REGULATION VIA SUPPLY FUNCTION EQUILIBRIA

A Master's Thesis

by

TURAL HUSEYNOV

THE DEPARTMENT OF
ECONOMICS
BILKENT UNIVERSITY
ANKARA
September, 2006

REGULATION VIA SUPPLY FUNCTION EQUILIBRIA

The Institute of Economics and Social Sciences
of
Bilkent University

by

TURAL HÜSEYNOV

In Partial Fulfilment of the Requirements for the Degree of
MASTER OF ARTS

in

THE DEPARTMENT OF
ECONOMICS
BİLKENT UNIVERSITY
ANKARA

September, 2006

I certify that I have read this thesis and have found that it is fully adequate,in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Prof. Dr. Semih Koray
Supervisor

I certify that I have read this thesis and have found that it is fully adequate,in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assist. Prof. Dr. Tarık Kara
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate,in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assoc. Prof. Dr. Azer Kerimov
Examining Committee Member

Approval of the Institute of Economics and Social Sciences:

Prof. Dr. Erdal Erel
Director

ABSTRACT

REGULATION VIA SUPPLY FUNCTION EQUILIBRIA

Tural Hüseynov
M.A. in Economics
Supervisor: Prof. Dr. Semih Koray
September, 2006

In this study we attempt to analyze the delegation problem via supply function equilibria in the intermanagerial game. We obtain that in a duopoly where firms play a supply function game an infinite number of equilibrium outcomes exists. Due to this fact one should first make a selection among these equilibrium outcomes in order to apply to delegation games. An artificial selection of an a priori intended outcome leads us to designing mechanisms. We demonstrate several regulatory mechanisms. Moreover, we show that the socially ε-best outcome can be implemented via supply function equilibria as well.

Keywords: Regulation, delegation, supply function equilibrium, social welfare, duopoly.

ÖZET

ARZ FONKSİYONU DENGELERİ İLE REGÜLASYON

Tural Hüseynov
Ekonomi, Yüksek Lisans
Tez Yöneticsi: Prof. Dr. Semih Koray
Eylül, 2006

Bu çalışmada delegasyon problemi yöneticilerarası oyunda arz fonksiyonu dengeleri ile ele alınmaktadır. Arz fonksiyonu oyununun oynandığı bir düopolde sonsuz denge çıktısının olduğu elde edilmektedir. Bundan dolayı delegasyon oyunlarına uygulamadan önce bu denge çıktıları arasından bir seçim yapmak gerekmektedir. Önceden istenilen bir çıktının suni olarak seçilmesi mekanizma tasarımı problemine götürmektedir. Bazı önceden belirlenmiş çıktılara yol açan regülasyon mekanizmaları tasarlanmıştır. Ayrıca toplumsal açından ε-en iyi çıktının arz fonksiyonu dengeleri ile elde edilebileceği gösterilmiştir.

Anahtar sözcükler: Regülasyon, delegasyon, arz fonksiyonu dengesi, sosyal refah, düopol.

Acknowledgement

I would like to express my deepest gratitude to Prof. Semih Koray for his invaluable guidance and positive personal effects on me in the last three years of my life.

I would also like to thank Prof. Tarık Kara And Prof. Azer Kerimov for reading and evaluating this thesis.

I am grateful to my friend Mehmet Karakaya for his useful comments, moral support and close friendship in the last couple of months while I was working on the preparation of this thesis. Without his help I would never be able to complete this study.

I especially would like to thank my classmates at Bilkent. Their friendship and continuous support was extremely important for my motivation.

Finally, I owe special thanks to my family members for their love and patience.

Contents

List of Figures

Chapter 1

Introduction

Standard economic theory literature treats economic agents with the sole goal of utility maximization. A certain utility maximization problem identified by a particular economic agent may seem quite reasonable at a first glance. However, in some economic environments, where strategic interdependence across agents is present, this may not always be the case. The Delegation Problem, first analyzed in the literature by Vickers (1987) deals with this situation for firms competing in the market strategically. In theory, firms' behavior is interpreted as to maximize their profits. Vickers (1987) gives a simple but very elegant example to demonstrate that a firm setting an objective function other than its profits may gain more profits than simply acting as a profit maximizer. We borrow his example of entry deterrence to motivate the subject and give an idea of delegation.

Suppose firm A is to decide whether or not to enter a market which currently is monopolized by firm B. If A enters the market, then B must decide whether to respond aggressively or in an accommodating fashion. Entry for A is profitable if and only if B does not fight. Faced with entry it is more profitable for B to accommodate rather than to fight, but B's profits are still greater if there is no entry at all. Consider, now, how the game would end up in each of the following two cases (which are assumed to be common knowledge).

i) B's managers are only concerned to maximize profits

ii)B's managers are principally concerned to maintain the firm's dominance over the market, while profit considerations are secondary.

In the first case of usual profit or utility maximization, clearly the game is resolved by backward induction, yielding that entry will take place and will be accommodated. If B's managers were to fight entry, they would be failing to maximize profits in the prevailing circumstances. Anticipating this fact, A will enter the market. In case ii), however, entry will be deterred, since A knows that B's managers would fight the entry. Thus the known presence of managers of type ii) whose objective is not principally maximize profits has the effect of earning greater profits for B than would be the case if B's managers had profit-maximization as their objective.

"In games it is almost always the case that u-maximizers do not maximize u" continues Vickers.

A number of authors, such as Koray and Sertel(KS)(1986,1988,1989a), Fershtman and Judd (1987) and Sklivas (1987) (FJS) analyzed the delegation problem for symmetric linear oligopolies with a linear cost function for one-owner one-manager case. The common framework in all these works is a two-stage game, where at the first stage the owners simultaneously delegate some maximands (objective functions) to their managers, and at the second stage of the game, managers choose either quantity or price, while accruing profits go to the owners. The managers are rewarded according to the incentives chosen by their owners. In all these studies managers should be regarded as "robots", as they do nothing other than playing a (Nash) equilibrium strategy for the given solution concept, be it Cournot (quantity competition), Bertrand (price competition) or some other. Cournot competition in the intermanagerial game (at the industry floor) in this setting is that firms now do not maximize directly their true profits, and the outcome materialized in the market is closer to perfect competition than the ordinary Cournot equilibrium outcome is. Bertrand competition at industry floor, on the other hand, leads to an outcome which is closer to collusive behavior than does ordinary Bertrand competition.

Since we will be using the results of FJS throughout our analysis we first summarize them now.

1.1 Summary of Sklivas (1987)

Consider a linear duopoly where each firm has one-owner and one-manager. The inverse demand and cost functions are known to both firms with certainty and are given by $D^{-1}(q) = a - q$ and $C_i(q) = cq$, for $i \in \{1, 2\}$, with $a > c$, respectively. The delegation game is described as a two-stage game, in first stage of which owners delegate maximands to their managers simultaneously. Both managers observe the delegated maximands and the game is resolved by the managers according to Nash equilibrium concept at the second stage where each manager sets a quantity so as to maximize the maximand delegated to himself. It is assumed that the maximands which can be delegated by owners are all functions of the form $\lambda\pi + (1 - \lambda)R$ for some positive λ, where π is the profit function of the firm, i.e. $\pi = pq - cq$, and R is the revenue or sales volume, i.e. $R = pq$. The goal of Sklivas (1987) is to identify the subgame perfect equilibria of this two-stage game. The method is backward or forward induction. Before proceeding any further, we want to make some modifications in the terminology employed by Sklivas. Consider the type of maximands that owners can delegate. Noting that, $\lambda\pi + (1-\lambda)R = \lambda(pq - cq) + (1-\lambda)pq = pq - \lambda cq$, it is seen that reporting λ is equivalent to reporting some cost parameter λc. From now on we will adopt the interpretation that "*firms report their cost parameters*", rather than giving weights to profits and revenues which sum up to one.

