNS
Prodhon (2011) [92]	Predefined	✘	✘	Hybrid Evolutionary Algorithm
Hemmelmayr (2015) [94]	Predefined	✘	✘	LNS
Koç (2016) [95]	Predefined	✘	✘	U-ALNS
This thesis	Decided by the model	✔	✔	IP & Cluster First, Route Second Algorithm

Table 4.4: PLRP Summary

It could be observed from the table that the limited number of studies do not integrate exact solution methodologies and they utilize a set of alternative visiting combinations for the frequencies. Additionally, none of them assign dedicated vehicles to the demand points or enforce certain visiting rules.

It is mentioned that the study in this thesis can be classified as a PLRP as it includes multiple visits to certain villages and location decisions of the base hospitals of the practitioners. The unique characteristics of this problem lie under the requirements of the Ministry of Health. Because of the rules of mobile healthcare services, this problem has to have dedicated doctors (vehicles) while satisfying explained visiting rules. Therefore, these change the dynamics of the classical PLRP and generate a necessity for a novel solution approach.

In addition to these unique characteristics determined by the problem specific requirements, this study aims to find the optimal solutions by generating the schedules of the practitioners with a mathematical model without defining any alternative visit combinations as a set of parameters. If the schedules of the practitioners are determined via predetermined set of visiting combinations, the chances of obtaining suboptimal solutions highly increase which may result in deviations from the global optimum. With the approach that will be utilized in this thesis, this can be prevented and the global optimum can be reached.

To the best of our knowledge, there does not exist any study that considers creating the schedules with a mathematical formulation, satisfying visiting rules as well as having dedicated vehicles. Even though these characteristics are taken into account separately in various studies, the number of them is also very limited. Therefore, it can be claimed that this study approaches to the PLRP from a different perspective.

Chapter 5

Mathematical Formulation

The main purpose of this study is to develop a mathematical formulation that can generate the schedules of the practitioners without using a set of alternatives while satisfying the problem specific requirements. In order to do so, the constraints should be developed in a way that they could assure these requirements and generate the monthly schedules.

In this chapter, an integer programming model that is developed for this problem is introduced, which determines the schedules of the practitioners via its constraints. In the next stage, some valid inequalities are generated with the expectation of reducing the size of the solution space and obtaining the optimum solutions in shorter time. Since different combinations of valid inequalities may lead to different computational times, the best combination is determined via extensive computational studies, whose details are going to be discussed in the following chapter.

5.1 PLRP Formulation

Before presenting the optimization model for the PLRP, the following notation to be used hereafter is introduced:

Sets:

N	Set of all nodes, $N = I \cup H$.
I	Set of villages.
$I2, I4, I8, I12$	Set of villages with frequency 2, 4, 8, 12, respectively.
H	Set of hospitals.
D	Set of doctors (practitioners).
T	Set of time periods.
$NT1$	Set of time periods consisting of $\{11, 21, 31\}$
$NT01$	Set of time periods consisting of $\{10, 11, 20, 21, 30, 31\}$

Parameters:

$DIST_{nm}$:	distance between nodes $n \in N$ and $m \in N$.
DEM_i:	visiting frequency of village $i \in I$.
CAP:	maximum working time of doctors.
p:	number of base hospitals to be selected.

The decisions to be made can be represented by the following sets of binary variables:

Decision Variables:

$$x_{nm}^{dt} = \begin{cases} 1, & \text{if doctor } d \in D \text{ travels from node } n \in N \text{ to } m \in N \text{ at time} \\ & \text{period } t \in T, \\ 0, & \text{otherwise.} \end{cases}$$

$$y_i^{dt} = \begin{cases} 1, & \text{if doctor } d \in D \text{ visits village } i \in I \text{ at time period } t \in T, \\ 0, & \text{otherwise.} \end{cases}$$

$$u_n^d = \begin{cases} 1, & \text{if node } n \in N \text{ is assigned to doctor } d \in D, \\ 0, & \text{otherwise.} \end{cases}$$

$$z_h = \begin{cases} 1, & \text{if a hospital at } h \in H \text{ is selected as a base hospital,} \\ 0, & \text{otherwise.} \end{cases}$$

$$k_{ih}^{dt} = \begin{cases} 1, & \text{if doctor } d \in D \text{ who is assigned to the hospital at point } h \in H \\ & \text{is present at village } i \in I \text{ at time period } t \in T, \\ 0, & \text{otherwise.} \end{cases}$$

The following integer program for PLRP can now be proposed:

$$\text{minimize} \quad \sum_{n\in N}\sum_{m\in N}\sum_{d\in D}\sum_{t\in T} x_{nm}^{dt}\cdot DIST_{nm} - \sum_{n\in N}\sum_{m\in N}\sum_{d\in D}\sum_{t\in NT1} x_{nm}^{dt}\cdot DIST_{nm} \tag{5.1}$$

$$+\sum_{i\in I}\sum_{h\in H}\sum_{d\in D}\sum_{t\in NT01} k_{ih}^{dt}\cdot DIST_{ih},$$

subject to

$$\sum_{i\in I}\sum_{h\in H} x_{hi}^{d1} = 1, \qquad d \in D \tag{5.2}$$

$$\sum_{d\in D}\sum_{t\leq 40} y_i^{dt} = DEM_i, \qquad i \in I \tag{5.3}$$

$$\sum_{n\in N} x_{ni}^{dt} = y_i^{dt}, \qquad i \in I,\ d \in D,\ t \leq 40 \tag{5.4}$$

$$\sum_{n\in N} x_{in}^{dt+1} = y_i^{dt}, \qquad i \in I,\ d \in D,\ t \leq 40 \tag{5.5}$$

$$y_i^{dt} \leq u_i^d, \qquad i \in I,\ d \in D,\ t \leq 40 \tag{5.6}$$

$$\sum_{d\in D} u_i^d = 1, \qquad i \in I \tag{5.7}$$

$$\sum_{i\in I} y_i^{dt} \leq 1, \qquad d \in D,\ t \leq 40 \tag{5.8}$$

$$\sum_{n\in N}\sum_{m\in M} x_{nm}^{dt} \leq 1, \qquad d \in D,\ t \in T, \tag{5.9}$$

$$\sum_{i\in I}\sum_{t\leq 40} y_i^{dt} \leq CAP, \qquad d \in D \tag{5.10}$$

$$\sum_{i \in I} \sum_{h \in H} \sum_{t \in T} x_{ih}^{dt} = 1, \qquad d \in D \tag{5.11}$$

$$y_i^{d41} = 0, \qquad i \in I,\ d \in D \tag{5.12}$$

$$k_{ih}^{dt} \leq \frac{y_i^{dt} + u_h^d}{2}, \qquad i \in I,\ h \in H,\ d \in D,\ t \in T \tag{5.13}$$

$$k_{ih}^{dt} \geq y_i^{dt} + u_h^d - 1, \qquad i \in I,\ h \in H,\ d \in D,\ t \in T, \tag{5.14}$$

$$\sum_{h \in H} z_h = p \tag{5.15}$$

$$\sum_{h \in H} u_h^d = 1, \qquad d \in D \tag{5.16}$$

$$x_{hi}^{dt} \leq u_h^d, \qquad i \in I,\ h \in H,\ d \in D,\ t \in T \tag{5.17}$$

$$x_{ih}^{dt} \leq u_h^d, \qquad i \in I,\ h \in H,\ d \in D,\ t \in T \tag{5.18}$$

$$u_h^d \leq z_h, \qquad h \in H,\ d \in D \tag{5.19}$$

$$y_i^{d2} + y_i^{d21} \geq y_i^{d1}, \qquad i \in I2,\ d \in D \tag{5.20}$$

$$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt+20} \geq y_i^{dt}, \qquad i \in I2,\ d \in D, t \leq 20 : t \neq \{1, 10\} \tag{5.21}$$

$$y_i^{dt-1} + y_i^{dt+20} \geq y_i^{dt}, \qquad i \in I2,\ d \in D,\ t = \{10, 20\} \tag{5.22}$$

$$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt-20} \geq y_i^{dt}, \qquad i \in I2,\ d \in D,\ 21 \leq t \leq 39, \tag{5.23}$$

$$y_i^{dt-1} + y_i^{dt-20} \geq y_i^{dt}, \qquad i \in I2,\ d \in D,\ t = \{30, 40\} \tag{5.24}$$

$$\sum_{t \leq 20} y_i^{dt} \geq 2 \cdot u_i^d, \qquad i \in I4,\ d \in D, \tag{5.25}$$

$$y_i^{d2} + y_i^{d11} \geq y_i^{d1}, \qquad i \in I4,\ d \in D, \tag{5.26}$$

$$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt+10} \geq y_i^{dt}, \qquad i \in I4,\ d \in D,\ 2 \leq t \leq 20, \tag{5.27}$$

$$y_i^{dt+20} + y_i^{dt+30} \geq y_i^{dt} + y_i^{dt+10}, \qquad i \in I4,\ d \in D,\ 1 \leq t \leq 10, \tag{5.28}$$

$$y_i^{dt+20} + y_i^{dt+21} \geq y_i^{dt} + y_i^{dt+1}, \qquad i \in I4,\ d \in D, 1 \leq t \leq 19 : t \neq 10, \tag{5.29}$$

$$\sum_{t \leq 10} y_i^{dt} \geq 2 \cdot u_i^d, \qquad i \in I8,\ d \in D, \tag{5.30}$$

$$y_i^{d2} + y_i^{d6} \geq y_i^{d1}, \qquad i \in I8,\ d \in D, \tag{5.31}$$

$$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt+5} \geq y_i^{dt}, \qquad i \in I8,\ d \in D,\ 2 \leq t \leq 5, \tag{5.32}$$

$$y_i^{dt+1} + y_i^{dt-1} + y_i^{dt-5} \geq y_i^{dt}, \qquad i \in I8,\ d \in D,\ 6 \leq t \leq 10, \tag{5.33}$$

$$y_i^{dt+10} \geq y_i^{dt}, \qquad i \in I8,\ d \in D, 1 \leq t \leq 30, \tag{5.34}$$

$$\sum_{t \leq 10} y_i^{dt} \geq 3 \cdot u_i^d, \qquad i \in I12,\ d \in D, \tag{5.35}$$

$$y_i^{d2} + y_i^{d3} \geq 2 \cdot y_i^{d1}, \qquad i \in I12,\ d \in D, \tag{5.36}$$

$$y_i^{d1} + y_i^{d3} + y_i^{d4} \geq 2 \cdot y_i^{d2}, \qquad i \in I12,\ d \in D, \tag{5.37}$$

$$y_i^{dt-2} + y_i^{dt-1} + y_i^{dt+1} + y_i^{dt+2} \geq 2y_i^{dt} \qquad i \in I12,\ d \in D,\ 3 \leq t \leq 8 \tag{5.38}$$

$$y_i^{d7} + y_i^{d8} + y_i^{d10} \geq 2 \cdot y_i^{d9}, \qquad i \in I12,\ d \in D, \tag{5.39}$$

$$y_i^{d8} + y_i^{d9} \geq 2 \cdot y_i^{d10}, \qquad i \in I12,\ d \in D, \tag{5.40}$$

$$y_i^{dt+10} \geq y_i^{dt}, \qquad i \in I12,\ d \in D, 1 \leq t \leq 30, \tag{5.41}$$

$$x_{nm}^{dt}, y_i^{dt}, u_n^d, z_h, k_{ih}^{dt} \in \{0, 1\}, \qquad n, m \in N,\ h \in H,\ d \in D, t \in T, \tag{5.42}$$

The objective function (5.1) minimizes the total distance of all doctors. In the first part, the distances are simply summed up for each trip from village n to m. In the second part, the distances between the villages where a doctor visits at the end of a week and at the beginning of the succeeding week are subtracted and in the final part, traveling distance from the node doctor visits at the end of the week to his/her assigned hospital and from his/her assigned hospital to the village that s/he will visit at the beginning of the following week are summed up. By this way, a doctor's total distance is calculated by considering his/her trips within a week as well as his/her trips from/to the hospital at the end and beginning of each week.

Constraints (5.2) guarantee that each doctor visits a village at the beginning of the month. Via constraints (5.3) each village is visited according to the required frequencies. Constraints (5.4) and (5.5) are simply the flow balance constraints. Unless a doctor is assigned to a village, s/he could not be in that village at time period t in constraints (5.6). Constraints (5.7) ensure that each village is assigned to a dedicated doctor. A doctor can be at most in one village at a certain time period in constraints (5.8). Similarly, constraints (5.9) assert that a doctor can travel to at most one village at any time period. The capacities of the doctors are satisfied via constraints (5.10) and it is guaranteed that every doctor returns to his/her assigned hospital at the end of the month via constraints (5.11) and (5.12). Constraints (5.13) and (5.14) are the linearization constraints of the k variable which can be defined as the multiplication of y and u variables.

