NETWORK FORMATION WITH SYSTEMIC RISK AND DEFAULT INSURANCE

A Master's Thesis

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July 2016

To my family and for those who are curious

NETWORK FORMATION WITH SYSTEMIC RISK AND DEFAULT INSURANCE

Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

NETWORK FORMATION WITH SYSTEMIC RISK AND DEFAULT INSURANCE

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In this paper, we study the networks that arise in the equilibrium when risk averse investors play a network formation game for making joint investments with each other. The outcomes of these projects are stochastic and hence, depending on the shock realizations, investors may choose to default. However, counterparty defaults are damaging and investors have the option to buy default insurances against this damage. We show that in this model, equilibrium networks consist of complete components of a certain size, which maximize the expected utility of investors in that component. For a wide variety of parameters, it turns out that investors choose not to buy any insurance at the equilibrium.

Keywords: Default Insurance, Network Formation, Risk Aversion, Strong-Stability, Systemic Risk.

ÖZET

SISTEMIK RISK VE İFLAS SIGORTASI ILE AĞ OLUŞUMU

İdem, Mehmet Hamdi Berk

Yüksek Lisans, İktisat Bölümü

Tez Yöneticisi: Prof. Dr. Semih Koray

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Bu araştırmada, riskten kaçınan yatırımcılar birbirleriyle ortaklaşa yatırım yapmak için bir ağ oluşumu oyunu oynadığında dengede ortaya çıkan ağları çalışıyoruz. Projelerin neticeleri rastsal olduğu için, gerçekleşen şoklara bağlı olarak, yatırımcılar iflas etmeyi seçebilir. Ancak, projelerdeki ortakların iflas etmesi yatırımcılar için zararlıdır ve yatırımcıların bu zarara karşı iflas sigortası alma seçeneği vardır. Biz bu modelde, denge ağların o belli bir büyüklükteki tam bileşenlerden oluştuklarını ve bu büyüklüğün de bir bileşenin içindeki yatırımcıların beklenen faydalarını ençoklaştıracak şekilde olduğunu gösterdik. Geniş bir parametre çeşitliliği için, görünen odur ki yatırımcılar dengede sigorta almamayı tercih etmektedir.

Anahtar Kelimeler: Ağ Oluşumu, İflas sigortası, Riskten Kaçınma, Sıkı-Kararlılık, Sistemik risk.

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CHAPTER 1

INTRODUCTION

The term 'systemic risk' refers to the risk of an event that could cause the collapse of the whole system. The 2008 crisis has shown economists that the interconnectedness of a system is directly related to the aggregate risk that the system bears. Hence, the literature on systemic risk has been growing at an increasing pace after the 2008 crisis.

The failure of the institutions which were the intermediaries in most of the Credit Default Swap (CDS) agreements is accepted by many as one of the most important reasons of the 2008 crisis. These agreements are closely related to default insurances in our model. Default insurance is an instrument used to reduce an individual's risk against the default of his partners. So, CDSs and many other financial instruments can be considered as special types of default insurance. In contrast to the important role of these financial instruments in one of the biggest financial break-downs of the modern times, they received very little attention in the theoretical literature on systemic risk. Like any kind of insurance, default insurance is also sensible only in a set up where agents do not enjoy the risk. For this reason, we investigate the equilibrium networks in a model with risk averse investors who can buy default insurance.

Among those who contributed to the systemic risk literature, Erol and Vohra (2014) is almost unique in considering the endogenous network formation in a model of systemic risk. In their model, risk neutral investors make joint projects whose outcome is stochastic. After the outcomes of all projects realize, investors decide whether to default from all projects or not. In their model, the only core networks are those that have complete connected components of a certain size. Our model and results are closely related to theirs. Particularly, we add risk aversion and option of buying a default insurance to their model and compare our results to theirs. The details of the relation of our study to theirs will be made clear after the model is introduced. In a similar setup to theirs, Erol (2015) focuses on a model of endogenous network formation where bailout is possible and shows that intervention increases the interconnectedness of the system and create a center-periphery structure which is by nature more volatile.

Allen and Gale (2000) and Babus (2016), focus on a two region model with two types of costumers: early consumers and late consumers who fluctuate between the regions and causing the liquidity uncertainty for banks. Both of these studies find that the complete networks arise in the equilibrium and they are more robust to systemic risk. However, Babus (2016) notes that the completeness is only a sufficient condition but not a necessary one. Our results show that the stability of complete networks requires the assumption of a very high probability of a good shock.

Many studies (Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015a b: Glasserman and Young, 2015: Caballero and Simsek, 2013: Gai and Kapadia, 2010: Eboli, 2013: Elliott, Golub, and Jackson, 2014: Cabrales, Gale, and Gottardi, n.d.) investigate different aspects of systemic risk in network games where the networks are exogenously given. Consideration of these studies include but are not limited to dynamics of cross-holdings of firms, flow network analysis and liquidity shocks. For a very extensive survey of the literature, the reader can refer to Chinazzi and Fagiolo (2015).

Zawadowski (2013) shows that banks buy less than the socially desirable level of counterparty insurance because they do not consider the consequences of their own failure on others in the system. However, their model does not feature an endogenous network formation game; banks decide to work with exogenously determined potential counterparties only.

Our results also at least partially extend those of Wilson (1968). Wilson shows that when people's information and risk preferences are identical and they are making investment on their joint savings, the equilibrium investment decisions are Pareto-optimal. In our model, we show that at least for some parameter values, the outcome is Pareto-optimal even without the assumption of same risk preferences.

We show that when people are risk-averse and they can buy default-insurance, they form complete clusters in which everyone is directly connected to everyone else and no one has connection with someone outside his own cluster. The size of the cluster is determined such that it maximizes expected utility of investors. Since the insurer makes the investors indifferent, the same equilibria would arise when there is no insurance option. Hence, the insurance does not change the network structure. If the probability of a good shock is lower than 0.5, we show that everyone makes a single project and they do not buy insurance, no matter how risk-averse they are. However, we have been unable to find a general result for the existence of a functioning default insurance market or even illustrate it by an example.

