MONETARY - FISCAL JOINT POLICY ANALYSIS: A REGIME SWITCHING DSGE MODEL

A Master's Thesis

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JUNE 2016

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

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Recent literature on the macroeconomic theory examines the importance of the regime switching in macroeconomic dynamics. Using a regime switching structure, this paper studies a baseline New Keynesian model with fiscal block where regimes are defined as active monetary passive fiscal (AMPF) and passive monetary active fiscal (PMAF) regimes. In this paper, I demonstrate that the dynamics of aggregate variables differ markedly when non-linear regime switching solutions are considered. To be specific, output and inflation level are more sensitive to the monetary policy shock under PMAF regime and more sensitive to the technological shocks under AMPF regime.

Keywords: DSGE, Joint Policies, Non-linear Models, Perturbation Method, Regime Switching

ÖZET

PARA VE MALİYE POLİTİKALARI ORTAK ANALİZİ: DEĞİŞKEN REJİMLİ DSGD MODELİ

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Makroekonomi literatü son zamanlarda makroekonomik dinamiklerde rejim değişiminin önemini çalışmaktadır. Bu çalışma, temel yeni keynesyen modeli, modele maliye politikası ve rejimlerin aktif para pasif maliye (APPM) ve pasif para aktif maliye (PPAM) politikaları olarak tanımlandığı bir değişken rejim yapısı ekleyerek çalışmaktadır. Bu tez çalışmasında, doğrusal olmayan rejim değişikliği çözüm metodları kullanıldığında toplam makroekonomik dinamiklerin belirgin bir şekilde farklılaştığı gösterilmektedir. Bilhassa üretim ve enflasyon seviyeleri, PPAM rejimi düşünüldüğünde para politikalarındaki şoklara ve APPM rejimi düşünüldüğünde teknolojik şoklara karşı daha hassastır.

Anahtar Kelimeler: Değisken Rejim, Doğrusal Olmayan Modeller, DSGD, Ortak Politikalar, Pertürbasyon Metodu

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CHAPTER 1

INTRODUCTION

Is regime switching an important source of macroeconomic dynamics? Even though the macroeconomics literature has focused on scenarios in which monetary policy dominates fiscal policy, the seminal contributions of Leeper (1991), Leeper and Sims (1994) and Woodford (1995) demonstrated the importance of the reverse case which is now called "the fiscal theory of the price level". Considering monetary policy dominating and fiscal policy dominating regimes, Leeper (1991) is the first to define the terms of active and passive policies. In his study, "*active monetary*" policy is defined as the regime that fiscal policy is dominated by monetary policy whereas "*active fiscal*" policy is defined as the regime that monetary policy is dominated by the fiscal authority. Recently, along with the New Keynesian framework, Davig and Leeper (2011) studied joint policies with a Markov-switching structure in the New Keynesian model. Davig and Doh (2014) and Bianchi and Melosi (2014) build simple Markov-switching New Keynesian (MSNK) models to examine different questions about monetary-fiscal joint policies.

Regime switching structure in the DSGE models studies the responses of the general equilibrium models under the assumption that there might be different states where all the parameters can be changed from one regime to another. Using the regime switching idea, Farmer et al. (2009), Farmer et al. (2011), Foerster et al. (2014) and Foerster (2013) examined linearised New Keynesian models with an exogenously defined Markov-switching structure. Additionally, Barthelemy and Marx (2011) and Maih (2014) solved the regime switching DSGE models with non-linear solution methods. According to the literature, the main difference between linearised and non-linear solution approach is that using the linearised solution one cannot observe the volatility effect on shocks; consequently, effects of different magnitude of shocks can not be clearly studied. Furthermore, non-linear solution methods allow us to see the time difference of any model when responding to the exogenous shocks.

In this paper, I examined the non-linear responses of a baseline New Keynesian model with a fiscal block where there exists a regime switching structure between active monetary passive fiscal (AMPF) and passive monetary active fiscal (PMAF) regimes. Contribution of this paper is to solve the baseline New Keynesian model non-linearly like Barthelemy and Marx (2011) and Maih (2014) in order to study the macroeconomic dynamics when the regimes, i.e. AMPF and PMAF are considered as defined in Leeper (1991).

As a solution method, I followed the perturbation method used by Judd (1998). Schmitt-Grohe and Uribe (2004) solved their RBC model with the perturbation method up to second order approximation. Up to the works of Foerster (2013) and Maih (2014), models were firstly solved by perturbation method and Markov-switching structure examined at the end. As Foerster et al. (2014)

suggests, in order to get models' actual responses, one should build up the model such that the agents solve their problems knowing that there is a probability that regime can switch.

This paper studies the response of economy under monetary-fiscal joint policies in which agents are aware of the fact that for each period there should be active monetary passive fiscal or passive monetary active fiscal regime is dominating. While solving the model, a non-linear solution approach is applied in order to be able to see the effects on the volatility on the shocks. By using a non-linear solution method, this paper finds that one can capture the time difference between shock responses considering a linearised model's response and that the level of inflation and output are more sensitive to the monetary policy shocks under PMAF regime and more sensitive to the technological shocks under AMPF regime when agents know that regimes can switch from one to other in each period.

CHAPTER 2

MODEL

In this chapter, a closed economy is modelled with the baseline New Keynesian (NK) model of Gali (2008) with an additional fiscal block and time-varying volatility. The use of NK models is very standard in monetary policy analysis. Considering joint policy response, I added a fiscal policy block to the baseline New Keynesian model in order to show the impact of regime switching. Additionally, solution of the model is not linearised so that volatility effect of the shocks can be capured more clearly.

2.1 The Model

The model structure is designed as follow. Firstly, there is the problem of the household. Secondly, the firm's problem is solved. Thirdly, aggregate price dynamics and optimal price settings are clarified. Then, the market clearing conditions are obtained. Finally, the monetary policy and fiscal policy rules are settled.