The location decisions are made in constraints (5.15)-(5.19). In this set of constraints, p hospitals are selected as base hospitals, every doctor is assigned to a single hospital and it is prevented to assign doctors to hospitals that are not selected as bases. If a doctor is not assigned to a hospital, then any trip from that hospital is also banned.

The remainder of the formulation consists of specialized constraint sets for each frequency level. The explained visiting rules are satisfied via utilizing these constraints. For instance, for frequency level of 2, set of constraints (5.20)-(5.24) guarantee that if a village is visited at time period t, then it should be visited either at the preceding or succeeding time period (given that the village is visited two periods in a row), or 20 periods later (given that the village is visited at the same slot in every two weeks). Some of the instances are defined on separate constraints in order to prevent the undesired schedules. For instance as $t = 10$ and $t = 11$ define the end and the beginning of weeks, these two slots cannot be accepted as two periods in a row. In order to prevent this situation, constraints (5.22) are defined.

Similarly sets of constraints (5.25)-(5.29), (5.30)-(5.34) and (5.35)-(5.41) are defined to satisfy the visiting rules of frequency levels of 4, 8 and 12, respectively. For the frequency level of 4, two weekly schedules are constructed as the next two weeks of the month will be the repetition of the first two. Weekly schedules are determined for the frequency levels of 8 and 12 for the same reasons above-mentioned. Finally, we have the domain constraints of the decision variables in constraints (5.42).

Conjecture 1: Even though there are two alternative visiting rules for the villages with frequency 8, as long as triangular inequality is satisfied, only one day (2 consecutive half day) visits in each week will be active in an optimal solution, where the objective is to minimize the overall distance of traversing villages.

Proof: Assume to the contrary, there is an optimal solution A^* in which the village x with frequency 8 is visited 1 half-day in each 2.5 days as shown in Table 5.1. In addition, let $A^{'}$ another solution which is obtained by visiting village x 1 day per week while shifting the slots of villages within these 2.5 days by one slot as shown in Table 5.2. This move will not affect the visiting rules of the villages $a - \ell$ as they will remain to satisfy the previous visiting rules. Let's assume that the villages in empty slots are the same in both solutions so that they do not have any effect on distance comparisons.

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1		x	a			e	x	i		
Week-2		x	b			f	x	j		
Week-3		x	c			g	x	k		
Week-4		x	d			h	x	ℓ		

Table 5.1: Schedule of optimal solution A^*

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1		x	x	a			e	i		
Week-2		x	x	b			f	j		
Week-3		x	x	c			g	k		
Week-4		x	x	d			h	ℓ		

Table 5.2: Schedule of solution $A^{'}$

We remark here that in order to represent the most general case, each slot is assigned to a different letter. However, for villages with frequency 2, we may have $a = c$ or $f = h$. Similarly, if village i has the frequency 4, then $i = j = k = l$ might be the case. A village with frequency 12 can be visited during Tuesday morning, afternoon and Wednesday morning in all weeks. It should be noted that these cases will not have any impact on the distance comparisons.

Let d_{ij} denote the distance between villages i and j. Define the set G_S for solution S as the set of pairings in which the two villages are visited consecutively.

$$G_S = \{(i,j) \in S : \text{villages } i \text{ and } j \text{ are visited consecutively in solution } S\} \tag{5.43}$$

The total distance of optimal solution A^* can be represented with $OptADist$ which can be represented as:

$$OptADist = \sum_{(i,j) \in G_{A^*}} d_{ij} \tag{5.44}$$

Similarly, total distance of solution $A^{'}$ is denoted with $ADist$ is:

$$ADist = \sum_{(i,j) \in G_{A^{'}}} d_{ij} \tag{5.45}$$

Observe that the distance difference between the two solutions can be simplified to

$$\Delta = d_{ex} + d_{xi} + d_{fx} + d_{xj} + d_{gx} + d_{xk} + d_{hx} + d_{xl} - d_{ei} - d_{fj} - d_{gk} - d_{hl} \tag{5.46}$$

It is known that if there exist triangular inequality in the distance matrix, then a direct visit between two nodes is at least as good as visiting them via a third node. Therefore, the following inequalities are satisfied:

$$d_{ex} + d_{xi} \geq d_{ei} \qquad d_{fx} + d_{xj} \geq d_{fj} \qquad d_{gx} + d_{xk} \geq d_{gk} \qquad d_{hx} + d_{xl} \geq d_{hl}$$

This indicates that $\Delta \geq 0$. This implies that solution $A^{'}$ has a lower objective value than optimal solution A^*, which creates a contradiction. Thus, in an optimal solution village x will be visited in 2 consecutive slots each week.

We remark here that moving from solution A^* to $A^{'}$ may result in an infeasible solution if and only if there is another village y with frequency 8 which is visited 1 half-day in every 2.5 days. Let's assume that there is an optimal solution B^* with the defined characteristics as shown in the Table 5.3. If the above-described sliding procedure is performed for x, then the following solution $B^{'}$ will be obtained as in Table 5.4.

However, it could be easily seen that the village y with frequency 8 is visited only 2 days after the first visit of the week, which violates the visiting alternative

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	x	a	y	e	i	x	m	y	q	u
Week-2	x	b	y	f	j	x	n	y	r	v
Week-3	x	c	y	g	k	x	o	y	s	w
Week-4	x	d	y	h	ℓ	x	p	y	t	z

Table 5.3: Schedule of optimal solution B^*

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	x	x	a	y	e	i	m	y	q	u
Week-2	x	x	b	y	f	j	n	y	r	v
Week-3	x	x	c	y	g	k	o	y	s	w
Week-4	x	x	d	y	h	ℓ	p	y	t	z

Table 5.4: Schedule of solution $B^{'}$

rules of the Ministry of Health. Therefore, moving only village x creates an infeasibility. Instead of this approach, both the second visits of villages x and y can be moved and the remaining villages can be slided and solution $B^{''}$ can be obtained as in Table 5.5.

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	x	x	a	y	y	e	i	m	q	u
Week-2	x	x	b	y	y	f	j	n	r	v
Week-3	x	x	c	y	y	g	k	o	s	w
Week-4	x	x	d	y	y	h	ℓ	p	t	z

Table 5.5: Schedule of solution $B^{''}$

In this case, $B^{''}$ turns out to be a feasible solution. Therefore the distance value of this schedule can be compared with the optimal solution B^*. Similar to the previous approach, the distance difference between two solutions can be calculated as follows:

$$OptBDist - BDist = \Delta = \sum_{(i,j)\in G_{B^*}\setminus G_{B^{''}}} d_{ij} - \sum_{(i,j)\in G_{B^{''}}\setminus G_{B^*}} d_{ij}. \tag{5.47}$$

As a result, $\Delta \geq 0$ when there is triangular inequality as it is explained in the previous case. Thus, it can be claimed that for each village with frequency 8, visiting them as a 1 day a week provides a better solution than separating the visits by 2.5 days. ■

According to this, the set of equations that determines the schedules of the villages with frequency 8, namely (5.30)-(5.34), are updated as follows:

$$\sum_{t \leq 10} y_i^{dt} \geq 2 \cdot u_i^d, \qquad i \in I8,\ d \in D, \tag{5.48}$$

$$y_i^{d2} \geq y_i^{d1}, \qquad i \in I8,\ d \in D, \tag{5.49}$$

$$y_i^{dt+1} + y_i^{dt-1} \geq y_i^{dt}, \qquad i \in I8,\ d \in D,\ 2 \leq t \leq 10, \tag{5.50}$$

$$y_i^{dt+10} \geq y_i^{dt}, \qquad i \in I8,\ d \in D, 1 \leq t \leq 30, \tag{5.51}$$

Via the constraints (5.49) and (5.50), it is ensured that when a village is visited at time period t, then it could be visited either at the previous or the following time period. This indicates that only 1 day visits at each week can be generated for the villages with frequency 8. Taking these adaptations in the constraints, the final version of the mathematical formulation can be represented as follows:

$$\begin{aligned} \text{minimize} \quad & (5.1) \\ \text{subject to,} \quad & (5.2) - (5.29), \\ & (5.48) - (5.51), \\ & (5.35) - (5.42) \end{aligned}$$

5.2 Valid Inequalities for PLRP

Valid inequality can be defined as an inequality which restricts the solution space of a problem without excluding any feasible or the optimal solution. It is known that they are used to strengthen the IP or MIP formulation by decreasing the computational time since they cut and leave out non-integer solutions and decrease the size of the solution space. In order to achieve this, it is decided to develop some valid inequalities for the mathematical formulation of this study. These inequalities are actually logical derivations that are generated from the requirements of the problem. These constraints can be formulated as below:

$$\sum_{i \in I12} u_i^d \leq 3, \qquad d \in D \tag{5.52}$$

$$\sum_{i \in I12} u_i^d + \sum_{i \in I8} u_i^d \leq 5, \qquad d \in D \tag{5.53}$$

$$\sum_{i \in I} u_i^d \cdot DEM_i \leq CAP, \qquad d \in D \tag{5.54}$$

$$x_{ij}^{dt} \leq u_i^d, \qquad i \in I,\ j \in I,\ d \in D,\ t \in T \tag{5.55}$$

Before explaining the constraints, it should be reminded that 1 period of service corresponds to a half day. In this sense, there are 10 periods in a week (considering only weekdays) and 40 periods in a month. Therefore, a doctor's working capacity is determined as 40 periods in this study.

Keeping this in mind, it could be said that constraints (5.52) and (5.53) are defined according to the capacities of the doctors. In constraints (5.52), it is asserted that a doctor cannot be assigned to more than 3 villages with frequency level of 12 as the 4^{th} one will exceed the working hour capacity of the doctor. Similarly, in constraints (5.53) a doctor cannot be responsible for more than 5 villages with frequencies 8 and 12. This upper bound of 5 is determined considering the worst-case scenario, which is serving to villages with only frequency level of 8. In the original formulation, the capacity restriction of the doctors is defined via y variables. As a valid inequality, it is also defined with u variables in constraints (5.54). Instead of adding up the visited villages over time periods, frequencies of the villages that the doctor is assigned are summed up. Finally, the routes from a certain village are prevented for doctors who are not responsible for that village in constraints (5.55).

Since different combinations of valid inequalities' effects can alter, it is aimed to find the best possible combination. As there are 4 inequalities introduced, there exists 16 combinations. The extensive analyses performed on each of these 16 different combinations and the conclusions reached are presented in the next chapter in detail.

Chapter 6

Computational Analysis of the Mathematical Model

In this chapter, extensive computational studies of the mathematical formulation are performed and corresponding results are analyzed in detail. To begin with, the characteristics of the data set utilized in this study are explained. Afterwards, the steps of selecting the best possible valid inequality combination for this model are demonstrated and one of the combinations is picked. According to the selection, results of the model are discussed. Finally, in order to investigate the effects of the parameters on the computations, certain sensitivity analyses are performed.

6.1 Data Generation

In order to test our mathematical formulation's performance, it is decided to utilize a real life data set for the computational studies. Burdur, one of the southern cities of Turkey, was among the first cities that Ministry of Health provided mobile primary healthcare services. Thus, its data is utilized to test our proposed modeling approach. The study is restricted to the three main municipalities of the city, which in total have 50 villages. To evaluate the performance of the mathematical model in various cases, three data sets are formed from this data, depending on the number of villages. These three sets are going to be referred as small, medium and large data sets, which include 15, 30 and 50 nodes, respectively. Small sized data set includes the first 15 nodes of the Burdur data set, whereas first 30 nodes of it determines the medium sized data set. Consequently, the large sized data set corresponds to the Burdur data set itself with 50 nodes. The distribution of the number of villages and potential base hospitals for each data set are presented in Table 6.1.

Data Set	Number of Nodes	Number of Villages	Number of Potential Base Hospitals
Small Data Set	15	12	3
Medium Data Set	30	26	4
Large Data Set	50	45	5

Table 6.1: Specifications of the Data Sets

The exact coordinates of these villages are gathered using *Google Maps* and the distance between each pair is calculated by using *Euclidean* distances. In addition to the distance information, number of inhabitants of each village is also known. Based on these values, the frequencies are determined. However, since there exists only one instance with real frequency levels, alternative artificial frequencies are also generated. Therefore, it is possible to have additional instances and perform detailed computational analyses. As it was mentioned before, the capacity of each doctor is set to 40 periods since there are 2 periods (half-days) per day and the month consists of 4 weeks with 5 workdays in each week. Number of base hospitals to select varied according to the number of doctors required for each instance.

In order to evaluate the solution performances in a more detailed way, the frequency distributions, number of doctors and base hospitals to select are changed in a systematical manner. There are 30, 10 and 10 instances generated for small, medium and large data sets, respectively. The first 15 instances of the small data set and all 10 instances of the medium data set are used for valid inequality analysis. According to the results, an alternative valid inequality combination is selected as the most suitable one for the mathematical model. Afterwards, this selection is used to analyze the remaining 15 instances of the small data set and all instances of the large data set. The details of these instances of small, medium and large data sets, in terms of frequency distributions, number of doctors and base hospitals can be found in Appendix A.1, A.2 and A.3.