CHAPTER 2

PRELIMINARIES

Let $N = \{1, 2, \ldots, n\}$ denote the set of risk-averse investors. Both investors and the risk-neutral insurer have von Neumann - Morgenstern utility functions, $u_i(x_i)$ for each investor i. The investors try to maximize their utility which is a strictly increasing, concave function of their monetary gains x_i. The insurer tries to maximize his profit,
$\pi =$ (Total revenue) $-$ (Total amount of coverage he has to provide) by setting up prices for insurance.

We are going to represent joint projects with the edges of a graph, so we will need the following formal definitions for the later discussions.

2.1 Definitions

Definition 1. A *graph* G is an ordered pair of sets (N, E) such that N is the set of vertices (investors) and E is a subset of $N \times N$ (the set of all possible joint projects). We denote the set of all graphs on N by $\mathcal{G}^N$.

Let $G = (N, E)$ be a given graph for the following definitions.

Definition 2. $H = (V, E')$ is a *subgraph* of a graph $G = (N, E)$ if $V \subseteq N$ and $E' \subseteq E$.

Definition 3. We say that an edge $ij \in E$ is adjacent to a vertex $k \in N$ if $i = k$ or $j = k$.

Definition 4. The *degree* of a vertex is the number of edges adjacent to it.

Definition 5. A graph G is *k-regular* if every vertex in G has degree k.

Definition 6. A graph G is a *star* graph if there is a vertex i such that for any j, k in N other than i, ij is in E and there does not exist jk in E such that $j \neq i$ and $k \neq i$.

Definition 7. A finite sequence of edges $e^1, e^2, ..., e^k \in E$ is a *path of length k* in G if for any $i, j \in \{1, 2, ..., k\}$ with $i \neq j$, $e^i \cap e^{i+1} \neq \emptyset, i = 1, 2, ..., k$ and $e^i \neq e^j$. We will shortly say a *k-path* to a path of length k.

Definition 8. The graph G is *connected*, if for all v_i, v_j in N with $v_i \neq v_j$, there is a path from v_i to v_j.

Definition 9. A subgraph H of a graph G is a *component* of G if it is connected and there is no other connected subgraph H' of G which contains H and $H' \neq H$.

Definition 10. Let G, G' be graphs in $\mathcal{G}^N$. G' is said to be *reachable* from G via $T \subseteq N$ if

1. $\forall ij \in G : \{i, j\} \subseteq N \setminus T \Rightarrow e \in G'$ and

2. $\forall ij \in G' : \{i, j\} \cap N \setminus T \neq \emptyset \Rightarrow ij \in G$.

We denote the expected utility of an investor i when the materialized graph is G by $E^G\left[u_i\left(x_i\right)\right]$.

Definition 11. Let k be in $\{1, 2, \ldots, n\}$. A graph $G \in \mathcal{G}^N$ is said to be *k-stable* if there is no $T \subseteq N$ with $\mid T \mid \leq k$ such that there is some G' reachable from G via T with $\forall i \in T : E^{G'}[u_i(x_i)] > E^G[u_i(x_i)]$.

We say that a graph $G \in \mathcal{G}^N$ is *strong-stable* if it is n-stable and *Nash-Stable* if it is 1-stable.

2.2 The Process

The process works as follows:

1. Each investor i announces the set of investors whom he wants to join projects with. If two investors i and j announce each other, we say that the project ij is materialized. There is an independently, identically distributed shock ω_{ij} to each materialized project ij whose distribution is common knowledge. ω_{ij} has a Bernoulli distribution on $\Omega = \{\omega_1, \omega_2\}$ and we denote the probability of a good shock with p: $P(\omega_{ij} = \omega_2) = p$.

2. Knowing the distribution of shocks, insurer sets a price P_{ij}[1] which denotes the price i should pay to be insured against the default of j in materialized project ij or declares that he is not offering insurance for i against j.

3. Each investor i decides whether to buy default insurance for his materialized projects, if he is offered insurance.

4. Shocks to each materialized project ij are realized.

5. Observing realized shocks, each investor i decides whether to default or not.

[1]In our model, an investor cannot buy insurance for a single project. However, insurer still calculate the price of insurance of an investor by individually calculating the prices of insurances for each of the investors project and adding them up.

6. The insurer provides insurance coverage to each investor i if his partner j in the materialized project ij defaulted and i himself did not.

Payoffs: Given the realized shock ω_{ij} to ij, without the insurance, the monetary gains of each investor from project ij -depending on their default decisions- are as follows:

		Investor j: ND	Investor j: D
Investor i	ND	$(\omega_{ij}, \omega_{ij})$	$(\omega_{ij} - 1, 0)$
	D	$(0, \omega_{ij} - 1)$	$(0, 0)$

Payoffs from the joint project ij given the value of ω_{ij}

Clearly, depending on realized value of ω_{ij}, this “game” has a different “dominant strategy”. However, since each investor either defaults in all projects or not default at all, this is not actually a game in itself and defaulting in an individual project is not a strategy. So, we render the action of defaulting in a single project irrelevant by definiton[2]. Hence, investors will consider all of their projects when they make their decisions about defaulting.

We consider an insurer who offers a price p_{ij} only if it gives a strictly positive expected profit and in case there is no such price, he declares that he is not offering insurance for i against j.

Assumption 1. $\omega_1 < 0 < \omega_2$ with $0 < \omega_2 < min\left\{\frac{1}{n-1}, \frac{-\omega_1}{n-2}\right\}$.

Assumption 1 ensures that an investor has to default if he receives a negative shock to a project or if he is uninsured and one of his partners receive a negative shock. Hence, under this assumption, if a negative shock hits an edge in a

[2]However, as we mention in the conclusion, we hope to extend this assumption in the future

component, every investor in the component must default in an equilibrium. (See Erol and Vohra (2014) for a proof of this using rationalizable strategies.)

CHAPTER 3

THE MODEL

Here, we first illustrate by an example and then prove that if the probability of a good shock is less than 0.5, at the equilibrium, everyone has one edge and no one buys insurance. We then consider another example where the only strong-stable network is the complete network. Using insights from the results of this example, we find a general necessary and sufficient condition for having the complete network as the only strong-stable network. Lastly, we find a general characterization of strong-stable networks.

