ENDOGENOUS CURRENCY SUBSTITUTION

A Master's Thesis

by

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DEPARTMENT OF
ECONOMICS
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ENDOGENOUS CURRENCY SUBSTITUTION

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by

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A B S T R A C T

ENDOGENOUS CURRENCY SUBSTITUTION

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We introduce a simple two-country exchange model with money introduced via a cash-in-advance constraint with a variable money velocity. Currency substitution is endogenous to this model. While all purchases must be made in local currency, foreign exchange can be traded for local currency within a transactions period while other non-monetary assets, bonds and equalities, cannot. We find that consumers only hold foreign exchange when the local inflation rate is high enough to induce a sufficient desire to reduce the holding of real domestic money balances below the level required to facilitate purchases during the exogenously determined pay period. Foreign exchange

holding allows the consumer to reduce the initial level of real balances below the level required to facilitate purchases until the next pay period.

In general equilibrium allowing consumers to hold foreign exchange reduces the amount of resources dedicated to transacting, the shoe leather cost. As a result, currency substitution reduces the velocity of money, increasing the level of real domestic money balances. Therefore currency substitution increases seigniorage revenue in the home country as well as in the country providing foreign exchange. Given these results, foreign exchange holding is complementary to, rather than a substitute of, domestic real balances.

Currency substitution can enhance efficiency. We find that currency substitution can increase utility by reducing shoe-leather expense. The country supplying foreign exchange can appropriate this gain by increasing its own inflation rate.

ÖZET

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Bu tezde, mali kısıtlı, para hızının değişken olduğu, basit iki ülkeli bir değişim modeli üzerinde çalışıyoruz. Modelimizde para ikamesi içsel olarak alınmıştır. Tüm harcamalar yerel para birimi ile yapılırken, yabancı para belirli işlemler sürecinde yerel para ile ikame edilebilmektedir. Ancak parasal olmayan mal varlıkları, bono ve tahviller bu ikameye dahil olamamaktadır. Modelimizde görüyoruz ki tüketiciler yabancı parayı saadece, yerel parayı elde tutma dengesini dışsal olarak belirlenmiş ödeme periyodu süresince satın almaları kolaylaştırmak için gerekli olan seviyenin altında tutacak kadar yüksek yerel enflasyon oranı olduğunda tutmaktadırlar. Yabancı para tutma; tüketiciye,

bir sonraki ödeme periyoduna kadar, reel dengelerin ilk seviyesini, satın almaları kolaylaştıracak seviyenin altına düşürme olanağını sağlar.

Genel dengede, tüketicinin yabancı para tutması, alışveriş için ayrılan kaynak miktarının düşmesine yol açar (ayakkabı-deri masrafı). Sonuç olarak, para ikamesi paranın hızını düşürürken, reel yerel para dengelerinin seviyesini yükseltir. Bundan dolayı, para ikamesi yabancı parayı sağlayan ülkede olduğu kadar yerel ülkede de para vergisi gelirini arttırır. Bu sonuçlarla, yabancı para tutma, yerel para dengelerini ikame etmekten ziyade, tamamlayan bir unsurdur.

Para ikamesi verimliliği arttırabilir. Gördük ki, para ikamesi ayakkabı-deri masraflarını azaltarak faydayı arttırabilir. Yabancı parayı arz eden ülke, bu kazancı kendi enflasyon oranını arttırarak uygun hale getirebilir.

TABLE OF CONTENTS

ABSTRACT ..……..……...iii

ÖZET ……... v

TABLE OF CONTENTS …...……...vii

LIST OF TABLES ….……...ix

CHAPTER I: INTRODUCTION ….……... 1

1.1 Translation Services Approach …….……...6

1.2 Portfolio-Balance Approach ..……... 7

CHAPTER II: LITERATURE SURVEY …..……...8

2.1 Theory Papers ……... 8

2.2 Empirical Papers ……... 13

2.3 Findings …….……... 16

CHAPTER III: THE MODEL ……...……... 19

3.1 The Government ...…….……... 20

3.2 The Consumer's Problem …..……... 21

3.3 Optimalization ……...25

3.4 No Currency Substitution Equilibrium ……..……... 26

3.5 Equilibrium With Currency Substitution …...……... 28

3.6 Money Velocity And Seigniorage Revenue ..……... 32

CHAPTER IV: CALIBRATION EXERCISE 35

CHAPTER V: EXTENTIONS AND CONCLUSION ……....…..…….....41

REFERENCES ……..……... 43

LIST OF TABLES

Table 1: Parameter Values ……... 36

Table 2: Calibration Results ……..……. 38

Table 3: Effect of Currency Substitution …….…….. 39

CHAPTER I

Introduction

The term 'Currency Substitution' implies that foreign exchange holdings are a substitute for holding domestic currency. Therefore, the existence of currency substitution reduces real balances and domestic seigniorage revenue. We find that this intuitive result does not hold up in a general equilibrium model with endogenous currency substitution.

In our model foreign exchange cannot be used to directly buy goods, but it can be traded for local currency within the period, whereas other assets, bonds and equities, cannot be traded for currency within the transaction period. This model attempts to capture the reality of a country like Turkey where currency substitution is widespread; there are currency exchange houses throughout the country, but where it is very rare to observe the purchase of goods made with foreign exchange, outside of a few narrowly defined tourist areas. In this context foreign exchange provides a liquidity function. We

find that currency substitution is endogenous. Foreign exchange, like domestic currency, is dominated in return by bonds and equities. Foreign exchange will be held if and only if it helps the consumer to reduce the cost of providing liquidity necessary to purchase goods.

We find that introducing currency substitution allows consumers to reduce the intensity with which they utilize domestic currency. That is, by holding foreign exchange the consumer can reduce the shoe leather expense, reducing the velocity of money. In general equilibrium currency substitution increases the level of real domestic money balances, increasing the seigniorage revenue of the home country. This is a general equilibrium result, we cannot obtain this result from a partial equilibrium analysis.

The extent of currency substitution varies widely from country to country. In the United States it is rare to fine anyone holding foreign exchange, while in Turkey there are foreign exchange houses throughout the country. Most studies of currency substitution focus on the medium of exchange function of money to motivate currency substitution. Foreign exchange is either included in the consumer's utility function or there is a second cash-in-advance constraint for some purchases to be made in foreign exchange. However, in Turkey although holding foreign exchange is widespread, it is rarely used in purchases. We examine the function of currency substitution when foreign exchange is not used in local transactions and we examine the effect of currency

substitution on the both the country holding foreign exchange and the country supplying this.

The definition of currency substitution is a highly debated topic in economics. There are competing definitions of currency substitution. First, currency substitution is viewed as the use of foreign currency along with domestic money in transacting. McKinnon (1985) differentiates between direct and indirect currency substitution and defines the former as the use of foreign currencies as means of payment, and the latter as the use of foreign non-monetary financial assets in a country. In his paper Guidotti (1993) generates currency substitution by allowing foreign exchange to reduce the total amount of time devoted to transaction activities. Sturzenegger (1997) also describes currency substitution as the use of stable foreign currency for transaction purposes.