Denote $g_i = pq - c_i q$, instead of $\lambda\pi + (1-\lambda)R$ i.e. owner i simply chooses the cost parameter c_i to determine his manager's incentives.

Definition 1 (q_1^*, q_2^*) is a Nash equilibrium in the managers' subgame if and only if $q_i^* \in argmax\ g_i(q_i, q_j^*)$ with $\{i, j\} = \{1, 2\}$.

Definition 2 (c_1^*, c_2^*) is a Nash equilibrium in the owners' subgame if and only if $c_i^* \in argmax\ \pi_i(q_1^*(c_i, c_j^*), q_2^*(c_i, c_j^*))$ with $\{i, j\} = \{1, 2\}$, where $(q_1^*(\widetilde{c}_i, \widetilde{c}_j), q_2^*(\widetilde{c}_i, \widetilde{c}_j)$ stands for the Nash equilibrium of the managers' game with $(\widetilde{c}_i, \widetilde{c}_j)$ being the costs delegated by the owners.

Now we are ready to apply backward induction in order to find subgame perfect Nash equilibria. Let (c_1, c_2) be the vector of reported cost parameters in stage 1. The managers now will resolve their game via Cournot competition, i.e. reach the Cournot equilibrium of the Cournot duopoly, with costs (c_1, c_2). Now $\phi_i(q_j, c_i) = \frac{a-c_i-q_j}{2}$ is the best response of i's manager, when he is handed down c_i and j's manager chooses quantity q_j. As c_i decreases $\phi_i(.)$ shifts out. Hence decreasing cost parameter c_i prompts manager i to behave more aggressively.

The Nash equilibrium quantities in the managers' game are $q_i^* = \frac{a-2c_i+c_j}{3}$ with $\{i, j\} = \{1, 2\}$

The equilibrium price is $p^* = \frac{a+c_1+c_2}{3}$ and the profits for the owners are given by $\pi_i = (\frac{a+c_1+c_2}{3} - c)(\frac{a-2c_i+c_j}{3})$, $i = 1, 2$

Thus owner i's best response to c_j is computed as

$\varphi_i(c_j) = \frac{6c-a-c_j}{4}$.

Finally, we get that the Nash equilibrium cost declarations of the owners are found to be

$c_i^* = \frac{6c-a}{5}$ for $i = 1, 2$

Noticing that $\frac{6c-a}{5} < c$, we obtain the following result

Proposition 1 *In the owner-manager game, managers behave more aggressively than profit maximizers, i.e.* $c_i^* = \frac{6c-a}{5} < c$ *for* $i = 1, 2$. *This results in outputs that are higher than the naked Cournot quantities, yet still below the social optimum, i.e.* $\frac{a-c}{2} > q_i^*(c_1^*, c_2^*) > q_i(c, c)$, $i = 1, 2$.

We now turn to the price competition on the industry floor of the above model with product differentiation, linear demand and constant marginal cost, c.

Linear demand is given as $q_i = \alpha - p_i + \beta p_j$, $0 < \beta < 1$, $\{i, j\} = \{1, 2\}$, $0 < c < \frac{\alpha}{1-\beta}$, where p_i is firm i's price.

The game is solved as in the above quantity competition applying backward induction.

By maximizing g_i over p_i we find manager i's best response function, $\psi_i(p_j; c_i)$ We obtain $\psi_i(p_j; c_i) = \frac{\alpha+c_i+\beta p_j}{2}$. The Nash equilibrium prices as a function of (c_i, c_j) are $p_i^* = \frac{2\alpha+2c_i+\alpha\beta+\beta c_j}{4-\beta^2}$. This yields the following profit function for the owners, where $k = \frac{2\alpha+\alpha\beta+\beta c_j}{2\alpha+\alpha\beta+\beta c_j-4c+\beta^2 c}$ is a constant:

$$\pi_i(c_i, c_j) = \frac{[k+2\alpha\beta^2 c_i+\alpha\beta^3 c_i+\beta^3 c_i c_j-6\beta^2 cc_i+\beta^4 cc_i+8cc_i+2\beta^2 c_i^2-4c_i^2]}{(4-\beta^2)^2}.$$

The owner's best response functions and Nash equilibrium cost declarations are given by the following two equations, respectively.

$$c_i = \frac{2\alpha\beta^2+\alpha\beta^3+\beta^3 c_j-6\beta^2 c+\beta^4 c+8c}{c(8-4\beta^2)},\ c_i^* = \frac{2\alpha\beta^2+\alpha\beta^3-6\beta^2 c+\beta^4 c+8c}{c(8-4\beta^4-\beta^3)}.$$

Proposition 2 *In the owner-manager game firms that compete in prices behave less aggressively than profit maximizers, i.e.* $c_i^* > c$ *for* $i = 1, 2$.

We observe that firms act as profit maximizers with greater than true cost, with the result that prices are higher than the naked Bertrand price, c. Owners also receive higher profits than the Bertrand model.

1.2 Critique of FJS

While analyzing the delegation problem, FJS impose quite restrictive assumptions on firms' behavior, such as delegating only a special type of maximands to the managers, resolving the game on the industry floor according to the Cournot-Nash solution concept and delegating only once, but yet propose their results as a part of positive theory. Some natural questions which come to one's mind within the framework of the theory of firm are as follows: Why should strategic agents as owners here restrict themselves to a special class of objective functions (linear convex combination of profits and sales) to be delegated while there are some others which do a better job?, Why should the intermanagerial game be resolved by an a priori given solution concept as the Cournot-Nash equilibrium notion here?, Why should it be the case that the firms delegate only once?, or what prevents a manager to redelegate one step further if it is beneficial? Right after the appearance of FJS theory, KS (1989) was first in the literature to criticize its interpretation as part of a positive theory by just asking the above questions (which we actually borrowed from KS (1989)). However, they propose that results obtained by FJS may well serve for a regulatory purposes or more generally for institutional design. Regarding the first and second questions above KS (1986, 1988) propose their "Pretend but Perform Mechanism", which implements exactly the same outcomes as those of FJS, as regulatory design. The details of this mechanism can be found in KS (1988, 1989a). Considering the first question separately, KS (1989b) show (explicitly) that there exists a maximand not in the class offered by FJS, which owners would prefer to delegate to their managers to any function in the FJS class. As for the third question, in case when redelegation is permitted by increasing the delegation chain one step further, with the maximands of the type proposed by FJS, and without extraneous delegation costs, KS (1989b) obtain the following interesting results:

1) each owner has an incentive to redelegate, increasing the length of his delegation chain.

2) as the delegation chain grows beyond bound,

i) total output at the (Cournot) equilibrium on the industry floor converges in monotonically increasing fashion to the socially efficient one,

ii) the maximand delegated by each primal delegator converges in monotonically decreasing fashion to the (true) profit function.

The above results tell us that the results of FJS with the delegation chain length of one is not stable, since each owner will want to redelegate one step further increasing the length of this chain. The importance of the above result from the view point of regulatory theory is that it tells us that we can approximate the socially efficient outcome and truthful revelation of profit functions by applying the Pretend-but-Perform Mechanism sufficiently many times.

Our attempt in this study is to investigate what happens when the delegation game is resolved via supply function equilibria on the industry floor. By a supply function we mean any non-decreasing continuous function passing through the origin, which relates price to quantity.

As we have seen, managerial incentives are different for Cournot and Bertrand duopolies, actually being oppositely located in a certain sense. It then would be natural to ask what happens in a duopoly where firms' strategic variables are functions relating prices to quantities. That is each firm reports its supply schedule announcing what quantity it will produce at each given price. Once this announcement is made we assume that it becomes a commitment, and whatever price clears the market, the firm should produce its announced quantity and sell at that price.