3.1 Sparse Strong-Stable Networks

In this section we will first solve an example with four investors and then generalize the results of this example to an arbitrary number of agents.

3.1.1 Four investors example with $p < 0.5$

In this example, we have four investors and one insurer. Preference of investor i is represented by $u_i(x_i)$[1] which is a strictly concave and strictly increasing function, where x_i is the total monetary gain of i from the game. We denote the probability of a good shock as p, i.e., $P(\omega = \omega_2) = p$. The preference of the insurer is represented by his profit function. Investors can either buy insurance for all of their projects or none of them. The insurer sets the price so as to make the investors indifferent between buying the insurance and not. We consider equilibria in which an agent who is indifferent between buying and not buying the insurance buys the insurance. We have one crucial assumption here.

Assumption 2. $p < 0.5$.[2]

3.1.1.1 Expected Utilities

Since, the insurer makes the investors indifferent between buying the insurance and not, we can find the equilibrium networks without taking the insurance decisions into account. In the case with four investors, the expected utilities of an investor in each network and each position are as given below.

3-regular network: Everyone has the same expected utility due to symmetry:

$$p^6 u(3\omega_2) + \left(1 - p^6\right) u(0).$$

[1] We drop the indices when there is no ambiguity.

[2] Since we have strict concavity, weak inequality here and in the rest of this analysis is sufficient for all the results. But when this assumption is made with strict inequality, all of the results are correct for all concave utility functions, not only for strictly concave ones.

This calculation uses the fact that, even if a single negative shock hits an edge in a component, everyone in the component has to default. So, if every edge receives a positive shock, which happens with probability p^6, everyone gets the utility $u(3\omega_2)$ and if there is at least one negative shock, then everyone defaults and they receive $u(0)$. In the following networks, we only provide the end-result of this calculation.

2-regular network: Everyone has the same expected utility due to symmetry:

$$p^4 u(2\omega_2) + \left(1 - p^4\right) u(0).$$

1-regular network: Everyone has the same expected utility due to symmetry:

$$pu(\omega_2) + (1-p) u(0).$$

3-cycle: Everyone in the cycle has the same expected utility due to symmetry:

$$p^3 u(2\omega_2) + \left(1 - p^3\right) u(0)$$

and the agent with degree 0 gets $u(0)$.

1-path: Everyone in the path has the same expected utility due to symmetry:

$$pu(\omega_2) + (1-p) u(0)$$

and the vertices with degree 0 gets $u(0)$.

2-path: Agents with degree 1 have the same expected utility due to symmetry:

$$p^2 u(\omega_2) + \left(1 - p^2\right) u(0)$$

and the vertex with degree 2 gets the following:

$$p^2 u(2\omega_2) + \left(1 - p^2\right) u(0)$$

and the vertex out of the path gets $u(0)$.

3-path: Vertices with degree 1 have the same expected utility due to symmetry:

$$p^3 u(\omega_2) + \left(1 - p^3\right) u(0)$$

and the vertices with degree 2 get the following:

$$p^3 u(2\omega_2) + \left(1 - p^3\right) u(0).$$

Star network: Vertices with degree 1 have the same expected utility due to symmetry:

$$p^3 u(\omega_2) + \left(1 - p^3\right) u(0)$$

and the vertex with degree 3 gets the following:

$$p^3 u(3\omega_2) + \left(1 - p^3\right) u(0)$$

Complete network minus an edge: Vertices with degree 3 have the same expected utility due to symmetry:

$$p^5 u(3\omega_2) + \left(1 - p^5\right) u(0)$$

and the vertices with degree 2 get:

$$p^5 u(2\omega_2) + \left(1 - p^5\right) u(0) .$$

Other networks: In all other networks with four agents, there is one agent with degree 1, two agents with degree 2 and one agent with degree 3. In this network, the vertices with degree 2 get the same expected utility due to symmetry:

$$p^4 u(2\omega_2) + \left(1 - p^4\right) u(0)$$

the vertex with degree 1 gets the following:

$$p^4 u(\omega_2) + \left(1 - p^4\right) u(0)$$

and the vertex with degree 3 gets:

$$p^4 u\left(3\omega_2\right) + \left(1 - p^4\right) u\left(0\right).$$

3.1.1.2 Equilibria

Clearly, all expected utilities are convex combinations of $u\left(0\right)$ and $u\left(k\omega_2\right)$ where k is the degree of the vertex. Expected utilities of the vertices with the same degree can easily compared with each other by checking which one has greater coefficient for $u\left(0\right)$. So, the three candidates for maximal expected utility are any vertex in 1-regular network, the center vertex in the path of length 2 and the center vertex in the star network. Notice that these are all vertices who do not inquire any indirect risk, meaning they are adjacent to all edges in their component. Now, we claim that the 1-regular network gives the highest possible expected utility under a certain condition on p. In order to check this, we only need to compare the expected utility of a vertex in 1-regular network with the expected utilities of other natural candidates mentioned above.

Comparison with the center vertex in the path of length 2 We claim:

$$p^2 u\left(2\omega_2\right) + \left(1 - p^2\right) u\left(0\right) < pu\left(\omega_2\right) + \left(1 - p\right) u\left(0\right).$$

Equivalently,

$$p\left(pu\left(2\omega_2\right) - u\left(\omega_2\right) + \left(1 - p\right) u\left(0\right)\right) < 0.$$

Or

$$p\left(p\left(u\left(2\omega_2\right)-u\left(0\right)\right)+u\left(0\right)-u\left(\omega_2\right)\right)<0.$$

Now, since u is strictly concave, we have:

$$u\left(\omega_2\right)-u\left(0\right)>\frac{u\left(2\omega_2\right)-u\left(0\right)}{2}$$

From above assumption, we have $p<1/2$, so we conclude that our claim is correct, since we also know $0<p$.

Comparison with the center vertex in the star network: We claim:

$$p^3u\left(3\omega_2\right)+\left(1-p^3\right)u\left(0\right)<pu\left(\omega_2\right)+\left(1-p\right)u\left(0\right)$$

.