6.2 Valid Inequalities

The first 15 instances of the small data set are used to test the performance of the valid inequalities. For each instance, 16 different valid inequality combinations are tested, which generates 240 different settings. The optimal solutions, computational times and optimality gaps of each setting are recorded for the analyses.

According to the results of the above-mentioned criteria, the best and worst solution times are determined for each of the 15 instances. For each valid inequality combination, the improvements in solution times compared to the original model (without any valid inequality) are calculated in percentages. In addition to that, the gaps from the best possible solution time are also determined in percentages. From these results, certain analyses are performed, whose outcomes can be found in Tables 6.2 and 6.3.

In Table 6.2, valid inequality combination performances are classified according to their solution times. First, for each instance, solution time of the combinations are listed in the ascending order. Then, the fastest and slowest combinations are determined. While doing this, 1% deviations from the best/worst solution time are disregarded. In other words, even though a combination does not provide the best solution time in an instance, it could be accepted as the fastest combination if it is only 1% slower than the best possible solution time. According to this, number of instances where each combination performs the fastest or slowest are counted. For instance using only constraint (5.54) performs either the fastest or almost as fast as the best combination in 4 instances out of 15, whereas it is only once the slowest one. However, when constraints (5.52), (5.53) and (5.55) are used together in the formulation, the model is solved the fastest twice, whereas it provides the worst or almost as bad as the worst solution time in 4 instances out of 15. In the second part of the table, the worst and average gaps from the best solution time are calculated taking 15 instances into account for each valid inequality combination. For instance using only constraint (5.55) gives the worst computation time at least in 1 instance as the worst gap from the best solution time is 100% where the average gap from the best results over 15 instances is 40.40%. However, combination of constraints (5.53) and (5.54) perform 53.52% off from the best computation time in their worst instance and 21.81% off from the top on the average.

Valid Inequality (V.I.) Combinations	Number of Instances with Best Solution Time	Number of Instances with Worst Solution Time	Worst Gap from the Best Solution Time (%)	Average Gap from the Best Solution Time (%)
Without V.I.	3	1	91.09	29.24
Only (5.52)	2	1	99.58	26.42
Only (5.53)	1	1	51.97	26.66
Only (5.54)	4	1	78.08	16.47
Only (5.55)	1	5	100.00	40.40
(5.52), (5.53)	2	1	100.00	28.08
(5.52), (5.54)	2	2	100.00	28.79
(5.52), (5.55)	2	2	100.00	37.35
(5.53), (5.54)	4	0	53.52	21.81
(5.53), (5.55)	2	3	89.54	35.91
(5.54), (5.55)	1	1	99.64	33.98
(5.52), (5.53), (5.54)	2	0	43.02	19.96
(5.52), (5.53), (5.55)	2	4	100.00	43.16
(5.52), (5.54), (5.55)	4	2	100.00	30.68
(5.53), (5.54), (5.55)	2	6	100.00	48.75
(5.52), (5.53), (5.54), (5.55)	4	3	100.00	34.54

Table 6.2: Valid Inequality Analyses Based on Solution Times

In this sense, the valid inequality combinations which are the slowest in majority of the settings are not considered to be selected as the best combination. Additionally, higher average and worst gap values from the best solution time hinder the selection of that combination for the final representation of the mathematical formulation.

In Table 6.3, analyses are based on the comparisons of valid inequality combinations with the original model, i.e., the one which does not utilize any valid inequalities. First, the number of instances where the combinations have better or worse solution times than the original model are counted over the 15 instances. As it can be seen from the table, especially the combination of only (5.54) seem to outperform other combinations as it provides better solution time values than the original model in 11 instances. Then, the best and average gaps of solution times from the original case without any valid inequalities are calculated for each combination. For instance, it could be read from the table that, combination of (5.52), (5.53), (5.54), (5.55) performs at most 43.66% better than the original model. On the average, this combination is 36.32% worse than the model which do not use valid inequalities.

According to these outcomes, combinations which are mostly worse than the original model, i.e., the ones with less than or equal to 7 better instances, are eliminated. The lower average gaps from the original model's solution time, the higher the chances that a combination is selected. Similarly, higher improvements in the best case also affect the selection decision positively.

Valid Inequality (V.I.) Combinations	Number of Instances Better than without V.I.	Number of Instances Worse than without V.I.	Best Gap from without V.I. Solution Time (%)	Average Gap from without V.I. Solution Time (%)
Only (5.52)	9	6	-37.02	45.42
Only (5.53)	7	8	-30.41	45.62
Only (5.54)	11	4	-64.42	137.75
Only (5.55)	4	11	-11.44	36.86
(5.52), (5.53)	8	7	-36.72	81.15
(5.52), (5.54)	8	7	-41.51	126.55
(5.52), (5.55)	7	8	-46.79	23.42
(5.53), (5.54)	9	6	-60.27	84.42
(5.53), (5.55)	5	10	-18.65	22.74
(5.54), (5.55)	8	7	-59.93	169.83
(5.52), (5.53), (5.54)	8	7	-43.22	65.82
(5.52), (5.53), (5.55)	7	8	-25.36	42.93
(5.52), (5.54), (5.55)	6	9	-58.07	168.50
(5.53), (5.54), (5.55)	4	11	-62.96	57.10
(5.52), (5.53), (5.54), (5.55)	8	7	-43.66	36.32

Table 6.3: Valid Inequality Analyses Based on Without V.I. Settings

As a result of all of these analyses, a dominating combination cannot be found as none of them outperformed remaining combinations in all of the criteria. Therefore, it is decided to select 3 combinations out of 16 and continue working with these in the medium data set. These combinations are only (5.54) and (5.53), (5.54) and (5.52), (5.53), (5.54), (5.55). The main reasoning behind these selections is that each of the three combinations provide 4 instances with the best solution time according to Table 6.2 and the first two combinations provide relatively smaller worst gaps from the best solution time. In addition, these instances perform generally better than the original model and have the highest best improvement percentages as it can be observed from Table 6.3.

In the next step, these three combinations are tested on 10 instances of the medium data set. Additionally, the original model that does not utilize any valid inequalities is also tested since all of the combinations provided positive average gaps from the original model in the previous studies, which indicates that none of the alternative combinations perform better than the original model on the average. Therefore, the case where using no valid inequalities is still considered to be an alternative formulation. It is observed that, it gets significantly harder to solve the mathematical model optimally when data set size increases. Hence, each experiment is limited with a 7200 seconds time bound. At the end of this duration, the value of the objective function and the remaining optimality gap are recorded for each setting. The results of these computational studies are presented in Table 6.4.

Based on the computational studies on the medium data set, it is observed that the original model provides the best solution only in one of the instances. The three combinations find the minimum distance value in 2, 3 and 4 instances with respect to the order they are presented in Table 6.4. The first combination of only (5.54) provides better results than the without valid inequality case at the end of 2 hours in 4 instances out of 10. This combination improves the objective value by 0.60% on average when it is compared with the original model. Similarly, combination of (5.53), (5.54) outperforms the original model in 4 instances; however, its solutions are 0.14% worse than it on average. Combination of (5.52), (5.53), (5.54), (5.55) generates more preferable outcomes in 6 instances compared

		Only (5.54)	(5.53), (5.54)	(5.52), (5.53), (5.54), (5.55)	Without V.I.
Instance 1	Obj. Value	9,075.3	9,155.5	**8,625.2**	9,030.8
	Opt. Gap (%)	79.97%	81.72%	**60.46%**	87.02%
Instance 2	Obj. Value	9,151.7	8,914	**8,743.9**	9,568.1
	Opt. Gap (%)	87.53%	86.57%	**83.36%**	88.04%
Instance 3	Obj. Value	13,359.6	**11,452.8**	12,600.2	13,121.0
	Opt. Gap (%)	86.47%	85.29%	**84.26%**	87.70%
Instance 4	Obj. Value	12,279.4	**10,685.5**	12,118.3	12,279.1
	Opt. Gap (%)	83.30%	**77.78%**	84.60%	82.65%
Instance 5	Obj. Value	7,976.7	7,765.3	7,848.4	**7,748.5**
	Opt. Gap (%)	79.59%	**33.64%**	75.16%	80.25%
Instance 6	Obj. Value	8,001.6	**7,464.9**	7,791.1	7538.3
	Opt. Gap (%)	82.62%	82.61%	77.74%	**74.19%**
Instance 7	Obj. Value	10,419.6	12,440.8	**9,874.5**	11155
	Opt. Gap (%)	82.97%	85.88%	**73.48%**	86.64%
Instance 8	Obj. Value	10,677.4	10,416.9	**10,307.9**	10407.9
	Opt. Gap (%)	81.77%	81.40%	81.04%	**80.45%**
Instance 9	Obj. Value	**9,964**	11,041.7	10,115.9	10087.9
	Opt. Gap (%)	81.43%	82.38%	**73.69%**	79.82%
Instance 10	Obj. Value	**11,028.1**	13,421.6	13,828.4	11960
	Opt. Gap (%)	**84.63%**	85.01%	87.54%	85.27%

Table 6.4: Valid Inequality Analyses on Medium Data Set

to the case where no valid inequality is used. Moreover, it decreases the distance value by 1.03% on the average. When these points are analyzed, it is observed that although each combination provides more or less similar results, combination of (5.52), (5.53), (5.54), (5.55) performs slightly better than the remaining ones in larger instances. Therefore, it is decided to select this combination as the most suitable one for the mathematical model. Consequently, the model can be represented as follows:

$$\begin{aligned} \text{minimize} \quad & (5.1) \\ \text{subject to,} \quad & (5.2) - (5.29), \\ & (5.48) - (5.51), \\ & (5.35) - (5.42), \\ & (5.52), (5.53), (5.54), (5.55) \end{aligned}$$

6.3 Results of the Mathematical Model

After determining the best valid inequality combination, the new formulation is tested on all data sets and their final results are obtained. The summary of the results of the model that utilize all four valid inequalities are provided in Tables 6.5 and 6.6 for all data sets with solution values, computational times or the remaining gaps in a predefined time bound.

Table 6.5 indicates that the computational times show significant variations from each other in 30 instances. The mean of the solution times is 2,811 seconds which is slightly more than 45 minutes. The standard deviation of the instances is quite high with 2,984 seconds. The 90% confidence interval of the solution time of this data set is determined via using the following equation with z-distribution, where $\overline{x}$ represents the sample mean, σ represents the standard deviation, $1-\alpha$ represents the confidence interval and n represents the number of instances.

$$\overline{x} \pm z_{\alpha} \cdot \frac{\sigma}{\sqrt{n}}$$

Consequently, the expected solution time of an instance with 15 nodes turns out to be within 1915 and 3707 seconds, i.e., 32 and 62 minutes.

Small Data Set					
Instance	Objective Value	Solution Time (sec)	Instance	Objective Value	Solution Time (sec)
Instance 1	5,281.18	6,684	Instance 16	5,310.06	3,259
Instance 2	4,943.81	357	Instance 17	4,863.60	3,381
Instance 3	4,491.33	1,180	Instance 18	5,002.72	2,569
Instance 4	4,805.67	344	Instance 19	4,663.24	3,669
Instance 5	4,876.04	2,677	Instance 20	5,310.06	2,890
Instance 6	7,885.68	2,989	Instance 21	4,895.42	5,071
Instance 7	7,908.40	6,773	Instance 22	3,059.73	1,494
Instance 8	6,053.13	303	Instance 23	2,601.60	1,127
Instance 9	4,856.33	5,483	Instance 24	2,984.28	6,978
Instance 10	6,097.48	5,926	Instance 25	2,752.23	24
Instance 11	6,012.08	12,742	Instance 26	2,919.58	62
Instance 12	5,551.66	197	Instance 27	2,731.42	31
Instance 13	4,235.74	210	Instance 28	2,767.94	22
Instance 14	5,594.12	3,989	Instance 29	2,681.71	18
Instance 15	5,179.78	3,878	Instance 30	2,717.74	24

Table 6.5: Results of Mathematical Model on Small Data Set

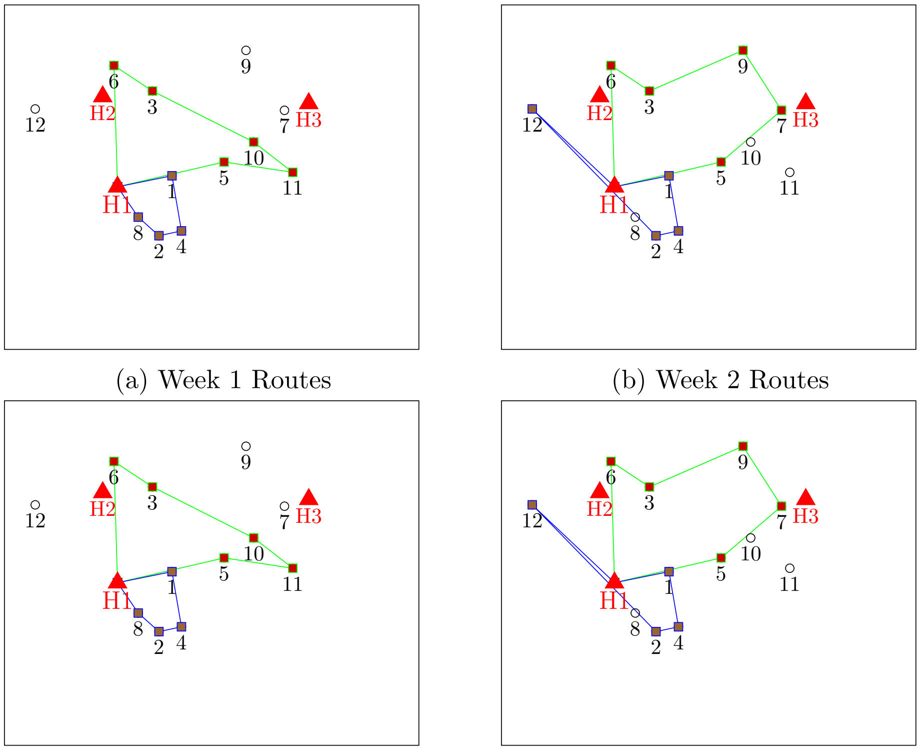
As it is mentioned above, the model cannot solve the instances in medium data set in reasonable times as the sizes of the instances increase. Hence, it is decided to determine time bounds on the instances of medium and large data sets. A 2-hour limitation is forced to the model that solves the medium data set, whereas this bound is increased to 4-hours for the large one.