Delegation game with supply functions then proceeds in two stages just as in the above Cournot case; in the first stage both owners simultaneously report their cost parameters. Managers then choose ech a supply function at the second stage, consequently the market outcome materializes and owners get their profits. So what are the best incentives for the owners to delegate to their managers with such strategic variables? The next chapter is devoted to the analysis of symmetric duopoly with supply function strategies. Analysis of an industry with asymmetric costs and implications for delegation and regulation are studied in chapter

3. Clearly, the results obtained in the next chapter (chapter 2) will constitute special cases of those in chapter 3. However, we will first consider the symmetric case in order to avoid technical difficulties and to give a better idea of the supply function game. The existing literature almost always imposes upon supply functions technical assumptions such as "differentiability", (see e.g. Grossman (1980) Klemperer and Meyer (1989), Moreno and Delgado (2004)). The technique that we provide here makes this differentiability assumption superfluous. Actually, by virtue of the fact that we can approximate every continuous function by differentiable functions (even with polynomials!) uniformly on a compact domain, using standard techniques such as Langrange Interpolation makes outcomes of the games with continuous and with differentiable functions to coincide. As we will see in the next chapter all these technical details are actually not needed in the analysis of the supply function game.

Chapter 2

Supply Function Game and its Equilibrium Outcomes for Symmetric Linear Duopoly

In this chapter we analyze symmetric linear duopoly and fully characterize its (Nash) equilibrium outcomes in case firms' strategies are supply functions.

We start by formally defining what we mean by a symmetric linear duopoly and a supply function game associated with it.

For any market for a homogenous good, variables p and q denote price and quantity, respectively. We assume that (p, q) is an element of $\mathbb{R}^2_+$.

Definition 3 *A symmetric linear duopoly* is a market for a homogenous good, which can be represented as a pair of functions $(D(p), C(q))$, where $D(p) = a - p$ and $C(q) = cq$, are the market demand and the common cost function of both firms, respectively, with $a, c \in \mathbb{R}_{++}$ and $a > c$.

The term *symmetric* underlines the fact that both firms have an access to the same production technology, i.e. have the same cost function.

In the standard literature, an inverse demand is given by $D^{-1}(q) = a - bq$ for some positive parameters a and b for which b does not necessarily equal to 1 contrary to our above definition. We immediately want to clarify why we omitted the coefficient b, actually setting it to 1. Since b is the coefficient of quantity in the above equation, it affects only the quantity units and thus rescaling it by some positive constant does not make any difference for theoretical analysis (such as comparison with other models, etc.). Mathematically, this means that we can make a linear transformation $\overline{q} = bq$, viewing $\overline{q}$ as our new quantity (variable) in the market. Observe that an analysis of the market with this new $\overline{q}$ has no qualitative effect on prices, profits, social welfare, etc. Summing up, we assume, *without loss of generality*, that $b = 1$ in the above definition.

Definition 4 The *Cournot outcome* of a linear symmetric duopoly is a Nash equilibrium outcome of the quantity competition game of the duopoly, denoted by (p^c, q_1^c, q_2^c).

We have $p^c = \frac{a+2c}{3}, q_1^c = q_2^c = \frac{a-c}{3}$. Moreover, profits of firms are $\pi_1(q_1^c, q_2^c) = \pi_2(q_1^c, q_2^c) = \frac{(a-c)^2}{9}$.

Definition 5 The *Bertrand outcome* of the linear symmetric duopoly is a Nash equilibrium outcome of the price competition gameof the duopoly, denoted by (p^b, q_1^b, q_2^b).

We have $p^b = c$ and $(q_1^b, q_2^b) \in \mathbb{R}_+^2$ is s.t. $q_1^b + q_2^b = a - c$. Moreover, profits of firms are $\pi_1(q_1^b, q_2^b) = \pi_2(q_{1b}^c, q_2^b) = 0$.

Definition 6 We call (p^m, q^m) a *monopoly outcome* of the linear symmetric duopoly if $p^m = argmax_p(a-p)(p-c)$ and $q^m = a - p^m$.

We have $p^m = \frac{a+c}{2}$ and $q^m = \frac{a-c}{2}$. Moreover, profits of monopolist are given by $\pi^m = \frac{(a-c)^2}{4}$.

Definition 7 By a *supply function* we mean any non-decreasing continuous function $s : [0, a] \to \mathbb{R}_+$ such that $s(0) = 0$.

The set of all supply functions will be denoted by $\mathcal{S}$.

Definition 8 By a *supply function game* for symmetric linear duopoly we mean a normal form game $\Gamma = (N, (X_1, X_2), u)$, where $N = \{1, 2\}$ is the agent set, $X_i = \mathcal{S}$ is the common strategy space and $u_i(s_1, s_2) = \pi_i(s_1, s_2) = \overline{p}s_i(\overline{p}) - c(\overline{p})$ is a payoff (profit) function for $i \in \{1, 2\}$, $(s_1, s_2) \in \mathcal{S}^2 = \mathcal{S} \times \mathcal{S}$, where $\overline{p}$ is a solution to the equation $s_1(p) + s_2(p) = a - p$, i.e. the market clearing price.

Henceforth, when we say *symmetric linear duopoly*, we will mean the above defined game and two notions will be used interchangeably.

Notice that for each $(s_1, s_2) \in \mathcal{S}^2$ there always exists $p \in [0, a]$, such that $s_1(p) + s_2(p) = a - p$, as the function $D(p) = a - p$ is decreasing, $s_1(p) + s_2(p)$ is non-decreasing, $D(0) = a > s_1(0) + s_2(0) = 0$ and $D(a) = 0 \leq s_1(a) + s_2(a)$. Moreover, it is trivial to observe that this price is unique, since decreasing and non-decreasing functions can have a unique point of intersection. And so the supply function game is well defined.

The importance of the assumption $s(0) = 0$ is twofold. Firstly, it is an intuitive economic assumption, stating that a firm will make *no supply at zero price* and thus *total supply is zero at zero price*. Secondly, it is an important technical assumption making the supply function game well defined, as it guarantees the existence of a market clearing price.

Definition 9 A triple $(\overline{p}, \overline{q}_1, \overline{q}_2)$ is said to be an *equilibrium outcome* of the supply function game if there exists a pair $(s_1, s_2) \in \mathcal{S}^2$, where (s_1, s_2) is a Nash equilibrium of the supply function game with $s_1(\overline{p}) = \overline{q}_1, s_2 = \overline{q}_2$ and $\overline{q}_1 + \overline{q}_2 = a - \overline{p}$. In this case we also say that the pair (s_1, s_2) supports the triple $(\overline{p}, \overline{q}_1, \overline{q}_2)$ as an equilibrium.

In the supply function game, each firm i searches for a price which maximizes its profits when it acts as a monopolist on its *residual demand*, i.e. $a - p - s_j(p)$ for a given s_j, where $\{i, j\} = \{1, 2\}$. Stating in other words firm i solves the maximization problem $max_{p \in [0,a]} \, (a - p - s_j(p))(p - c)$. Observe that the function $(a - p - s_j(p))(p - c)$ is continuous as a composition of continuous functions, and so attains its maximum as its domain is compact.

Since we are interested in outcomes that will materialize in the market as a consequence of the supply function game, our next goal is to find the set of all equilibrium outcomes.

From above we observe that for $(\overline{p}, \overline{q}_1, \overline{q}_2)$ to be an equilibrium outcome it should satisfy the following conditions:

$\overline{p} = argmax_p (a - p - s_i)(p - c)$ for $i \in \{1, 2\}$

$\overline{q}_1 + \overline{q}_2 = a - \overline{p}$, where $\overline{q}_i = s_i(\overline{p})$ for $i \in \{1, 2\}$.

Observe that the *supply functions* generate an infinite dimensional vector space of *all* continuous functions on $[0, a]$, actually being a cone in this vector space. So the characterization of equilibrium outcomes is not an easy matter, and we do not have a general algorithm that yields Nash equilibrium strategies. However, our next few observations will make the analysis extremely simple for such a characterization. What we observe, as a combination of the following facts, is that we can restrict our attention to a certain subset of $\mathcal{S}$ the analysis of which is easy to handle. To this end, we formulate and prove a lemma and a proposition.

