EFFECTS OF ENDOGENOUS DEPRECIATION ON THE OPTIMAL TIMING OF TECHNOLOGY ADOPTION

A Master's Thesis

by

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To My Father...

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by

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

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In this thesis, we use two stage optimal control techniques to analyze optimal timing of technology adoption under embodied technical change taking into account the endogenous nature of depreciation. We show that a more efficient maintenance service reducing the depreciation rate postpones the optimal timing of technology adoption. In this respect, we study to what extent can maintenance of the existing capital stock be a substitute for adoption of new technologies.

Keywords: technology adoption, two stage optimal control, maintenance, investment.

ÖZET

BAKIM ONARIM HARCAMALARININ TEKNOLOJİ ADAPTASYONUNUN OPTİMAL ZAMANLAMASINA ETKİSİ

Ramazan Karaşahin
Yüksek Lisans, İktisat Bölümü
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Bu tezde iki aşamalı optimal kontrol tekniklerini kullanarak somutlaştırılmış bir teknik ilerleme ortamında teknoloji adaptasyonunun optimal zamanlaması sorununu, aşınma payının içsel yapısını dikkate alarak çözüyoruz. Aşınma payını düşürmede daha etkin bir bakım onarım servisinin teknoloji adaptasyonunun optimal zamanlamasını ertelediğini gösteriyoruz. Bu bağlamda var olan ekipmanın bakım onarımının yeni teknolojilere yapılacak yatırıma ne derecede bir ikame olabileceğini inceliyoruz.

Anahtar sözcükler: teknoloji adaptasyonu, bakım onarım, yatırım, iki aşamalı optimal kontrol.

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Contents

Chapter 1

INTRODUCTION

Technology adoption has been one of the most important research topics in recent years in macroeconomic studies. With his seminal study, Solow (1956) concluded that the increase in the technological level of the economy is the only way of guaranteeing the long term GDP per capita growth. However the adoption costs have a considerable weight (see Jovanovic, 1997) and this makes technology adoption an interesting research area.

To better understand the costs of technology adoption, consider an economy that realizes technology adoption. In order to adopt the technology, some specific physical and human capital is required. As Parente (1994) states, when technology upgrades, the preexisting human capital has efficiency loss due to the new technology. The labor resources need time in order to gain expertise on the new technology. Such a cost associated with technology adoption has been called learning cost in the existing literature.

Different scholars, considering the benefits and costs of technology adoption, have analyzed the determinants of technology adoption process. Growth advantages of switching, speed of learning and obsolescence costs are shown to be the main determinants of technology adoption by Boucekkine, Saglam and Vallee (2002). Even at the firm level, when the firms demonstrate greater ability in learning, they more frequently adopt higher technologies and will have more market values even if they have lower profitability according to Ahn (2003). The growth rate of the frontier technology level also increases the number of technology adoptions and shortens the durations between these subsequent adoptions according to Saglam (2002).

Besides these factors, with their empirical study on cross-country technology adoption of more than twenty technologies from 1788 to 2001, Comin and Hobjin (2003) state that the most crucial determinants for technology adoption are real income, human capital, adoption of preceding technologies, openness to trade and the type of the government. The real income is also shown to affect technology adoption decision by Khan and Ravikumar (2000). Poorer households, which start with a smaller level of capital stock are found to postpone adoption to later dates.

Nevertheless, in these studies maintenance activities are not taken into account in the technology adoption analysis. Like most of the optimal growth theories, these studies treat depreciation as constant and build their models with this approach. However, there is enough empirical evidence to conclude that the depreciation rate is not constant and affected by different factors (See Storchmann 2004, Gylfason and Zoega 2001, Mullen and Williams 2004). A well-known alternative to constant depreciation is depreciation in use hypothesis, which states that a higher level of economic activity causes a higher depreciation rate. However as Boucekkine, Martinez and Del Rio (2005) state, this hypothesis does not seem to be completely satisfactory. This is the first problem that will be addressed in this paper: How rational is to take the depreciation rates constant in the optimal growth models? Taking into account the embodied nature of technological progress, what are determinants of optimal maintenance and depreciation decisions? In this analysis, rather than using capital utilization to break the constant depreciation assumption, we will concentrate on the fact that depreciation rate depends on heavily the maintenance services. (See Mullen and Williams 2004, Nelson and Caputo 1997, Nickell 1975). In fact, maintenance spending has been a comparable part of the GDP. For instance, McGrattan and Schmitz (1999) find that maintenance spending is around 6 percent of the GDP for Canada where they also claim that maintenance and investment spending are gross substitutes to some extent.

The most trivial connection between the maintenance and adoption is that maintenance activities use the physical and human resources that can be used for adoption or investment. In this sense, they compete for taking the resources existing in the economy. Another reason for connecting adoption and maintenance is that while adopting the higher technology, countries or firms also consider the necessary maintenance spending related with this higher technology. If the new capital stock embodying a superior technology will cause an enormous increase in the maintenance spending, the decision to adopt this superior technology can be discarded. This is the second problem that will be addressed in this paper. We will analyze the role of maintenance in the adoption process and try to establish the mechanism that connects the investment and maintenance activities. Can maintenance of the existing capital stock be a substitute for investment in new technologies? If so, to what extent? Can maintenance services provide an alternative explanation for why are the new technologies adopted after a time lag? These questions become increasingly crucial in understanding the optimal timing of technology adoption if one takes into account the embodied technological progress. As Greenwood, Hercowitz and Krussel (1997) have found, around 60 percent of US productivity growth can be attributed to the embodied technological change.

In this paper, we provide the needed analysis on the role of maintenance in the technology adoption process. Among very few contributions on the role of maintenance in macro theory, Boucekkine, Martinez and Saglam (2001) characterize the balanced growth paths of their models both with and without maintenance options. As a result they find that although the maintenance option increases the technological gap since the labor resources are diverted from adoption, in the long run the output level increases with the maintenance option. Secondly, they find that the equilibrium maintenance and adoption decisions move in opposite directions for different policy or technology shocks.

At the firm level, Ruiz-Tamarit and Boucekkine (2001) concentrate on the relationship between maintenance and investment activities. They conclude that that investment and maintenance activities are gross complements rather than substitutes. However, their analysis does not take into account the embodied nature of technological progress and is not able to address whether maintenance can act as a substitute for the adoption of new technologies.

The organization of the paper is as follows. In Chapter 2, the benchmark model will be intoduced and solved with the two-stage optimal control approach of Tomiyama and Rossana (1989). The solution procedure of two-stage optimal control will also be clearly established in Chapter 2. In Chapter 3, we will extend our model by allowing the frontier technological level to increase throughout time. However, in Chapter 3, due to the complexity of the model, the open form solutions for the optimal adoption timing cannot be reached as the most of optimal adoption timing analyses. Thus, in Chapter 4, numerical analysis and comparative statistics will take place. The thesis will end with a brief conclusion.