2.1.1 Households

In this model, it is assumed that there is a representative household which is defined as infinitely-lived. This household gains utility from its consumption and leisure. Therefore, household chooses their consumption, C_t, and their hours of work, N_t, in order to maximize its lifetime utility function:

$$\max E_t \sum_{t=0}^{\infty} \beta^t U_t(C_t, N_t) \tag{2.1}$$

In which the utility function is defined as:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right] \tag{2.2}$$

The household maximizes their utility function by considering the following budget constraint:

$$P_t C_t + Q_t B_t \leq W_t N_t + T_t + B_{t-1} \tag{2.3}$$

Solving equation (2.2) with the budget constraint of equation (2.3) we obtain the following first order conditions (FOC's) for the household:

$$N_t^{\psi} C_t^{\sigma} = \frac{W_t}{P_t} \tag{2.4}$$

$$Q_t = \beta E_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right) \tag{2.5}$$

2.1.2 Firms

In this model, it is assumed that there exist a continuum of firms which is indexed by $i \in [0,1]$. Additionally, we assumed that we have Calvo pricing in the model as a friction. Hence, for each period only 1-θ portion of the firms can set their prices with optimal price setting and θ share of the firms keep their prices unchanged for the next period. Then for each firm i we have:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{2.6}$$

where A_t represents the technology in the model which is evolving according to the following equation.

$$log(\frac{A_t}{\bar{A}}) = \rho_a log(\frac{A_{t-1}}{\bar{A}}) + \sigma_a \epsilon_a \tag{2.7}$$

Note that for each firm, the term P_t^* is defined as the optimal price set for period t. And the term Π_t is the gross inflation rate which is defined as

$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{2.8}$$

2.1.3 Price Dynamics and Settings

The equation that defines the aggregate price changes and its dynamics is:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{2.9}$$

When the firms are maximizing their profit by choosing optimal price setting P_t^* we obtain the following first order condition (FOC).

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k,k} \left(\frac{P_t^*}{P_{t-1}} - \frac{\epsilon}{\epsilon - 1} MC_t \Pi_{t-1,t+k} \right) \right] \tag{2.10}$$

where $\Pi_{t-1,t+k} = P_{t+k}/P_t$ is the gross inflation in k period, MC_t is the real marginal cost and $Q_{t,t+k} = \beta^k$ is obtained. Writing equation (2.12) recursively, we obtain the following first order conditions of the optimal price setting:

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{H_t^1}{H_t^2} \tag{2.11}$$

where

$$H_t^1 = MC_t C_t^{1-\sigma} P_t^{\epsilon} + \beta\theta H_{t+1}^1 \tag{2.12}$$

and

$$H_t^2 = C_t^{1-\sigma} P_t^{\epsilon-1} + \beta\theta H_{t+1}^2 \tag{2.13}$$

where MC_t is solved as

$$MC_t = \frac{W_t}{P_t A_t} \tag{2.14}$$

2.1.4 Market Clearing Conditions

In order to capture market clearing conditions, the output equilibrium is stated first as

$$Y_t = C_t \tag{2.15}$$

The next part is to solve the market clearing in the labor market. The hours of work is evaluating with the following equation.

$$N_t = \int_0^1 N_t(i)di \tag{2.16}$$

Replace equation (2.6) and following demand curve to the equation (2.17)

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{2.17}$$

Then we will have

$$Y_t\Delta_t = N_tA_t \tag{2.18}$$

where the term Δ_t is defined as follow:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{1-2\epsilon} + \theta\Pi_t^{2\epsilon-1}\Delta_{t-1} \tag{2.19}$$

Additionally, we have the following equation of nominal interest rate, $R-t$, from definition.

$$R_tQ_t = 1 \tag{2.20}$$

2.1.5 Policy Rules

In this model, we have two policy tools that have control on aggregate model dynamics. The first policy is the monetary policy and the second policy is the fiscal policy. The regime switching occurs in the policy rules where some parameters of the policy rules can change when $s_t = 1$ or $s_t = 2$. More detailed explanation will be studied in section 2.2.

The monetary policy rule is the rule that gives interest rate taking the output gap, deviation from the inflation targeting and last period's interest rate into account. Therefore, following monetary policy rule is used. Note that the response of the monetary policy rule to the inflation deviation is defined as regime dependent variable.

$$log(\frac{R_t}{\bar{R}}) = \rho_r log(\frac{R_{t-1}}{\bar{R}}) + (1-\rho_r)\phi_y log(\frac{Y_t}{\bar{Y}}) + \phi_\pi(s_t) log(\frac{\Pi_t}{\bar{\Pi}}) + \sigma_r \epsilon_r \quad (2.21)$$

The second policy part comes from the fiscal policy block. The fiscal policy rule has its own constraint, which is the budget constraint of the simple government with no government expenditures. Following two equations defines the fiscal policy block; hence, fiscal rule. Remember that, Gali (2008) has no fiscal block in its baseline New Keynesian model. Therefore, the fiscal policy rule is used as Bianchi et al. (2015) considered.

The term B_t is the debt level, where the term T_t is defined as tax level in the model.

$$B_t = \beta^{(-1)}(B_{t-1} + b(R_{t-1} + \Pi_t + (Y_t - Y_{t-1}) - \Delta y_t^n)) - T_t \quad (2.22)$$

is the debt level constraint of the government. The following equation is the regime dependent fiscal rule.[1]

$$T_t = \delta_b(s_t) B_{t-1} + \delta_y(Y_t - \bar{Y}) + \sigma_t \epsilon_t \quad (2.23)$$

[1]Note that the tax term T_t is actually $T_t - G_t$ where, for the simplicity, the government expenditure term is taken as zero for all time periods t. Therefore, the main purpose of fiscal block is to ensure determinacy in the solution of the regime switching model. More complicated fiscal rules which allows to capture fiscal shock responses is left for future work.