Lemma 1 *If* $\widehat{p} \in [0, c]$, *then* $(\widehat{p}, q_1, q_2)$ *is not an equilibrium outcome for any* q_1, q_2.

Proof Let $\widehat{p}$ be as in the statement and suppose that $(\widehat{p}, q_1, q_2)$ is an equilibrium outcome. First assume $\widehat{p} < c$. Since $a - \widehat{p} = q_1 + q_2 > 0$ (as $a > c$), one of q_1 or q_2 is strictly positive. W.l.o.g. assume $q_1 > 0$. Now, there is a pair $(s_1, s_2) \in \mathcal{S}^2$ such that (s_1, s_2) constitutes a Nash equilibrium of the game supporting $(\widehat{p}, q_1, q_2)$. Then $\pi_1(s_1, s_2) = \widehat{p}q_1 - cq_1 = (\widehat{p} - c)q_1 < 0$ which contradicts that (s_1, s_2) is an equilibrium, as it is easily seen that firm 1 has an incentive to deviate to the supply function $s(p) \equiv 0$, as its profits will rise from some negative number to 0 in that case. Now assume $\widehat{p} = c$. Again assume, on the contrary, that (c, q_1, q_2) is an equilibrium outcome supported by $(s_1, s_2) \in \mathcal{S}^2$. Obviously, $\pi_1(s_1, s_2) = \pi_2(s_1, s_2) = 0$. W.l.o.g. assume $a - c \geq q_1 \geq q_2 \geq 0$, and $q_1 + q_2 = a - c$. Notice that $q_2 \leq a - c$. Let $\widehat{p}$ be such that $a - \widehat{p} = s_2(\widehat{p})$. It is easy to observe that such a $\widehat{p}$ exists and is unique as $a - p$ is decreasing and $s_2(p)$ is non-decreasing. Since $s_2(c) = q_2 < a - c$, one has $\widehat{p} > c$. Pick some $\widetilde{p} \in (c, \widehat{p})$. Evidently $a - \widetilde{p} - s_2(\widetilde{p}) > 0$, which implies $(\widetilde{p} - c)(a - \widetilde{p} - s_2(\widetilde{p}) > 0$. This means that firm 1 has an incentive to deviate from s_1 to $\widetilde{s_1} \in \mathcal{S}$, where $\widetilde{s_1}$ satisfies $\widetilde{s_1}(\widetilde{p}) + s_2(\widetilde{p}) = a - \widetilde{p}$, and so s_1 is not a best response to s_2, which completes the proof. $\square$

The above lemma states that no price lower than or equal to the Bertrand price (which is the competitive price in our case as well) can be part of an equilibrium outcome.

The next simple proposition plays a central role in the whole analysis to follow. It states that any equilibrium outcome can be supported by an equilibrium pair of supply functions belonging to a very special class. Therefore, in the sequel we will confine ourselves to this class of supply functions and work with them.

Proposition 3 *Let* $(s_1, s_2) \in \mathcal{S}^2$ *be a Nash equilibrium of the supply function game supporting* $(\overline{p}, \overline{q}_1, \overline{q}_2)$ *as an equilibrium outcome. Then* $(\overline{s}_1, \overline{s}_2) \in \mathcal{S}^2$ *is also a Nash equilibrium of the supply function game supporting the same outcome, where for* $i \in \{1, 2\}, \overline{s}_i(p)$ *is defined through*

$$\overline{s}_i(p) = \begin{cases} \frac{s_i(\overline{p})}{c} p & \text{if } p \in [0, c], \\ s_i(\overline{p}) & \text{if } p \in (c, \overline{p}], \\ s_i(p) & \text{if } p \in (\overline{p}, a]. \end{cases}$$

Proof Let $\{i, j\} = \{1, 2\}$ and $s_j(p)$ be given. Observe that $\overline{s}_i \in \mathcal{S}$ and passes through $(\overline{p}, s_i(\overline{p}))$ by construction. As $\overline{p}$ maximizes $(a - p - s_j)(p - c)$, $\overline{s}_i(p)$ is a best response to s_j. Now, as s_j is a best response to s_i, it is also a best response to $\overline{s}_i$. To see this observe that since $\overline{p}$ maximizes $(a - p - s_i(p))(p - c)$ it also maximizes $(a - p - \overline{s}_i(p))(p - c)$, as $\overline{s}_i(p) \geq s_i(p)$, for all p. Finally, $(\overline{s}_i, s_j)$ constitutes a Nash equilibrium inducing the outcome $(\overline{p}, \overline{q}_1, \overline{q}_2)$. Now, viewing the above (s_i, s_j) as $(s_j, \overline{s}_i)$ and applying the same proof, we get that $(\overline{s}_i, \overline{s}_j)$ is a Nash equilibrium of the game with outcome $(\overline{p}, \overline{q}_1, \overline{q}_2)$, and the proof is complete.

□

The above proposition is a simple observation. But it is quite powerful, as it

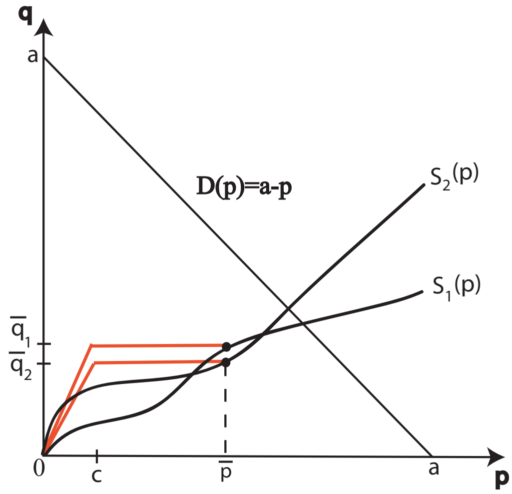

Figure 2.1: Geometric illustration of Proposition 3

tells us that we can confine ourselves to a very special subset of $\mathcal{S}$ while searching for the equilibrium outcomes of the supply function game. The speciality of this subset is namely that, each of its elements is a supply function which is constant on some interval $[c, p]$ for some $p > c$.

Definition 10 Let $\overline{\mathcal{S}} \subset \mathcal{S}$ be such that for all $s \in \overline{\mathcal{S}}$, there is p_s with $p_s > c$ and $s(p) = s(c)$ for all $p \in [c, p_s]$.

Obviously, $\overline{\mathcal{S}}$ is a proper subset of $\mathcal{S}$, as the function $s \in \mathcal{S}$ defined as $s(p) = p$ does not fit the above definition.

It is also a simple matter to observe that $\overline{\mathcal{S}}$ is a cone in a vector space of *all* continuous functions on $[0, a]$.

The following analysis will provide us with a full characterization of equilibrium outcomes.