Chapter 2

BENCHMARK MODEL

Our economy consists of a representative agent which has an intertemporal utility function

$$\int_0^\infty u(C(t))e^{-\rho t}dt.$$

Here $u(t)$ is assumed to be increasing and strictly concave. Throughout the paper, we will not analyze the labor dynamics, so for all of the paper we normalize the population to 1 and assume that there is no population growth. Our time horizon is infinite and we also have the discounting parameter ρ as usual. For the consumption sector, our production function is Ak where k(t) denotes the capital stock at time t and A denotes the overall efficiency in the production. The produced good can be used in three alternative ways: it is either consumed, used for investment or used for maintenance activities(We skip time subscripts wherever there is no confusion):

$$Y = C + I + M = F(k) = Ak$$

The capital stock in the overall economy evolves according to the following equation:

$$\dot{k} = qI - \delta k = (qi - \delta)k.$$

From this point on the lowercase i and m will denote the investment and maintenance spending per capital stock. In the capital stock evolution function q measures the efficiency of the investment and the technological progress is embodied with this variable. Unlike A, a rise in this variable will only affect the newest capital goods and investment specific. For the capital stock, we assume that the initial level of capital stock $k(0) = K0$ is given. The depreciation rate is affected by the maintenance spending throughout the following function:

$$\delta(t) = \bar{\delta} - am(t)^b.$$

Here $\bar{\delta}$ represents the maximum level of depreciation when there is no maintenance throughout the economy. Both a and b are positive variables and they together represent the efficiency of the maintenance spending. With increasing the maintenance spending, the depreciation rate can be decreased and by this way the capital stock on hand will be prevented from getting useless for the production.

For our problem, there are two options for the representative agent: (A_1, q_1) and (A_2, q_2). The level of technology which the economy has at initial point is q_1 and the available technology level for the agent to jump is q_2. Obviously, q_2 needs to be greater than q_1 for this technology adoption problem. This simply means that by switching to the new regime, the investment spending will be more efficient in terms of increasing production. However, to incorporate the expertise loss due to adopting a new technology, the marginal productivity of the capital in the new regime A_2 is smaller than the marginal productivity of the capital in the first regime A_1. Thus, the tradeoff between higher technologies but less efficiency is well reflected via this model.

Now, assume that the economy switches at date t_1 to the new technology regime, obviously the state equation for the capital stock will be different before t_1 and after t_1. Before adopting the new technology, i.e. when $0 \leq t \leq t_1$ the evolution will be:

$$\dot{k}(t) = [q_1 i(t) - \delta(t)]k(t) \tag{2.1}$$

On the other hand, after adopting the technology, i.e. when $t_1 \leq t$ the evolution of the capital stock will be:

$$\dot{k}(t) = [q_2 i(t) - \delta(t)]k(t) \tag{2.2}$$

As we see, there is a tradeoff between two consecutive regimes. The marginal productivity of the investment is higher in the higher technology regime, however as the marginal productivity of the capital stock decreases, the total production thus the resources that can be devoted to investment is negatively effected. After building up the model, in the next subsection we will briefly sketch the two-stage optimal control technique.

2.1 Two-stage optimal control approach

The problem of the central planner can be summarized as:

$$\max_{i,m,t_1} \int_0^\infty u(c(t))e^{-\rho t}dt.$$

Subject to (2.1), (2.2) and K_0 given. When we carefully look at the problem, it is obvious that the objective function can be rewritten as:

$$U(i,m,t_1) = \int_0^{t_1} u(c(t))e^{-\rho t}dt + \int_{t_1}^\infty u(c(t))e^{-\rho t}dt \tag{2.3}$$

Since the original problem's objective function can be separated into two periods and the welfares of the two possible regimes can be added to find the optimal welfare of the original problem, two-stage optimal control approach can be directly applied as it is clearly suitable for our case. The application of two-stage approach first divides the problem into two stages and operates as following:

1) The second stage problem: We start by assuming the economy realizes the switch at time t_1 and the initial capital stock at time t_1 is given as $k(t_1) = K_1$. For this stage the problem is to maximize the total welfare obtained from this period, i.e. $U_2(K_1,t_1) = \int_{t_1}^\infty u(c(t))e^{-\rho t}dt$, with subject to the state equation 2.2. For this stage let us denote the co-state variable as $\lambda_2(t)$, the corresponding Hamiltonian as $H_2(k,i,m,t,\lambda_2) = -u(c(t))e^{-\rho t} + \lambda_2(t)[q_2 i(t) - \delta(t)]k(t)$. Assume there exists a maximum for this problem. Let us denote the maximum welfare obtained in this stage as $U_2^*(K_1,t_1)$ and the minimum Hamiltonian as $H_2^*(K_1,t_1)$ for the given K_1 and t_1.

2) The first stage problem: Now we turn to the first stage of the model and maximize $U(C, t_1) = \int_0^{t_1} u(c(t))e^{-\rho t}dt + U_2^*(K_1, t_1)$ with subject to equation (2.1), K_0 given and finally $K(t_1) = K_1$ free. To solve this problem we first need to solve the first stage Pontryagin problem for fixed t_1 and K_1. We will denote the co-state variable as $\lambda_1(t)$ and the Hamiltonian as $H_1(k, i, m, t, \lambda_1) = -u(c(t))e^{-\rho t} + \lambda_1(t)[q_1 i(t) - \delta(t)]k(t)$. The resulting minimum Hamiltonian will be denoted as $H_1^*(K_1, t_1)$. The most essential point of the technique is linking the first stage and the second stage. For interior solutions and assuming $U_2(K_1, t_1)$ is twice continuously differentiable in K_1 and t_1, the optimal K_1^* and t_1^* should satisfy the following equations:

$$\lambda_1(t_1^*) = \lambda_2(t_1^*) \tag{2.4}$$

$$H_1^*(K_1^*, t_1^*) = H_2^*(K_1^*, t_1^*) \tag{2.5}$$

3) For all of the models we study hereafter, $U_2(K_1, t_1)$ is twice continuously differentiable in K_1 and t_1. However, although this assumption is satisfied, it can be the case that equations (2.4) and (2.5) has no solution for a t_1 satisfying $0 \le t_1$. In this case there can be two corner solutions possible: immediate switching and technological sclerosis. If, $H_1^*(K_0, 0) \ge H_2^*(K_0, 0)$, $t_1^* = 0$ and the economy immediately switches to the new technology regime. On the other hand, if $H_1^*(K_0, 0) < H_2^*(K_0, 0)$ for every $t_1 \ge 0$, the economy never switches to the new technology regime and technological sclerosis occurs.