2.1.6 First Order Conditions

Defining the model properties throughout the section 3.1 we have found the first order conditions above. In order to solve the model computationally without defining an extra steady state value, all price level dependent variables are represented in their *real* or *price normalised* forms. Hence, price level P_t is hidden for every price dependent variable. Additionally, equations (2.15) and (2.20) are replaced throughout the first order conditions. Note that, following *nonlinear* equations, (2.24)-(2.36), define the baseline New Keynesian model with fiscal block without any linearisation or approximation.

$$W_t = N_t^{\psi} Y_t^{\sigma} \tag{2.24}$$

$$(\frac{Y_t}{Y_{t+1}})^{\sigma} = \frac{\Pi_{t+1}}{\beta R_t} \tag{2.25}$$

$$Y_t = \frac{A_t N_t}{\Delta_t} \tag{2.26}$$

$$P_t^* = \frac{\epsilon H_t^1}{(\epsilon - 1) H_t^2} \tag{2.27}$$

$$H_t^1 = Y_t^{(1-\sigma)} MC_t + \theta\beta\Pi_{t+1}^{\epsilon} H_{t+1}^1 \tag{2.28}$$

$$H_t^2 = Y_t^{(1-\sigma)} + \theta\beta\Pi_{t+1}^{\epsilon-1} H_{t+1}^2 \tag{2.29}$$

$$(1-\theta) P_t^{*(1-\epsilon)} = \theta\Pi_t^{(\epsilon-1)} \tag{2.30}$$

$$\Delta_t = (1-\theta) P_t^{*(-\epsilon)} + \theta\Pi_t^{(\epsilon)} \Delta_{t-1} \tag{2.31}$$

$$MC_t = \frac{W_t N_t}{Y_t} \tag{2.32}$$

$$B_t = \beta^{(-1)}(B_{t-1} + b(R_{t-1} + \Pi_t + (Y_t - Y_{t-1}) - \Delta y_t^n)) - T_t \tag{2.33}$$

$$log(\frac{R_t}{\bar{R}}) = (1-\rho_r)\phi_y log(\frac{Y_t}{\bar{Y}}) + \phi_\pi(s_t) log(\frac{\Pi_t}{\bar{\Pi}}) + \rho_r log(\frac{R_{t-1}}{\bar{R}}) + \sigma_r \epsilon_r \quad (2.34)$$

$$T_t = \delta_b(s_t) B_{t-1} + \delta_y(Y_t - \bar{Y}) + \sigma_t \epsilon_t \quad (2.35)$$

$$log(\frac{A_t}{\bar{A}}) = \rho_a log(\frac{A_{t-1}}{\bar{A}}) + \sigma_a \epsilon_a \quad (2.36)$$

2.2 Regime Switching Structure

The idea of the regime switching is to change the responses of the model by changing the variables, consequently the FOC's that defines the model. Therefore, regime switching can be considered as changing parameter values in which parameter values are defined for each regimes distinctly. Considering model specific regimes for this model, firstly we will define four different regimes that can be considered in regime switching literature. Terms of *active* and *passive* policy follow the literature started with Leeper (1991).

The monetary rule in which the response of the interest rate to the deviation of inflation from inflation target is more than one-for-one is called as *active monetary rule* as Taylor (1993) stated. Similarly, if the response of the interest rate to the deviation from the inflation target is less than one-for-one; then, we can call this monetary rule as *passive monetary rule.*

In the fiscal block, if the tax ratio is more independent than the debt level of government, we call this type of fiscal rule as *active fiscal rule*. Similarly, if the tax level is more dependent to the debt level of government, than we call this type of fiscal rule as *passive fiscal rule.*

Considering four types of regime defined above, we can talk about four different regimes for the baseline New Keynesian model with fiscal block.

2.2.1 Active Monetary Passive Fiscal Regime (AMPF)

In this regime, monetary policy rule responses to the deviation from the inflation target more aggressively. In the same time, fiscal rule tries to have a lower debt level; hence, the influence of debt level on tax ratio will be higher. For any time t, the state variable s_t =1 for this regime. Considering the model specific parameters:

- $\rho_t(1) \geq 1$
- $\delta_t(1) \geq \beta^{-1} - 1$

will be called as AMPF regime.

2.2.2 Passive Monetary Active Fiscal Regime (PMAF)

In this regime, monetary policy rule responses to the deviation from the inflation target less aggressively. In the same time, fiscal rule does not try to have a lower debt level; hence, the influence of debt level on tax ratio will be smaller. For any time t, the state variable s_t =2 for this regime. Considering the model specific parameters:

- $\rho_t(2) < 1$
- $\delta_t(2) < \beta^{-1} - 1$

will be called as PMAF regime.

2.2.3 Active Monetary Active Fiscal Regime (AMAF)

In this regime, monetary policy rule responses to the deviation from the inflation target more aggressively. In the same time, fiscal rule does not try to have a lower debt level; hence, the influence of debt level on tax ratio will be smaller. For any time t, the state variable s_t =3 for this regime. Considering the model specific parameters:

- $\rho_t(3) \geq 1$
- $\delta_t(3) < \beta^{-1} - 1$

will be called as AMAF regime.

2.2.4 Passive Monetary Passive Fiscal Regime (PMPF)

In this regime, monetary policy rule responses to the deviation from the inflation target less aggressively. In the same time, fiscal rule tries to have a lower debt level; hence, the influence of debt level on tax ratio will be smaller. For any time t, the state variable s_t =4 for this regime. Considering the model specific parameters:

- $\rho_t(4) < 1$
- $\delta_t(4) \geq \beta^{-1} - 1$

will be called as PMPF regime.

2.2.5 Existence And Determinacy of the Solutions

As Farmer et al. (2009) and Leeper (1991) discussed, not all the regimes provide unique solutions. Intuitively, it is easy to understand that there will not be any solution when both of the policy makers follow active policies, meaning that, both fiscal and monetary policy rules concern about their objectives only and not other dynamics. Therefore, there will be no solution for AMAF regime. As further discussions, there will be determinacy in solution when regimes are AMPF and PMAF; whereas, PMPF regime will provide indeterminacy in the solutions. Considering the regime that none of the policy makers acting actively, there can be infinitely many solution to the model.