A second definition is that a change in the relative cost of holding one currency induces a change in the ratio of domestic to foreign money holdings demanded (Calvo and Rodriguez 1977). One of the related definitions is that it is the propensity of residents of one country to vary their holdings of domestic and foreign money in response to changes in the associated opportunity costs. A related definition is the practice in an era of convertible currencies of changing the composition of international monetary holdings with a view to making gains or avoiding losses from expected changes in currency values. Also Khan and Ramirez-Rojas (1986) attest to the lack of consensus mentioning that currency substitution could correspond to a number of

different magnitudes such as foreign exchange deposits and / or foreign currency notes in circulation within national boundaries and deposits held abroad by domestic residents.

An extreme view of currency substitution, also known as the replacement of domestic currency in its traditional roles by foreign currencies or the replacement of domestic currency by a foreign currency (frequently U.S. dollar) as a store of value, unit of account and even means of payment, is dollarization. This process has been dubbed ‘‘dollarization’’, because the U.S. dollar has primarily assumed this role in the post-World War 2 economy. According to Pozo and Wheeler (2000), ‘‘dollarization’’ refers to the substitution of foreign money for domestic money and thought to take place when local currency doesn’t adequately fulfill the functions of money. Other phenomenon says that the degree to which an economy’s total transactions are conducted in U.S. dollars rather then domestic currency is referred to as ‘dollarization’. According to Calvo and Vegh (1992) dollarization occurs when the store of value and the unit of account properties of the domestic currency are transferred to foreign monies. When the medium of exchange property is assumed by foreign monies as well, the process is defined as currency substitution referred to as ‘‘dollarization’’.

There are two separate strands to the current discussions of currency substitution issues. The first is concerned with how currency substitution affects money demand and monetary independence (Chen 1973; Miles 1978; McKinnon 1982). Direct as well as indirect empirical tests seem to support the contention that substitutability does exist among the currencies of leading industrial countries (Laffer and Miles 1982). The

second is focused on the dynamics of real exchange rates. Existing literature on currency substitution, almost without exception, ignores inflation abroad and assumes a fixed foreign price for traded goods or imports. This convenient assumption leads to a strong conclusion that an increase in the rate of monetary expansion will not affect the real exchange rate or the terms of trade in the long run. Currency substitution also presents a serious challenge to our understanding of inflationary finance because it implies that some agents choose to evade the inflation tax while others choose to pay it. The term currency substitution is undoubtedly more in accord with modern monetary theory then the traditional one in models of flexible exchange rates in which domestic residents hold only their own currency, because the same arguments of portfolio diversification and transaction costs, which can be used to justify the demand for domestic money, are also applicable to foreign exchange.

Currency substitution has been studied extensively because of its implications for policy. Existing analyses have not been uniform, however, in their treatments of incentives for altering money holdings in response to changes in opportunity costs. Some studies allow substitution among all assets whereas others restrict substitution possibilities to money alone. The different treatments have translated into different approaches to measuring currency substitution and the differing findings have resulted in disagreement about the relevance of currency substitution to monetary policy. There are two basic motives for currency substitution in the literature.

1.1 Transaction Services Approach

In this group, aside from the overlapping generations models (McCallum, 1983; Samuelson, 1958; Wallace, 1980), which emphasize the store of value role of money, two other approaches to the demand for money are very popular. The first one, adopted by Sidrausky (1967), Brock (1974), Calvo (1979), Obstfeld (1981), and Danthine and Donaldson (1986), among many others, introduces real cash balances as an argument in the utility function to account for the real liquidity services provided by money compared non-marketable income streams. Appealingly, the money in the utility approach (MIUF) provides a simple motive demand for money which depends on the wealth and nominal interest rate, consistent both with empirical evidence and with widely used money demand functions not explicitly derived from microeconomic behavior (Laidler, 1985). The second approach, adopted by Lucas (1980), Lucas and Stokey (1983), and Helpman (1981) among others introduces money through a cash in advance constraint proposed by Clower (1967). The transactions role is emphasized by stating that goods can only be purchased with money. While, on the one hand the cash-in-advance (CIA) approach is particularly appealing because it minimizes 'implicit theorizing' (Karaken and Wallace, 1980); on the other hand, in its simplest version, it has the drawback of generating a money demand function with zero interest elasticity or unitary income velocity.

1.2 Portfolio-Balance Approach

The portfolio-balance approach, first proposed by Cuddington (1983), emphasizes the allocation of wealth between different types of money and other assets simultaneously. In accordance with financial equilibrium in the presence of currency substitution, individuals hold both domestic and foreign money. Under the assumption of perfect capital mobility, individuals may also hold domestic and foreign bonds. The portfolio share of each asset is determined by the expected return on all assets and real income. One branch of portfolio-balance approach, the sequential portfolio-balance approach, to explaining currency substitution, assumes that money holders diversify the currency denomination of their cash balances in several stages. In the first stage, they find the optimal balance between nonmonetary assets and money (domestic and foreign money combined). In the second stage, they achieve the optimal balance between foreign and domestic money by changing money balances until the gains in monetary services equal to their opportunity cost (Miles 1978; Ramirez-Rojas 1985; Marquez 1987).

CHAPTER 2

Literature Survey

There have been extensive studies about currency substitution in the economic literature. There are theoretical approaches and empirical approaches. Also different motives are used in these approaches.

2.1 Theory papers

In theoretical approaches some economists, in their research, chose money in the utility function motives in which money directly enters the preferences.

Calvo and Rodriguez (1977) analyze a two-sector perfect foresight model, with endogenous currency substitution-where residents are assumed to hold foreign exchange in addition to their own currency. Foreign inflation is ignored and domestic and foreign

exchange are the only assets. The real exchange rate is shown to depend on monetary variables in the short run while it is fully determined by real variables in the long run. A higher rate of monetary expansion results in an instantaneous increase in both the exchange rate and the price level, the jump in the former exceeding the latter. The higher rate of monetary expansion is followed by a transition period which the economy accumulates foreign exchange.

Chen and Tsaur (1983) show how an open economy, in which there is currency substitution, will be affected by a change in the rate of inflation abroad and other monetary shocks and also discusses how the domestic rate of inflation and the balance of trade are affected by the inflation abroad. They use a ''British I''[1] small open economy model for their study.

Following Chen and Tsaur's paper (1983) Chan, Tsaur and Liu (1989) incorporate rational expectations, full price flexibility and currency substitution to the usual 'British' economy model, taking explicit account of inflation abroad. They find that the impact effect and the adjustment path of the terms of trade engendered by an increase in the rate of domestic monetary expansion depend crucially on the size of import demand elasticities.

[1] Too small to affect the world price of imports, but large enough to affect the price of its exportables. Domestic currency and foreign currency are the only available assets in the economy. Since the country is small, its currency is not held by foreigners. The stock of domestic currency is exogenously determined by the government; whereas the domestic stock of foreign currency is endogenously determined.

Rogers (1990) demonstrates that foreign inflation may be negatively transmitted under flexible exchange rates and currency substitution. In his model, currencies are assumed to be imperfect substitutes by entering each currency into the utility function individually instead of entering aggregate real balances.

Another motive in the theoretical approaches is cash-in-advance motive. In this motive it is simply assumed that money accumulated in the previous period is necessary to finance current period transactions.