Here we need to add that linking the first stage and second stage, i.e. the optimality condition changes when adoption timing t_1 explicitly affect the evolution of capital stock or not. In other words, if t_1 exists in any of equations (2.1) and (2.2), then equation (2.5) is not used to find the optimal adoption timing. Instead we use:

$$\frac{\partial U_2^*(K_1, t_1)}{\partial t_1} = H_1^*(K_1, t_1) + \int_0^{t_1} \frac{\partial H_1^*}{\partial t_1} dt \tag{2.6}$$

This equation is also same as the following equation by Tomiyama and Rossana(1989):

$$H_2^*(K_1, t_1^*) - H_1^*(K_1, t_1^*) = \int_0^{t_1} \frac{\partial H_1^*}{\partial t_1} dt + \int_{t_1}^{\infty} \frac{\partial H_2^*}{\partial t_1} dt \tag{2.7}$$

After clearly introducing the two-stage optimal control procedure, in the next subsection we will apply this methodology by not getting stuck on the algebraic details.

2.2 Solving the model

We will closely apply the approach defined above for our model. For making the calculations easier, we will take the logarithmic utility function. We start by considering the second stage problem:

$$\max_{i,m} U_2(c, t_1) = \int_{t_1}^{\infty} \ln(c(t)) e^{-\rho t} dt \tag{2.8}$$

Subject to K_1 given and state equation (2.2). For this problem Hamiltonian is $H_2(k, i, m, t, \lambda_2) = -\ln(c(t))e^{-\rho t} + \lambda_2[q_2 i(t)k(t) - \delta(m(t))k(t)]$ and we can directly write first order conditions:

$$H_i(k, i, m, t, \lambda_2) = \frac{e^{-\rho t} k(t}{c(t)} + \lambda_2(t) q_2 k(t) = 0 \tag{2.9}$$

$$H_m(k, i, m, t, \lambda_2) = \frac{e^{-\rho t} k(t)}{c(t)} - \lambda_2(t) k(t) \delta'(m(t)) = 0 \tag{2.10}$$

$$-\dot{\lambda}_2(t) = H_k(k, i, m, t, \lambda_2) = \lambda_2(t)[q_2 m(t) + \delta(m(t)) - q_2 A_2] \tag{2.11}$$

$$\dot{k}(t) = H_\lambda(k, i, m, t, \lambda_2) = [q_2 i(t) - \delta(m(t))]k(t) \tag{2.12}$$

And the transversality condition:

$$\lim_{t \to \infty} \lambda_2(t) k(t) = 0 \tag{2.13}$$

When we solve (2.9) and (2.10) together, we finally reach that:

$$\delta'(m(t)) = -q_2 \;\Rightarrow\; m(t) = \big(\frac{ab}{q_2}\big)^{\frac{1}{1-b}} \;\Rightarrow\; \delta(m(t)) = \bar{\delta} - a\big(\frac{ab}{q_2}\big)^{\frac{b}{1-b}}$$

Here we see that for the whole period, in this framework the optimal resources allocated to maintenance, thus the depreciation rate does not change and equal to the above values. Then we denote these constant maintenance and depreciation rate as m_2 and δ_2 respectively. The integration of the necessary conditions from (2.9) to (2.12) yield the following results:

$$k(t) = \frac{e^{-(q_2m_2+\delta_2-q_2A_2+\rho)t}\big(1 - e^{\rho(t-t_1)}\big)}{-a_0\rho} + K_1e^{-(q_2m_2+\delta_2-q_2A_2)(t-t_1)} \tag{2.14}$$

$$c(t) = -\frac{1}{q_2a_0}e^{-(q_2m_2+\delta_2-q_2A_2+\rho)t} \tag{2.15}$$

$$\lambda_2(t) = a_0e^{(q_2m_2+\delta_2-q_2A_2)t} \tag{2.16}$$

$$i(t) = A_2 - m_2 - \frac{c(t)}{k(t)} \tag{2.17}$$

By using transversality condition we find the constant of integration as:

$$a_0 = -\frac{1}{K_1\rho}e^{-(q_2m_2+\delta_2-q_2A_2+\rho)t_1} \tag{2.18}$$

By incorporating this constant of integration to the equations, we can find the optimal welfare as:

$$U_2^*(K_1, t_1) = \frac{e^{-\rho t_1}}{\rho}\big(\ln[\frac{K_1\rho}{q_2}] - \frac{q_2m_2 + \delta_2 - q_2A_2 + \rho}{\rho}\big) \tag{2.19}$$

As we see the optimal value of the objective function is twice differentiable both with respect to K_1 and t_1. After solving the second stage, we need to turn to the first stage and solve the following problem:

$$\max_{i,m,t_1}U_1(C, t_1) = \int_0^{t_1} \ln(c(t))e^{-\rho t}dt + U_2^*(K_1, t_1)$$

Obviously subject to state equation (2.1), K_0 given and $K_1 = k(t_1)$ free. We first treat the problem as K_1 and t_1 fixed. This allows us to write the first order conditions of the problem straightforwardly as in the second stage. Solution of these first order conditions yield:

$$\delta'(m(t)) = -q_1 \;\Rightarrow\; m_1 = \Big(\frac{ab}{q_1}\Big)^{\frac{1}{1-b}} \;\Rightarrow\; \delta_1 = \bar{\delta} - a\Big(\frac{ab}{q_1}\Big)^{\frac{b}{1-b}} \tag{2.20}$$

$$k(t) = \frac{e^{-(q_1 m_1 + \delta_1 - q_1 A_1 + \rho)t}(1 - e^{\rho t})}{-\bar{a}_0 \rho} + K_0 e^{-(q_1 m_1 + \delta_1 - q_1 A_1)t} \tag{2.21}$$

$$c(t) = -\frac{1}{q_1 \bar{a}_0} e^{-(q_1 m_1 + \delta_1 - q_1 A_1 + \rho)t} \tag{2.22}$$

$$\lambda_1(t) = \bar{a}_0 e^{(q_1 m_1 + \delta_1 - q_1 A_1)t} \tag{2.23}$$

$$i(t) = A_1 - m_1 - \frac{c(t)}{k(t)} \tag{2.24}$$

After this point, for finding the optimal t_1 value, we will use the equations (2.4) and (2.5). By using equation (2.4), we find the constant of integration for the first stage as:

$$\bar{a}_0 = \frac{e^{-(q_1 m_1 + \delta_1 - q_1 A_1 + \rho)t_1}}{K_1 \rho}$$

Using this result we find the consumption, investment, co-state and capital stock levels for the first period and t_1. Finally in order to determine the optimal t_1, we use equation (2.5) and find:

$$H_2^*(K_1, t_1) - H_1^*(K_1, t_1) = 0 \;\Rightarrow\; \rho \ln\Big(\frac{q_2}{q_1}\Big) + [q_1 A_1 - q_2 A_2] + [q_2 m_2 + \delta_2 - q_1 m_1 - \delta_1] = 0$$

From these results it is obvious that there is no interior solution for the model since in the equation t_1 does not exist. Therefore under this framework there can be only corner solutions. If we analyze under which conditions the corner solutions arise, as we stated in the explanation of the two-stage optimal control technique, the solution is either immediately switching or technological sclerosis. If $\rho \ln(\frac{q_2}{q_1}) + [q_1 A_1 - q_2 A_2] + [q_2 m_2 + \delta_2 - q_1 m_1 - \delta_1] \leq 0$, the economy immediately switches to the new technology. Otherwise, the economy sticks to the old technology and does not switch.