Table 2.1: Existence and Determinacy of the Solution

	Active Fiscal Regime (AF)	Passive Fiscal Regime (PF)
Active Monetary Regime (AM)	No Solution	Determinacy
Passive Monetary Regime (PM)	Determinacy	Indeterminacy

Table 2.1 summarizes the determinacy of the solution for each regime. Hence, in our model only the AMPF and PMAF regimes will be used. Note that, for further cases PMPF regime can be also considered. Therefore:

- $s_t \in \{1{,}2\} \; \forall \; t$

Regime switching is an exogenous process that for each period $t+1$ there is a probability defined so that the regime at period t switches. Therefore, for our model we have a $2x2$ probability transition matrix. Using the same probability transition matrix of Bianchi et al. (2014), if the regime is defined as AMPF, which is $s_t = 1$ for time period t, it will stay in the same regime with probability $p_1 = 0.75$ and it will switch to the PMAF, which is $s_{t+1} = 2$, with probability $1 - (p_1) = 0.25$ in the period $t+1$. If the regime is stated as PMAF, which is $s_t = 2$ for time period t, it will stay in the same regime with probability $p_1 = 0.5$ and it will switch to the AMPF, which is $s_{t+1} = 1$, with probability $1 - (p_1) = 0.5$ in the time period $t+1$.

Considering those, following state transition matrix, H^t, is defined so that in each period t the regime and consequently the policy rule parameters can switch from one value to another. Note that, this transition probability matrix is not determined by any endogenous process.

$$H^t = \begin{bmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{bmatrix}$$

Main purpose of this thesis is to solve the baseline New Keynesian model with fiscal block by taking the matrix H^t into account while setting the problem. Even though this transition matrix is defined exogenously, that does not mean that agents do not know about the regime switching process. Instead, each agent is aware that going from period t to $t+1$ there is a possibility that policy maker that rules the regime changes. Hence, their decisions will be shaped considering the exogenously given transition matrix.[2]

[2] Calibrating the probability values of the transition matrix H^t with Turkish data is another enhancement of this paper as a future work.

CHAPTER 3

METHODOLOGY

In order to solve the model discussed in the previous chapter with an exogenously regime switching mechanism defined, perturbation method is used as Foerster et al. (2014) and Maih (2014) suggested. Recall that their algorithm of perturbation method allows one to find non-linear responses around steady state.

3.1 Perturbation Method

For a given transition probability matrix H^t and discrete regimes indexed as $s_t \in \{1, 2, ..., n_s\}$ in which the probabilities in the transition matrix give the possibility that state switches or stays same while time period is going from t to $t+1$.

Assuming that all regime-dependent parameters are contained in the vector $\theta(s_t)$, all the exogenous shock terms are contained in the vector ϵ_t, x_t is the vector that contains all predetermined variables where $x_t \in R^{n_x}$ and y_t is the vector that contains all non-predetermined variables where $y_t \in R^{n_y}$; the equilibrium conditions, i.e. first order conditions, that define the model is represented in the general form as:

$$E_t f(y_{t+1}, y_t, x_t, x_{t-1}, \epsilon_{t+1}, \epsilon_t, \theta(s_{t+1}), \theta(s_t)) = 0 \tag{3.1}$$

Note that the function f is assumed to be infinitely differentiable with respect to all of the vectors and integration and differentiation of f are exchangeable. Defining the steady state values of non-predetermined variables as y_{ss} and the steady state values of predetermined variables as x_{ss}, we need to have following equation holds at the steady state.

$$E_t f(y_{ss}, y_{ss}, x_{ss}, x_{ss}, 0_{n_\epsilon}, 0_{n_\epsilon}, \theta_{ergodic}, \theta_{ergodic}) = 0 \tag{3.2}$$

where $\theta_{ergodic}$ is the ergodic mean of $\theta(s_t)$ considering the probability transition matrix H^t.

Perturbation solution is the method that allows us to find solution around steady state. therefore, it is a local solution approach and not a global solution method. Following the same representation with Foerster et al. (2014) it is stated that:

$$y_t = g(x_{t-1}, \epsilon_t) \tag{3.3}$$

$$x_t = h(x_{t-1}, \epsilon_t) \tag{3.4}$$

where their steady state vectors are in the form of:

$$y_{ss} = g(x_{ss}, 0_{n_\epsilon}) \tag{3.5}$$

$$x_{ss} = h(x_{ss}, 0_{n_\epsilon}) \tag{3.6}$$

As distinct from Blanchard and Kahn (1980)'s definition for state and control variables, forward looking variables are considered as non-predetermined while parameters which appears at both period t and $t-1$ are considered as predetermined variables. From the definition, the parameters appears as both forward looking and predetermined are classified as predetermined variables. Hence, we have:

- Non-predetermined variables: Those are variables that appears in model at time periods t and $t+1$.
- Predetermined variables: Those are variables that appears in model at time periods t and $t-1$. Any jump variables and any variables that are both forward looking and backward looking are also considered as predetermined variables.

3.2 Order of Approximation

Note that the first order conditions defined (2.24) - (2.36) are non-linear equations. Considering the perturbation method, an n^{th} order approximation will be used. As stated earlier, main purpose of this thesis to cover non-linear responses of the regime switching baseline New Keynesian model with fiscal block. Therefore, using first order approximation, one cannot observe the non-linear responses of the model to any source of exogenous (or endogenous) shocks.

Using second order approximation for the solution of the model, the effects of different magnitudes of shocks in the model can be observed. Hence, volatility parameters will not be ignored while we are having the impulse responses. Remember that, shock parameters of equations (2.34) - (2.36) have σ terms along with ϵ terms. Having the σ terms in the monetary policy rule, fiscal policy rule and technology process we can observe different magnitude of shocks for each period due to the variance in the shocks. Therefore, a second order approximation is more suitable to capture the non-linear behaviour of the model than the first order approximation. [1]

3.3 Model Solution

For the model defined at section 2, the vectors of non-predetermined variables y_t, the vector of predetermined variables x_t, the parameters of exogenously defined shocks ϵ_t and the regime dependent variables $\theta(s_t)$ are defined as follow.

$$\epsilon_t = \begin{bmatrix} \sigma_r \epsilon_r & \sigma_t \epsilon_t & \sigma_a \epsilon_a \end{bmatrix}$$

$$\theta(s_t) = \begin{bmatrix} \phi_\pi(s_t) & \delta_b(s_t) \end{bmatrix}$$

$$y_t = \begin{bmatrix} \Pi_t & H_t^1 & H_t^2 \end{bmatrix}$$

$$x_t = \begin{bmatrix} W_t & N_t & Y_t & P_t^* & \Delta_t & MC_t & B_t & R_t & T_t & A_t \end{bmatrix}$$

Replacing the first order conditions defined in equations (2.24) - (2.36) into the equation (4.1) we have following vector.