According to the results presented in Table 6.6, the model does not seem to be able to perform well with the medium data set. The remaining optimality gaps after 2 hours are rather high with the average of 74.37%. This lack of performance quality can also be observed in the large data set as the optimality gaps after 4 hours remain between 93% and 95% rates. It can be claimed that the formulation can almost only determine an initial solution in 4 hours for the instances of large data set; thus, the optimality gaps remain significantly high. Even, certain instances, such as 7 and 10, the model cannot determine an initial feasible solution in that period of time. Thus, these results indicate to the necessity of a fast and efficient algorithm.

Medium Data Set			**Large Data Set**		
Instance	Objective Value	% Gap (in 2 hours)	Instance	Objective Value	% Gap (in 4 hours)
Instance 1	8,625.2	60.46%	Instance 1	28,775.6	94.01%
Instance 2	8,743.9	83.36%	Instance 2	29,883.9	93.03%
Instance 3	11,452.8	85.29%	Instance 3	33,787.1	93.66%
Instance 4	10,685.5	77.78%	Instance 4	34,438.5	94.15%
Instance 5	7,765.3	33.64%	Instance 5	37,147.7	94.82%
Instance 6	7,464.9	82.61%	Instance 6	39,227.4	95.11%
Instance 7	9,874.5	73.48%	Instance 7	-	-
Instance 8	10,307.9	81.04%	Instance 8	33,501.4	94.30%
Instance 9	9,964.0	81.43%	Instance 9	33,660.6	94.03%
Instance 10	11,028.1	84.63%	Instance 10	-	-

Table 6.6: Results of Mathematical Model on Medium and Large Data Sets

In order to be able to visualize the results of the mathematical formulation, a graphical representation of an instance from each data set is provided in this section as well as the monthly schedules of each doctor and relevant analyses. To begin with, the routes of instance 1 from the small data set are provided in Figure 6.1. Recall from Appendix A.1 that instance 1 has 2 doctors for 2 villages with frequency 12, 4 villages with frequency 8 and 6 villages with frequency 4.



(a) Week 1 Routes (b) Week 2 Routes

(c) Week 3 Routes (d) Week 4 Routes

Figure 6.1: Small Data Set - Instance 1, Weekly Routes of Model

As it can be observed, hospital 1 is selected as the base hospital for both doctors. The first doctor is responsible for villages 3, 5, 6, 7, 9, 10 and 11, while the second doctor visits only villages 1, 2, 4, 8 and 12 as their populations are higher than the others, thus they have higher frequency levels. When the routes are examined, it is found out that the 3^{rd} and 4^{th} weeks are the repetition of the first two weeks for both doctors. Within these two weeks, small variations in the routes are observed. For instance first doctor visits villages 7 and 9 in the second week instead of 10 and 11. Similarly, second doctor visits village 12 instead of 8, but the remainder of the routes stay the same. The exact monthly schedules of the doctors can be found in Table 6.7.

In this table, the columns correspond to the 4-hour periods and represent the first and second halves of each workday. It can also be observed here that the routes of both doctors in weeks 1 and 2 are repeated in the remainder of the month. The villages that have frequency of 8, namely 3, 4, 5 and 6 are visited

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	5	5	11	11	10	10	3	3	6	6
Week-2	5	5	7	7	9	9	3	3	6	6
Week-3	5	5	11	11	10	10	3	3	6	6
Week-4	5	5	7	7	9	9	3	3	6	6

(a) Schedule of Doctor 1

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	1	1	1	4	4	2	2	2	8	8
Week-2	1	1	1	4	4	2	2	2	12	12
Week-3	1	1	1	4	4	2	2	2	8	8
Week-4	1	1	1	4	4	2	2	2	12	12

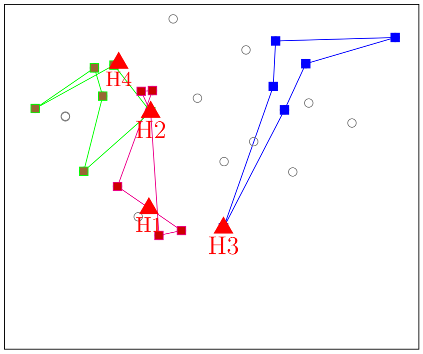
(b) Schedule of Doctor 2

Table 6.7: Small Data Set - Instance 1, Doctor Schedules

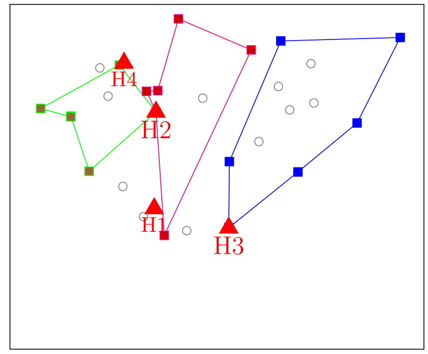
1 day in each week in the optimal solution. The villages with frequency 4 are chosen to be visited 1 day in every other week instead of half-day periods at each week. When the overall solution is analyzed, it can be claimed that the schedules of the doctors are satisfying the problem specific requirements with the minimum travel distances.

For the medium data set, the results of second instance with 2-hour time bound are presented in Figure 6.2. According to the graphical representation, hospitals 2 and 3 are selected as base hospitals for the three practitioners, where two of them are assigned to the second base hospital and the remaining doctor is dispatched from the third one. The first and second doctors are responsible for 8 and 7 villages, respectively, whereas the third doctor visits 11 of them in a month. The routes of the first two doctors follow a similar pattern over the weeks; however, the last doctor travels through different routes at each week. The main reasoning behind is that the villages assigned to that doctor mostly have the frequency levels of 1 and 2, so that they can be visited in one week and no more services are required to be provided to them in the following ones. The schedules of each doctor can be explicitly observed in Table 6.8.

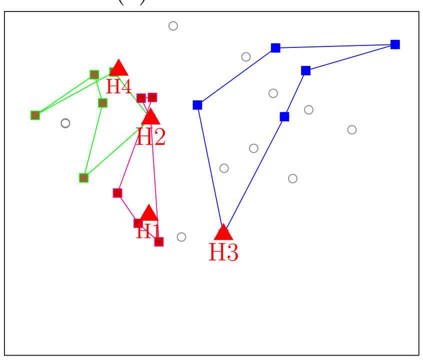
As it can be observed from the schedules, the villages with frequency 12 are visited 1.5 days and the ones with frequency 8 are visited 1 day at each week.



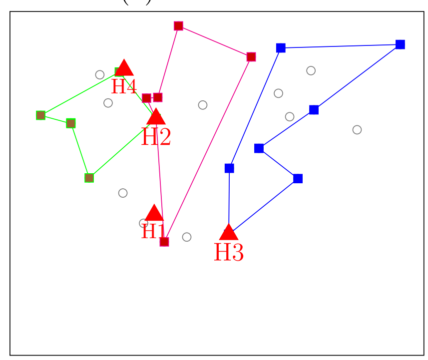
(a) Week 1 Routes



(b) Week 2 Routes



(c) Week 3 Routes



(d) Week 4 Routes

Figure 6.2: Medium Data Set - Instance 2, Weekly Routes of Model

Villages such as 12, 13 or 18 with frequency level 4 are selected to be visited 1 day in every two week periods. The other visiting alternative, which is to visit these villages a half-day in every week, is not observed in the schedule of this instance. On the other hand, both visiting alternatives can be encountered in this solution. In the third doctor's schedule, it can be observed that the village 23 is visited on Tuesday in the second week, whereas village 4 received service on the Friday afternoons of 2^{nd} and last week. Finally, the villages with frequency 1 are visited on the remaining available slots during the month.

For the large data set, the graphical representation of the instances are not provided as the integer solutions at the end of 4 hours are only initial solutions and they do not form reasonable routes at that point. Instead, the monthly schedules of each of the 5 doctors are provided in Table 6.9 for the first instance of this data set.

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	1	1	1	3	12	12	2	2	16	16
Week-2	1	1	1	8	8	15	2	2	16	16
Week-3	1	1	1	7	12	12	2	2	16	16
Week-4	1	1	1	8	8	15	2	2	16	16

(a) Schedule of Doctor 1

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	25	25	25	13	13	22	11	11	5	5
Week-2	25	25	25	21	21	20	11	11	5	5
Week-3	25	25	25	13	13	22	11	11	5	5
Week-4	25	25	25	21	21	20	11	11	5	5

(b) Schedule of Doctor 2

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	6	6	18	18	24	24	19	19	19	17
Week-2	10	10	23	23	24	24	19	19	19	4
Week-3	6	6	18	18	24	24	19	19	19	26
Week-4	10	10	9	14	24	24	19	19	19	4

(c) Schedule of Doctor 3

Table 6.8: Medium Data Set - Instance 2, Doctor Schedules

When the monthly schedules of each doctor is examined individually, it is observed that in most of the cases 1^{st} and 3^{rd}, and 2^{nd} and 4^{th} weeks have quite similar routes. Since the frequency levels 12 and 8 are visited every week and frequency 4 is visited in every two weeks in the worst case, majority of the routes consist of the same villages. As it is the case in the medium data set solution, villages with 12 and 8 are visited consecutively in each week. This time the alternative visiting rules can be observed for the villages with frequency 4. For instance, village 12 is visited at every Tuesday afternoon by the first doctor, whereas village 19 is visited on Fridays of 2^{nd} and 4^{th} weeks by the fifth doctor. Both of the visiting alternatives for the frequency level 2 can also be observed in the schedules. Second doctor visits village 18 on week 3, while village 32 is visited on Thursday afternoon in every other two weeks by the same doctor.

According to all these solutions, it can be said that the mathematical model is capable of satisfying the requirements of the problem and generates the schedules of the doctors in the desired manner.

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	31	31	31	12	11	21	24	24	35	35
Week-2	31	31	31	12	25	25	24	24	35	35
Week-3	31	31	31	12	1	21	24	24	35	35
Week-4	31	31	31	12	25	25	24	24	35	35

(a) Schedule of Doctor 1

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	44	44	44	41	40	3	3	32	26	29
Week-2	44	44	44	8	8	3	3	15	15	9
Week-3	44	44	44	18	18	3	3	32	7	29
Week-4	44	44	44	8	8	3	3	15	15	9

(b) Schedule of Doctor 2

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	23	23	23	42	43	45	45	13	38	38
Week-2	23	23	23	17	17	2	2	13	38	38
Week-3	23	23	23	42	43	45	45	13	38	38
Week-4	23	23	23	17	17	2	2	13	38	38

(c) Schedule of Doctor 3

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	27	27	27	37	37	39	39	6	6	28
Week-2	27	27	27	37	37	39	39	33	33	20
Week-3	27	27	27	37	37	39	39	34	34	28
Week-4	27	27	27	37	37	39	39	33	33	20

(d) Schedule of Doctor 4

	Mon-1	Mon-2	Tue-1	Tue-2	Wed-1	Wed-2	Thu-1	Thu-2	Fri-1	Fri-2
Week-1	14	14	14	30	4	4	36	36	5	16
Week-2	14	14	14	30	4	4	36	36	19	19
Week-3	14	14	14	30	4	4	36	36	5	16
Week-4	14	14	14	30	4	4	36	36	19	19

(e) Schedule of Doctor 5

Table 6.9: Large Data Set - Instance 1, Doctor Schedules

6.4 Analysis on the Parameters

In this section, the effects of the parameters on the solution times are analyzed. As a result of these performed analyses, the parameters that increase or decrease the solution time of the mathematical model are determined. Since only the small data set instances can be solved to optimality in reasonable times, 30 instances of this data set are used for the analysis purposes. On the other hand, in order to increase the size of the data set, each instance is triplicated by changing the assignment of the visit frequencies to the villages. In other words, the number of villages with each frequency level remains the same; however, the distribution of frequencies among all villages change in the other two variants of an instance.