In Boyer and Kingston's paper (1987), which is identical to the money in the utility function framework, is a perfect foresight, continuous time, two currency cash in advance model. The model shows the consequences that currency substitution has for a simple shock by a delayed step increase in the quantity of one money. This shock evokes responses which have three notable features: there is negative transmission of inflation (price movements) as McKinnon (1982) described; there is less systematic movement of the exchange rate than there is of price levels, as Frenkel and Mussa (1980) have found and there is the random walk properties of exchange rates in an open economy. As a result, high elasticity of substitution lends to instability in exchange rate (Like Boyer 1978, Karaken and Wallace 1981, Lapen and Enders 1980, Nickelsburg 1984).

Sturzenegger (1997) describes currency substitution as the use of a stable foreign currency for transaction purposes (transaction motive). His paper develops a model of

endogenous currency substitution which some purchases will be made with foreign exchange like Calvo and Rodriguez (1977).

Liviaton(1981), Calvo (1985), Bufman and Leiderman. On the contrary Karaken and Wallace (1981) and Lucas (1985) relied on cash in advance constraint to justify the use of money. Sturzenegger tries to merge the original intuition of currency substitution models-that both currencies are used to some extent-with Lucas cash-in-advance specification. He shows that currency substitution increases inflation and there is a process of currency substitution which lowers the quantity of real domestic money holdings at each rate of inflation. Also it was shown that currency substitution is welfare improving by shifting the incidence of taxation towards inelastically demanded commodities.

The other motives that we include in theoretical approaches are portfolio balance motive, overlapping generations motive or the motives with risk and uncertainty.

Miles (1978) explains currency substitution in a different way. In his paper there exists a group of individuals within a country who diversify their real cash balance holdings between domestic and foreign currency-denominated balances. So that the mechanism of currency substitution can work. These diversified portfolios imply that monetary policy will produce changes in the interest rate that induces offsetting money flows even under flexible exchange rates. The importance of these offsetting flows will be directly proportional to the degree of substitution between currencies. In the case

where currencies are perfect substitutes, monetary independence is impossible, even where central banks do not intervene in the foreign exchange market. He also uses a constant elasticity of substitution (CES) production function for the services of money to derive a testable model of currency substitution. This model is then tested on Canadian data. It is found that a high degree of currency substitution exists in Canada, especially during floating rate periods.

The portfolio balance model of Thomas (1985) shows that the usual portfolio-theoretic justification for currency substitutability, such as foreign interest rates, foreign inflation rates, or expected exchange rate changes in the list of arguments of the domestic money demand function fail if borrowing opportunities exist in all currencies. Foreign and domestic money monies need not to be substitutes when transactors hold both. He finds that portfolio theory is useful for explaining capital mobility, but not currency substitution. The major implication of the model for currency substitution is that cross-interest elasticity of money demand may be zero even when domestic residents hold foreign monies.

An endogenous currency substitution OLG model with multiple currencies is Chang's paper (1994).In his paper there is an exogenously given fiscal deficit that must be financed with inflation. Flexible exchange rates and free trade imply that foreign currency has a nominal return equal to the domestic rate of inflation. People then choose which currency to hold by comparing the cost of holding foreign currency against the inflation rate: If inflation is high enough currency substitution emerges.

Although people take inflation as given, in equilibrium, the more people decide to hold the foreign currency, the smaller he base of the inflation tax and the higher the inflation rate needed to finance deficit. Thus inflation causes currency substitution but currency substitution also causes inflation. An important finding is that currency substitution may emerge as a purely expectational result.

Theory of exchange rate determination has been studied mostly as the relative price of two assets: domestic money and foreign money. However, most of the research has dealt with models in which there is no uncertainty. The study of Stulz (1984) focuses on questions that cannot be addressed in a word of certainty. It investigates how purchasing power risk affects the determination of the exchange rate. He uses an optimizing model where money and foreign exchange enter the utility function and the risk premium incorporated in the forward exchange rate depends only on the relative riskiness of the domestic and foreign monetary policies. As a result, the paper offers the finding that, ceteris paribus, a change in the purchasing power risk of two currencies affects today's exchange rate between the two countries.

2.2 Empirical Papers

There are also various empirical approaches to the currency substitution subject in the literature.

Joines (1985) studies the potential destabilizing domestic money demand due to currency substitution. Joines finds that a large elasticity of substitution is required, but is not sufficient, to destabilize domestic money demand. Domestic residents must have large amounts of foreign exchange holdings for an important effect of a larger elasticity of substitution for destabilizing domestic money demand. Joines also mentions that changes in the cost of holding foreign exchange might affect the demand for and the income velocity of domestic money where currencies are substitutes in demand.

An empirical study of Holmes (2000), where co-integration of European Union countries over long period is studied, interest rates are found to be integrated exogenously by the influences of different exchange regimes and usage of capital controls. The new technique employed in this paper, developed by Snell (1996), is an extension of the principal component methodology, based on testing for the stationarity of the first largest principal component (LPC) using data on differences in velocity growth rates among EU countries. In the light of the results found in the paper, the debate over the stability of a European money demand function has been considered. Under currency substitution, money demand and money velocity are inversely related. From the results in the paper, the increased integration of money velocity differentials imply that movements in national money demands would also be at their most integrated. Thus in a relative sense, money demand would be at their most stable. However, with less integration among velocities money demands are less stable against a background of no capital controls and increased currency substitution.

In Ilyas Sıklar's paper of "Currency Substitution and Seignorage Revenue in a developing country: Turkish Case" the error correction method is used to estimate short and long-term money demand functions modified for expectations on exchange rate. Sıklar finds that currency substitution is a partial replacement of domestic money by foreign money. Thus, the standard money demand function that is used is Cagan's semi-log equation. In this study it is found that currency substitution affects the real balances and, via its effects on real money balances, it affects the inflation rate that maximizes seignorage revenue. It is also verified that currency substitution reduces seignorage revenue collected by empirical results. The latter curve is shifted downward by currency substitution.

Another empirical study on dollarization in Turkey is by Akcay, Alper, Karasulu (1997). The theoretical models of exchange rate determination indicate that exchange rate instability increases with the degree of currency substitution. In this paper they test for the presence of dollarization as well as its effect on the volatility of the exchange rate. An expected depreciation (of the TL/USD exchange rate) series is obtained in the first stage of estimation where exchange rate volatility and the impact of dollarization on the very same volatility are investigated by means of an Exponential Generalized Autoregressive Conditional Heteroscedasticity in Mean (EGARCH-M) model. The resultant series is subsequently employed in the money demand equation to test for the presence of dollarization. Furthermore they test for the extent of currency substitution in determining short-run dynamics of real money balances utilizing the expected depreciation series obtained from the E-GARCH model. They find evidence in support

of dollarization and their findings are in conformity with the assertion that the higher the level of dollarization, the higher is the volatility of exchange rate.

The empirically based model of Brittain (1981) explains the movements in the velocity of circulation across countries and shows that it is related to exchange rate movements. Unexpectedly low levels of velocity in early 1970's and unexpectedly high values after 1975 in the United States have been a reason for adding portfolio variables to money demand function (Hamburger (1977)).