Equivalently,

$$p\left(p^2u\left(3\omega_2\right)-u\left(\omega_2\right)+\left(1-p^2\right)u\left(0\right)\right)<0$$

.

Or

$$p\left(p^2\left(u\left(3\omega_2\right)-u\left(0\right)\right)+u\left(0\right)-u\left(\omega_2\right)\right)<0$$

.

Now, since u is strictly concave, we have:

$$u\left(\omega_2\right)-u\left(0\right)>\frac{u\left(3\omega_2\right)-u\left(0\right)}{3}$$

.

Since we assumed $p < 1/2$ above, we also have $p^2 < \frac{1}{3}$, so that we can conclude that our claim is correct, since we also know $0 < p$.

Then, 1-regular network gives the highest possible expected utility for all vertices when we have $p < 1/2$. So, it is the only 4-stable or strong-stable network structure in this context.

3.1.2 A Sufficient Condition for Sparse Strong-Stable Networks

Now, suppose there are arbitrary number of investors, say n. The state space is $\Omega = \{\omega_1, \omega_2\}$ with $\omega_1 < 0 < \omega_2$ and $0 < \omega_2 < min\left\{\frac{1}{n-1}, \frac{-\omega_1}{n-2}\right\}$. Consider a vertex with k links in a component with m links. His expected utility will be:

$$EU_1 = p^m u\left(k\omega_2\right) + \left(1 - p^m\right) u\left(0\right)$$

.

However, when $m \neq k$, clearly this cannot be the maximum achievable expected utility, since

$$EU_2 = p^m u\left(m\omega_2\right) + \left(1 - p^m\right) u\left(0\right)$$

.

would be reachable in a network where he has links with everyone in his component and EU_2 will be strictly higher than EU_1, since u is a strictly increasing function. Hence, in order to find the maximum achievable expected utility for a vertex, we can restrict our attention to vertices who have links with every other vertices in their component. Our claim is again the same: Vertices in a 1-regular network receive the highest possible expected utility, when $p < 0.5$. We refer to expected utility of a vertex in a 1-regular network by EU_m. The comparisons for the case with 4 vertices clearly apply here. Now, let $m > 3$ and consider.

$$EU = p^m u\left(m\omega_2\right) + \left(1 - p^m\right) u\left(0\right)$$

.

Now, our claim is that:

$$EU_m - EU = p\left(p^{m-1} u\left(m\omega_2\right) - u\left(\omega_2\right) + \left(1 - p^m\right) u\left(0\right)\right) < 0$$

.

Equivalently,

$$p\left(p^{m-1}\left(u\left(m\omega_2\right) - u\left(0\right)\right) + u\left(0\right) - u\left(\omega_2\right)\right) < 0$$

.

From strict concavity of u,

$$u\left(\omega_2\right) - u\left(0\right) > \frac{u\left(m\omega_2\right) - u\left(0\right)}{m}$$

.

So, for our claim to be true, we need

$$p^{m-1} < \frac{1}{m}$$

.

Equivalently,

$$p < m^{\frac{-1}{m-1}}$$

.

Define $g(m) =: m^{\frac{-1}{m-1}}$ and check the first derivative of g:

$$g'(m) = \frac{\frac{\ln(m)}{(m-1)^2} - \frac{1}{(m-1)m}}{m^{\frac{1}{m-1}}} > 0, \forall m > 3$$

.

So, g is an increasing function of m on relevant domain.

Now, since $p < 0.5 = g(2)$, we have $p < g(2) < g(m)$ for all $m > 3$ so that our claim is proven. Hence, the following theorem.

Theorem 1. *Under assumptions 1 and 2, the only strong-stable networks are the 1-regular networks.*

Proposition 1. *If everyone has a strictly concave and strictly increasing utility functions, then, no one sells or buys any insurance at the strong-equilibrium.*

Proof. From Theorem 1, we know that in a strong-stable equilibrium, the network is 1-regular. When the insurer sets the price for insurance of investor i against j, he finds the price P_{ij} which solves the following problem:

$$pu(\omega_2) + (1-p)\,u(0) = p^u(\omega_2 - P_{ij}) + (1-p)\,u(0 - P_{ij})$$

.

Now, suppose some $P_{ij} > 0$ solves above equation. Then, we have $u(\omega_2) > u(\omega_2 - P_{ij})$ and $u(0) > u(0 - P_{ij})$ since u is strictly increasing. So, $pu(\omega_2) > pu(\omega_2 - P_{ij})$ and $(1-p)\,u(0) > (1-p)\,u(0 - P_{ij})$ contradicting with P_{ij} being a solution to above equation. Thus, $P_{ij} = 0$ since offering a negative price for a service to maximize the profit would be absurd. But then, the expected profit of the insurer from insuring i is non-positive.

$$\pi_i = pP_{ij} + (1-p)(P_{ij}) = P_{ij} = 0.$$

But then, he would declare that he is not offering insurance for i against j. So, there is no agent who buys or sells any insurance in the strong equilibrium.

□

Remark 1. If $p < 0.5$, then the strong-stable networks are "efficient" in the sense that everyone is receiving the maximum possible utility for themselves in this game.

3.2 Complete Strong Stable Networks

Here we will show that with certain parameter values and preferences, the only strong-stable network is the complete network and then characterize the conditions for strong-stability of the complete network.

3.2.1 Four investors example with $p = 0.9$

Now, let $|N| = 4$ and $u_i(x_i) = \sqrt{x_i + 6}$, for any i in N and suppose the probability of a good shock is 0.9, i.e. $p = P(\omega_2) = 0.9$. The reason we add 6 to the monetary gain in the utility function is to ensure that the discriminant of the square root is nonnegative, since the price of an insurance cannot exceed 1 and there can be at most 6 edges. Clearly, u_i is a strictly increasing and strictly concave function of x_i and it is always well-defined.

Now, we will show that the only strong-stable network is the complete network. Here, as the reader can check, every network is a Nash-stable so Nash-stability concept does not eliminate any network. For this reason, we choose strong-stability as our equilibrium notion which is stronger than Nash-stability.