In the following parts of this section, the effects of increasing the number of doctors and base hospitals are going to be discussed. Afterwards, increasing the occurrence of each frequency level as well as changing the assignment of these frequencies to the villages are going to be analyzed.

6.4.1 Number of Doctors

According to the frequency distribution over the villages, the number of doctors required to visit all villages varies. When number of villages with frequency of 12 is higher, it directly increases the need for the practitioners. On the other hand, when the low-populated villages which have frequency of 1 or 2 constitute the majority, then scheduling less practitioners can be enough.

As it could be observed from Appendix A.1, there exists 6 instances which have 1 doctor, whereas 18 instances with 2 doctors. The remaining 6 instances utilize 3 doctors. When the variants of each instance are considered, then it could be said that there are 18, 54 and 18 instances with 1, 2 and 3 doctors, respectively. All of these 90 instances are compiled and their solution times are recorded. Taking the outcomes into account, the minimum, average and the maximum solution times of the instances with 1, 2 and 3 doctors are determined. Relevant results can be found in Table 6.10 and Figure 6.3.

Number of Doctors	Number of Instances	Minimum Solution Time (sec)	Average Solution Time (sec)	Maximum Solution Time (sec)	Standard Deviation of Solution Time	90% Confidence Interval of Solution Time
1	6	17	44	180	48.41	[23.59, 63.29]
2	18	197	2,724	8,948	2,320.27	[2,204.25, 3,242.97]
3	6	2,065	7,120	18,893	4,320.39	[5,348.34, 8,891.32]

Table 6.10: Solution Time Analysis Based on Number of Doctors

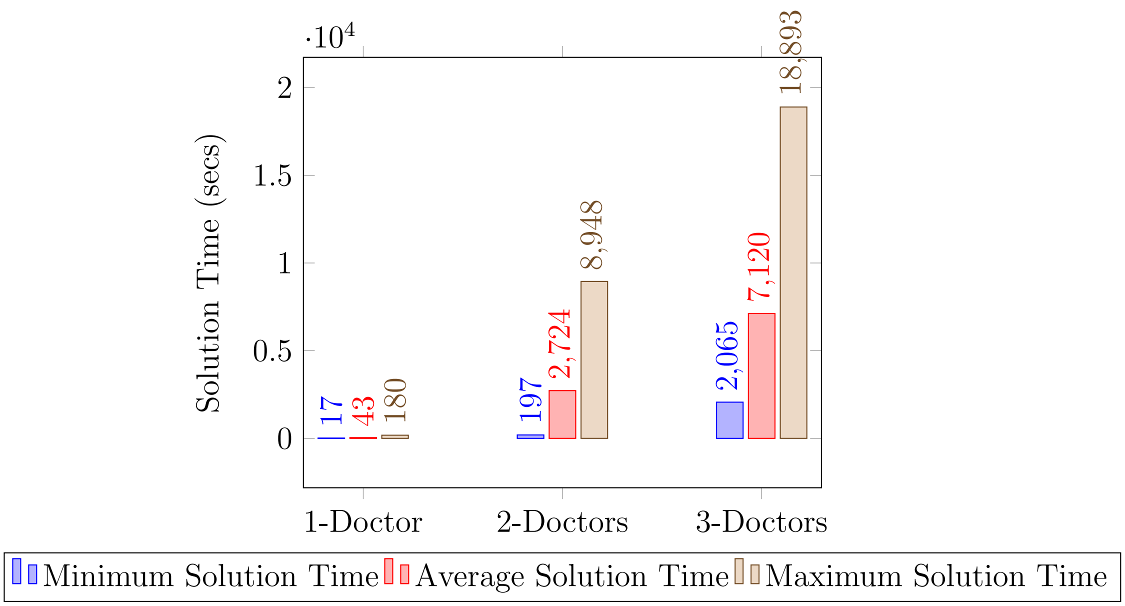


Figure 6.3: Solution Time Comparisons Based on Number of Doctors

It can be observed from the results that increasing number of schedules (doctors) naturally increase the complexity of the problem, thus, the solution times increase dramatically. While the mobile healthcare service schedules of the less dense villages with low levels of frequencies can be determined to optimality in less than a minute on average, the schedules of multiple doctors can be generated in 45 minutes (2 doctors) and approximately 2 hours (3 doctors) on average. In addition to the average solution time values, the minimum and maximum solution times over all instances also have the same increasing pattern when there are more schedules to be generated, which can be seen from Table 6.10. This result was expected since increasing the number of doctors also increases the number of possible schedules which result in computational complexities.

Besides the average solution time values, the standard deviations of them are also calculated. Then, 90% confidence intervals are determined by using these standard deviation values and t-distribution when the sample size is small ($n < 30$) and z-distribution when the sample size is large ($n \geq 30$). According to these results, it can be claimed with 90% confidence that the solution time is expected to be around 24 and 63 seconds when there is only 1 doctor. Similarly, the average solution time lies within 36 to 54 minutes for 2 doctors and 99 minutes to 148 minutes for 3 doctors with 90% confidence interval.

6.4.2 Number of Base Hospitals

The number of selected base hospitals, which corresponds to the parameter p of the mathematical formulation, is varied over the 30 instances of the small data set. In fact, certain instances are actually determined as the duplicates of each other by changing the p values. For instance, instances 2 and 3 have the same frequency distribution and 2 doctors; however, the previous one selects only 1 base hospital, whereas the latter one selects two. When small data set is examined, it is observed that there are 18 instances which pick only 1, 10 instances that pick 2 and 2 instances which pick 3 base hospitals. In order to make more reliable comparisons, the instances are categorized for each number of base hospital and doctor combinations. The respective solution time analysis can be found in Table 6.11.

Number of Doctors	Number of Base Hospitals	Number of Instances	Minimum Solution Time (sec)	Average Solution Time (sec)	Maximum Solution Time (sec)	Standard Deviation of Solution Time	90% Confidence Interval of Solution Time
1	1	6	17	43	180	48.41	[23.59, 63.29]
2	1	10	197	2,579	8,948	2,786.87	[1,741.98, 3,415.82]
2	2	8	210	2,905	5,679	1,597.39	[2,345.66, 3,463.34]
3	1	2	2,065	5798	10,933	3,686.23	[2,765.89, 8,830.77]
3	2	2	3,989	6,917	12,850	3,124.06	[4,346.52, 9,486.48]
3	3	2	3,878	8,645	18,893	5,931.28	[3,765.36, 13,523.98]

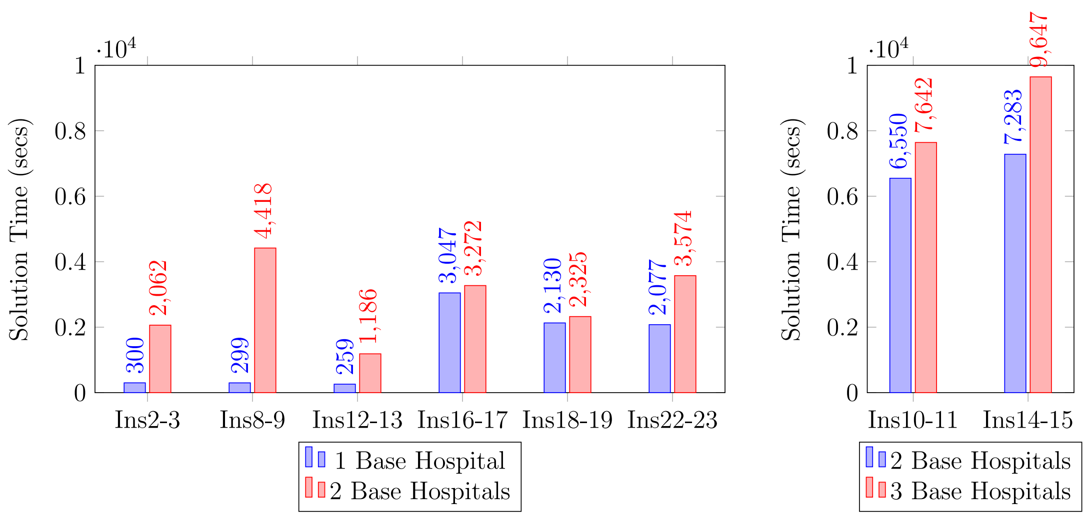
Table 6.11: Solution Time Analysis Based on Number of Base Hospitals

In Table 6.11, it could be observed that selecting less base hospitals results mostly in shorter solution times. On the other hand, as it was observed in the previous analyses, an increase on the number of doctors causes increase on the computational times independent from the number of base hospitals to be selected. When there is only 1 doctor, it is not reasonable to select more than 1 base hospital; therefore, this setting cannot be compared with the others. However, it could be observed that it provides the shortest solution times on average.

In the case of scheduling 2 doctors, selecting a single base has a lower average solution time value than selecting two base hospitals. On the other hand, the standard deviation of the setting with 2 doctors and 1 base hospital is relatively higher than the setting with 2 doctors and 2 base hospitals, which is caused by an extreme solution time of 8,948 seconds. When the 90% confidence intervals of both settings are determined, it is observed that both have similar upper bounds, while selecting 1 base hospital provides a better lower bound. Therefore, it can be claimed that selecting 1 base hospital provides better average solution times in 90% of the cases; however, it cannot be guaranteed that there would not be any exceptions, as it is observed in these computational studies.

Similar outcomes to the previous ones are obtained when the mobile services are provided with 3 doctors. When there is only one base hospital for all doctors, the model can be solved around in 1.5 hours on average. Approximately 18 additional minutes are required to solve the setting which selects one more base hospitals for 3 doctors. When 3 base hospitals are determined, then the solution time becomes almost 2.5 hours. According to the 90% confidence intervals, it can be claimed that selecting 1 base hospital will provide the best results in terms of computational times in the majority of the cases. Since the minimum solution time of selecting 3 base hospitals is better than the minimum solution time of selecting 2, the lower bound of the confidence interval of that setting is 10 minutes better. However, as the upper bound is worse more than 1 hour, it can be claimed that the models with less number of base hospitals will be solved in shorter times in the majority of the cases with minor exceptions.

As it is mentioned above, certain instances are duplicates of each other with different number of base hospitals. These instance pairs are also compared with each other in terms of solution times in order to observe the effects of varying number of base hospitals for the exact same settings. It should be reminded that there are 3 variants for each instance, thus, the average solution times of these 3 cases are determined and compared with each other. These comparisons can be found in Figure 6.4.



(a) Solution Times of Duplicate Instances with 2 Doctors

(b) Solution Times of Duplicate Instances with 3 Doctors

Figure 6.4: Solution Time Analysis Based on Duplicate Instances

When the figure is examined, it can be observed that there are 8 instance pairs that are duplicates of each other. Among all of these instance pairs, it is found out that increasing the number of base hospitals by one unit increases the solution times slightly or dramatically, depending on the instance. For example, instance 17 requires only 7% more solution time than instance 16, whereas the solution time of instance 9 is almost 14 times higher than instance 8's computational time on average. For the remaining settings, such increases between 7% and 1400% are observed. Therefore, depending on the computational studies that are performed here, it can be said that higher p values for the same instances increase the complexity of the problem and results in higher the solution times.

6.4.3 Frequency Distribution

As a final analysis on the computational times of the mathematical formulation, the frequency distributions are evaluated. While doing this, first the number of villages with each frequency level is altered and impacts of having a frequency more than others are investigated. Then, the assignment of these frequencies to the villages are changed so that the effects of positioning the frequencies are tried to be determined. According to the 90 settings of the small data set, average solution times of 3 variants for each 30 instances, whose details can be found in Appendix A.1, are provided in Table 6.12.

Instance Number	Average Solution Time (sec)	Instance Number	Average Solution Time (sec)
Instance 1	4,478	Instance 16	3,046
Instance 2	299	Instance 17	3,272
Instance 3	2,062	Instance 18	2,130
Instance 4	310	Instance 19	2,325
Instance 5	2,150	Instance 20	4,597
Instance 6	2,670	Instance 21	4,247
Instance 7	8,926	Instance 22	2,077
Instance 8	298	Instance 23	3,574
Instance 9	4,417	Instance 24	8,921
Instance 10	6,549	Instance 25	29
Instance 11	7,642	Instance 26	86
Instance 12	258	Instance 27	80
Instance 13	1,186	Instance 28	24
Instance 14	7,283	Instance 29	18
Instance 15	9,647	Instance 30	22

Table 6.12: Average Solution Times of Small Data Set

While analyzing these instances, they are categorized according to the frequency level that constructs the majority, i.e., the frequency level that is assigned to the villages of an instance 6 or more times. According to this, the instances that have majority of frequency 12 are 6, 7, 10, 11, 14 and 15. The common characteristic of these 6 instances besides having majority of frequency of 12s, is scheduling 3 doctors. Since having more highly populated villages increases the need for mobile services, more practitioners are required in the system. Therefore, as it was observed in the previous sections, the solution times of these instances are the highest with the exception of instance 6. On the contrary, the instances that contain mostly frequency of 2 and 1, which are instances 25-30, require only 1 doctor, thus, they are solved in less than 100 seconds on the average.