2.3 Findings

Studies on currency substitution have brought some common findings near. According to Sturzenegger (1997), in the case of representative agent, currency substitution is found to be welfare improving. He also finds that as inflation increase, currency substitution lowers the domestic money holdings at each rate of inflation. Miles (1978), Girton and Roper (1981) and McKinnon (1982) also argued that if there is currency substitution, variations in foreign inflation rates or expected future exchange rates can make domestic money function unstable. Sıklar's study about currency substitution also supports the hypothesis that currency substitution effects the real money demand and, itself, is influenced by the monetary instability in the economy. Currency substitution is also found to affect the monetary stability. According to Joines (1985), large elasticity of substitution and large initial foreign money holdings are

needed for currency substitution to destabilize local money demand. Studies also had findings about the effects of currency substitution on inflation. Chang (1994) finds that, under flexible exchange rates and free trade, inflation causes currency substitution and currency substitution causes inflation. In Sıklar's paper a similar finding is seen. Currency substitution affects real balances and via its effects on real balances, it affects the inflation rate that maximizes the seignorage revenue. In high inflation countries like Turkey currency substitution is found to reduce the seignorage revenue and decrease in seignorage revenue accelerates inflation. High inflation and depreciation of domestic currency strengthen the currency substitution. Selcuk's paper (2001) about Turkey shows that if there is currency substitution, Turkish government cannot collect more seignorage revenue by simply setting the growth rate of monetary base to a higher level. Boyer and Kingston (1987) finds that there is negative transmission of inflation when there is a shock in currency substitution as McKinnon (1982) found. Also according to Rogers (1990) under flexible exchange rates and currency substitution foreign inflation may be transmitted negatively. Miles (1978) says that under flexible exchange rates without currency substitution there is complete monetary independence. With currency substitution some independence is lost. Other conclusions from currency substitution studies are about exchange rates. Within an optimizing framework, Stulz (1984) finds that a change in the purchasing power risks of two currencies affects today exchange rate between the two currencies. Akcay, Alper and Karasulu (1997) shows that currency substitution causes exchange rate destabilizing. Boyer and Kingston (1987) have a similar finding that high elasticity of substitution lends to instability in exchange rate. In the study of Calvo and Rodrigues (1977) it is found that, under currency substitution,

higher money growth rate causes a depreciation of the currency, and an increase in the real exchange rates, expected rate of inflation and the price levels. According to Chen, Tsaur (1983) flexible exchange rates insulate a country from foreign monetary shocks and when there is currency substitution, this result changes.

CHAPTER 3

The Model

We consider a two-country world. Each country specializes in the production of one good which can be consumed domestically or sold in the other country, while consumers consume both goods. This is a Lucas' tree type model, meaning that production is exogenous in each country. Money demand is motivated by a cash-in-advance constraint. We include a variable velocity of money following Baumol (1952) and Tobin (1956). The consumer faces a trade-off between holding currency, which pays no interest return, and obtaining currency through costly transactions. Higher inflation leads to higher nominal interest rates, causing the consumer to hold fewer real money balances and to spend more on transactions services. Local currency, only, can be used to facilitate purchases of the consumption goods.

The innovation of this paper is to add a second liquidity constraint. At the beginning of a transactions period the consumer receives his income and allocates his resources.

Within the transactions period, equities and bonds cannot be used to obtain currency. The consumer must enter the period with enough currency, either domestic or foreign, to facilitate purchases throughout the period. While foreign exchange cannot be used directly to purchase goods, it can be used be traded for domestic currency within the transactions period. We view the length of the transactions period as exogenously determined and fixed.

To preview the results, we find that currency substitution is endogenous. When the domestic inflation rate is low, consumers prefer not to hold foreign exchange because it is dominated in return by equities and bonds and it is dominated in return by the local currency. This is consistent with the observation that most low inflation countries exhibit no currency substitution. If the domestic inflation rate is high enough, then consumers elect to hold some foreign exchange, allowing them to reduce their level of real domestic cash balances.

We examine the stationary equilibrium of a non-stochastic competitive economy.

3.1 The Government

The government supplies money via flat rate transfer payments to consumers. The money supply evolves according to $\overline{M}' = \omega \cdot \overline{M}$ [2]. The government also supplies nominal

[2] We follow the notation in which the time subscript is suppressed for variables evaluated at time *t*, and a prime denotes a variable evaluated at time *t + 1*.

bonds. We assume a zero supply of bonds without the loss of generality. The money added (withdrawn) to circulation is given to the consumer as a flat rate transfer payment (tax) $TR = \overline{M}' - \overline{M} = (\omega - 1) \cdot \overline{M}$. Since we are interested in the steady-state equilibrium, we assume that money is introduced to the economy by the government at a steady growth rate.

3.2 The Consumer's Problem

The consumer is subject to a budget constraint and two liquidity constraints. The consumer's budget constraint is:

$$P_1 \cdot (c_1 + \tau) + P_2 \cdot c_2 + M' + B' + Q \cdot z' + \frac{FX' - FX}{e} \leq \left(Q + P_1 \cdot (y - x) + \frac{P_1^B}{e} \cdot x \right) \cdot z + M + (1 + i) \cdot B + TR + FX'^B - FX^B \quad (1)$$

The right hand side of equation 1 shows the resources available to the consumer. The first term represents the value of equities plus the return to equity which consists of the value of domestic sales and foreign sales, translated into domestic currency. The ownership share of equity, *z*, is unitary in equilibrium. Next is money balances selected in the previous period, then government bonds, plus interest,. *TR* represents the flat rate tax or subsidy from the government. The last term (*FX'-FX*) represents the change in foreign exchange in country B. If consumers in the foreign country are increasing their

holdings of country A's currency, there will be a trade surplus, increasing the resources available domestically.

The left hand side of equation 1 shows how the consumer allocates his wealth. The consumer consumes the domestically produced good, expends resources in transacting, τ. To simplify the model, we assume that 'shoe leather' comes out of domestic production for both countries. The consumer also consumes the foreign good which is priced in domestic currency. The consumer can carry wealth into the future by holding domestic currency, by purchasing a government bond or by owning a share, *z*, of the total equity, *Q*. Finally, the consumer may chose to carry foreign exchange, *FX*, into the future. Of these three assets, money is dominated in return by both bonds and equities. We will see that money is held because of the liquidity service which it provides.

Equation 2 represents a cash-in-advance constraint with variable velocity:

$$P_1 \cdot c_1 + P_2 \cdot c_2 \leq M \cdot n \tag{2}$$

Total purchases, represented on the left hand side of equation 2, must be made with non-interest bearing domestic cash which has been obtained in advance. The choice variable *n* represents how often cash balances, μ, are acquired during the period. The production function for transactions is given in equation 3:

$$n = \phi \cdot \tau . \tag{3}$$

Here τ represents the resources, taken from the domestic production good, which are devoted to facilitating transactions. The fixed parameter, ϕ, represents the productivity

of τ in producing transactions. Increasing τ allows the consumer to reduce his holding of money, but it reduces the resources available for consumption.

Between the exogenously determined pay periods, bonds and equities cannot be used to obtain additional currency. Equation 4 shows that, at a pay period, either the consumer's initial level of domestic currency holding must be sufficient to last until the next pay period, or the consumer must hold foreign exchange which can be used to obtain additional domestic currency within the transaction period:

$$P_1 \cdot c_1 + P_2 \cdot c_2 \leq \eta \cdot M + \frac{FX}{e}. \tag{4}$$

Here η an exogenous fixed parameter which states how many pay periods there are per data collection period. For instance, if the consumer is paid monthly and data is reported annually, then $\eta = 12$. This parameter is not restricted to integer values because we are dealing with averages over individuals and time.