Let $(\overline{p}, \overline{q}_1, \overline{q}_2)$ be an equilibrium outcome of the linear symmetric duopoly. W.l.o.g. let $\overline{q}_1 \geq \overline{q}_2$. Lemma 1 implies that $\overline{p} > c$, and proposition 1 implies that there exists a pair $(s_1, s_2) \in \overline{\mathcal{S}}$ with $s_i(\overline{p}) = \overline{q}_i$ and $s_i(p) = \overline{q_i}$, for all $p \in [c, \overline{p}]$, $i \in \{1, 2\}$. Consider the intervals $[0, c]$, $[c, \overline{p}]$, and $[\overline{p}, a]$. Set $s_i(p) = \frac{\overline{q}_i}{c} p$ on the first interval and $s_i(p) = \overline{q}_i$ on $[c, \overline{p}]$. Then firm i solves $max_p\ (a - p - \overline{q}_j)(p - c)$, and by the definition of $\overline{p}$, we get $\overline{p} \in argmax\{(a - p - \overline{q}_j)(p - c)\}$. Observe that the real-valued function $(a - p - \overline{q}_j)(p - c)$ of a real variable is nothing but the downward parabola, vanishing at $p = c$ and $p = a - q_j$, with the unique maximizer at $p = \frac{a - q_j + c}{2}$. We immediately obtain that for $\overline{p}$ to maximize this function on $[c, \overline{p}]$, we should have $\overline{p} \leq \frac{a - \overline{q}_j + c}{2}$, which in turn implies that $\overline{q}_j \leq a + c - 2\overline{p}$. Since $a - \overline{p} = \overline{q_i} + \overline{q_j}$, we have $\overline{q_i} \geq \overline{p} - c$. So the maximum and minimum values coincide, i.e. $a + c - 2\overline{p} = \overline{p} - c$, at $p = p^c = \frac{a+2c}{3}$, which is nothing but the Cournot price. We also have $a + c - 2p \geq p - c$ for $p \leq p^c$. Moreover, observe that we should have $\overline{q}_i \leq a + c - 2\overline{p}$ from j's maximization problem. Adding the two inequalities, $\overline{q}_i \leq a + c - 2\overline{p}$, $\overline{q}_j \leq a + c - 2\overline{p}$ and using $a - \overline{p} = \overline{q}_1 + \overline{q}_2$ we get that $\overline{p} \leq \frac{a+2c}{3} = p^c$, which leads us to the following lemma.

Lemma 2 *If $\widehat{p} \in (p^c, a]$, then $(\widehat{p}, q_1, q_2)$ is not an equilibrium outcome for any q_1, q_2.*

We assert that any outcome $(\overline{p}, \overline{q}_1, \overline{q}_2)$ can be supported as an equilibrium outcome such that $p \in (c, p^c]$ and $(\overline{q}_1, \overline{q}_2)$ is such that $\overline{q}_1 \leq a + c - 2\overline{p}$ and $\overline{q}_2 \geq \overline{p} - c$. By the above analysis, we already know that there is $(s_1, s_2) \in \overline{\mathcal{S}}^2$ with $s_1(p) = \overline{q}_1, s_2(p) = \overline{q}_2$, for $p \in [c, \overline{p}]$ and $s_1(p) = \frac{\overline{q}_1}{c} p, s_2(p) = \frac{\overline{q}_2}{c} p$, for $p \in [0, c]$, such that s_1 and s_2 are best responses to each other on $[0, \overline{p}]$. It remains only to extend functions s_1 and s_2 to the domain $[\overline{p}, a]$ in such a way that they remain best responses to each other supporting $(\overline{p}, \overline{q}_1, \overline{q}_2)$ as an equilibrium outcome over this domain. Consider firm 1 to extend its function s_1 in such a way that $\overline{p} \in argmax\ (a - p - s_1(p))(p - c)$. We prove that it is possible to do this without giving an explicit functional form, though the latter can be demonstrated easily as well. Suppose $s_1(p) = n\sqrt{p-\overline{p}} + \overline{q}_1$ on $[\overline{p}, a]$ for some integer n. Obviously, s_1 is continuous. In order to support $(\overline{p}, \overline{q}_1, \overline{q}_2)$ as an equilibrium outcome, it remains to have $(a - p - n\sqrt{p-\overline{p}} - \overline{q}_2)(p - c) \leq \overline{q}_1(\overline{p} - c)$ on $[\overline{p}, a]$. Since $p \geq \overline{p}$ the result will follow if we show that $(a - \overline{p} - n\sqrt{p-\overline{p}} - \overline{q}_2)(p - c) \leq \overline{q}_1(\overline{p} - c)$ or $\overline{q}_1 - n\sqrt{p-\overline{p}} \leq \frac{\overline{q}_1(\overline{p}-c)}{p-c}$, as $a - \overline{p} = \overline{q}_1 + \overline{q}_2$. This implies $n\sqrt{p-\overline{p}} \geq \overline{q}_1(1 - \frac{\overline{p}-c}{p-c})$ Now consider any sequence $\{p_k\}_{k\in\mathbb{N}} \subset (\overline{p}, a]$ converging to $\overline{p}$. There obviously exists $\overline{k} \in \mathbb{N}$, such that for all $k > \overline{k}$, above inequality holds. Now set n in such a way that the solution of equation $n\sqrt{p-\overline{p}} + \overline{q}_2 = a - p$ is less than $\overline{p}_{\overline{k}+1}$. It should now be clear that with a such supply function of firm 1, firm 2 will maximize its profits at $\overline{p}$. Of course, a similar construction works for firm 2 for supporting the outcome $(\overline{p}, \overline{q}_1, \overline{q}_2)$ as an equilibrium outcome. Let us summarize this analysis in the following theorem.

Theorem 1 *In a symmetric linear duopoly (with supply function strategies), the set of equilibrium outcomes consists of $(p, q_1, q_2) \in \mathbb{R}^3_+$ such that $p \in (c, p^c]$ and for any $p \in (c, p^c], (q_1, q_2)$ is such that $q_1 + q_2 = a - p$ with $q_i \leq a + c - 2p$ for $i \in \{1, 2\}$.*

So the set of equilibrium outcomes for such a duopoly is continuum. As shown in figure 2.2 every point of the shaded triangle (excluding its boundary) corresponds to exactly one equilibrium outcome.

We close this section with the following proposition.

Proposition 4 *Among all equilibrium outcomes of the symmetric linear duopoly the joint profit maximizing one is $((p^c, q^c, q^c)$, i.e. the Cournot outcome.*

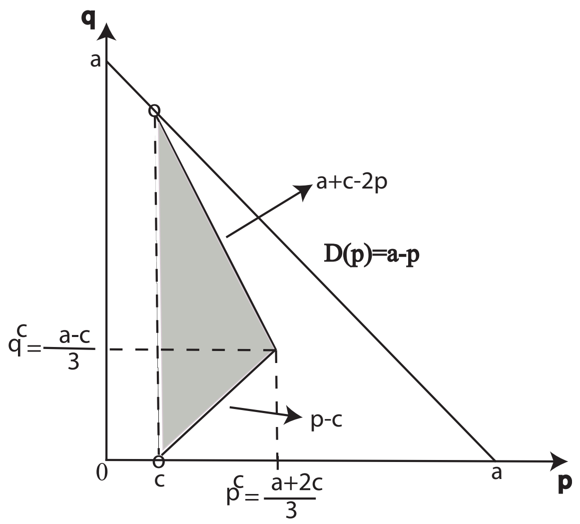

Figure 2.2: Geometric illustration of equilibrium outcomes for symmetric duopoly

Proof For any equilibrium outcome (p, q_1, q_2), $\pi_1 + \pi_2 = q_1(p-c) + q_2(p-c) = (a-p)(p-c)$, but *argmax* $(a-p)(p-c)$ is $\frac{a+c}{2}$, which is greater than $p^c = \frac{a+2c}{c}$. So the function $(a-p)(p-c)$ achieves its maximum at p^c on the domain $(c, p^c]$ as it is a downward winging parabola, which establishes the result. $\square$

Chapter 3

Delegation Game and Regulatory Implications

3.1 Asymmetric Duopoly Equilibria

In this chapter we analyze the delegation games where the game on the industry floor is resolved according to the Nash solution concept with strategies of firms being supply functions, $\mathcal{S}$.

As we have seen in the previous chapter the set of equilibrium outcomes of the supply function game is very large. So, one should first specify what equilibrium outcome will materialize in order to analyze the problem in a proper manner. Here we are faced with a situation, where positive theory is not of much help. In terms of regulatory theory, however, we can make a selection of any equilibrium outcomes of our own choice by an appropriate design of a regulatory mechanism. However, we may also assume that firms can coordinate in such a way that the profit maximizing outcome will be chosen in case of multiple equilibria, which seems natural in this case. The last proposition of the previous chapter states that this outcome is the *Cournot outcome*.