[1] In order to find second order approximated solutions for the model, RISE toolbox of Maih (2014) is used

$$0 = E_t f(y_{t+1}, y_t, x_t, x_{t-1}, \epsilon_{t+1}, \epsilon_t, \theta(s_{t+1}), \theta(s_t)) =$$

$$E_t \begin{bmatrix} W_t = N_t^{\psi} Y_t^{\sigma} \\ (\frac{Y_t}{Y_{t+1}})^{\sigma} = \frac{\Pi_{t+1}}{\beta R_t} \\ Y_t = \frac{A_t N_t}{\Delta_t} \\ P_t^* = \frac{\epsilon H_t^1}{(\epsilon-1) H_t^2} \\ H_t^1 = Y_t^{(1-\sigma)} MC_t + \theta \beta \Pi_{t+1}^{\epsilon} H_{t+1}^1 \\ H_t^2 = Y_t^{(1-\sigma)} + \theta \beta \Pi_{t+1}^{\epsilon-1} H_{t+1}^2 \\ (1-\theta) P_t^{*(1-\epsilon)} = \theta \Pi_t^{(\epsilon-1)} \\ \Delta_t = (1-\theta) P_t^{*(-\epsilon)} + \theta \Pi_t^{(\epsilon)} \Delta_{t-1} \\ MC_t = \frac{W_t N_t}{Y_t} \\ B_t = \beta^{(-1)}(B_{t-1} + b(R_{t-1} + \Pi_t + (Y_t - Y_{t-1}) - \Delta y_t^n)) - T_t \\ log(\frac{R_t}{R}) = (1-\rho_r)\phi_y log(\frac{Y_t}{Y}) + \phi_\pi(s_t) log(\frac{\Pi_t}{\Pi}) + \rho_r log(\frac{R_{t-1}}{R}) + \sigma_r \epsilon_r \\ T_t = \delta_b(s_t) B_{t-1} + \delta_y (Y_t - \bar{Y}) + \sigma_t \epsilon_t \\ log(\frac{A_t}{\bar{A}}) = \rho_a log(\frac{A_{t-1}}{\bar{A}}) + \sigma_a \epsilon_a \end{bmatrix}$$

3.4 Parametrization

In order to solve the model with Maih (2014) solution algorithm, parameter values have to be given. Due to the fact that calibration is not in the scope of this thesis, parameter values are taken from the paper of Bianchi and Melosi (2014). All parameter values are summarized in table 3.1.

Table 3.1: Parameter Values for Regime Switching Baseline New Keynesian Model with Fiscal Block

Parameter	Value
β	0.99
σ	1
ψ	3
ϵ	6
θ	0.75
ρ_r	0.8
ϕ_y	0.5
δ_y	0.5
ρ_a	0.8
σ_r	0.01
σ_a	0.01
σ_ϵ	0.01
$\phi_\pi(1)$	2
$\phi_\pi(2)$	0.9
$\delta_b(1)$	0.03
$\delta_b(2)$	0.005

Note that the regime $s_t = 1$, is the AMPF regime in which the response of interest rate to the deviation from the inflation target is more than one-for-one where the tax level is highly dependent on the debt level. On the other hand, for the regime $s_t = 2$, which is the PMAF regime, where the response of interest rate to the deviation from the inflation target is less than one-for-one where the tax level is less dependent on the debt level; therefore, fiscal policy is active.[2]

[2]In this paper, the parameter values of Bianchi and Melosi (2014) is used in order to make the results comparable with the results of regime switching models in the recent literature.

CHAPTER 4

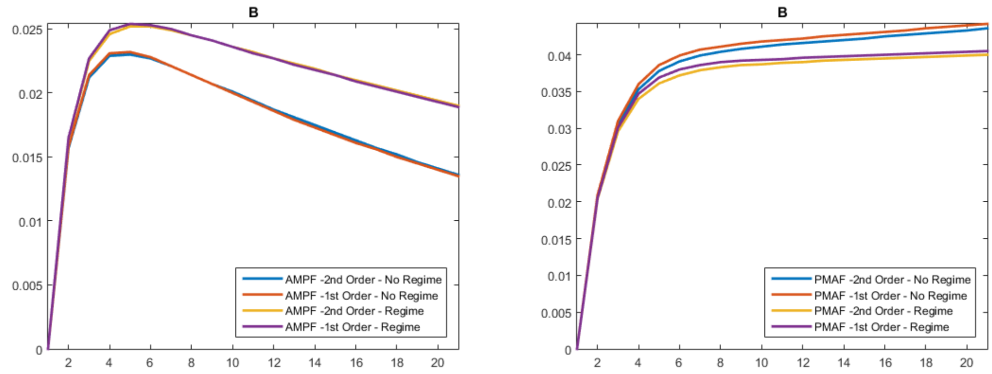
RESULTS

The model summarized by equations (2.24) - (2.36) is solved via perturbation method approach developed by Maih (2014).

In order to clarify the non-linear response of the model, both first order and second order approximated impulse responses are obtained and compared by using exogenous shocks whose magnitudes are one standard deviation. Observing the impulse responses to the monetary policy shock and technology shock one can see that second order approximated results carry significant information for different cases.[1]

For each impulse response, agents are aware of the fact that regimes can switch between ant two periods t and $t+1$. Considering the probabilities, we check the responses of the regimes while regimes did not change during the response to any exogenous shock. Hence, even though regimes kept fixed for a short-run, agents solve their problems by taking the probability into account while solving their problems.

[1]For this baseline New Keynesian model, fiscal policy shock has no effect on the model dynamics due to explicit assumption discussed on the footnote 1 of section 2. Recall that, assuming net transfer term as T_t with zero government expenditure, any shock in the government spending will have no effect on the constraint of the household.