We divide equations 1, 2 and 4 by a nominal term to make the problem stationary. We chose to divide all terms by the level of locally held domestic currency, $\left(1 - s'^{B}\right) \cdot \overline{M}'$. We define the following real variables:

$$m' = \frac{M'}{\left(1 - s'^{B}\right) \cdot \overline{M}'}, \quad p_1 = \frac{P_1}{\left(1 - s'^{B}\right) \cdot \overline{M}'}, \quad p_2 = \frac{P_2}{\left(1 - s'^{B}\right) \cdot \overline{M}'}, \quad q = \frac{Q}{\left(1 - s'^{B}\right) \cdot \overline{M}'},$$

$$p_1^B = \frac{P_1^B}{\left(1 - s'^{A}\right) \cdot \overline{M}'^{B}}, \quad \hat{e} = e \cdot \frac{1 - s'^{B}}{1 - s'} \cdot \frac{\overline{M}'}{\overline{M}'^{B}}, \quad s^A = \frac{FX^A}{\overline{M}^B}, \quad s^B = \frac{FX^B}{\overline{M}^A}, \quad b' = \frac{B'}{\left(1 - s'^{B}\right) \cdot \overline{M}'}.$$

These definitions imply the following transformations:

$$\frac{FX'}{e\cdot\left(1-s'^{B}\right)\cdot\overline{M}'}=\frac{s'}{\hat{e}\cdot\left(1-\bar{s}'\right)},\quad \frac{FX}{e\cdot\left(1-s'^{B}\right)\cdot\overline{M}'}=\frac{s}{\hat{e}\cdot\omega^{B}\cdot\left(1-\bar{s}'\right)},\quad \frac{M}{\left(1-s'^{B}\right)\cdot\overline{M}'}=\frac{1-s^{B}}{1-s'^{B}}\cdot\frac{m}{\omega},$$

$$\frac{TR}{\left(1-s'^{B}\right)\cdot\overline{M}'}=\frac{\omega-1}{\omega}\cdot\frac{1-s^{B}}{1-s'^{B}},\quad \frac{P_1^{B}}{e\cdot\left(1-s'^{B}\right)\cdot\overline{M}'}=\frac{p_1^{B}}{\hat{e}},\quad \frac{FX'^{B}}{\left(1-s'^{B}\right)\cdot\overline{M}'}=\frac{s'^{B}}{1-s'^{B}},$$

$$\frac{FX^{B}}{\left(1-s'^{B}\right)\cdot\overline{M}'}=\frac{s^{B}}{\left(1-s'^{B}\right)\cdot\omega},\quad \frac{B}{\left(1-s'^{B}\right)\cdot\overline{M}'}=\frac{1-s^{B}}{1-s'^{B}}\cdot\frac{b}{\omega}.$$

Making these substitutions we obtain the following real representations of the consumer's budget and liquidity constraints. Throughout the rest of the paper we only utilize the steady-state variables::

$$\begin{aligned}p_1\cdot(c_1+\tau)+p_2\cdot c_2+m'+b'+q\cdot z'+\frac{\omega^{B}\cdot s'-s}{\hat{e}\cdot\omega^{B}\cdot\left(1-\bar{s}'\right)}&=\left(q+p_1(y-x)+\frac{p_1^{B}}{\hat{e}}\cdot x\right)\cdot z\\&+\frac{m-1+\omega+(1+i)\cdot b}{\omega}\cdot\frac{1-s^{B}}{1-s'^{B}}+\frac{s'^{B}\cdot\omega-s^{B}}{\left(1-s'^{B}\right)\cdot\omega}\end{aligned}\tag{5}$$

$$p_1\cdot c_1+p_2\cdot c_2=\frac{m\cdot n}{\omega}\cdot\frac{1-s^{B}}{1-s'^{B}}\tag{6}$$

$$p_1\cdot c_1+p_2\cdot c_2=\eta\cdot\frac{m}{\omega}\cdot\frac{1-s^{B}}{1-s'^{B}}+\frac{s}{\hat{e}\cdot\omega^{B}\cdot\left(1-\bar{s}'\right)}.\tag{7}$$

In equilibrium, the following market clearing conditions apply:

$$\begin{aligned}&z'=z=1\\&m'=m=1\\&0\le s'<1\end{aligned}\tag{8}$$

The share of equity owned by the representative consumer is unity, as is the ratio of money demanded to money supplied. The share of foreign exchange held domestically is less than one.

3.3 Optimization

Both the domestic and foreign produced goods enter the utility function, providing a motive for foreign trade. We assume the following utility function:

$$u(c_1, c_2) = \alpha \ln(c_1) + (1-\alpha)\ln(c_2). \tag{9}$$

The consumer's problem can be posed as a value function problem:

$$v(S) = \max_{c_1, c_2, x, \tau, m', s', z'} \{u(c_1, c_2) + \beta \cdot v(S')\} \tag{10}$$

The state of the world is summarized by the vector $S = \left\{ s, \omega, s^B, \omega^B \right\}$. The maximization is conducted subject to the constraints in equations 5-7.

We use the first order and equilibrium conditions to solve for the choice and price variables. There are two possible cases, depending upon the values of the fixed parameters. The choice of equilibrium is especially dependent upon the rate of domestic inflation. In both cases, the nominal interest rate obeys the Fisher equation:

$$i = \frac{\omega}{\beta} - 1. \tag{11}$$

One plus the nominal interest rate is equal to one plus the inflation rate, ω, multiplied by one plus the real interest rate, β^{-1}. The equity price is given by the discounted value of future profits, revenues in this case:

$$q = \frac{p_1 \cdot y}{\beta^{-1} - 1}. \tag{12}$$

3.4 No Currency Substitution Equilibrium

In the first case, the initial holding of domestic cash is sufficient to facilitate all purchases during the period, $\phi \cdot \tau \leq \eta$. This occurs when the number of transactions per year is less than, or equal to, the number of pay periods per year, η. The consumer holds no foreign exchange because it is not needed for liquidity, and foreign exchange is dominated in return by government bonds and equities. This occurs when the domestic inflation rate is low, or when the transactions technology is very productive, ϕ is big.

In this equilibrium the consumer holds no foreign exchange:

$$s' = 0 . \tag{13}$$

We focus on the stationary equilibrium so there must be balanced foreign trade:

$$p_2 \cdot c_2 = p_1 \cdot c_1^B . \tag{14}$$

The value of imports must equal the value of exports since there are no foreign exchange holdings. This is obtained by placing the market clearing conditions within the budget constraint, equation 5.

Closed form solutions for all variables are obtained. First we present the equilibrium level of shoe leather expense:

$$\tau = \frac{\sqrt{i^2 + 4\phi y i} - i}{2 \cdot \phi} . \tag{15}$$

This is a time-dynamic variation of Baumol (1952). The amount of resources devoted to transacting is an increasing function of the nominal interest and output.