We address the following question: Is there any symmetric Nash equilibrium of the delegation game when the intermanagerial game is the supply function game. To be able to deal with this problem, we now turn to the analysis of the duopoly with asymmetric costs.

All definitions and results of this part are generalizations of those in the previous chapter, to which we will often make a reference. We start by modifying our definitions for the asymmetric case.

Definition 11 *The asymmetric linear duopoly* is the market for a homogenous good which can be represented as a triple of functions $(D(p), (C_i(q))_{i\in\{1,2\}})$, where $D(p) = a - p$ and $C_i(q) = c_i q$, are the market demand and the cost functions of firms, respectively, with $a, c \in \mathbb{R}_{++}$ and $a > c$.

Denote $c = (c_1, c_2)$, and w.l.o.g assume that $c_1 < c_2$ throughout our analysis.

Definition 12 We call (p^c, q_1^c, q_2^c) the *Cournot outcome* of a linear asymmetric duopoly if it is the Nash equilibrium outcome of the quantity setting game resolved according to the Nash equilibrium concept.

We have $p^c = \frac{a+c_1+c_2}{3}, q_1^c = \frac{a-2c_1+c_2}{3}, q_2^c = \frac{a-2c_2+c_1}{3}$ if $a - 2c_i + c_j \geq 0$ for $\{i, j\} = \{1, 2\}$ and $p^c = \frac{a+c_i}{2}, q_i^c = \frac{a-c_i}{2}, q_j^c = 0$ for $i \in \{1, 2\}$ and $i \neq j$ if $a - 2c_i + c_j \leq 0$

Definition 13 We call (p^b, q_1^b, q_2^b) the *Bertrand outcome* of a linear asymmetric duopoly if it is the Nash equilibrium outcome of the price setting game resolved according to the Nash equilibrium concept.

We have $p^b = c_2$ and $q_1^b = a - c_2, q_2^b = 0$.

Definition 14 By the *supply function game* for asymmetric linear duopoly we mean a normal form game $\Gamma = (N, (X_1, X_2), u)$, where $N = \{1, 2\}$ is the agent set, $X_i = \mathcal{S}$ is the common strategy space and $u_i(s_1, s_2) = \pi_i(s_1, s_2) = \overline{p}s_i(\overline{p}) - c_i(\overline{p})$ is the payoff (profit) function for $i \in \{1, 2\}$, $(s_1, s_2) \in \mathcal{S}^2 = \mathcal{S} \times \mathcal{S}$, where $\overline{p}$ is a solution to the equation $s_1(p) + s_2(p) = a - p$, i.e. the market clearing price.

Our first result is an analogue of lemma 1 of the previous chapter.

Lemma 3 *If $\widehat{p} \in [0, c_1]$, then $(\widehat{p}, q_1, q_2)$ is not an equilibrium outcome for any q_1, q_2.*

Proof Same as lemma 1. $\square$

Proposition 2 of the previous chapter clearly applies to the case of asymmetric duopoly. So, every equilibrium outcome of an asymmetric duopoly can be supported by Nash equilibrium pairs of supply functions which are in $\overline{\mathcal{S}}$. So, we will restrict ourselves to this space while searching for equilibrium outcomes of an asymmetric duopoly. We will also borrow a method of extending functions to the interval $[\overline{p}, a]$ from the previous chapter in order to support given outcome $(\overline{p}, \overline{q}_1, \overline{q}_2)$ as an equilibrium outcome. Notice that in extending the functions to the mentioned domain, cost asymmetry plays no role and so the method is applicable here as well.

Our next lemma is about equilibrium outcomes, when price is between the two cost parameters.

Lemma 4 *If $\widehat{p} \in (c_1, c_2]$, then the set of all outcomes which can be supported as an equilibrium outcome is equal to $\{(\overline{p}, \overline{q}_1, \overline{q}_2) : \overline{q}_1 = a - \overline{p}$ and $\overline{q}_2 = 0\}$*

Proof Let $\widehat{p} \in (c_1, c_2)$. If $\overline{q}_2 > 0$ then firm 2 gets negative profits, but negative profits can never arise at equilibrium since an agent would deviate, e.g. to the identically zero function to increase profits. Let $\widehat{p} = c_2$. Again suppose $\overline{q}_2 > 0$. Clearly the profit of firm 2 is zero. Assuming the contrary, denote by (s_1, s_2) the Nash equilibrium strategy pair supporting $(c_2, \overline{q}_1, \overline{q}_2)$ as an equilibrium outcome. Now, since $\overline{q}_1 < a - c_2$, there exists $\widehat{p}$ such that $a - p > s_1(p)$, for all $p \in [c_2, \widehat{p})$. Pick some $\widetilde{p} \in (c_2, \widehat{p})$. Moreover, let $s \in \mathcal{S}$ be such that $s(\widetilde{p}) + s_1(\widetilde{p}) = a - \widetilde{p}$. Since $s(\widetilde{p}) > 0$ and $\widetilde{p} > c_2$, by deviating to s, firm 2 will get positive profits. So $q_2 = 0$. Fix some $\overline{p} \in (c_1, c_2]$. Let $s_1(p) = a - \overline{p}$ for all $p \in [0, a]$. Evidently firm 2 has no reason to deviate. Moreover, observe that profits of firm 2 are positive. In order to support $(\overline{p}, a - \overline{p}, 0)$ as an equilibrium outcome it remains to extend $s_2(p)$, where $s_2(p) = 0$ for all $p \in [0, \overline{p}]$ to $[\overline{p}, a]$ in such a way that $\overline{p}$ remains a maximizer of $(a - p - s_2(p))(p - c_1)$. And this extension is obviously possible by the construction of the previous chapter, and the proof of lemma is complete. $\square$

We now analyze the question of what outcomes $(\overline{p}, \overline{q}_1, \overline{q}_2)$ can be supported as equilibrium outcomes, when $\overline{p} > c_2$. Let $(s_1, s_2) \in \overline{\mathcal{S}}^2$ be a joint equilibrium strategy supporting the given outcome as an equilibrium outcome. Firm 1 solves $max_{p \in [0,a]}(a - p - s_2(p))(p - c_1)$. For $\overline{p}$ to be the maximizer of $(a - p - s_2(p))(p - c_1)$ on $[0, \overline{p}]$, we should have $\overline{p}$ to be less than or equal to the maximizer of the function $(a - p - \overline{q}_2(p))(p - c_1)$ as it is downward looking parabola. The maximizer of $(a - p - \overline{q}_2(p))(p - c_1)$ is $\frac{a + c_1 - \overline{q}_2}{2}$, so we have $\overline{p} \leq \frac{a + c_1 - \overline{q}_2}{2}$ or $\overline{q}_2 \leq a + c_1 - 2\overline{p}$, which implies $\overline{q}_1 \geq p - c_1$. The maximization problem of firm 2 is similar, implying that $\overline{q}_1 \leq a + c_2 - 2\overline{p}$ and $\overline{q}_2 \geq p - c_2$. For $\overline{p}$ to be an equilibrium price, these inequalities should be consistent. Adding up $\overline{q}_1 \geq p - c_1$ and $\overline{q}_2 \geq p - c_2$ side by side, we get $c_1 + c_2 \geq 2\overline{p} - (\overline{q}_1 + \overline{q}_2)$. Finally adding up a to both sides and using the fact that $a - (\overline{q}_1 + \overline{q}_2) = \overline{p}$, we obtain $\overline{p} \leq \frac{a+c_1+c_2}{3}$, i.e. no price greater than the Cournot price can be an equilibrium price, which is an analogue of lemma 2 for the symmetric case. Moreover, observe that the outcome when $\overline{p} = p^c$ is unique, yielding $\overline{q}_1 = q_1^c = \frac{a-2c_1+c_2}{3}$ and $\overline{q}_2 = q_2^c = \frac{a-2c_2+c_1}{3}$. One may ask what happens if $\frac{a-2c_2+c_1}{3} < 0$. In this case, notice that the lines $a + c_2 - 2\overline{p}$ and $p - c_1$ do not intersect and firm 1 gets its monopoly outcome, i.e. $\frac{a-c_1}{2}$ at its monopoly price $\overline{p} = \frac{a+c_1}{2}$. An extension of functions to support the given $(\overline{p}, \overline{q}_1, \overline{q}_2)$ as an equilibrium outcome can be accomplished as in the previous chapter.