3.2.1.1 Expected Utilities

First, let us find the expected utilities of an agent in all possible networks and positions.

3-regular network: Everyone has the same expected utility due to symmetry:

$$p^6 u(3\omega_2) + \left(1 - p^6\right) u(0) \cong 2.5284.$$

2-regular network: Everyone has the same expected utility due to symmetry:

$$p^4u\left(2\omega_2\right)+\left(1-p^4\right)u\left(0\right)\cong 2.5151.$$

1-regular network: Everyone has the same expected utility due to symmetry:

$$pu\left(\omega_2\right)+\left(1-p\right)u\left(0\right)\cong 2.4949.$$

3-cycle: Everyone in the cycle has the same expected utility due to symmetry:

$$p^3u\left(2\omega_2\right)+\left(1-p^3\right)u\left(0\right)\cong 2.5224$$

and the person out of the cycle gets $u\left(0\right)\cong 2.4494$.

1-path: Everyone in the path has the same expected utility due to symmetry: a

$$pu\left(\omega_2\right)+\left(1-p\right)u\left(0\right)\cong 2.4949$$

and the vertices out of the path gets $u\left(0\right)\cong 2.4494$.

2-path: Vertices with degree 1 have the same expected utility due to symmetry:

$$p^2u\left(\omega_2\right)+\left(1-p^2\right)u\left(0\right)\cong 2.4904$$

the vertex with degree 2 gets the following:

$$p^2 u\left(2\omega_2\right) + \left(1 - p^2\right) u\left(0\right) \cong 2.5305$$

and the vertex with degree 0 $u\left(0\right) \cong 2.4494$.

3-path: Agents with degree 1 have the same expected utility due to symmetry:

$$p^3 u\left(\omega_2\right) + \left(1 - p^3\right) u\left(0\right) \cong 2.4863$$

and the vertices with degree 2 get the following:

$$p^3 u\left(2\omega_2\right) + \left(1 - p^3\right) u\left(0\right) \cong 2.5224.$$

Star network: Vertices with degree 1 have the same expected utility due to symmetry:

$$p^3 u\left(\omega_2\right) + \left(1 - p^3\right) u\left(0\right) \cong 2.4863$$

and the vertex with degree 3 get the following:

$$p^3 u\left(3\omega_2\right) + \left(1 - p^3\right) u\left(0\right) \cong 2.5578$$

Complete network minus an edge: Vertices with degree 3 have the same expected utility due to symmetry:

$$p^5 u\left(3\omega_2\right) + \left(1 - p^5\right) u\left(0\right) \cong 2.5085$$

and the vertices with degree 2:

$$p^5 u\left(2\omega_2\right) + \left(1 - p^5\right) u\left(0\right) \cong 2.5372.$$

Other networks: In all other networks with four vertices, there is one vertex with degree 1, two vertices with degree 2 and one vertex with degree 3. In this network, the vertices with degree 2 get the same expected due to symmetry:

$$p^4 u\left(2\omega_2\right) + \left(1 - p^4\right) u\left(0\right) \cong 2.5151$$

the vertex with degree 1 gets the following:

$$p^4 u\left(\omega_2\right) + \left(1 - p^4\right) u\left(0\right) \cong 2.4826$$

and the vertex with degree 3 links gets:

$$p^4 u\left(3\omega_2\right) + \left(1 - p^4\right) u\left(0\right) \cong 2.5469.$$

3.2.1.2 Equilibria

Now, we will note the profitable deviations in all networks but the complete network and show that there is no profitable deviation for any coalition in the complete network.

2-regular network: Any coalition of 3 agents can deviate to a *3-cycle* and strictly increase their expected utilities. So, it is not 3-stable.

1-regular network: Similar to 2-regular networks, any 3 agents can form a *3-cycle* and improve their expected utilities. So, this is also not 3-stable.

3-cycle: This networks are not 4-stable since by moving to the *complete network*, everyone strictly improves their expected utility.

1-path: It is not 3-stable since a *3-cycle* strictly improves everyone's expected utility in the cycle.

2-path: It is not 2-stable since the vertices with degree 1 can improve their expected utility by creating a new edge and moving to a *3-cycle*.

3-path: These networks are also not 2-stable since the vertices with degree 1 can again make a new edge and move to *2-regular network* and strictly increase their expected utilities.

Star network: These networks are not 2-stable since any two vertices with degree 1 can make a new edge between each other and improve their expected utilities strictly.

Complete network minus an edge: Since the vertices with degree 2 can make a new edge and strictly improve their expected utilities, this network is not 2-stable.

Other networks: Vertices with degree 1 and 2 can turn these networks in to the *complete network* and strictly increase their expected utilities. Hence, these networks are not 3-stable.

3-regular network: Now, since this network is complete, they can only drop edges but cannot make any new one. One agent can drop 1, 2 or 3 edge(s) and all of these moves would strictly decrease his expected utility. Clearly, there is no other coalition which can drop some of their edges and strictly increase all of their members' expected utilities. Hence, this network is 4-stable. So, it is strong-stable network. Moreover, since all other networks are shown to have coalitions that have some profitable deviations, the complete network is the unique strong-stable network.

3.2.2 A Necessary and Sufficient Condition

In this section, we find a necessary and sufficient condition for strong-stability of the complete network in a four agent setup.

Condition 1.

$$p^3 u\left(3\omega_2\right) + \left(1 - p^3\right) u\left(0\right) > u\left(2\omega_2\right)$$

$$p^5 u\left(3\omega_2\right) + \left(1 - p^5\right) u\left(0\right) > u\left(\omega_2\right)$$

Proposition 2. *The complete network is strong-stable if and only if p, u and ω_2 satisfy Condition 1.*

Proof. ($\Rightarrow$) Suppose the complete network is strong-stable.