Consumption is a constant share of output, net of the shoe leather expense, with the share of output given by the utility function parameter α:

$$\left.\begin{aligned} c_1 &= \alpha\cdot(y-\tau) \\ c_1^B &= (1-\alpha)\cdot(y-\tau) \\ c_2 &= \alpha\cdot\left(y^B-\tau^B\right) \\ c_2^B &= (1-\alpha)\cdot\left(y^B-\tau^B\right) \end{aligned}\right\} \tag{16}$$

Goods' prices are given by:

$$\left.\begin{aligned} p_1 &= \frac{\phi\cdot\tau}{\omega\cdot(y-\tau)} \\ p_1^B &= \frac{\alpha}{1-\alpha}\cdot\frac{\phi\cdot\tau^B}{\omega^B\cdot(y-\tau)} \\ p_2 &= \frac{1-\alpha}{\alpha}\cdot\frac{\phi\cdot\tau}{\omega\cdot(y-\tau)^B} \\ p_2^B &= \frac{\phi\cdot\tau^B}{\omega^B\cdot(y-\tau)^B} \end{aligned}\right\}. \tag{17}$$

The price of each good is decreasing in output, net of shoe leather expense. The effect of inflation, ω, on the price level is non-linear, it includes a direct effect and an indirect effect through inflation's effect on τ. Using these prices, we can present the exchange rate as a function of given parameters:

$$\hat{e} = \frac{p_1^B}{p_1} = \frac{p_2^B}{p_2} = \frac{\alpha}{1-\alpha}\cdot\frac{\omega}{\omega^B}\cdot\frac{\tau^B}{\tau}. \tag{18}$$

Finally, we present the Lagrange multipliers for the three constraints. The marginal value of relaxing the consumer's real budget constraint is:

$$\lambda = \frac{\omega\cdot(y-\tau)}{\phi\cdot\tau\cdot y}. \tag{19}$$

The liquidity value of local currency is:

$$\mu = \frac{\omega}{\phi \cdot y}. \tag{20}$$

The third constraint is slack because the consumer has not been induced to hold any foreign exchange:

$$\gamma = 0. \tag{21}$$

3.5 Equilibrium with Currency Substitution

In equation 15 we see that the amount of resources devoted to transacting is increasing in the inflation rate. When the inflation rate is high enough, the number of desired transactions exceeds the number made possible by the exogenously determined number of pay periods, $\phi \cdot \tau > \eta$. A simpler interpretation is that the initial level of real balances is insufficient to take the consumer to the next paycheck. Since bonds and equities cannot be traded for cash in-between pay periods, the consumer must hold foreign exchange to trade for local currency. Only in the case in which the foreign interest rate exceeds the domestic rate, $i^B > i$, will the consumer elect to hold enough real domestic balance to satisfy the second liquidity constraint. In this case the cash-in-advance constraint is slack. We focus only on the case in which country B is a low inflation country so $i^B < i$.

In steady state, there must be an imbalance in the trade flow in order for the consumer to replace real foreign exchange balances eroded by inflation in the foreign country. The trade balance is related to the foreign inflation rate and share of foreign exchange held locally by the following equation:

$$p_2 \cdot c_2 = p_1 \cdot c_1^B - \left(1 - \frac{1}{\omega^B}\right) \cdot \frac{s}{1-s} \cdot \frac{1}{\hat{e}}. \tag{22}$$

Again, this is found by substituting the market clearing values of endogenous variables in the budget constraint, equation 5.

Combining the first order equations, the market clearing conditions and trade balance allows us to obtain closed-form, although somewhat complicated, solutions for the model. First, the level of resources used in transacting is:

$$\tau = \frac{\sqrt{\left(i - i^B\right)^2 + 4\,\phi\, y\, \alpha \frac{\Theta}{\Omega}\left(i - i^B\right)} - \left(i - i^B\right)}{2\,\alpha\,\phi\frac{\Theta}{\Omega}} \tag{23}$$

where,

$$\Omega \equiv \frac{\alpha \cdot \phi \cdot \tau^B}{\omega^B} - \frac{s}{1-s} \cdot \left(1 - \frac{1}{\omega^B}\right) \tag{24}$$

and

$$\Theta \equiv \frac{\phi \cdot \tau^B}{\omega^B} - \frac{s}{1-s} \cdot \left(1 - \frac{1}{\omega^B}\right). \tag{25}$$

For concreteness and simplicity we will pay special attention to the case in which there is zero inflation in the foreign country, $\omega^B = 1$. In this case foreign exchange does not lose value over time so the trade of goods in balanced in stationary equilibrium. The shoe leather cost becomes:

$$\tau = \frac{\sqrt{\left(i - i^B\right)^2 + 4\phi y\left(i - i^B\right)} - \left(i - i^B\right)}{2 \cdot \phi}. \tag{26}$$

Comparing this to the expression for τ in case 1, we see that *currency substitution reduces the amount of resources devoted to transacting*. This is the key result of this paper. A lower shoe-leather expenditure implies that the level of domestic real balances is greater in the face of currency substitution, than it would be without currency substitution. Foreign exchange allows the consumer to use domestic currency less intensely than he would if he could not hold foreign exchange; the ability to hold foreign exchange increases the demand for domestic currency. There is a caveat to this result. The first is that this model adds the possibility currency substitution jointly with an additional liquidity constraint. Without the additional constraint, the consumer would not choose to hold foreign exchange since it is dominated in return by bonds and equities and it is dominated by domestic currency in providing liquidity services.

The prices are:

$$\left.\begin{aligned} p_1 &= \frac{\alpha\cdot\phi\cdot\tau}{\omega\cdot(y-\tau)}\cdot\frac{\Theta}{\Omega} \\ p_1^B &= \frac{\alpha}{1-\alpha}\cdot\frac{\Theta}{y-\tau} \\ p_2 &= \frac{(1-\alpha)\cdot\phi\cdot\tau}{\omega\cdot\left(y^B-\tau^B\right)}\cdot\frac{\Theta}{\Omega} \\ p_2^B &= \frac{\Theta}{y^B-\tau^B} \end{aligned}\right\} \tag{27}$$

The consumption levels are:

$$\left.\begin{aligned} c_1 &= \frac{\Omega}{\Theta}\cdot(y-\tau) \\ c_1^B &= (1-\alpha)\cdot\frac{\phi\cdot\tau^B}{\omega^B\cdot\Theta}\cdot(y-\tau) \\ c_2 &= \frac{\Omega}{\Theta}\cdot\left(y^B-\tau^B\right) \\ c_2^B &= (1-\alpha)\cdot\frac{\phi\cdot\tau^B}{\omega^B\cdot\Theta}\cdot\left(y^B-\tau^B\right) \end{aligned}\right\} \tag{28}$$

When there is no foreign inflation, the price and consumption levels are identical, as functions of τ, as in the risk-free level outlined in the previous section.

These prices imply the exchange rate:

$$\hat{e} = \frac{p_1^B}{p_1} = \frac{\omega\cdot\Omega}{(1-\alpha)\cdot\phi\cdot\tau}. \tag{29}$$

The share of foreign exchange held locally is:

$$s = \frac{\alpha\cdot\phi\cdot\tau^B\cdot(\phi\cdot\tau-\eta)}{\left(\alpha\cdot\phi\cdot\tau^B+\omega^B-1\right)\cdot(\phi\cdot\tau-\eta)+(1-\alpha)\cdot\phi\cdot\tau\cdot\omega}. \tag{30}$$

This share is found by combining the consumer's first order condition with respect to money demand with the second liquidity constraint.