The instances with majority of frequency of 8's are 16-20 and 4's are instances 1, 21-24. It is observed that these instances mostly provide results within 2,000 and 4,500 seconds, which indicate that they cannot be solved easily. On the other hand, instances 2-5, 8, 9, 12 and 13 do not have any majority of frequency level, i.e., the frequencies are evenly distributed among all villages. When these instances are analyzed, it is observed that they provide results in shorter times compared to others with the exception of instance 9. Moreover, among these 8 instances, 2, 4, 8 and 12 select only one base hospital and they can be solved under 5 minutes.

Overall, it can be said that having majorly frequencies 1 and 2 solves the model faster whereas dominance of frequency 12 increases the solution times. This situation can be easily associated with the varying number of doctor schedules. Besides that, it can be claimed that having more even frequency distributions with the selection of less base hospitals results in shorter computational times and eases the complexity of problem significantly.

It should be noted that, when the variants of each instance are generated, it was aimed to determine the effects of the assignment of frequencies to the villages; however, over 90 different settings, no correlation between the solution times and assignment of frequencies can be observed.

Chapter 7

A Two Stage Heuristic for PLRP

The computational studies of the mathematical model pointed out that the smaller instances can be solved to optimality in reasonable durations; however, increasing the size of the instances results in significantly higher solution times. Moreover, the optimality gaps remaining at the end of 2 hours for the medium data set and 4 hours for the large data set indicated the necessity of an efficient algorithm that can provide qualified results in much shorter times. This was an expected result since the PLRP is an *NP-hard* problem as it is explained in Chapter 4. Therefore, it is decided to develop a heuristic approach.

It is known that the problem considered has different dynamics and characteristics compared to the classical PLRPs. Therefore, existing heuristic approaches are not adapted to this problem, but instead a new methodology is developed. It can be said that this methodology consists of two stages which can be classified as a *"Cluster First, Route Second"* approach.

Construction Phase

In the first stage of the heuristic, it is aimed to determine the set of villages for each doctor without any scheduling decisions. In order to achieve that, certain clusters for each doctor have to be generated which satisfy their working hour capacity. The aim here is to determine the clusters in a way that they could possibly lead to optimal solutions when scheduling decisions are made. In the second stage, each doctor's assignments are taken into consideration individually and the routings to those villages are determined separately. Since it is observed from the computational studies of the mathematical model that generating the schedule of a single doctor is computationally easy, it is decided to adapt the model in Chapter 5 for the routing decisions.

In order to determine the clusters, it is decided to use an Integer Programming (IP) formulation which takes the p-median model as a basis and adds some constraints to it in order to satisfy the problem specific requirements. This model simply determines the clusters of villages for each doctor, including their base hospital, without exceeding the working hour capacities. The aim of the model is to assign every village to a cluster while minimizing the overall distance within each cluster.

The notation used in the IP has similarities with the original model. Only the sets of villages I and base hospitals H are utilized in this formulation. The definitions of the parameters representing distance, frequency, capacity and number of base hospitals are valid in this model too. The only decision variable that is also used in this IP is the z variable. The additional parameters and decision variables determined for the first stage of the heuristic algorithm are defined below:

Additional Parameters and Decision Variables:

$cluster$: number of clusters, i.e. number of doctors

$$x_j = \begin{cases} 1, & \text{if village } j \in I \text{ is selected as a cluster origin,} \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{ij} = \begin{cases} 1, & \text{if village } i \in I \text{ is assigned to cluster origin } j \in I, \\ 0, & \text{otherwise.} \end{cases}$$

The following IP for the clustering stage can now be proposed:

$$\text{minimize} \quad \sum_{i \in I} \sum_{j \in I} DIST_{ij} \cdot DEM_i \cdot y_{ij} \tag{7.1}$$

$$\sum_{j \in I} x_j = cluster, \tag{7.2}$$

$$\sum_{j \in I} y_{ij} = 1, \qquad i \in I, \tag{7.3}$$

$$\sum_{i \in I} DEM_i \cdot y_{ij} \leq CAP, \qquad j \in I \tag{7.4}$$

$$y_{ij} \leq x_j, \qquad i \in I,\ j \in I, \tag{7.5}$$

$$x_j = y_{jj}, \qquad j \in I, \tag{7.6}$$

$$\sum_{h \in H} z_h = p, \tag{7.7}$$

$$\sum_{h \in H} y_{hj} = x_j, \qquad j \in I \tag{7.8}$$

$$y_{hj} \leq z_h, \qquad j \in I,\ h \in H, \tag{7.9}$$

$$x_j, y_{ij}, z_h, \in \{0, 1\}, \qquad i, j \in I,\ \ h \in H, \tag{7.10}$$

In the objective function (7.1), the total weighted distance within the clusters is minimized. By multiplying the terms with demand levels, more emphasis is given to the villages which require more visits than less populated villages. Therefore, a village with frequency of 12 has more importance than the one with frequency of 1. In constraint (7.2), it is aimed to divide the overall village set into clusters according to the number of doctors, so that each doctor is assigned to a single cluster. The number of doctors determines the number of villages to be selected as the cluster origins. It is noted here that these points are dummy location points defined for ease of determining the clusters via village assignments to the origin locations. Consequently, constraints (7.3) make sure that each village is assigned to a cluster origin, thus, to a doctor. The capacity restriction of a cluster (doctor) is attained by constraints (7.4). In constraints (7.5) and (7.6) assigning a village to a non-selected cluster origin is prevented and assigning a village to itself when that location is selected as a cluster origin is guaranteed. p base hospitals are selected in constraint (7.7) and each cluster is assigned to exactly one hospital via the remaining set of constraints. Similarly, it is prevented to assign a cluster to non-selected base hospital via constraints (7.9). Constraints (7.10), represent the domain restrictions of the decision variables.

According to the results of the above explained IP, y_{ij} variables that take value of 1 are grouped based on their j index where each group represents the cluster of villages that a doctor is responsible for. Then, these clusters are taken into consideration individually and the index sets and parameters of the mathematical model in Chapter 5 are updated according to these clusters. In addition to these updates, the doctor index d is removed from the formulation since each doctor's schedule is generated individually. Moreover, the location decision is also eliminated from the formulation as the base hospitals of each doctor are determined in the first stage of the heuristic. Thus, it can be said that the original model with certain alterations is executed for each doctor separately, which greatly reduced the complexity of the model. After determining every doctor's schedule, the distances that are found are summed up and the overall distance traveled is obtained.

Improvement Phase

After determining an initial solution for the problem, it is aimed to improve the overall distance value if it is possible. Since the routing stage is solved to optimality by means of the mathematical modeling approach, the stage that can be improved is the clustering one. Thus, an iterative approach for this stage is implemented in order to decrease the total distance of the solution.

In this methodology, it is aimed to find the next best cluster at each iteration. In other words, the clustering of villages that provides the minimum distance which is strictly greater than the previous iteration's objective value is found at every step. The routings of these clusters are generated in the same manner for each iteration and overall distance values for each step are calculated. At the end of the improvement phase, the iteration that provides the minimum objective value is determined to be the solution of the heuristic algorithm.

In order to determine the next best cluster, p-median based IP model is utilized with an additional constraint, which is as follows:

$$\sum_{i \in I} \sum_{j \in I} DIST_{ij} \cdot DEM_i \cdot y_{ij} \geq PrevIter + k \tag{7.11}$$

In this constraint, $PrevIter$ represents the objective function value of the previous iteration and k is a small enough number that guarantees no-skip of any feasible solutions. The purpose of this constraint is simply to determine a lower bound for the objective function of IP. When constraint (7.11) is included in the model, a new set of clusters will be found with an objective value that is strictly greater than the previous iteration's. It should be noted that $PrevIter$ value is set to 0 at the first iteration, thus, the constraint is ensured to be redundant.

At this point, there is a potential issue that can arise at each iteration. It is highly possible to obtain the exact same clustering structure with different cluster origins at the next iteration. As it is explained above, determining cluster origins is only a dummy decision, therefore, these iterations would not have any difference than the previous ones. In order to prevent these repetitions, a new constraint is included in the formulation at each step that can be represented as follows:

$$\sum_{s \in S} y_{sj} \leq |S| - 1, \qquad S = \{i \in I : y_{i1} = 1\} \tag{7.12}$$

The set S can be defined as the villages that are assigned to the first cluster. Via constraint (7.12), it is indicated that the villages in the first cluster cannot be in the same cluster in the next iteration; there must be at least one different village in that cluster. At each iteration, this constraint it added to the formulation while keeping the previously determined ones.

It should be noted here that, including this constraint might result in missing the optimal solution of the problem as it is only stated for the first cluster of the clustering model. When there are 3 or more doctors to be scheduled, then it is possible to find the optimal solution with the same first cluster but differences in the remaining ones. For the case where there are 2 doctors, the optimal solution might contain the same villages in a cluster with an additional village. However, when it is indicated that only $|S| - 1$ of them can be in the same cluster, this case is also prevented. Therefore, it is decided to execute the heuristic algorithm with two different settings: one with and the other without constraint (7.12).

According to all these information, the representation of the heuristic algorithm is provided in Algorithm 1.

Algorithm 1 Heuristic Approach for PLRP

Require: $iter$: Number of predetermined iterations
p-median($prevIter$): The IP formulation explained in Chapter 7.
routing(D_j): The IP formulation explained in Chapter 5.

1: **for** $i = 1 : iter$ **do**
2:     **if** i=1 **then**
3:         $prevIter = 0$
4:     **else**
5:         $prevIter = solution$
6:     **end if**
7:     Solve p-median($prevIter$)
8:     $solution$ = p-median($prevIter$).objective
9:     Add new constraint (7.12)
10:     $size$ = number of clusters (i.e. number of doctors)
11:     Record clusters in $Doctors(size)$
12:     **for** $j = 1 : size$ **do**
13:         Solve routing($Doctors_j$)
14:         $distance_j$ = routing($Doctors_j$).objective
15:         $j = j + 1$
16:     **end for**
17:     $Sum(i) = \sum_{j=1}^{size} distance_j$
18:     $i = i + 1$
19: **end for**
20: $Result = \min_{i=1..iter} Sum(i)$

Note here that this representation is valid for the first setting, which will be referred as Heuristic-1 in the following chapters. The only difference in the representation of the second setting is the removal of step 9. This setting will be referred as Heuristic-2 in the remainder of this thesis.

Chapter 8

Computational Analysis of the Heuristics

The main goal of the heuristic approach is to determine solutions in quite shorter times than the mathematical model without compromising from the solution quality significantly. With this purpose, first, the instances of the small data set are solved with both heuristic settings and their outcomes are compared with the optimal results. While performing these computational studies, only the original variants of the instances are taken into consideration. It should be noted here that solving instances through 25 to 30 of the small data set with heuristic approaches cannot provide any insights about the heuristic performances as they schedule only one doctor. Since there will be only one cluster and its route will be determined by the mathematical model, the first iteration of the heuristic will be exactly same with solving this instance with the original model. Hence, only the first 24 instances of the data set are utilized to test the performance of the heuristic algorithm. The number of iterations to be performed for this data set is set to 20.

The results of the three solution methodologies are presented in Table 8.1. In this table, first, the optimal objective values are given with the computational times of the mathematical formulation. Afterwards, the minimum distance values that are found by Heuristic-1 and Heuristic-2 are provided as well as their solution times and gap percentages from the optimal values. The iteration that provided the best result is also given in the table under the "Iteration" column for each instance.