1. Since the complete network is strong stable, there is no 2-person coalition with a profitable deviation. So, two agents in the complete network cannot be better off by cutting edges with all other agents. That is,

$$p^6 u\left(3\omega_2\right) + \left(1 - p^6\right) u\left(0\right) > pu\left(1\omega_2\right) + \left(1 - p\right) u\left(0\right).$$

Equivalently,

$$p^5 u\left(3\omega_2\right) + \left(1 - p^5\right) u\left(0\right) > u\left(\omega_2\right).$$

2. Similarly, no 3-person coalition has a profitable deviation. So, three agents in the complete network would not be better off by cutting their edges with the forth agent:

$$p^6 u\left(3\omega_2\right) + \left(1 - p^6\right) u\left(0\right) > p^3 u\left(2\omega_2\right) + \left(1 - p^3\right) u\left(0\right).$$

Equivalently,

$$p^3 u\left(3\omega_2\right) + \left(1 - p^3\right) u\left(0\right) > u\left(2\omega_2\right).$$

So, if the complete network is strong-stable, Condition 1 is satisfied.

($\Leftarrow$) Now, suppose Condition 1 is satisfied. As shown above, it is equivalent to

$$p^6 u\left(3\omega_2\right) + \left(1 - p^6\right) u\left(0\right) > pu\left(1\omega_2\right) + \left(1 - p\right) u\left(0\right).$$

and

$$p^6 u(3\omega_2) + \left(1 - p^6\right) u(0) > p^3 u(2\omega_2) + \left(1 - p^3\right) u(0).$$

Consider the complete network.

Now, Condition 1 directly implies that the complete network is better than a 1-regular network or a 3-cycle for everyone. Since expected utilities are convex combinations of $u(k\omega_2)$ and $u(0)$, and since u is strictly increasing, decreasing the coefficient of $u(1\omega_2)$ would decrease the expected utility. Since $p < 1$, this means 1-regular network is better than all other networks and vertices with degree 1 since in all other networks, the power of p is greater. Similarly, 3-cycle is better than all other networks and vertices with degree 2. So, no one would deviate to those vertices from the complete network. Then, we should only consider the deviations to the rest of the vertices which are a degree 3 agent in star network, complete network minus an edge, or the other networks or the degree 2 agent in a 2-path. However, deviations to all of these vertices require that some other vertices cut edges to make the deviation possible and it is not individually rational for them to do so.

In star network, vertices with degree 1 would be worse off:

$$p^6 u(3\omega_2) + \left(1 - p^6\right) u(0) > pu(1\omega_2) + (1 - p) u(0) > p^3 u(1\omega_2) + \left(1 - p^3\right) u(0).$$

In the complete network minus an edge, vertices with degree 2 would be worse off:

$$p^6 u(3\omega_2) + \left(1 - p^6\right) u(0) > p^3 u(2\omega_2) + \left(1 - p^3\right) u(0) > p^5 u(2\omega_2) + \left(1 - p^5\right) u(0)$$

In the other network, agent with degree 1 would be worse off:

$$p^6 u(3\omega_2) + \left(1 - p^6\right) u(0) > pu(1\omega_2) + (1 - p) u(0) > p^4 u(1\omega_2) + \left(1 - p^4\right) u(0).$$

Lastly, in 2-path, vertices with degree 2 would be worse off:

$$p^6 u(3\omega_2) + \left(1 - p^6\right) u(0) > pu(1\omega_2) + (1 - p) u(0) > p^2 u(1\omega_2) + \left(1 - p^2\right) u(0).$$

Since, without these agents' contributions it would not be possible for an agent to be in one of the possibly profitable positions, we conclude that there is no profitable deviation for any coalition. So, the complete network is strong-stable.

□

Proposition 3. *Complete network is the unique strong-stable network if and only if p, u and ω_2 satisfies Condition 1.*

Proof. ($\Leftarrow$) Now, suppose the Condition 1 is satisfied. Then, the complete network is strong-stable. Now, we will show that the other networks are not strong-stable.

2-regular network: Any coalition of 3 agents can deviate to a *3-cycle* and strictly increase their expected utilities since they would still have degree 2 each but they would eliminate the extra risk as can be seen below. So, it is not 3-stable.

$$p^3 u(2\omega_2) + \left(1 - p^3\right) u(0) > p^4 u(2\omega_2) + \left(1 - p^4\right) u(0).$$

1-regular network: By the second part of the condition, everyone would prefer the complete network over this one. So, this is not 4-stable.

3-cycle: These networks are not 4-stable since by moving to the *complete network*, everyone strictly improves their expected utility by the first part of the condition.

1-path: Similar to 1-regular networks, these networks are not 4-stable since the *complete network* strictly improves everyone's expected utility in the cycle.

2-path: It is not 2-stable since the vertices with degree 1 can improve their expected utility by cutting their existing edges and creating a new edge and moving to a *1-path.*

$$pu(1\omega_2) + (1 - p) u(0) > p^2 u(1\omega_2) + \left(1 - p^2\right) u(0).$$

3-path: These networks are also not 2-stable since the vertices with degree 1 can improve their expected utility by cutting their existing edges and creating a new edge and moving to a *1-path.*

$$pu\left(1\omega_2\right)+\left(1-p\right)u\left(0\right)>p^3u\left(1\omega_2\right)+\left(1-p^3\right)u\left(0\right).$$

Star network: These networks are <u>not</u> 2-stable since any two vertices with degree 1 can cut their existing edges and create a new edge between each other and improve their expected utilities strictly.

$$pu\left(1\omega_2\right)+\left(1-p\right)u\left(0\right)>p^3u\left(1\omega_2\right)+\left(1-p^3\right)u\left(0\right).$$

Complete network minus an edge: Since the vertices with degree 2 can make a new edge and strictly improve their expected utilities, this network is <u>not</u> 2-stable.

$$p^6u\left(3\omega_2\right)+\left(1-p^6\right)u\left(0\right)>p^3u\left(2\omega_2\right)+\left(1-p^3\right)u\left(0\right)>p^5u\left(2\omega_2\right)+\left(1-p^5\right)u\left(0\right)$$

Other networks: Vertices with degree 1 and 2 can turn these networks into the *complete network* and strictly increase their expected utilities. Hence, these networks are <u>not</u> 3-stable.

$$p^6u\left(3\omega_2\right)+\left(1-p^6\right)u\left(0\right)>p^3u\left(2\omega_2\right)+\left(1-p^3\right)u\left(0\right)>p^4u\left(2\omega_2\right)+\left(1-p^4\right)u\left(0\right)$$

$$p^6u\left(3\omega_2\right)+\left(1-p^6\right)u\left(0\right)>pu\left(1\omega_2\right)+\left(1-p\right)u\left(0\right)>p^4u\left(1\omega_2\right)+\left(1-p^4\right)u\left(0\right).$$

These exhaust all possible networks with 4 agents. Then, there is no other strong-stable network.