The Lagrange multipliers are given below:

$$\lambda = \frac{\omega}{\phi \cdot \tau} \cdot \left[1 + \frac{\alpha \cdot \tau}{y - \tau} \cdot \frac{\Theta}{\Omega} + i^B \right]^{-1} \tag{31}$$

$$\mu = \frac{\alpha \cdot \omega}{\phi \cdot (y - \tau)} \cdot \left[1 + \frac{\alpha \cdot \tau}{y - \tau} \cdot \frac{\Theta}{\Omega} + i^B \right]^{-1} \tag{32}$$

$$\gamma = \frac{\omega \cdot i^B}{\phi \cdot \tau} \cdot \left[1 + \frac{\alpha \cdot \tau}{y - \tau} \cdot \frac{\Theta}{\Omega} + i^B \right]^{-1} . \tag{33}$$

Even in the case in which there is zero inflation in the foreign country, the Lagrange multipliers differ from the solution given in case 1 by the inclusion of the foreign interest rate. The clearest discernable effect of currency substitution on these multipliers is to reduce the degree with which the cash-in-advance constraint, equation 6, binds. Currency substitution reduces the shadow price of domestic currency.

These results hold up in the case in which the inflation rate in country B is not zero.

3.6 Money Velocity and Seigniorage Revenue

With the prices in hand we can determine the expressions for money velocity and seigniorage revenue. We can express the velocity of money as:

$$\frac{P_1 \cdot y}{\overline{M}} = \omega \cdot p_1 \cdot y \tag{34}$$

in country A, and it is:

$$\frac{P_2^B \cdot y^B}{\overline{M}^B} = (1 - s') \cdot \omega^B \cdot p_2^B \cdot y^B \tag{35}$$

in country B.

Currency substitution reduces the velocity of money in both countries. In country A the presence of foreign exchange reduces τ reducing the level of domestic prices.[3] In country B the fact that a portion of the country's currency is not held domestically reduces the observed money velocity.

A lower velocity of money is associated with an increased level of real balances. This increases seigniorage in both countries. Seigniorage in country A is:

$$(\omega - 1) \cdot \frac{\overline{M}}{P_1} = \frac{\omega - 1}{\omega \cdot p_1} \tag{36}$$

in country B seigniorage is:

$$(\omega - 1) \cdot \frac{\overline{M}}{P_1} = \frac{\omega^B - 1}{(1 - s') \cdot \omega^B \cdot p_2^B}. \tag{37}$$

In the case in which there is no foreign inflation, currency substitution increases welfare in both countries by reducing the resources lost to transacting. This is a second

[3] At, unrealistically, high levels of inflation of the price level in the equations 27 is decreasing in τ.

order effect. Country B can appropriate this gain, and more, by increasing its inflation rate which allows country B to collect seigniorage revenue on the foreign exchange holdings of country A.

An increase in the inflation rate in country B increases the threshold level of inflation in country A for currency substitution to occur. However, once currency substitution occurs, an increase in country B's inflation rate marginally increases the degree of currency substitution and the level of real balances in country A. This is understood by examining equations 23 and 30. Country B inflation reduces the relative opportunity cost of holding domestic currency, so the consumer holds more domestic real balances.

Welfare could be increased in country A by eliminating the underlying cause of currency substitution: inflation.

CHAPTER IV

CALIBRATION EXERCISE

In the previous section we presented closed-form solutions for the endogenous variables. To give a better 'feel' for the nature of these results, we also conduct a calibration excursive. In this excursive we view Country B as the United States. Country A is an aggregation of countries which *may* hold foreign exchange. To conserve notation as much as possible, we assign identical parameter values to each country with the exception of the output level and inflation rate. As in the general model presented above, we focus on the steady-state equilibrium with no real growth and a constant inflation rate. We consider a variety of Country A inflation rates. Table 1 contains the base-line parameters.

Table 1: Parameter Values

β	= 0.96
α	= 0.50
ϕ	= 3.05
η	= 12
ω^B	= 1.0
y	= 250
y^B	= 100

The discount rate, β, implies a real interest rate of 4%. For symmetry we assume an equal weight, α, for both goods in the utility function. The productivity of resources devoted to transacting, ϕ, is set at 3.05. This has been calibrated to match the M1 velocity the United States, which is standing in for the low inflation country, country B, and to match the M1 velocity of Turkey, which is standing in for a high inflation country, country A. We calculate an average M1 velocity of money in Turkey[4] to be 23.5 for the period 1996-2002. The M1 velocity of money in the U.S. is found to be 7.1 for the period 1981-2000.. We assume annual data and a monthly pay period, so $\eta = 12$. To eliminate steady-state trade imbalance, we focus on the case of zero inflation in the country supplying foreign exchange for currency substitution. We assume that the output level is 250 in country A and 100 in country B. These, relative, values also assist in calibrating the velocities of money in both countries. In calibrating, we have selected values of the parameters which match the observed velocities of money, and we pick

values which achieve a ratio of 28% of country B cash held in country A when A's domestic inflation rate is 25%. Doyle (2000) finds that up to 30% of U.S. currency has been overseas while Porter and Judson (1996) find that more than 50% of U.S. currency in circulation outside of banks was outside of the U.S. in 1996.

Table 2 presents the main endogenous variables for a variety of domestic inflation rates. For the parameter values given above, the consumer begins to hold foreign exchange when the domestic inflation rate rises above 14.42%. This threshold does not depend upon the inflation rate in country B, as long as the inflation rate in country B is less than the inflation rate in country A. We do not consider the case in which there is no country with a lower rate of inflation than the domestic country, country A. Table 2 shows the reaction of variables to domestic inflation. Inflation increases the shoe-leather expense, hence the velocity of money.

Table 3 presents a comparison of the equilibrium values of endogenous variables with currency substitution to the values of these variables in a model without the second liquidity constraint. With the exception of the inflation rate, all variables in Table 3 are presented as percentage changes between the model without the second liquidity constraint, to the model with this constraint and with currency substitution. This table only includes inflation rates which correspond to currency substitution. We see that currency substitution relaxes the cash-in-advance constraint, reducing the shoe-leather

[4] Obtained from the Republic of Turkey Central Bank web cite at www.TCMB.gov.tr.

expenditure; μ is much smaller under currency substitution. This increases real balances, so seigniorage in increased in both countries. In country A this is because of

Table 2: Calibration Results (Equilibrium Values as a Function of the İnflation Rate)