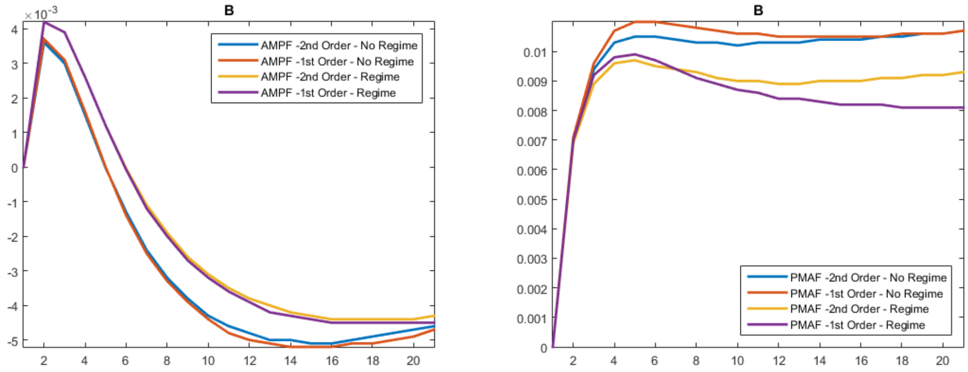
(a) AMPF (b) PMAF

Figure 4.1: Debt Level, B_t, Response to the Monetary Policy Shock Under Fixed Regime and Regime Switching Models

4.1 Impact of the Regime Switching Structure and Non-linear Solution

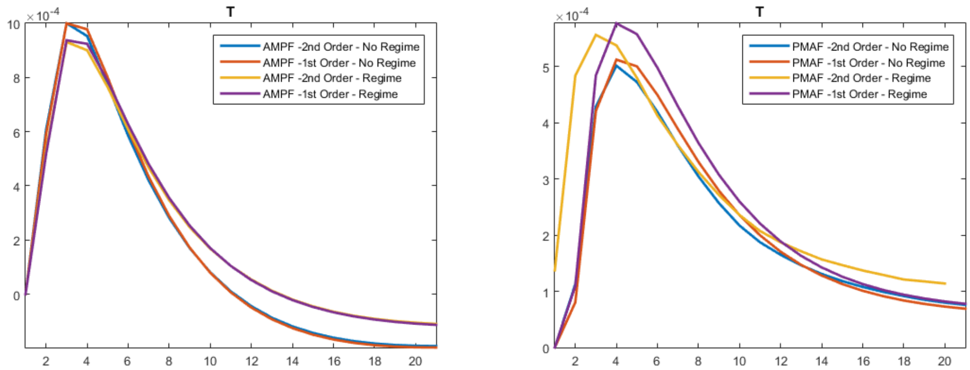
In this section model responses are compared with those which has no regime switching mechanism introduced. For this purpose, responses of the model with regime switching are compared with the fixed regime model responses of the same FOC's. In order to show the effect of the order of solution both first order and second order approximated model responses are given. Figure 4.1 clearly shows that when an exogenous monetary shock applied to the model; the debt level, B_t, response separates significantly when a regime switching structure is applied. Note that, regime switching responses are closer to each other than non-regime switching responses.

Considering the figure 4.2, it is obvious that regime switching model has significantly different response under technology shock which is coherent with the monetary policy shock case. Also note that, second order approximation results give a slower response in recovery period compared to the first order



(a) AMPF (b) PMAF

Figure 4.2: Debt Level, B_t, Response to the Technology Shock Under Fixed Regime and Regime Switching Models

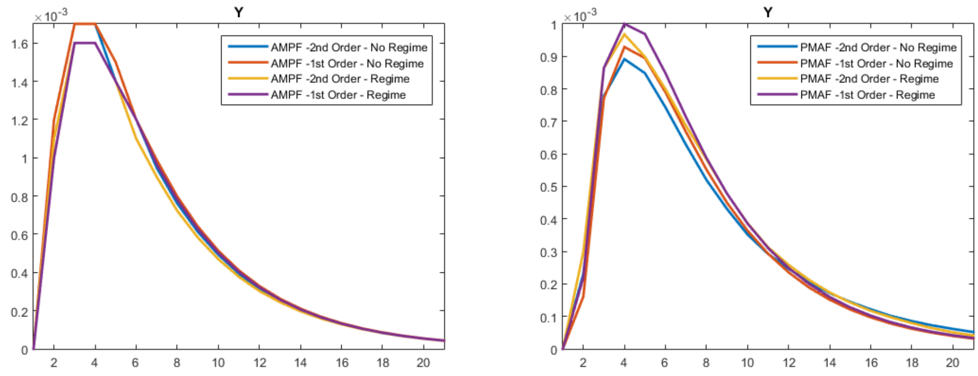


(a) AMPF (b) PMAF

Figure 4.3: Tax Level, T_t, Response to the Technology Shock Under Fixed Regime and Regime Switching Models

responses under PMAF.

Responses of the output, Y_t, and tax level, T_t are also markedly different when a regime switching structure is defined. Figure 4.3 and 4.4 displays that difference for the exogenously defined technology shock. Checking the first order and second order approximated model responses, it is obvious that one can not capture the genuine response timing of the model by using a linearised model.



(a) AMPF (b) PMAF

Figure 4.4: Output, Y_t, Response to the Technology Shock Under Fixed Regime and Regime Switching Models

4.2 Impulse Responses of the Regime Switching New Keynesian Model

This section illustrates the responses of the model under different shocks when a regime switching structure is defined. As explained earlier, exogenously defined monetary policy and technology shocks are examined with both linearised and non-linear solutions in order to observe the effect of non-linear solutions more clearly.

4.2.1 Monetary Policy Shock Responses of the Model

Applying exogenous shock to the monetary policy rule with a magnitude of one standard deviation, we observe the following responses to the model. Note that, following impulse response figures have both first order and second order approximated results in order to understand the reasoning behind second order approximation. Comparing the first order and second order approximated model responses, we observe only small differences between different order of approximations in the monetary policy shock responses. The future work will consider a more elaborate model to see whether this is due to the simplicity of the current model.