Let us interpret this analytical result. The increase in the embodied technology level results with a decrease in the relative price of capital at the date of switch in our model. Thus the consumption goods have higher relative prices in the second stage and the economy reallocates the resources by increasing capital stock and decreasing consumption at the date of switch. This consumption decrease also causes the welfare decrease and this can be expressed as $\rho \ln(\frac{q_2}{q_1})$ since the utility function is logarithmic here. This cost can be called as obsolescence cost.

On the other hand, when the economy switches to new regime although the expertise level decreases, the efficiency of capital goods can compensate this loss and even can supply the economy a greater growth rate. The growth advantage of switching can be expressed as $q_2 A_2 - q_1 A_1$.

Besides the obsolescence cost ($\rho \ln(\frac{q_2}{q_1})$) and growth advantage of switching ($q_2 A_2 - q_1 A_1$) we have the additional term ($q_2 m_2 + \delta_2 - q_1 m_1 - \delta_1$) when compared with Boucekkine, Saglam and Vallee (2002). When we consider this term carefully, we see that in fact this term reflects the comparative or additional maintenance cost in terms of capital goods when the switching is realized. When the economy switches and realizes the optimal maintenance spending, the spending in terms of capital goods is $q_2 m_2$, since instead of spending m_2 units of consumption good, the economy could have increased the capital stock by $q_2 m_2$ with investing this spending. Also, with switching the capital stock in the economy depreciates with δ_2 and this is also in terms of capital goods. Thus ($q_2 m_2 + \delta_2 - q_1 m_1 - \delta_1$) reflects the comparative maintenance cost of switching. As a result, the economy switches if and only if the growth advantage of switching exceeds the obsolescence cost and comparative maintenance cost of switching.

Together with this result, we also have an important result that will stay unchanged throughout our models. The relationship between maintenance spending and the technology level of the economy can be summarized as:

$$m(t) = \left(\frac{ab}{q_t}\right)^{\frac{1}{1-b}}$$

This equation shows the relationship between the maintenance spending m_t and the technology level q_t. With this equation we reach the following result. If the depreciation function is convex with respect to maintenance, i.e. $b < 1$, as accepted by Boucekkine, Saglam and Martinez 2002 (this is the dominant view of the existing literature), then increase in the technology level is followed by a decrease in the maintenance spending since the derivative of maintenance with respect to technology level is negative then. However, if the depreciation function is concave, then they are positively correlated.

This result is extremely important since this issue has gained a great attention especially in the empirical studies. It confirms Storchmann's finding that the developed countries have a greater depreciation level than the undeveloped countries. The technology level is much greater in developed countries and this increases the maintenance costs in terms of investment. Thus, instead of maintaining their old cars, they buy new cars and with this obsolescence effect, the depreciation gets higher values in developed countries.

Now in order to find an interior solution and analyze the factors affecting the optimal timing of adoption, in the next section we will extend our benchmark model by incorporating technological progress.

Chapter 3

INCREASING FRONTIER TECHNOLOGY CASE

In our benchmark model, we started with assuming a constant level of higher technology for switching. Absolutely, in real life the situation is quite different. The frontier level of technology also increases with time and the economy can jump to a higher level of technology by waiting.

To include this reality in our model, we assume that the frontier level of technology increases with a constant speed γ, thus the available level of technology the economy can switch equals $e^{\gamma t}$. Thus in this case, the evolution equation of the capital stock in the second stage of this model is:

$$\dot{k}(t) = [e^{\gamma t_1} i(t) - \delta(t)]k(t) \tag{3.1}$$

Other assumptions and variables also continue to be valid in this enlarged model. To see the effects of the maintenance option, in the next subsection we firstly solve the model without maintenance option and then incorporate the maintenance to our model.

3.1 Without Maintenance Control

When there is no maintenance, the depreciation is constant at $\bar{\delta}$. Moreover, without maintenance spending, the resources can be allocated for consumption and investment. Thus, the resource allocation constraint is:

$$Y = C + I = F(k) = Ak$$

After taking into consideration these changes in the benchmark model, we again start with the second stage. The corresponding Hamiltonian is:

$$H_2(k,c,\lambda_2,t) = -\ln(c(t))e^{-\rho t} + \lambda_2[e^{\gamma t_1}i(t)k(t) - \bar{\delta}k(t)] \tag{3.2}$$

The solution of the first order conditions and doing the necessary algebraic manipulations we obtain:

$$c(t) = -\frac{1}{\lambda_0 e^{\gamma t_1}} e^{-(\bar{\delta}-e^{\gamma t_1}A_2+\rho)t} \tag{3.3}$$

$$k(t) = e^{-(\bar{\delta}-e^{\gamma t_1}A_2)t}\big[K_1 e^{(\bar{\delta}-e^{\gamma t_1}A_2)t_1} - \frac{e^{-\rho t}-e^{-\rho t_1}}{\lambda_0\rho}\big] \tag{3.4}$$

$$\lambda_2(t) = \lambda_0 e^{(\bar{\delta}-e^{\gamma t_1}A_2)t} \tag{3.5}$$

When we solve the transversality condition, we find the constant of integration as:

$$\lambda_0 = -\frac{e^{-(\bar{\delta}+\rho-e^{\gamma t_1}A_2)t_1}}{K_1\rho} \tag{3.6}$$