π	i	τ	λ	μ	γ	p_1	p_2	c_1	c_2	Utility	q	e	velocity	seigniorage /GDP	s	Country B velocity
-4%	0%	0.0	- - -	0.00126	- - -	0.00	0.00	125.0	49.4	4.364	0	- - -	0.0	- - -	0.00	3.59
-2%	2%	1.3	0.25	0.00129	0.0000	0.02	0.04	124.3	49.4	4.362	98	0.87	4.0	-0.005	0.00	3.59
0%	4%	1.8	0.18	0.00131	0.0000	0.02	0.06	124.1	49.4	4.361	136	0.63	5.7	0.000	0.00	3.59
1%	5%	2.1	0.16	0.00132	0.0000	0.03	0.06	124.0	49.4	4.360	150	0.57	6.3	0.002	0.00	3.59
2%	6%	2.3	0.15	0.00134	0.0000	0.03	0.07	123.9	49.4	4.360	163	0.53	6.9	0.003	0.00	3.59
3%	8%	2.5	0.14	0.00135	0.0000	0.03	0.07	123.8	49.4	4.359	177	0.48	7.6	0.004	0.00	3.59
5%	9%	2.8	0.12	0.00138	0.0000	0.03	0.08	123.6	49.4	4.359	194	0.44	8.5	0.006	0.00	3.59
10%	15%	3.4	0.10	0.00144	0.0000	0.04	0.10	123.3	49.4	4.357	232	0.37	10.6	0.009	0.00	3.59
15%	20%	3.6	0.10	0.00047	0.0042	0.04	0.10	123.2	49.4	4.357	229	0.38	11.0	0.014	0.00	3.59
20%	25%	4.1	0.09	0.00044	0.0038	0.04	0.11	123.0	49.4	4.356	254	0.34	12.7	0.016	0.11	3.21
25%	30%	4.6	0.08	0.00043	0.0035	0.05	0.11	122.7	49.4	4.355	273	0.32	14.2	0.018	0.28	2.56
30%	35%	5.0	0.08	0.00043	0.0033	0.05	0.12	122.5	49.4	4.354	288	0.30	15.6	0.019	0.37	2.26
35%	41%	5.4	0.08	0.00042	0.0032	0.05	0.12	122.3	49.4	4.353	300	0.29	16.9	0.021	0.42	2.09
40%	46%	5.8	0.07	0.00043	0.0031	0.05	0.13	122.1	49.4	4.353	309	0.28	18.0	0.022	0.45	1.98
45%	51%	6.1	0.07	0.00043	0.0030	0.05	0.13	121.9	49.4	4.352	317	0.28	19.1	0.024	0.47	1.91
50%	56%	6.4	0.07	0.00044	0.0030	0.05	0.13	121.8	49.4	4.351	323	0.27	20.2	0.025	0.48	1.87
55%	61%	6.8	0.07	0.00045	0.0029	0.05	0.13	121.6	49.4	4.351	328	0.27	21.2	0.026	0.49	1.83
60%	67%	7.1	0.07	0.00046	0.0029	0.06	0.14	121.5	49.4	4.350	332	0.26	22.1	0.027	0.49	1.81
65%	72%	7.3	0.07	0.00047	0.0029	0.06	0.14	121.3	49.4	4.349	335	0.26	23.1	0.028	0.50	1.80
70%	77%	7.6	0.07	0.00048	0.0028	0.06	0.14	121.2	49.4	4.349	338	0.26	23.9	0.029	0.50	1.79
80%	88%	8.1	0.07	0.00050	0.0028	0.06	0.14	120.9	49.4	4.348	342	0.26	25.6	0.031	0.50	1.78
90%	98%	8.6	0.07	0.00053	0.0028	0.06	0.14	120.7	49.4	4.347	344	0.26	27.2	0.033	0.50	1.78
100%	108%	9.1	0.07	0.00056	0.0028	0.06	0.14	120.5	49.4	4.346	345	0.26	28.7	0.035	0.50	1.79

Table 3: Effect of Currency Substitution (Compared to Model without Second Liquidity Constraint)

π	τ	λ	μ	p_1	p_2	c_1	c_2	Utility	q	e	velocity	country B velocity	seigniorage /GDP
15%	-11%	8%	-69%	-11%	-11%	0.180%	0%	0.021%	-11%	12%	-11%	0%	13%
20%	-9%	5%	-72%	-9%	-9%	0.158%	0%	0.018%	-9%	9%	-9%	-11%	10%
25%	-7%	4%	-74%	-7%	-7%	0.142%	0%	0.016%	-7%	8%	-7%	-28%	8%
30%	-6%	2%	-75%	-6%	-6%	0.131%	0%	0.015%	-6%	6%	-6%	-37%	7%
35%	-5%	1%	-76%	-5%	-5%	0.122%	0%	0.014%	-5%	5%	-5%	-42%	6%
40%	-5%	1%	-77%	-5%	-5%	0.114%	0%	0.013%	-5%	5%	-5%	-45%	5%
45%	-4%	0%	-77%	-4%	-4%	0.108%	0%	0.012%	-4%	4%	-4%	-47%	4%
50%	-4%	0%	-78%	-4%	-4%	0.103%	0%	0.012%	-4%	4%	-4%	-48%	4%
55%	-3%	0%	-78%	-3%	-3%	0.098%	0%	0.011%	-3%	4%	-3%	-49%	4%
60%	-3%	-1%	-78%	-3%	-3%	0.094%	0%	0.011%	-3%	3%	-3%	-49%	3%
65%	-3%	-1%	-78%	-3%	-3%	0.090%	0%	0.010%	-3%	3%	-3%	-50%	3%
70%	-3%	-1%	-79%	-3%	-3%	0.087%	0%	0.010%	-3%	3%	-3%	-50%	3%
80%	-2%	-1%	-79%	-2%	-2%	0.082%	0%	0.009%	-2%	2%	-2%	-50%	3%
90%	-2%	-2%	-79%	-2%	-2%	0.077%	0%	0.009%	-2%	2%	-2%	-50%	2%
100%	-2%	-2%	-79%	-2%	-2%	0.073%	0%	0.008%	-2%	2%	-2%	-50%	2%

the increase in domestic real balances, in country B this is because some of the currency in held in country A. Velocity in both counties decreases due to currency substitution.

The effect is much stronger in country B. The surprising result is that currency substitution increases seigniorage revenue in country A. This effect diminishes as the inflation rate increases. Since we have assumed there is no inflation in country B, there is no seigniorage revenue in that country. Currency substitution increases utility in both countries because fewer resources are devoted to transacting in country A. This is a second order effect.

These results in table 3 are not very sensitive to the parameter values. The productivity of shoe-leather in producing transactions, ϕ, determines when country A begins holding foreign exchange. The inflation rate in country B determines whether currency substitution takes place, and the level of real balances and currency substitution in country A.

To better see the effect of currency substitution, we provide graphs of a few of the endogenous variables. To get a better separation in the equilibria with currency substitution and that without, we construct the first five graphs using a calibrated inflation rate in country B of 5%. The other parameters, and all parameters in the sixth graph, are based upon the parameter values states in Table1.

CHAPTER V

EXTENSIONS AND CONCLUSION

This paper is based upon the assumption that, if there is a cash-in-advance constraint with a variable velocity of money, then the consumer must hold another liquid asset in order to obtain additional domestic currency between pay periods. In this model the only asset which can serve this function, besides domestic currency, is foreign exchange. A natural extension of this model would be to introduce a domestic asset which can also serve this liquidity function, such as interest earning bank deposits, inside money. While the existence of such an asset may increase the threshold rate of domestic inflation required for currency substitution to occur, if there are frictions associated with providing inside money, then we may still see currency substitution at high rates of inflation.

The primary innovation of this paper is that holding of foreign exchange is endogenous; it occurs when the domestic inflation rate is high , and it allows the

consumers to use domestic currency less intensively. Therefore, currency substitution increases the demand for domestic real balances, so it is associated with an increase in domestic seigniorage revenue.

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