($\Rightarrow$) Suppose the complete network is the only strong-stable network. Then, since the complete network is strong-stable, by Proposition 2, Condition 1 is satisfied.

$\square$

3.2.3 A Characterization of Strong-Stable Complete Networks

Lemma 1. *Let there be n agents. In any strong-stable network, all components are complete.*

Proof. Let G be a strong-stable network. Suppose there are two non-adjacent vertices v^1, v^2 connected via a path in G. Denote the degrees of v^1, v^2 by d^1, d^2 respectively and denote the number of edges in their initial component be k. Now, their expected utilities are as follows:

$p^k u\left(d^1\omega_2\right) + \left(1 - p^k\right) u\left(0\right)$ and $p^k u\left(d^2\omega_2\right) + \left(1 - p^k\right) u\left(0\right)$.

Since they are connected, there is a path from $v^1 = v_1, v_2, ..., v_m = v^2$. Consider their expected utilities when v^1 cuts his edge with v_2 and v_2 cuts his edge with v_{m-1} and they make a new edge v^1v^2:

$p^l u\left(d^1\omega_2\right) + \left(1 - p^l\right) u\left(0\right)$ and $p^l u\left(d^2\omega_2\right) + \left(1 - p^l\right) u\left(0\right)$ where $l < k - 1$ since their initial component may not continue to be connected after they cut aforementioned edges so that their new component may include less edges.

Obviously, both of these investors strictly increase their expected utility, contradicting that G is a strong-stable network. Hence, in a strong-stable network, every component is complete.

□

Idea of the above proof is very similar to another proof in Erol and Vohra (2014).

Assumption 3. Let there be n agents.

$$p^{\frac{n(n-1)}{2}}u\left(\left(n-1\right)\omega_2\right)+\left(1-p^{\frac{n(n-1)}{2}}\right)u\left(0\right)>$$
$$p^{\frac{k(k-1)}{2}}u\left(\left(k-1\right)\omega_2\right)+\left(1-p^{k\frac{(k-1)}{2}}\right)u\left(0\right), \text{ for every } k<n.$$

Assumption 3 is the mathematical expression which communicates that the expected utility of an agent in the complete network is higher than that of an agent in any complete component with k agents for $0<k<n$.

Theorem 2. *Let there be n agents. Assumption 3 holds if and only if the complete network is the only strong-stable network.*

Proof. ($\Rightarrow$) Let the Assumption 3 hold.

Consider the complete network G. Assume it is not strong-stable. Everyone has an expected utility of $p^{\frac{n(n-1)}{2}}u\left(\left(n-1\right)\omega_2\right)+\left(1-p^{\frac{n(n-1)}{2}}\right)u\left(0\right)$. Since G is not strong-stable, a coalition can profitably deviate to some other network. This would mean some agent v^1 cuts some edges and decrease his degree to k for some $k<n-1$. Notice that due to Assumption 3, we know that they would not deviate to a network where they form a complete component. If $k=0$, he would be isolated and be worse off.

Let $k\neq 0$. Now, his expected utility will be $p^a u\left(k\omega_2\right)+\left(1-p^a\right)u\left(0\right)$ for some $a<\frac{(n-1)(n-2)}{2}$.

Suppose $a\geq\frac{k(k+1)}{2}$. Then

$p^{\frac{n(n-1)}{2}}u\left((n-1)\,\omega_2\right)+\left(1-p^{\frac{n(n-1)}{2}}\right)u\left(0\right)>$
$p^{\frac{k(k+1)}{2}}u\left(k\omega_2\right)+\left(1-p^{\frac{k(k+1)}{2}}\right)u\left(0\right)\geq p^a u\left(k\omega_2\right)+\left(1-p^a\right)u\left(0\right)$ so that v^1 would be worse off and would not deviate to this new network.

Then, we must have $a<\frac{k(k+1)}{2}$. But then, there must be some other agent v^2 who is also cutting edges and decreasing his degree to some $l<k$ since there are at least $k+1$ vertices in this component and at least one vertex, v^1 has degree higher than maximum average degree in the component. Following the same logic, we must also have $a<\frac{l(l+1)}{2}$. So, there must be another agent v^3 with a degree $m<l$ and we must have $a<\frac{m(m+1)}{2}$. Continuing in this fashion, we see that there must be some agent v^t with degree 0, a contradiction. Hence, no deviation is profitable for all of its members. So, the complete network is strong-stable.

Now, in order to show that the complete network is the only strong-stable network, we will argue that the other networks are not strong-stable. From Lemma 1, we know that any network with an incomplete component is not strong-stable. Now, suppose G is not the complete network but in G, every component is complete. Then, every agent has a lower expected utility than they would have in the complete network by Assumption 3. So, they would deviate to the complete network. Thus, G is not strong-stable. So, no network other than the complete network is strong stable.

($\Leftarrow$) Now, suppose the complete network is the only strong-stable network. Since, it is strong stable, no coalition has a profitable deviation. So, for any size $k<n$, a member of a complete component of size k must be worse than a member of the complete network. Hence, Assumption 3 must be satisfied.

$\square$

3.3 General Characterization of Strong-Stable Networks

In the following theorem, we say that the strong-stable networks consist of complete components of a certain size. However, if the total number of investors is not divisible by this size, naturally, not all the components can be of this size. In this case, the number of vertices which cannot form a component of that size is less than that size and we call them remainders in the following results.

Theorem 3. *Let there be n agents. Let $U(k)$ denote the expected utility of an agent in a complete component with k agents. Suppose κ is the unique maximizer of U over $\{1, 2, ..., n\}$. Then, any strong-stable network consists of complete components of size κ and the remainders*[3].