We replace this constant of integration and obtain the optimal consumption, investment, capital stock, maintenance and total welfare obtained in the second period. Later on, we turn to the first stage. As in section 2, we start by assuming K_1 and t_1 fixed. This allows us to treat the first stage problem as an ordinary Pontryagin problem. Writing the relevant first order conditions and integrating them gives us the following results for the first stage problem:

$$c(t) = -\frac{1}{\bar{\lambda}_0 q_1} e^{-(\bar{\delta}-q_1A_1+\rho)t} \tag{3.7}$$

$$k(t) = e^{-(\bar{\delta}-q_1A_1+\rho)t}\big[\frac{-1+e^{\rho t}(1+K_0\rho\bar{\lambda}_0)}{\rho\bar{\lambda}_0}\big] \tag{3.8}$$

$$\lambda_1(t) = \bar{\lambda}_0 e^{(\bar{\delta} - q_1 A_1)t} \tag{3.9}$$

Now we need to apply the continuity and optimality conditions. The continuity condition is same as the Section 2. Co-state variable and the capital stock cannot have a distinct jump at t_1. Thus considering this continuity condition and using the results we found for the two stages, we find:

$$\bar{\lambda}_0 = -\frac{e^{-(\bar{\delta} - q_1 A_1 + \rho)t_1}}{K_1 \rho} \tag{3.10}$$

$$K_1 = K_0 e^{-(\rho + \bar{\delta} - q_1 A_1)t_1} \tag{3.11}$$

Finally, in order to find the optimal timing of adoption, we need to apply optimality condition. However, the optimality condition we will apply in this model is different than Section 2. To understand this optimality condition change, let us consider the evolution equation of the capital stack in the second period for the benchmark model and this model respectively:

$$\dot{k}(t) = [q_2 i(t) - \delta(t)]k(t) \tag{3.12}$$

$$\dot{k}(t) = [e^{\gamma t_1} i(t) - \delta(t)]k(t) \tag{3.13}$$

As we see, in the benchmark model the capital stock evolution does not depend on t_1. On the other hand, equation (3.13) has t_1 and here the capital stock evolution is not independent of t_1. In other words, t_1 explicitly effects capital stock evolution and as Tomiyama and Rossana (1989) states, we need to apply equation (2.7) as the optimality condition.

However, when we try to solve this equation we obtain that the optimal t_1 should satisfy the following equation:

$$\frac{e^{-\rho t_1}\left(2A_2 e^{\gamma t_1}(\gamma - \rho) + \rho^2 \ln[e^{-t_1(-A_1 + \rho + \bar{\delta})} K_0 \rho] - \rho(-2A_1 + \gamma + \rho \ln[K_0 \rho e^{-t_1(\rho + \bar{\delta} + \gamma - A_1)}])\right)}{\rho^2} = 0 \tag{3.14}$$

Obviously, this equation has no algebraic solution and thus we need to do numerical analysis and try to see the effects of maintenance and other variables to adoption timing. However, we can prove the existence of a t_1 satisfying this equation.

Proposition 3.1.1 *Assume $A_2 < A_1$ and $\gamma < \rho < A_2$. If we normalize $q_1 = 1$, then there exists a t_1 satisfying equation (3.14).*

Proof: When we consider left side of equation (3.14), we see a continuous function of t_1. When t_1 is 0, this function takes the value $\frac{(2A_1-2A_2)\rho+(2A_2-\rho)\gamma}{\rho^2}$ and this value is greater than 0 under these assumptions. When t_1 is not equal to 0, this function becomes $e^{-\rho t_1}\left(\frac{(2A_2e^{\gamma t_1}(\gamma-\rho)+2A_1\rho-\gamma\rho+\rho^2\gamma t_1}{\rho^2}\right)$. Obviously $e^{-\rho t_1}$ and ρ^2 are always positive. On the other hand while t_1 goes to infinity, $(2A_2e^{\gamma t_1}(\gamma-\rho)+2A_1\rho-\gamma\rho+\rho^2\gamma t_1$ goes to minus infinity. Thus for sufficiently great values of t_1, the function takes negative values. Following intermediate value theorem we can conclude that there exists a t_1 where the function takes the value 0 and obviously for this value of t_1, optimality equation (3.14) is satisfied.

□

To visualize the proof assume $\gamma = 0.02$, $\rho = 0.04$, $A_1 = 1$ and $A_2 = 0.8$. Now we need to have $2A_2e^{\gamma t_1}(\gamma-\rho)+2A_1\rho-\gamma\rho+\rho^2\gamma t_1 = 0$. Here $\rho^2\gamma t_1$ refers to obsolescence cost and $(2A_2e^{\gamma t_1}(\rho-\gamma)-2A_1\rho+\gamma\rho$ refers to growth advantage. In the optimal time of adoption, growth advantage needs to start exceeding obsolescence cost. When we draw the graph, we see that at 0, growth advantage is below obsolescence cost but however as time passes growth advantage increases faster.

— Insert Figure 1 Here—

In the next section we will include maintenance and obtain the condition for optimal timing.

3.2 With Maintenance Control

Unlike section 3.1, the model we will solve when there is maintenance option has the only difference as replacing equation (3.13) instead of equation (3.12). As defined in the two-stage technique, we start with the second period. The corresponding Hamiltonian is:

$$H_2(k,i,m,\lambda_2) = -\ln(c(t))e^{-\rho t} + \lambda_2[e^{\gamma t_1} i(t)k(t) - \delta(m(t))k(t)] \tag{3.15}$$

Directly writing the first order conditions and doing the necessary algebraic operations yield the following results for the second stage:

$$c(t) = -\frac{1}{\lambda_0 e^{\gamma t_1}} e^{-(e^{\gamma t_1} m_2 + \delta_2 - e^{\gamma t_1} A_2 + \rho)t} \tag{3.16}$$

$$k(t) = e^{-(e^{\gamma t_1} m_2 + \delta_2 - A_2 e^{\gamma t_1})t} \big[K_1 e^{(e^{\gamma t_1} m_2 + \delta_2 - e^{\gamma t_1} A_2)t_1} - \frac{e^{-\rho t} - e^{-\rho t_1}}{\rho \lambda_0}\big] \tag{3.17}$$

$$\lambda_2(t) = \lambda_0 e^{(e^{\gamma t_1} m_2 + \delta_2 - e^{\gamma t_2} A_2)t} \tag{3.18}$$

Incorporating the transversality condition, we find the constant of integration as:

$$\lambda_0 = -\frac{e^{-(e^{\gamma t_1} m_2 + \delta_2 - e^{\gamma t_1} A_2 + \rho)t_1}}{K_1 \rho} \tag{3.19}$$