Observing the figure 4.5, we can see that PMAF regime is more sensitive to

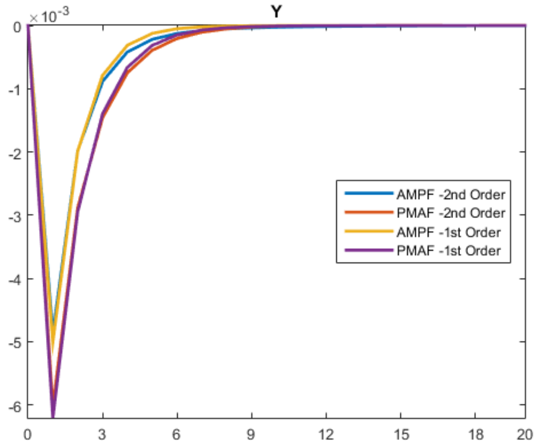


Figure 4.5: Output, Y_t, Response to the Monetary Policy Shock

monetary policy shock than AMPF regime. This response is quite intuitive considering that if the monetary policy is more *actively* practising then the output deviation for any monetary policy shock can be observed in a lower magnitude than the case with *passively* monetary policy practise.

Similar with output responses, the response of the interest rate R_t to the monetary policy shock is more sensitive when the regime is in PMAF which can be seen in the figure 4.6, $s_t = 2$; hence, interest rate deviates from the steady state in a lower magnitude in AMPF regime than PMAF regime.

Apparent from the figure 4.7 that the debt level evolves into higher values in the PMAF regime due to the fact that in *active* fiscal policy, policy maker does not care about debt level that much comparing with *passive* fiscal regime. Hence, the PMAF regime debt level response is more sensitive to the monetary

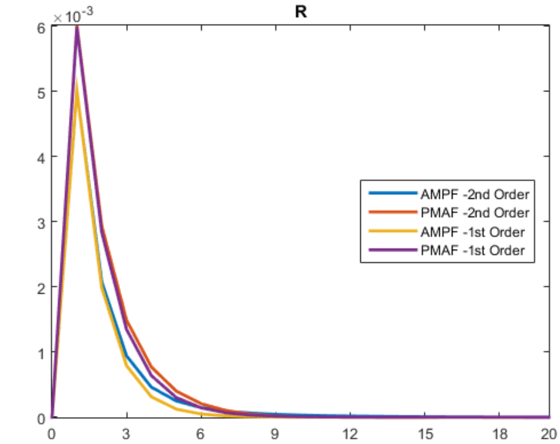


Figure 4.6: Interest Rate, R_t, Response to the Monetary Policy Shock

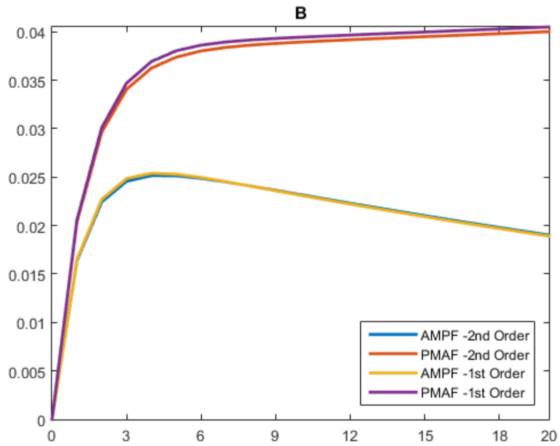


Figure 4.7: Debt Level, B_t, Response to the Monetary Policy Shock

policy shock comparing with AMPF.

Clearly, the figure 4.8 states that the tax level, T_t deviates more in the PMAF regime than AMPF regime. Additionally, due to the fact that in the *active* fiscal policy, policy maker does not care about debt level B_t that much, tax level goes to steady state more slowly.

The inflation rate, Π_t responses are very similar with the output responses when we have monetary policy shock exogenously given as it can be clearly seen from the figure 4.9.

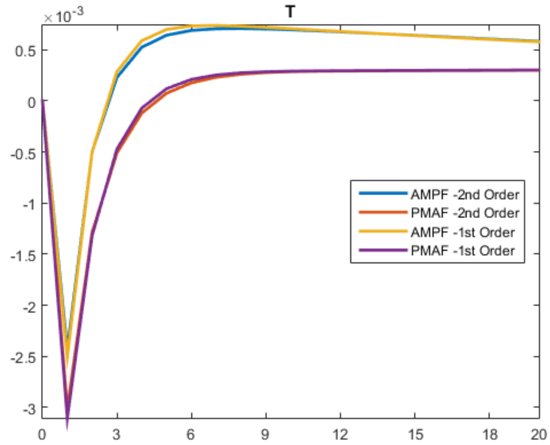


Figure 4.8: Tax Level, T_t, Response to the Monetary Policy Shock

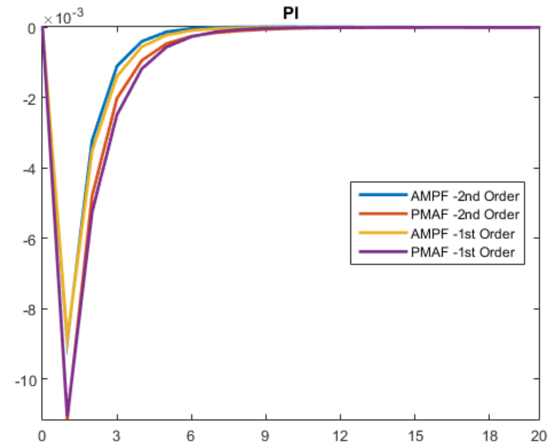
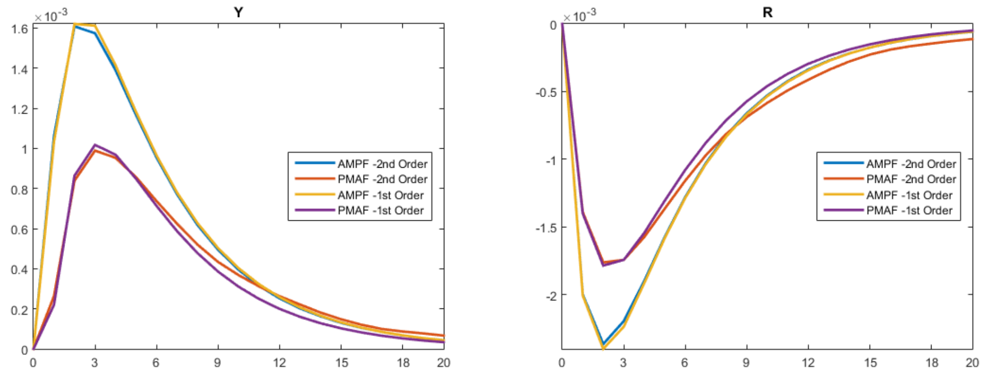


Figure 4.9: Inflation Rate, Π_t, Response to the Monetary Policy Shock

4.2.2 Technology Shock Responses of the Model

Applying an exogenously defined technology shock to the model, we observe the following impulse responses. Note that, comparing with the monetary policy shocks, effect of second order approximation is studied more clearly with the technology shock.