Proof. Suppose G is a strong stable network and it includes components of size more than κ. We know from Lemma 1 that this component is complete. Then, by our assumption, its members have an expected utility lower than that of members of components of size κ. So, κ investors would deviate and form a component of size κ and all of them would strictly increase their expected utilities. So, it cannot include a component of size higher than κ. So, suppose there are components of sizes less than κ such that sum of their sizes are higher than or equal to κ. Again, by Lemma 1, those components are complete and hence, they yield an expected utility less than it would be in a complete component of size κ. So, κ investors would deviate to a complete component of size κ. So, G cannot include a component of size greater than κ or components of sizes less than κ with their sum of sizes more than or equal to κ. So, if n is divisible by κ, G consists of complete components of size κ. If it is not divisible, then G consists of $\frac{n}{\kappa}$ components of size κ and the remainders would form

[3]What we mean by remainders and what happens to them is explained in great detail in the proof.

components of size λ where λ maximizes U over the set $\{1, 2, ..., \kappa\}$. If, κ is not divisible by λ, the remainder repeats the same procedure recursively. $\square$

Theorem 4. *Let there be n agents. Let $U(k)$ denote the expected utility of an agent in a complete component with k agents. Suppose M is the set of all maximizers of U over $\{1, 2, ..., n\}$. A network consists of complete components of sizes $\kappa_1, \kappa_2, ..., \kappa_m \in M$ and the remainders if and only if it is strong stable.*

Proof. Suppose W.L.O.G. κ_1, κ_2 are the minimum and the maximum in M.

($\Rightarrow$) From above proof, it is clear that in a strong stable network, there cannot be components of sizes not in M unless they consist of the remainders, i.e. the sum of sizes of those components is less than κ_1. So, if G consists of complete components of sizes $\kappa_1, \kappa_2, ..., \kappa_m \in M$ and the remainders, it is strong-stable.

($\Leftarrow$) Now, suppose G is a strong stable network. Now, suppose the sum of sizes of components, whose sizes are not members of M is greater than or equal to κ_1. From above lemma, we know that in a strong stable network, all components are complete. Since, their sizes are not maximizers of $U(x)$, κ_1 investors would be strictly better off by forming a complete component of size κ_1. So, the sum of sizes of components, whose sizes are not members of M is less than κ_1. So, they are the remainders whose strategy was described in the above proof.

$\square$

CHAPTER 4

CONCLUSION

In this paper, we analyzed the equilibrium networks where investors are risk-averse and they are able to buy default insurances. We have one major result which has two corollaries. However, since these corollaries have been independently discovered before the more general result, we presented them in the order of discovery. Our major result is that, the stable network consists of complete components of a certain size, which maximizes expected utilities of its members. If the probability of a good shock to a project is less than 0.5, the strong-stable networks are 1-regular. Moreover, no one buys insurances in these networks. On the other hand, the complete network gives the highest expected utility among all k-regular networks if and only if the unique strong stable network is the complete network. It should also be noted that, in our model, the default insurance does not change the network structure. Moreover, for a wide variety of parameters, it turns out that investors choose not to buy any insurance at the equilibrium. Erol and Vohra (2014) show in their model that the core networks are k-regular. Here, we also have this feature.

In a future study, we wish to move our framework to a dynamic one where the shocks are not identically and independently distributed and the agents learn these over time. We also wish to relax the assumption that an investor can either buy insurance for all of his projects or not buy any insurance at all.

The model introduced here can be used in comparison of different insurance procedures. Many different rules and types of insurance can be implemented, e.g. the insurance might be contingent to the shocks or the insurance can be sold separately for projects. However, it is easy to choose the efficient policy, the policy with the lowest systemic risk or according to any other criteria.

BIBLIOGRAPHY

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. 2015a. *Networks, shocks, and systemic risk.* Retrieved from http://www.nber.org/papers/w20931.

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. 2015b. Systemic risk and stability in financial networks. *The American Economic Review*, *105*(2), 564–608.

Allen, F., and Gale, D. 2000. Financial contagion. *Journal of Political Economy*, *108*(1), 1–33.

Babus, A. 2016. The formation of financial networks. *The RAND Journal of Economics*, *47*(2), 239–272.

Caballero, R. J., and Simsek, A. 2013. Fire sales in a model of complexity. *The Journal of Finance*, *68*(6), 2549–2587.

Cabrales, A., Gale, D., and Gottardi, P. n.d.. Financial contagion in networks. Retrieved from http://econpapers.repec.org/paper/euieuiwps/eco2015_2f01.htm.

Cabrales, A., Gottardi, P., and Vega-Redondo, F. 2014. *Risk-sharing and contagion in networks.* Retrieved from ssrn.com/abstract=2425558.

Chinazzi, M., and Fagiolo, G. 2015. *Systemic risk, contagion, and financial networks: A survey.* Retrieved from http://ssrn.com/abstract=2243504.

Eboli, M. 2013. A flow network analysis of direct balance-sheet contagion in financial networks. *Kiel Institute for the World Economy, Working Paper*(1862).

Elliott, M., Golub, B., and Jackson, M. O. 2014. Financial networks and contagion. *The American Economic Review*, *104*(10), 3115–3153.

Erol, S. 2015. *Network hazard and bailouts.* Retrieved from https://economics.sas.upenn.edu/graduate-program/candidates/selman-erol.

Erol, S., and Vohra, R. 2014. *Network formation and systemic risk, second version.* Retrieved from https://economics.sas.upenn.edu/graduate-program/candidates/selman-erol.

Gai, P., and Kapadia, S. 2010. Contagion in financial networks. In *Proceedings of the royal society of london a: Mathematical, physical and engineering sciences* (p. rspa20090410).

Glasserman, P., and Young, H. P. 2015. How likely is contagion in financial networks? *Journal of Banking & Finance*, *50*, 383–399.

Wilson, R. 1968. The theory of syndicates. *Econometrica: journal of the Econometric Society*, *51*(5), 119–132.

Zawadowski, A. 2013. Entangled financial systems. *Review of Financial Studies*, *26*(5), 1291–1323.