	Mathematical Model		**Heuristic-1**				**Heuristic-2**			
	Obj. Value	Solution Time (sec)	Obj. Value	Itera-tion	Solution Time (sec)	Gap (%)	Obj. Value	Itera-tion	Solution Time (sec)	Gap (%)
Ins 1	5,281.18	6,684	5,281.18	4	179	0.00%	5,281.18	7	219	0.00%
Ins 2	4,943.81	357	4,943.81	1	106	0.00%	4,943.81	1	124	0.00%
Ins 3	4,491.33	1,180	4,491.33	1	104	0.00%	4,491.33	1	122	0.00%
Ins 4	4,805.67	344	4,805.67	1	95	0.00%	4,805.67	1	130	0.00%
Ins 5	4,876.04	2,677	4,876.04	2	140	0.00%	4,876.04	2	173	0.00%
Ins 6	7,885.68	2,989	7,908.40	2	94	0.29%	7,908.40	2	119	0.29%
Ins 7	7,908.40	6,773	7,908.40	1	93	0.00%	7,908.40	1	106	0.00%
Ins 8	6,053.13	303	6,053.13	1	120	0.00%	6,053.13	1	121	0.00%
Ins 9	4,856.33	5,483	4,856.33	1	122	0.00%	4,856.33	1	118	0.00%
Ins 10	6,097.48	5,926	6,097.48	2	80	0.00%	6,097.48	3	116	0.00%
Ins 11	6,012.08	12,742	6,012.08	2	78	0.00%	6,012.08	3	119	0.00%
Ins 12	5,551.66	197	5,551.66	14	102	0.00%	5,635.70	1	124	1.51%
Ins 13	4,235.74	210	4,235.74	1	108	0.00%	4,235.74	1	126	0.00%
Ins 14	5,594.12	3,989	5,594.12	16	78	0.00%	5,978.96	1	115	6.88%
Ins 15	5,179.78	3,878	5,179.78	5	80	0.00%	5,196.60	17	99	0.32%
Ins 16	5,310.06	3,259	5,447.26	1	166	2.58%	5,447.26	1	195	2.58%
Ins 17	4,863.60	3,381	4,926.29	1	158	1.29%	4,926.29	1	214	1.29%
Ins 18	5,002.72	2,569	5,002.72	3	148	0.00%	5,002.72	4	206	0.00%
Ins 19	4,663.24	3,669	4,724.97	3	152	1.32%	4,663.24	10	195	0.00%
Ins 20	5,310.06	2,890	5,310.06	6	173	0.00%	5,310.06	14	203	0.00%
Ins 21	4,895.42	5,071	4,895.42	1	153	0.00%	4,895.42	1	193	0.00%
Ins 22	3,059.73	1,494	3,353.17	2	198	9.59%	3,287.80	6	225	7.45%
Ins 23	2,601.60	1,127	2,601.60	1	214	0.00%	2,601.60	1	228	0.00%
Ins 24	2,984.28	6,978	3,103.14	17	242	3.98%	3,179.24	17	315	6.53%

Table 8.1: Results of Solution Methodologies for Small Data Set

One of the things that can be observed is that both heuristic settings provide significantly better computational times than the mathematical model at each instance. It can be said that the model is solved in 3507 seconds, which corresponds to almost 1 hour, on average. On the other hand, Heuristic-1 performs within 1 and 4 minutes with the average of 132 seconds and Heuristic-2 solves the problem within 1.5 and 5 minutes while its average is slightly less than 3 minutes; 2 minutes 42 seconds to be exact.

Another observation that can be made is about the solution qualities. Among these 24 instances, Heuristic-1 is able to find the optimal solution in 18 instances, whereas Heuristic-2 can find it in 16 of them. The average gaps from the optimal solutions are only 0.79% for the first setting and 1.12% for the second one. Thus, it can be claimed that both approaches are eligible to find qualified results in the majority of the instances and performing quite well on the small data set.

When two heuristic settings are compared with each other, it can be said that Heuristic-1 seems to perform better in terms of both solution times and qualities. On the other hand, there are certain instances where Heuristic-2 dominates the first one. In order to illustrate both cases, certain instances can be given as examples. When instance 14 is examined, it can be observed that Heuristic-1 finds the optimal solution; however, Heuristic-2 fails to do so and finds a solution that falls 6.88% behind the optimal one. The main reasoning behind this is that the first setting achieves this solution at the 16^{th} iteration. Since repetition of the clusters are not allowed in this approach, it determines different cluster sets and finds the best at the 16^{th} step. However, the clusters of Heuristic-2 repeats itself multiple times and cannot reach to the 16^{th} set in the 20 iterations, hence cannot find the optimal solution. On the contrary, instance 19 implies the exact opposite result when Heuristic-2 reaches to the optimality but Heuristic-1 does not. Since adding the constraint (7.12) might eliminate the optimal clustering structure as it was explained in Chapter 7, this setting finds a solution with an 1.32% optimality gap. Thus, it can be said that both approaches has its own drawbacks in terms of finding the optimal solution.

According to these results, it is decided that the heuristic algorithm requires sufficiently less computational times and is able to find solutions that does not compromise from the quality. Hence, it is tested with the medium data set in the next step. It is observed that the best results are found within the first 10 iterations of the small data set in 88% of the instances. Since the schedules are determined with the mathematical model, it is decided to decrease the number of iterations to 10 in the medium data set in order not to increase the solution times dramatically. The exact same data are recorded for this data set with the exception of the remaining optimality gaps of the mathematical model after 2 hours and the results can be found in Table 8.2.

It is known that the mathematical formulation is not providing well performing results as the optimality gaps are considerably high, which is 74.37% on average.

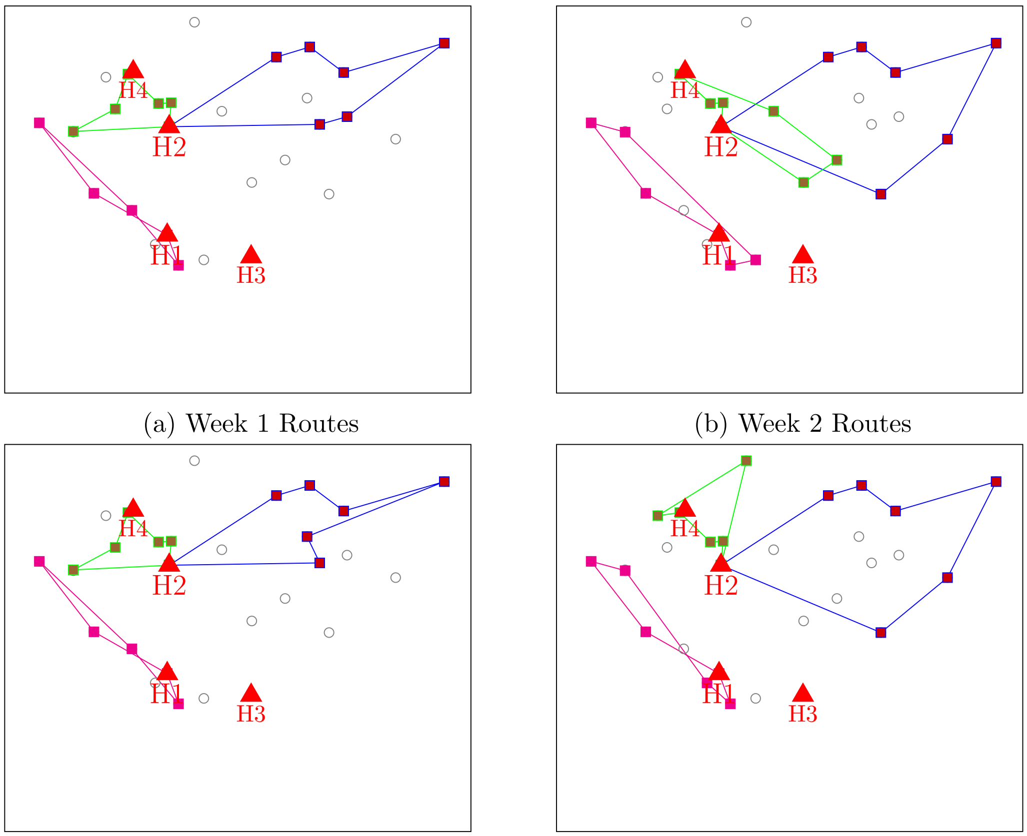
	Mathematical Model		Heuristic-1				Heuristic-2			
	Obj. Value	% Gap (in 2 hours)	Obj. Value	Itera-tion	Run Time (sec)	Gap (%)	Obj. Value	Itera-tion	Run Time (sec)	Gap (%)
Ins 1	8,625.2	60.46%	8,382.0	2	174	-2.82%	8,382.0	3	225	-2.82%
Ins 2	8,743.9	83.36%	7,872.4	1	173	-9.97%	7,872.4	1	215	-9.97%
Ins 3	11,452.8	85.29%	10,349.7	1	157	-9.63%	10,349.7	1	183	-9.63%
Ins 4	10,685.5	77.78%	9,958.0	1	153	-6.81%	9,958.0	1	192	-6.81%
Ins 5	7,765.3	33.64%	7,738.4	6	207	-0.35%	7,907.2	1	269	1.83%
Ins 6	7,464.9	82.61%	6,884.4	7	218	-7.78%	7,439.9	5	276	-0.33%
Ins 7	9,874.5	73.48%	9,457.4	6	164	-4.22%	9,561.2	5	201	-3.17%
Ins 8	10,307.9	81.04%	10,492.9	2	168	1.79%	9,781.1	3	233	-5.11%
Ins 9	9,964.0	81.43%	10,303.6	2	161	3.41%	9,598.5	3	211	-3.67%
Ins 10	11,028.1	84.63%	8,725.3	1	139	-20.88%	8,546.7	6	134	-22.50%

Table 8.2: Results of Solution Methodologies for Medium Data Set

Thus, it is clear that the necessity of a fast and efficient algorithm significantly increases. When the solution times of Heuristic-1 are examined, it is found out that the instances can be solved in 171 seconds on average. It takes around 40 seconds more for Heuristic-2 to solve the same instances, where the average computational time is 213 seconds. In addition to the speed of the algorithm, it improves the solutions that the model finds in most of the instances. 5.73% and 6.22% improvements on the solutions are observed on the average by the first and second settings, respectively. Both heuristic approaches provided the same results for the first 4 instances, and among the remaining 6, each setting outperformed the other in three of them. Similar to the small data set results, there are certain cases where one methodology provides better results than the other, hence, a strict domination between two algorithms cannot be determined. Yet, it can be claimed that the heuristic approach is performing well and it provides better solutions in 3 to 4 minutes than the model generates in 2 hours.

A remark on the performance of the model can be made at this point. Based on the results that are obtained from the small data set, it could be claimed that the solutions provided by the heuristic approach are mostly optimal or near optimal. In the medium data set, heuristic and model solutions are not extremely distant than each other. This may indicate that the incumbent solution that the model provides at the end of 2 hours is actually close to the optimal value; however, as the gap is significantly large, the lower bound cannot be improved by the model in this period of time.

In order to illustrate the results, the monthly schedules of instance 2 found by Heuristic-1 are shown on the graphics in Figure 8.1. Each graphic represents a week while each color identifies a single doctor's routes. It should be noted that multiple visits in a row are not represented in the figures.



(a) Week 1 Routes

(b) Week 2 Routes

(c) Week 3 Routes

(d) Week 4 Routes

Figure 8.1: Medium Data Set - Instance 2, Weekly Routes of Heuristic-1

In this instance hospitals 1 and 2 are selected as base hospitals for the doctors and one of them is assigned to the first one while the remaining two are routed from the second hospital. Two first two doctors have the exact same schedule in weeks 1 and 3. On the other hand, the remaining one utilizes the same route in weeks 2 and 4 while the first two doctors travel different villages during these times. According to the graphical representation, it can be inferred that reasonable tours are generated via Heuristic-1. There does not seem to be an anomalous village in any of the routes that can be visited by a different doctor. Thus, the results of the heuristic approach appears to be applicable.

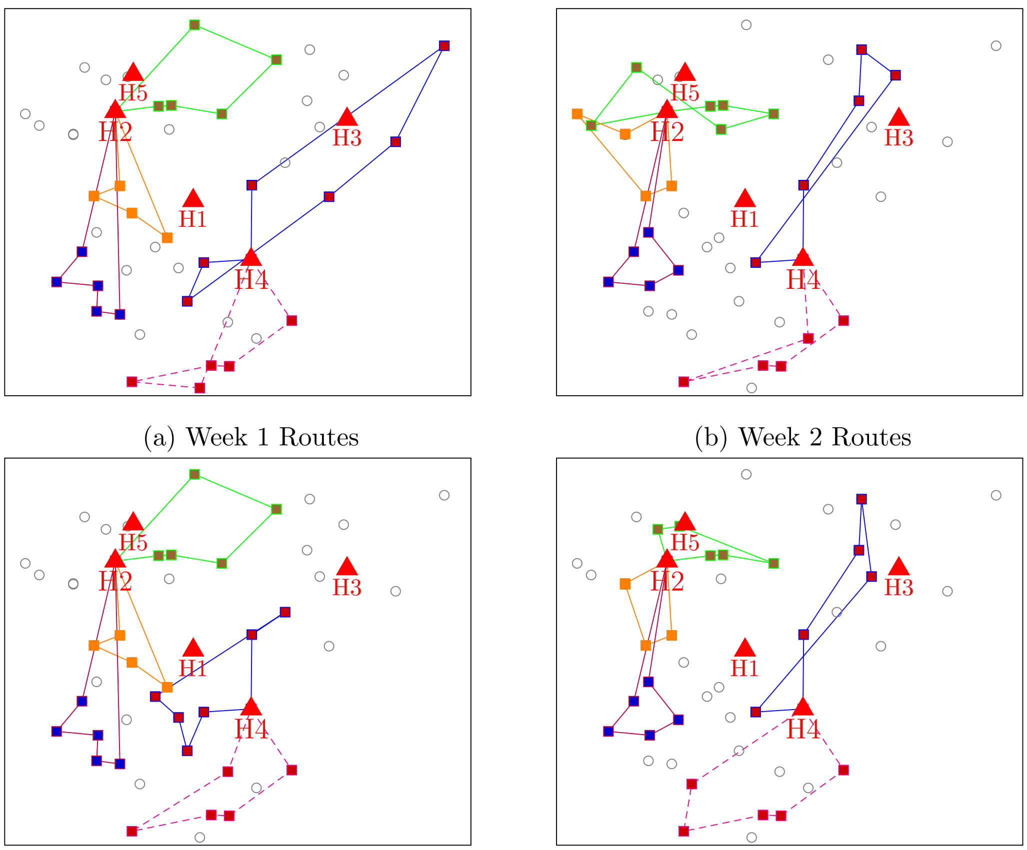
Finally, the large data set is solved with the heuristic approach. This time the mathematical formulation's results after 4 hours are provided with the remaining optimality gaps in addition to the both heuristic settings' minimum distance values, solution times and gaps from the model's solution. The outcomes of three solution approaches can be found in Table 8.3.