The figure 4.10a shows that AMPF acts more sensitive to the technology shock than PMAF regime. we can clearly see that second order approximated results acts more slowly when going back to steady state under PMAF regime. Similar with output response, the interest rate R_t is more sensitive under



(a) Output, Y_t, Response to the Technology Shock

(b) Interest Rate, R_t, Response to the Technology Shock

Figure 4.10: Output, Y_t, and Interest Rate, R_t, Responses to the Technology Shock

AMPF regime which can be seen from the figure 4.10b. Additionally, interest rate goes back to its steady state more slowly under PMAF regime when we observe the second order approximated results rather than the first order one.

Observing the figure 4.11, debt level goes higher values quickly in the PMAF

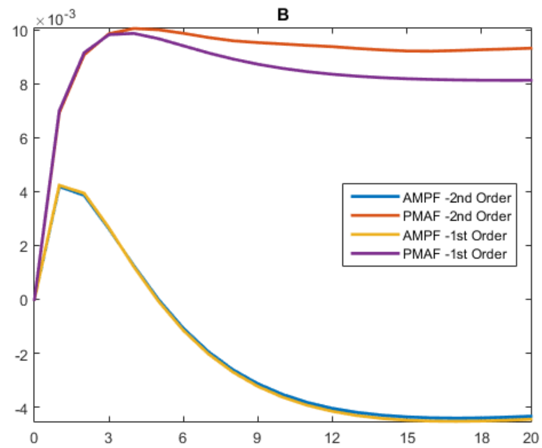


Figure 4.11: Debt Level, B_t, Response to the Technology Shock

regime comparing with AMPF regime. Most important point in that impulse response is that, second order approximation gives a slower behaviour to reach steady state under PMAF which is consistent with other variables such as output and interest rate.

Different from tax level, T_t, response under monetary policy shock, tax level

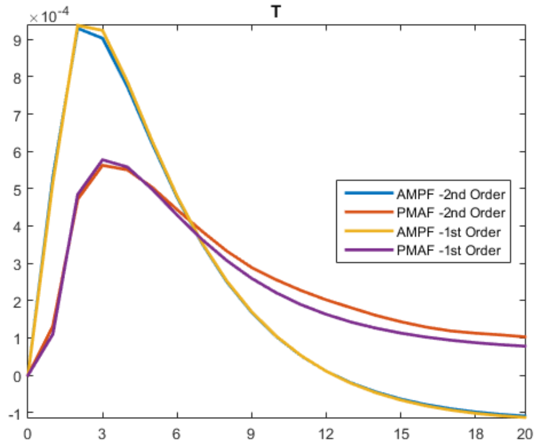


Figure 4.12: Tax Level, T_t, Response to the Technology Shock

is more sensitive under AMPF regime and it moves quickly to the steady state which can be clearly observed in the figure 4.12. However, we observe a slower response under PMAF regime where higher order approximations give even slower responses.

The inflation rate, Π_t, response is very similar to the one with monetary policy

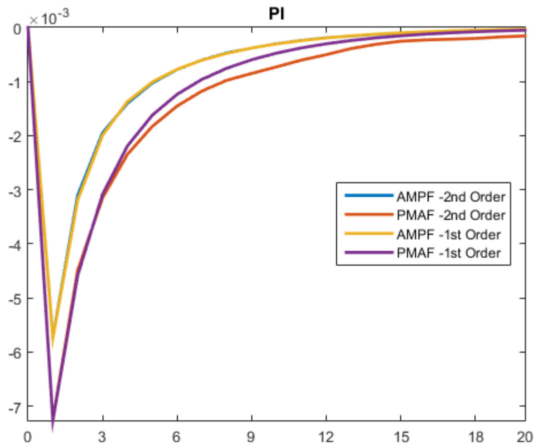


Figure 4.13: Inflation Rate, Π_t, Response to the Technology Shock

shock. As the figure 4.13 shows, in the technology shock, we observe the slower behaviour of impulse response function under PMAF which is consistent with other variables.

CHAPTER 5

CONCLUSION

In this paper, a baseline New Keynesian model with fiscal block is structured in order to study the behaviour of the economy when agents are aware that regimes can go from AMPF to PMAF or vice versa. The model solved around steady state with a local solution methodology which is perturbation method. The main contribution of this paper is to set regime switching structure before solving the model. In other words, agents solve their problems by taking the regime switching mechanism into account. Even if the impulse responses are observed with the assumption that regimes kept fixed during the short run, the solution to the model always captured the probability that regime can switch next period.

Considering the non-linear structure of the model, it is suggested that a minimum second order approximation in the perturbation method should be used in order to obtain more accurate solutions and cover the time difference between recovery to steady state. Note that, both first order and second order approximated results give the dynamics of regime switching structure. However, comparing the second order approximation and the first order approximation clearly stated that non-linear behaviour of the model is acting more slowly while going back to steady state. Hence, using a first order linearisation as simple IS and Phillips curve, we can not capture that speed of recovery correctly.

The model suggested that output, interest rate and inflation rate are more sensitive to the monetary policy shocks under PMAF regime. Pacifying the monetary policy rule, it is intuitively correct to assume a fragile interest rate and inflation along with output under PMAF. On the other hand, considering the structural shocks as technology shock, economy acts more sensitive under AMPF.

Considering the non-linear behaviour of the model with the feature that agents aware of the possibility of regime switching; this paper studied the response of the economy under passive monetary active fiscal (PMAF) and active monetary passive fiscal (AMPF) regimes. As a future work enhancement, higher order solutions with the parameter calibration with Turkish data is aimed. Another extend is to solve the model endogenously defined transition matrix probabilities.

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