After solving the second stage problem, now we turn to the first stage problem. With again taking t_1 and K_1 fixed, we take the first stage problem as ordinary Pontryagin problem and directly write the first order conditions. Integrating these conditions give us the following results for the first stage:

$$c(t) = -\frac{1}{\bar{\lambda}_0 q_1} e^{-(q_1 m_1 + \delta_1 - q_1 A_1 + \rho)t} \tag{3.20}$$

$$k(t) = e^{-(q_1 m_1 + \delta_1 - q_1 A_1 + \rho)t} \big[\frac{-1 + e^{\rho t}(1 + K_0 \rho \bar{\lambda}_0)}{\rho \bar{\lambda}_0}\big] \tag{3.21}$$

$$\lambda_1(t) = \lambda_0 e^{(q_1 m_1 + \delta_1 - q_1 A_1)t} \tag{3.22}$$

After finding these results, now we need to apply continuity and optimality conditions to find the optimal t_1. As usual the continuity condition states that the capital stock and co-state variable can not have a distinct jump at adoption time. Applying this condition gives us the following:

$$\bar{\lambda}_0 = -\frac{e^{(q_1 m_1 + \delta_1 - q_1 A_1 + \rho)t_1}}{K_1 \rho} \tag{3.23}$$

$$K_1 = K_0 e^{-(q_1 m_1 + \delta_1 - q_1 A_1 + \rho)t_1} \tag{3.24}$$

Finally to find the optimal adoption timing, we need to apply optimality condition. Solving equation (2.7), we find the condition that optimal t_1 needs to satisfy as:

$$\begin{aligned}
&\frac{1}{-b\rho^2}\Big[e^{-\rho t_1}\big(-(1-b)(\frac{ab}{q_1})^{\frac{1}{1-b}} q_1\rho - b\rho(A_1 q_1 - \gamma) - A_2 b e^{\gamma t_1}(\gamma-\rho) \\
&+ e^{\gamma t_1}(abe^{-\gamma t_1})^{\frac{1}{1-b}}(\rho - b\rho + b\gamma) - b\rho^2(-\ln\big[K_0 \rho e^{-t_1(a(1-b)(\frac{ab}{q_1})^{\frac{b}{1-b}} - A_1 q_1 + \rho + \delta + \gamma)}\big] \\
&+ \ln\big[e^{-t_1(a(1-b)(\frac{ab}{q_1})^{\frac{b}{1-b}} - A_1 q_1 + \rho + \delta)}\frac{K_0\rho}{q_1}\big])\big)\Big] = 0
\end{aligned} \tag{3.25}$$

Again this equation can not be solved algebraically and we need to do numerical analysis. However, before proceeding to numerical analysis, we need to show the existence of the optimal t_1.

Proposition 3.2.1 *Assume $\gamma < \rho$, $A_2 \leq A_1$ and $\rho + (ab)^{\frac{1}{1-b}} \leq A_2$. Moreover assume $b < 1$, i.e. depreciation function is convex. If we normalize $q_1 = 1$, then there exists a $t_1 > 0$ satisfying equation (3.25).*

Proof: When we consider the left side of equation (3.25), we see a continuous function of t_1. When t_1 equals 0, this function takes the value:

$$-\frac{1}{b\rho^2}[-b\rho(A_1 - A_2) - b\gamma(A_2 - \rho - (ab)^{\frac{1}{1-b}})]$$

and under these assumptions this value is greater than 0. When t_1 does not equal to 0, with algebraic manipulations it can be shown that the left hand side becomes:

$$-\frac{1}{b\rho^2}[-A_2b(\gamma-\rho)e^{\gamma t_1}+e^{\frac{-b\gamma t_1}{1-b}}(ab)^{\frac{1}{1-b}}(\rho-b\rho+b\gamma)-b\rho^2\gamma t_1-(1-b)(ab)^{\frac{1}{1-b}}\rho].$$

Obviously, $-\frac{1}{b\rho^2}$ is always negative. On the other hand, with given assumptions:

$$\lim_{t_1\to\infty}[-A_2b(\gamma-\rho)e^{\gamma t_1}+e^{\frac{-b\gamma t_1}{1-b}}(ab)^{\frac{1}{1-b}}(\rho-b\rho+b\gamma)-b\rho^2\gamma t_1-(1-b)(ab)^{\frac{1}{1-b}}\rho]=+\infty$$

This simply means that the function takes negative values for sufficiently great values of t_1. Following intermediate value theorem, we can conclude that there exists a t_1 satisfying optimality condition equation (3.25).

□

To visualize the proof assume $A_1 = 1$, $A_2 = 0.8$, $q_1 = 1$, $\gamma = 0.02$, $\rho = 0.04$, $a = 0.117$ and $b = 0.24$. When we closely look at (3.25), we see that for existence of t_1, growth advantage of switching at t_1, $(e^{\gamma t_1}A_2b(\rho-\gamma)-(q_1A_1-\gamma)b\rho)$ must exceed the sum of obsolescence cost $(b\rho^2\gamma t_1)$ and additional maintenance cost $(q_2m_2(\rho-b\rho+b\gamma)-q_1m_1(1-b)\rho)$. When we graph, we see that although at time 0 the growth advantage is smaller than this sum, as the time passes it exceeds this sum.

— Insert Figure 2 Here—

The numerical analysis and comparative statistics for the models with and without maintenance control will be done in the next chapter.

Chapter 4

NUMERICAL ANALYSIS

Since up to this point, analytical analysis does not yield a basis for analyzing the comparative statistics, we need to study numerically. For the benchmark case, we use Boucekkine, Martinez and Saglam (2003) numerical analysis.

$\bar{\delta}$ is taken as 0.12. This value reflects the depreciation rate in the capital stock when there is no maintenance activity. This value is in the range of the values for depreciation rates used by the existing literature. a and b are taken as 0.117 and 0.24 respectively. These values satisfy the following requirements:

i)$\delta(m(t)) > 0$, $\delta'(m(t)) < 0$ and $\delta''(m(t)) > 0$

ii)$\lim_{m\to 0} \delta(m) = \bar{\delta}$

Obviously, technology acceleration parameter cannot be greater than the discount rate since otherwise technology adoption is delayed till infinity. We took γ as 0.02, ρ as 0.04, q_1 and k_0 as 1. In most of the literature the discount rate is taken as 0.04 and also the frontier technology acceleration parameter is taken smaller than this value.

While finding the optimal technology adoption timing, one thing to be kept in mind is that whether this timing minimizes or maximizes the total welfare. For this reason we also calculated the total welfare when the technology adoption is realized at t_1. The optimal t_1 must maximize this total welfare. Our results are consistent since they maximize the total welfare and second derivatives are negative. To visualize, assume $A_1 = 1$, $A_2 = 0.8$, $q_1 = 1$, $\gamma = 0.02$, $\rho = 0.04$, $a = 0.117$ and $b = 0.24$. Then we find optimal t_1 as 46.87 and when the total welfare with respect to adoption timing is considered this value maximizes the total welfare and second derivative is negative.

— Insert Figure 3 Here—

After finding the optimal adoption timing as t_1 and also calculating the optimal welfare, we applied different changes in the parameters. In table 1, the effects of an increase in the frontier technology acceleration parameter; in table 2, the effects of production efficiency change or learning cost change; in tables 3 and 4, the effects of a depreciation function or maintenance efficiency changes; in table 5, the effects of a initial capital stock level and in table 6, the effects of discount parameter change are presented in Appendix. Main results driven out of these simulations can be interpreted as following:

Maintenance Option: Maintenance option increases the total welfare (the value of the objective function, (U) in all cases. The overall economy reaches a greater level of satisfaction when the maintenance option is used efficiently with regarding all of the constraints.