	Mathematical Model		**Heuristic-1**				**Heuristic-2**			
	Obj. Value	% Gap (in 4 hours)	Obj. Value	Itera- tion	Solution Time (sec)	Gap (%)	Obj. Value	Itera- tion	Solution Time (sec)	Gap (%)
Ins 1	28,775.6	94.01%	11,821.4	2	459	-58.92%	11,995.6	4	483	-58.31%
Ins 2	29,883.9	93.03%	12,009.2	3	596	-59.81%	12,036.4	8	561	-59.72%
Ins 3	33,787.1	93.66%	12,996.8	3	374	-61.53%	13,059.2	1	414	-61.35%
Ins 4	34,438.5	94.15%	12,996.8	3	360	-62.26%	13,059.2	1	420	-62.08%
Ins 5	37,147.7	94.82%	13,845.2	1	351	-62.73%	13,845.2	1	372	-62.73%
Ins 6	39,227.4	95.11%	13,917.7	9	349	-64.52%	13,845.2	5	359	-64.71%
Ins 7	-	-	14,216.4	3	271	-	14,222.7	1	277	-
Ins 8	33,501.4	94.30%	10,958.5	1	432	-67.29%	10,774.9	3	502	-67.84%
Ins 9	33,660.6	94.03%	12,114.3	2	326	-64.01%	12,364.2	3	325	-63.27%
Ins 10	-	-	12,355.1	2	298	-	12,299.7	6	325	-

Table 8.3: Results of Solution Methodologies for Large Data Set

Recall that mathematical formulation is not working efficiently on large instances, so that optimality gaps above 90% are observed at all instances. Because of this, considerable improvements on the distance values can be observed in both Heuristic-1 and Heuristic-2 solutions. The average improvement percentages of respective methodologies are 62.6% and 62.5%, hence, they are indifferent in terms of improvement rates. As it is observed in both small and medium data set analyses, Heuristic-1 provides the results in slightly shorter times, namely in 6 minutes 21 seconds on average whereas Heuristic-2's average computational time corresponds to 6 minutes 43 seconds. Among the 10 instances of the large data set, Heuristic-1 provides better solutions in 6 of them while Heuristic-2 outperforms the former approach in 3 instances.

One of the solutions from the large data set, that is the solution of instance 1 generated by Heuristic-2, is also represented graphically in Figure 8.2. It can be observed that base hospitals are determined as hospitals 2 and 4. First three doctors are assigned to base hospital 2 and the remaining two doctors are scheduled from hospital 4. Similar to the instance from the medium data set, certain doctor's routes are the same in certain weeks, while others may have different routes each week. For instance, all of the three doctors that are assigned to base hospital 2 have the same weekly schedules in weeks 1 and 3. On the other hand both doctors of base hospital 4 visit different villages at every week. Based on the graphical representation of the schedules, the clusterings of the villages make sense as the closest ones are grouped together. The routes of the doctors also seem logical and well coordinated. Hence, it could be claimed that satisfying results are obtained from the heuristic approach.



(a) Week 1 Routes

(b) Week 2 Routes

(c) Week 3 Routes

(d) Week 4 Routes

Figure 8.2: Large Data Set - Instance 1, Weekly Routes of Heuristic-2

In the end, according to the computational studies performed on three different sized data sets, it can be said that neither of the heuristic approaches can be determined as the better one in terms of providing smaller distance values. Depending on the instance, either methodology can generate a solution that the other one skips. Nevertheless, it can be claimed that Heuristic-1 finds a better solution in slightly more cases than Heuristic-2. In addition to that, the solution times of the first approach turn out to be better than the latter in almost all of the instances. Therefore, it can be easily said that Heuristic-1 is preferable in terms of computational times. Even though Heuristic-2 is developed because initial setting is expected to miss solutions that might be optimal with the introduction of a constraint at each iteration, it is found out that clustering performs well enough to find qualified solutions in shorter times via Heuristic-1.

Chapter 9

Conclusions

In this thesis, periodic location routing problem is addressed in the context of providing mobile healthcare services in rural areas. Lack of healthcare services in rural areas is a common problem faced by many developed or developing countries, hence, it requires the development of an effective solution methodology. Among several alternative solutions, providing mobile healthcare services seem to be the most applicable where the practitioners are responsible for visiting the villages that do not have medical centers. Motivated by this, the problem of determining schedules of the doctors is addressed with the help of certain OR approaches.

In Chapter 2, the healthcare policies of several countries are examined in detail. In addition to the general and primary healthcare service policies, principles of mobile healthcare services of these countries are compared with each other. As a result of these analyses, it is observed that mobile services are commonly used throughout the world with different beneficiaries; however, they have been recently started to be provided and are not developed fully. Hence, it is concluded that mobile services could be improved in terms of service quality and effectiveness.

In the next chapter, the problem of mobile healthcare services is defined considering the application of it in Turkey. The problem specific requirements that are enforced by Ministry of Health are explained in detail and the aim of this study is determined as to generate monthly schedules of the practitioners and their base hospitals in the most cost and manpower efficient way. Based on the characteristics and requirements of the problem, it is classified as a PLRP and the relevant literature on this research area as well as PVRP and LRP are analyzed in detail in Chapter 4. At the end, the unique characteristics of the problem considered are indicated and the contributions of this work are emphasized.

In order to solve the problem defined, an integer program is developed. The main difference of this model than the existing studies is that it does not generate the solutions by finding out the best schedule among a predefined visiting combination set. It is clear that unless all possible visiting combinations are defined to the model, the global optimal solutions might not be reached. Therefore instead of this approach, the developed model determines the schedules of the doctors via its constraints. Subsequently, four valid inequalities are generated with the goal of improving solution times and one of their combinations is aimed to be determined as the most suitable one.

In the computational studies of the mathematical model, first the valid inequality combination that improves the model is determined by the analyses performed on small and medium data sets. Afterwards, based on the results that are obtained by the inclusion of the selected combination to the original model, it is observed that the computational times of the small data set show high variations but can be solved in reasonable times. On the other hand, as the size of the instances increase, it is found out that the model is not able to adjust itself well enough to higher dimensions, thus, considerably high optimality gaps are obtained at the end of the predetermined time bounds. In order to determine the effects of different parameters on computation times, extended studies are performed on the small data set. According to the analyses, it is discovered that increasing the number of doctors or base hospitals to select also increases the time to obtain an optimal solution. On the other hand, having more balanced frequency distributions among the villages provides optimal results in shorter times. At the end of the computational analyses of the mathematical formulation, it is decided that there is a need for a fast and efficient algorithm to solve the larger instances.

In Chapter 7, a heuristic approach is developed in order to solve the large instances in reasonable durations without compromising from the solution qualities. Hence, an iterative heuristic methodology based on a “Cluster First, Route Second” approach is introduced. In this sense, the clusters, which correspond to the villages that one doctor can be responsible for, are generated via a p-median based integer programming model and then the routes of each doctor are determined separately via using the simplified version of the mathematical model that is developed in this thesis. After obtaining an initial solution, it is tried to be improved by utilizing an iterative approach. While performing the iterations, two variants are constructed and drawbacks of each are discussed.

According to the computational studies performed on the heuristic approach, it is found out that the solution methodology improves the computational times drastically and generates optimal or near optimal solutions in the small data set. For the medium and large data sets, improvements on the objective values of the mathematical model are observed since they cannot be solved at optimality and the solutions of the heuristic can be obtained in much less period of times. Therefore, it can be claimed that an algorithm which is both fast and efficient is developed for the considered problem.

In this thesis, mobile healthcare services are studied under the context of providing healthcare in rural areas; however, these services also have different application fields such as routing specialists (eg: cardiologists, dermatologists, geriatrists) to provide secondary healthcare services to the patients that require continuous care and chronic treatment. Besides the healthcare applications, other services such as education or hygiene can also be provided via mobile services. Recently, as a result of arising need throughout the world, the doctors and teachers started to travel through the refugee camps to provide services to those refugees. Therefore, it can be said that the problem considered here can be adapted to various cases in the world and with the variations on the problem specific requirements, they can be solved effectively and efficiently.

As a future research direction, certain aspects can be included and the problem can be adapted accordingly. First of all, the assumption of practitioners staying at villages during weekdays can be eliminated and the model can be constructed in a way that they return to their base hospitals at the end of every day. Another extension to the model can be about the maximum distance covered between two villages in the middle of the day, so that the practitioners can use their midday breaks more efficiently. Finally, in order to maintain a fairness between the practitioners where each of them is responsible for equal number of patients, it could be aimed to distribute the workload equally to the service providers in a multi-objective context.

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Appendix A

Data Sets

Instance (INS)	Number of Nodes with Freq 12	Number of Nodes with Freq 8	Number of Nodes with Freq 4	Number of Nodes with Freq 2	Number of Nodes with Freq 1	Number of Doctors	Number of Base Hospitals
INS 1	2	4	6	0	0	2	1
INS 2	4	2	3	1	2	2	1
INS 3	4	2	3	1	2	2	2
INS 4	5	1	2	0	4	2	1
INS 5	3	3	4	2	0	2	2
INS 6	7	4	1	0	0	3	1
INS 7	8	2	2	0	0	3	1
INS 8	3	3	3	1	2	2	1
INS 9	3	3	3	1	2	2	2
INS 10	7	3	1	1	0	3	2
INS 11	7	3	1	1	0	3	3
INS 12	4	1	2	3	2	2	1
INS 13	4	1	2	3	2	2	2
INS 14	7	2	1	2	0	3	2
INS 15	7	2	1	2	0	3	3
INS 16	1	6	3	2	0	2	1
INS 17	1	6	3	2	0	2	2
INS 18	1	7	1	2	1	2	1
INS 19	1	7	1	2	1	2	2
INS 20	0	8	2	2	0	2	1

Instance (INS)	Number of Nodes with Freq 12	Number of Nodes with Freq 8	Number of Nodes with Freq 4	Number of Nodes with Freq 2	Number of Nodes with Freq 1	Number of Doctors	Number of Base Hospitals
INS 21	1	5	6	0	0	2	2
INS 22	0	1	7	0	4	2	1
INS 23	0	1	7	0	4	2	2
INS 24	0	0	8	2	2	2	1
INS 25	1	1	1	6	3	1	1
INS 26	0	2	2	7	1	1	1
INS 27	0	2	1	8	1	1	1
INS 28	1	1	2	2	6	1	1
INS 29	2	0	1	2	7	1	1
INS 30	1	1	2	0	8	1	1

Table A.1: Details of the Small Data Set

Instance (INS)	Number of Nodes with Freq 12	Number of Nodes with Freq 8	Number of Nodes with Freq 4	Number of Nodes with Freq 2	Number of Nodes with Freq 1	Number of Doctors	Number of Base Hospitals
INS 1	3	5	7	5	6	3	1
INS 2	3	5	7	5	6	3	2
INS 3	7	6	4	3	6	4	2
INS 4	7	6	4	3	6	4	3
INS 5	3	4	7	7	5	3	1
INS 6	3	4	7	7	5	3	2
INS 7	5	7	6	4	4	4	2
INS 8	6	4	6	6	4	4	2
INS 9	6	4	6	6	4	4	3
INS 10	8	7	4	5	2	5	3

Table A.2: Details of the Medium Data Set

Instance (INS)	Number of Nodes with Freq 12	Number of Nodes with Freq 8	Number of Nodes with Freq 4	Number of Nodes with Freq 2	Number of Nodes with Freq 1	Number of Doctors	Number of Base Hospitals
INS 1	5	8	11	11	10	5	2
INS 2	5	8	11	11	10	5	3
INS 3	7	11	10	11	6	6	3
INS 4	7	11	10	11	6	6	4
INS 5	5	12	11	11	6	6	3
INS 6	5	12	11	11	6	6	4
INS 7	9	12	10	8	6	7	5
INS 8	3	7	13	15	7	5	3
INS 9	5	12	7	11	10	6	3
INS 10	5	12	7	11	10	6	4

Table A.3: Details of the Large Data Set