We summarize the result of our analysis in the following theorem, and the figure 3.1 below illustrates it graphically.

Lemma 5 *The set of all equilibrium outcomes of the supply function game for the asymmetric duopoly is equal to the union of the following two sets:*

$\{(p, q_1, q_2) : p \in (c_1, c_2]$ *and* $q_1 = a - p, q_2 = 0\}$,

$\{(p, q_1, q_2) : p \in (c_2, p^c]$ *and* $p - c_1 \leq q_1 \leq a + c_2 - 2p$, $p - c_2 \leq q_2 \leq a + c_1 - 2p$ *with* $q_1 + q_2 = a - p\}$.

3.2 Delegation Game

We see that the equilibrium outcomes of the game discussed again comprise a large set. So what outcome is going to be selected turns out to be an arbitrary procedure which depends on the regulatory mechanism instituted or the kind of coordination available to the firms which we mentioned earlier. But this shows

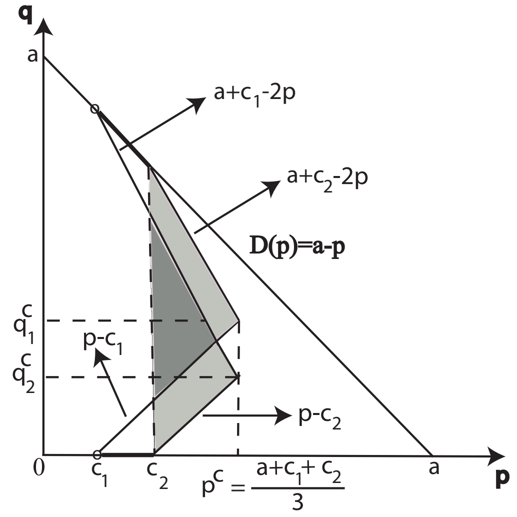

Figure 3.1: Geometric illustration of equilibrium outcomes for asymmetric duopoly

us that FJS's argument to view the delegation problem as part of the positive theory of firm is not acceptable in this setting either. Arguing that the delegation game is a part of positive theory, what FJS fail to observe is that the game they set and analyze is constructed artificially. And the reason that KS(1989) criticize is just based on this. Actually, before FJS(1987), KS(1986) studied the problem and treated it as a regulatory mechanism. A mechanism constructed by them implements the same outcome as the delegation game of FJS.

For example, selection of an equilibrium outcome for an asymmetric duopoly can be made by implementing certain bargaining solutions, viewing them as contracts among firms. Observe that the choice of a bargaining solution can also be viewed as a regulatory mechanism for the duopoly in question. Possible bargaining solutions may be Nash solution, joint utility maximizing among many others. We assume that the point of disagreement is $(0, 0)$. The most natural among these in case of symmetric duopoly is the joint profit maximizing solution. The reason for this is that this solution gives rise to the unique outcome where firms receive equal profits. If we intend to carry the same solution over the asymmetric case, it is not always the case that the Cournot outcome gets selected.

Let us provide a brief analysis of this. From now on when we say a delegation game, we mean a one-shot game played by firms where strategy spaces are $[0, a]$ and payoffs are obtained by first finding joint profit maximizing outcome in the game induced on the industry floor, and then calculating the profits with respect to true cost parameter c (which is equal among agents). At the resulting outcome this one-shot game obviously leads to the two-stage game discussed so far.

Now, suppose a joint strategy $c = (c_1, c_2)$ is played. Assume $c_1 \leq c_2$. To find what payoffs the agents get, we have to find the joint profit maximizing outcome of the resulting duopoly. For a given outcome (p, q_1, q_2), the joint profit of firms is $pq_1 - c_1q_1 + pq_2 - c_2q_2$. We maximize this function over the domain described in the theorem above, i.e. over $\mathcal{D}$=$\{\{(p, q_1, q_2) : p \in (c_1, c_2]$ and $q_1 = a - p, q_2 = 0\} \cup \{(p, q_1, q_2) : p \in (c_2, p^c]$ and $p - c_1 \leq q_1 \leq a + c_2 - 2p$, $p - c_2 \leq q_2 \leq a + c_1 - 2p$ with $q_1 + q_2 = a - p\}\}$. Rearranging the terms and using $a - p = q_1 + q_2$ we have $pq_1 - c_1q_1 + pq_2 - c_2q_2 = p(a - p) - c_1q_1 - c_2q_2$. Notice that if $\overline{p}$ is the maximizer of this function then we should necessarily have $q_1 = a + c_2 - 2\overline{p}$ and $q_2 = p - c_2$, as these values minimize $c_1q_1 + c_2q_2$, by $c_1 \leq c_2$. So the function to be maximized is $p(a-p) - c_1(a + c_2 - 2p) - c_2(p - c_2)$. This function is maximized at $p^* = \frac{a+2c_1-c_2}{2}$. Now if $p^* \geq \frac{a+c_1+c_2}{3} = p^c$, or $a + 4c_1 - 5c_2 \geq 0$, then the Cournot outcome is the joint profit maximizing outcome. If $p^* < \frac{a+c_1+c_2}{3} = p^c$, or $a + 4c_1 - 5c_2 < 0$ then we obtain another outcome which is given by $q_1 = a + c_2 - 2p^*, q_2 = p^* - c_2$. We observe that the outcomes materializing at this game are discontinuous in prices.

The parameters a and c may be related in such a way that the joint strategy $(c^*, c^*) = (\frac{6c-a}{5}, \frac{6c-a}{5})$ is no longer an equilibrium. We will not give an explicit relationship for which this statement is valid, since it is not essential in our analysis.

Consider an industry where $a = 6c$. Then $(c^*, c^*) = (0, 0)$. Assume that firm 1 deviates to $\widetilde{c} = c$, where c is the true cost parameter. Then $a - 5c > 0$ and we have that firm 1's profits in this case is $(\frac{a-3c}{2})^2$. The profit it gets at $(c^*, c^*) = (0, 0)$ is $\frac{a(a-3c)}{9}$. Comparing the two profits, we get that $27c < 5a$, which holds as $a = 6c$. So $(\frac{a-3c}{2})^2 > \frac{a(a-3c)}{9}$, and thus (c^*, c^*) is no longer an equilibrium.

3.3 Regulatory Mechanisms

If we in addition want to preserve the continuity of the function $f : [0, a]^2 \mapsto \mathbb{R}^3_+$, such that $f(c_1, c_2) = (p^c, q^c_1, q^c_2)$, then the outcome to be chosen is (p^c, q^c_1, q^c_2), and we obtain the results of Sklivas(1987). Such artificial restrictions amount in fact to designing a mechanism which implements *a priori* intended outcomes. By artificially designing mechanisms we can implement several equilibrium outcomes. One trivial example of this is the following. Suppose a regulator intends to implement a collusive outcome which respects symmetry, i.e. (p^m, q^m_1, q^m_2), where $q^m_1 = q^m_2 = \frac{q^m}{2}$. Then the mechanism is that among all supply function equilibria it selects the joint profit maximizing one with equilibrium quantities being equal among firms. In this case the strategy $(\bar{c}, \bar{c}) = (\frac{a+3c}{4}, \frac{a+3c}{4})$, which leads to the monopoly price is the equilibrium joint strategy. Obviously no firm has any reason to deviate, since by doing so it will make price to materialize at a level different than the monopoly price and by the designed mechanism lowers its profits.