With and Without Maintenance: The technology adoption timing t_1 is delayed when there is maintenance option in all cases. The economy adopts the technology in later times by allocating resources to maintenance of existing capital stock and by this way can increase the benefits of adopting a higher technology in a near future. Analytically, additional maintenance cost of switching $q_2m_2 + \delta_2 - q_1m_1 - \delta_1$ delays the adoption timing and makes the economy wait in order to have a greater growth advantage $q_2A_2 - q_1A_1$ exceeding obsolescence cost and additional maintenance cost.

Efficiency of Maintenance: When the depreciation function parameters a and b change in order to increase the efficiency of maintenance activity on decreasing depreciation, the technology adoption time t_1 is delayed. Although the delaying time is so small as it can be seen from tables 3 and 4, it is certain. Analytically, when the maintenance activities become more efficient on decreasing depreciation rates, optimal maintenance spending increases and again additional maintenance cost $q_2m_2 + \delta_2 - q_1m_1 - \delta_1$ tends to increase. This causes the economy wait until growth advantage compensates this increase. Again, the total welfare increases as a result of this increase in the efficiency of the maintenance activity.

Initial Capital Stock: The initial level of capital stock does not change the optimal technology adoption timing whereas any increase in the initial level of capital stock yields a greater level of total welfare certainly. In fact when we carefully consider the sufficient condition for the optimal technology adoption timing, there does not exist the initial capital stock level.

Discount Rate: Any increase in the discount rate makes the consumption in the future valueless and thus pulls the technology adoption timing back. Moreover, it decreases the total welfare as expected.

Technology Acceleration: Surprisingly, any technology acceleration parameter increase does not result with an adoption timing decrease. In smaller values of γ, an increase in the technological progress rate results in decrease in the adoption timing. Instead of waiting, the economy can reach same level or greater level of technology in a relatively shorter time with adopting the technology sooner. However, as the technological progress rate γ increases and gets closer to the discount rate, the economy delays the adoption mainly for two reasons. Firstly, as the adoption timing gets smaller, the maintenance spending increase and the consumption, thus the total welfare decreases. Secondly, the economy can reach much greater levels of technology by waiting and the cost of waiting $(\rho - \gamma)$ decreases as the technology progress rate gets closer to the discount rate.

Analytically, obsolescence cost is linearly associated with respect to γ since $\rho \ln(\frac{q_2}{q_1}) = \rho \ln(\frac{e^{\gamma t_1}}{q_1}) = \rho\gamma t_1$ when q_1 is normalized to 1. On the other hand both the growth advantage and additional maintenance cost include $e^{\gamma t_1}$. Thus the optimal adoption timing can be thought as the time where the difference of growth advantage than additional maintenance cost starts to exceed obsolescence cost. In small values of γ, an increase in γ causes a greater increase in the linearly associated obsolescence cost than the difference of growth advantage than maintenance cost. This causes optimal adoption timing decrease. However, in great values of γ, an increase in γ causes a greater increase in the exponentially associated difference of growth advantage than maintenance cost and this causes optimal adoption timing increase.

Actually this means that there exists a threshold level for optimal technology adoption timing associated with the technology acceleration parameter. After the optimal timing decreases below a threshold level as a result of increasing technology acceleration, the adoption timing starts to increase with the increase in technology acceleration. The following proposition gives the sufficient condition for the adoption timing to increase in the without maintenance control framework.

Proposition 4.0.2 *Assume $A_1 > \rho + \gamma$. Normalize $q_1 = 1$. Then, when the optimal technology adoption timing satisfies $t_1^* < \frac{1}{\rho-\gamma}$, the adoption timing increases when the technology acceleration parameter γ increases.*

Proof: When we consider the optimality condition equation (3.14), this equation gives us t_1 as an implicit function of γ. Doing the necessary algebraic manipulations, we find that $\frac{\partial t_1}{\partial \gamma} \geq 0$ when the condition $t_1 \leq \frac{1}{\rho-\gamma} + \frac{\gamma}{(\rho-\gamma)(A_1-\rho-\gamma+\rho\gamma t_1)}$ is met. With the given assumptions, it is sufficient for t_1 to satisfy $t_1 \leq \frac{1}{\rho-\gamma}$ in order to make $\frac{\partial t_1}{\partial \gamma} \geq 0$ which means an increase in γ will yield an increase in t_1.

□

Finally, we need to analyze the relationship between maintenance and investment activities. For this reason, we must consider an increase in the technology level. By algebraic manipulations one can reach the following relationship between the investment in the second stage and the switched technology level:

$$i_2(t) = A_2 - m_2 - \frac{\rho}{q_2} \tag{4.1}$$

We know that $\frac{\partial m_2}{\partial q_2} < 0$ from the beginning. Together with this fact, it is easy to see that $\frac{\partial i_2(t)}{\partial q_2} > 0$. Thus we can conclude that investment and maintenance are gross substitutes when the effects of a technology level increase considered.

When the economy switches to a greater technology level, the older machines or the machines with older technology becomes less efficient compared to newer machines. Thus the economy increases the investment rate since when compared to the first stage, marginal benefit to marginal cost ratio for investment exceeds marginal benefit to marginal cost ratio of the maintenance activity. The economy thus increases the investment spending and decreases the maintenance spending respectively.

This relationship can be also observed in numerical analysis. m_2 and i_2 move in opposite directions in all of the simulations except an increase in A_2. But this is simply caused by the direct effect of A_2 on i_2.

Chapter 5

CONCLUSION

In this study we have applied two stage optimal control technique to analyze the effects of endogenous depreciation on optimal adoption problems under embodiment. Firstly, we solved a benchmark model without increasing frontier technology. In this case, in the optimal adoption the consumption level drops at the time of adoption because of the fall in the relative price of capital. This is obsolescence cost. Growth advantage of the technology adoption must exceed this obsolescence cost and the additional maintenance cost caused by the adoption for an immediate switch. Otherwise technological sclerosis occurs. For the benchmark case delaying adoption is never optimal. Another result we find is that under convex depreciation assumption, the optimal maintenance level decreases with adopting the higher technology.

Secondly, we introduced the increasing frontier technology. Although we could not derive the open form optimal adoption timing, we stated the conditions that the optimal adoption timing must satisfy and under which assumptions there exists such solutions for both exogenous and endogenous depreciation cases. We also analyzed numerically to understand the dynamics of adoption process. We find that maintenance and investment activities are gross substitutes in adoption process. When maintenance activity becomes more efficient by a change in parameters, the adoption time is delayed. Another striking result is that, any technology acceleration parameter increase does not result with an adoption timing decrease. In smaller values, an increase in the technological progress rate results in decrease in the adoption timing. However, as the technological progress rate increases and gets closer to the discount rate, the economy delays the adoption.