A central authority regulating a duopoly with unknown costs would wish to implement a socially best outcome, i.e. to achieve $p = c, q_1 + q_2 = a - c$ as its objective. A natural question to ask now is whether the regulator can implement this outcome by an appropriate design of mechanism via supply function equilibria? Obviously the regulator can not implement this outcome, as c does not arise as a part of equilibrium at any supply function equilibrium outcome. However, he can implement an outcome which is close to c to any degree desired. Let us demonstrate this.

Let the regulator's mechanism be designed as follows (step by step)

1)First it fixes some positive ε;

2)At any pair (c_1, c_2) declared by the firms as their cost parameters, it sets the price $p = \frac{c_1+c_2}{2}$ and quantities being $q_i = \frac{2a-c_1-c_2}{2}, q_j = 0$ if $c_i < c_j$ with $\{i, j\} = \{1, 2\}$, and he sets price as $p = c_1+\varepsilon$ in case if $c_1 = c_2$ and distributes the quantities equally among firms, i.e. a central authority chooses $q_1 = q_2 = \frac{a-(c_1+\varepsilon)}{2}$.

Observe that all the above outcomes are supply function equilibrium outcomes.

We now assert that the above mechanism implements ε-socially best outcome with firms revealing their common true cost parameter.

We use the term *ε-socially best outcome* in a sense that the price obtained as a result of the mechanism is at most ε away from the socially best one, i.e. c.

Proof of the assertion: First assume that both c_1 and c_2 are greater than c, the true cost parameter. We have two possible cases, either one of c_1 or c_2 is greater than the other or they are equal. First suppose w.l.o.g. that $c_1 > c_2$. Then $\pi_1(c_1, c_2) = 0$ by the mechanism described, where π_i stands for the profit function of firm i as usual. But then firm 1 has an incentive to deviate to from c_1 to c, in which case its profits will rise from 0 to some positive amount. Next suppose that $c_1 = c_2$. We have $\pi_1(c_1, c_2) = \pi_2(c_1, c_2) = (\frac{a-c_1-\varepsilon}{2})(c_1+\varepsilon-c)$. Then firm 1 has an incentive to deviate from c_1 to $c_1 - \delta$ for some positive δ iff $(\frac{a-c_1-\varepsilon}{2})(c_1+\varepsilon-c) < (c_1-c-\frac{\delta}{2})(a-c_1+\frac{\delta}{2})$, as $(c_1-c-\frac{\delta}{2})(a-c_1+\frac{\delta}{2})$ stands for the profit of firm i after deviating to $c_1-\delta$. Straightforward calculations yield that δ should be an element of the interval $(-(a-c_1+c)-\sqrt{(\varepsilon+(a-2c_1+c))^2-\varepsilon^3+(a-c_1)(c_1-c)}, -(a-c_1+c)+\sqrt{(\varepsilon+(a-2c_1+c))^2-\varepsilon^3+(a-c_1)(c_1-c)})$ for the above inequality to hold. Observe that setting ε sufficiently small (depending on parameters a and c), we can always assure that no symmetric equilibrium where both the coordinates of joint strategy greater than the true cost parameter exist. Non-existence of asymmetric equilibrium of the similar type easily seen by the above observation. Consider the case where strategies of both firms are less than the true cost parameter c. First consider the asymmetric case, i.e. $c_i \neq c_j$ for $\{i, j\} = \{1, 2\}$. Obviously, the firm with the smaller cost declaration has an incentive to deviate to c. Now consider the case where exactly one firm, say firm 1, has c as its reported declaration. Now, if $c_2 > c$, then firm 2 has an incentive to deviate to c, and if $c_2 < c$, then firm 2 has an incentive to deviate to c as well. Consider the case where $c_i < c < c_j$ for $\{i, j\} = \{1, 2\}$. Then firm i has an incentive to deviate to c. All cases except the symmetric $(\widetilde{c}, \widetilde{c})$, where $(\widetilde{c}, \widetilde{c}) \leq (c, c)$ such that $\widetilde{c} + \varepsilon \geq c$, are exhausted. To see that every such $(\widetilde{c}, \widetilde{c})$ is an equilibrium joint strategy simply observe that firm i by deviating to some $\overline{c} < \widetilde{c}$ will get negative profits while at each $(\widetilde{c}, \widetilde{c})$ it gets non-negative profits. Moreover, by deviating to $\overline{c} > \widetilde{c}$ it gets 0 profits, so it has no reason to do so.

Moreover observe that among all these joint equilibrium outcomes truthfully revealing true cost parameter, i.e. (c, c) is a joint profit maximizer and since profits are equal for each equilibrium strategy it also maximizes firm's profits over all such equilibrium outcomes.

Summing up, we demonstrated a mechanism that implements socially best outcome to any degree desired.

Chapter 4

Conclusion

In this study we tried to answer the question of what outcomes are obtained as a result of the delegation game when managers resolve their game according to supply function equilibrium concept. We formally defined our supply function game and analyzed its equilibrium outcomes. While doing so, we did not impose any technical assumptions upon our functions, other than the intuitive "continuity" assumption. One of the contributions of our study is the characterization of supply function equilibrium outcomes of linear symmetric as well as asymmetric duopolies. We obtained that the of equilibrium outcomes formed a continuum for both symmetric and asymmetric duopolies, except one trivial case for an asymmetric duopoly where the equilibrium outcome is unique. These results lead to make some conclusions connecting our study to the delegation literature studied so far. As we mentioned earlier FJS proposed their delegation results as a contribution to positive theory, while KS (1988,1989a,1989b) treated the problem as a regulatory mechanism. The results obtained in this study regarding the equilibrium outcomes of a duopoly are in line with the critiques of KS. Due to the fact that there are an infinite number of equilibrium outcomes one should first make a selection among them in order have a well defined delegation game. This can, for example be accomplished by utilizing a bargaining solutions as a theoretical device, taking us to the world of designing some artificial mechanisms. We explicitly give examples of mechanisms that implement various important outcomes such as collusive one. Moreover, by appropriately designing a regulatory mechanism, we demonstrated that we can implement an outcome, which can be made arbitrarily close to the socially best one via supply function equilibria. It is worth to note that by applying "Pretend-but-Perform" sequentially we can approximate the socially best outcome to any degree desired, which we can also get via supply function equilibria, but now in one shot.

Bibliography

Delgado, J. and D. Moreno, (2004) "Coalition-Proof Supply Function Equilibria in Oligopoly", *Journal of Economic Theory*, 114: 231-257.

Delgado, J. and D. Moreno, (2004) "Coalition-Proof Supply Function Equilibria in Oligopoly", *Journal of Economic Theory*, 114: 231-257.

Fershtman, C., and Judd K.L. , (1987) "Equilibrium Incentives in Oligopoly", *The American Economic Review*, 77: 927-940.

Klemperer, P.D. and M.A. Meyer, (1989) "Supply Function Equilibria in Oligopoly Under Uncertainty", *Econometrica*, 57: 1243-1277.

Koray, S. and M.R. Sertel, (1989) "Regulating Duopoly by a Pretend-but-Perform Mechanism", *European Journal of Political Economy*, 4: 95-115.

Koray, S. and M.R. Sertel, (1989) "Limit Theorems for Recursive Delegation Equilibria", *Research Paper Mathematics*, 89/7, Middle East Technical University, Ankara, June 1989.

Koray, S. and M.R. Sertel, (1989), "Meta-Cournotic Equilibrium in Oligopoly: Positive or Regulatory Theory?", *CARESS Working Paper*, University of Pennsylvania, January 1989.

Sklivas, S.D., (1987) "The Strategic Choice of Managerial Incentives", *Rand Journal of Economics*, 18: 452-458.

Vickers, J., (1985) "Delegation and Theory of the Firm", *The Economic Journal*, 95: 138-147.