Further extensions may be done by applying multistage optimal control technique to technology adoption decision. Also, instead of homogeneous capital stock assumption, heterogeneity in capital stock can be incorporated to the model. This heterogeneity can be supplied by a vintage capital model considering technology adoption decision. Lastly, instead of a constant efficiency loss in expertise, a finer learning algorithm can be incorporated to the model.

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Appendix

Table 1: Effect of a change in technology acceleration

ρ	q1	γ	A1	A2	δ	a	b	Ko	t1 with m	t1 without m	m2 with m	i2 with m
0.04	1	0.03	1	0.8	0.12	0.117	0.24	1	55.07	53.64	0.0010	0.7913
0.04	1	0.025	1	0.8	0.12	0.117	0.24	1	49.33	48.15	0.0018	0.7866
0.04	1	0.02	1	0.8	0.12	0.117	0.24	1	46.87	45.81	0.0026	0.7817
0.04	1	0.015	1	0.8	0.12	0.117	0.24	1	47.29	46.2	0.0036	0.7767
0.04	1	0.01	1	0.8	0.12	0.117	0.24	1	52.41	51.08	0.0046	0.7718
0.04	1	0.005	1	0.8	0.12	0.117	0.24	1	73.69	71.33	0.0056	0.7667

Table 2: Effect of a change in expertise loss

ρ	q1	γ	A1	A2	δ	a	b	Ko	t1 with m	t1 without m	m2 with m	i2 with m
0.04	1	0.02	1	0.95	0.12	0.117	0.24	1	37.85	37.22	0.0034	0.9279
0.04	1	0.02	1	0.9	0.12	0.117	0.24	1	40.69	39.92	0.0031	0.8792
0.04	1	0.02	1	0.85	0.12	0.117	0.24	1	43.69	42.78	0.0029	0.8304
0.04	1	0.02	1	0.8	0.12	0.117	0.24	1	46.87	45.81	0.0026	0.7817
0.04	1	0.02	1	0.75	0.12	0.117	0.24	1	50.26	49.04	0.0024	0.7329
0.04	1	0.02	1	0.7	0.12	0.117	0.24	1	53.88	52.49	0.0022	0.6842

Table 3: Effect of a change in a

ρ	q1	γ	A1	A2	δ	a	b	Ko	t1 with m	m2 with m	i2 with m
0.04	1	0.02	1	0.8	0.12	0.12	0.24	1	46.89	0.0027	0.7816
0.04	1	0.02	1	0.8	0.12	0.117	0.24	1	46.87	0.0026	0.7817
0.04	1	0.02	1	0.8	0.12	0.1	0.24	1	46.84	0.0022	0.7822
0.04	1	0.02	1	0.8	0.12	0.06	0.24	1	46.76	0.0011	0.7832
0.04	1	0.02	1	0.8	0.12	0.02	0.24	1	46.70	0.0003	0.7840
0.04	1	0.02	1	0.8	0.12	0.01	0.24	1	46.68	0.0001	0.7842

Table 4: Effect of a change in b

ρ	q1	γ	A1	A2	δ	a	b	Ko	t1 with m	m2 with m	i2 with m
0.04	1	0.02	1	0.8	0.12	0.117	0.99	1	46.67	0.0000	0.7843
0.04	1	0.02	1	0.8	0.12	0.117	0.7	1	46.68	0.0000	0.7843
0.04	1	0.02	1	0.8	0.12	0.117	0.4	1	46.80	0.0013	0.7830
0.04	1	0.02	1	0.8	0.12	0.117	0.24	1	46.87	0.0026	0.7817
0.04	1	0.02	1	0.8	0.12	0.117	0.2	1	46.88	0.0028	0.7815

Table 5: Effect of a change in initial capital stock

ρ	q1	γ	A1	A2	δ	a	b	Ko	t1 with m	t1 without m	m2 with m	i2 with m
0.04	1	0.02	1	0.8	0.12	0.117	0.24	0.25	46.87	45.81	0.0026	0.7817
0.04	1	0.02	1	0.8	0.12	0.117	0.24	0.5	46.87	45.81	0.0026	0.7817
0.04	1	0.02	1	0.8	0.12	0.117	0.24	1	46.87	45.81	0.0026	0.7817
0.04	1	0.02	1	0.8	0.12	0.117	0.24	2	46.87	45.81	0.0026	0.7817
0.04	1	0.02	1	0.8	0.12	0.117	0.24	4	46.87	45.81	0.0026	0.7817
0.04	1	0.02	1	0.8	0.12	0.117	0.24	8	46.87	45.81	0.0026	0.7817

Table 6: Effect of a change in discount rate

ρ	q1	γ	A1	A2	δ	a	b	Ko	t1 with m	t1 without m	m2 with m	i2 with m
0.06	1	0.02	1	0.8	0.12	0.117	0.24	1	32.51	31.43	0.0039	0.7648
0.05	1	0.02	1	0.8	0.12	0.117	0.24	1	37.73	36.69	0.0034	0.7731
0.04	1	0.02	1	0.8	0.12	0.117	0.24	1	46.87	45.81	0.0026	0.7817
0.03	1	0.02	1	0.8	0.12	0.117	0.24	1	67.39	66.08	0.0015	0.7907

Figure 1: Visualization of Proposition 3.1.1

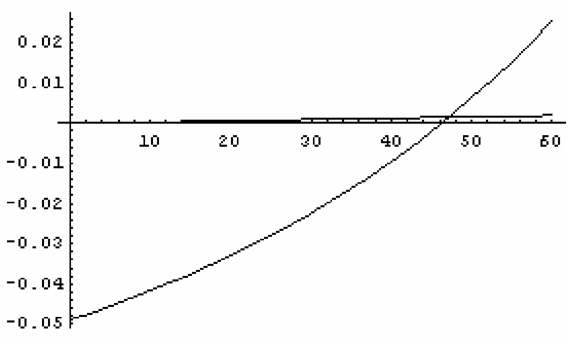

Figure 2: Visualization of Proposition 3.2.1

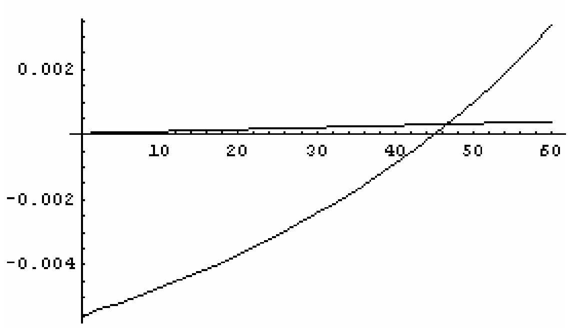

Figure 3: Maximization of Total Welfare