AN APPLICATION OF CLUSTERING R&D PROJECTS BY USING THE ANALYTIC HIERARCHY PROCESS

A THESIS
SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING
AND THE INSTITUTE OF ENGINEERING AND SCIENCE
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

By
Mehmet Tunç
July, 2004

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Prof. Dr. Ülkü Gürler (Supervisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Asst. Prof. Dr. Oya Ekin Karaşan

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Asst. Prof. Dr. Yavuz Günalay

Approved for the Institute of Engineering and Science:

Prof. Dr. Mehmet B. Baray
Director of the Institute Engineering and Science

ABSTRACT

AN APPLICATION OF CLUSTERING R&D PROJECTS BY USING THE ANALYTIC HIERARCHY PROCESS

Mehmet Tunç
M.S. in Industrial Engineering
Supervisor: Prof. Dr. Ülkü Gürler
July, 2004

Due to imprecision or uncertainty that is inherent in the design process, the management of research and development projects is very challenging. With the growing complexity of the design process and need for different specializations, it is getting even more tougher. Especially, in an organization where tens of high-tech, military R&D projects are carried out concurrently, management should be supported with the state-of-the-art decision and operations research methods.

In this thesis, we consider an application of the classification of R&D projects. More specifically, the problem discussed in this study is grouping N different projects into groups based on a predetermined set of features. Since some features are fuzzy, expert knowledge is needed to quantify the features. In order to quantify the features successfully, Analytic Hierarchy Process is used. Finally, by using various clustering algorithms the projects are clustered.

Keywords: Analytic Hierarchy Process, clustering projects, quantification, multi-criteria clustering, project evaluation.

ÖZET

ANALİTİK HİYERARŞİ YÖNTEMİNİ KULLANARAK AR&GE PROJELERİNİN SINIFLANDIRILMA UYGULAMASI

Mehmet Tunç
Endüstri Mühendisliği, Yüksek Lisans
Tez Yöneticisi: Prof. Dr. Ülkü Gürler
Temmuz, 2004

Tasarım sürecinin doğasından kaynaklanan belirsizlikler sebebiyle, araştırma ve geliştirme projelerinin yönetimi oldukça zordur. Tasarım çalışmalarının artmakta olan karmaşıklığı ve farklı alan uzmanlıkları gereksinimi yönetilmelerini daha da zorlaştırmaktadır. Özellikle, onlarca yüksek teknoloji içeren askeri Ar-Ge projelerinin bir arada yönetildiği bir organizasyonda yönetimin, modern karar verme ve yöneylem araştırması yöntemleri ile desteklenmesi gerekmektedir.

Bu tezde, Ar-Ge projelerinin sınıflandırılmasına ilişkin bir uygulama incelenmiştir. Bu çalışmada anlatılmakta olan problem, N farklı projenin önceden belirlenmiş nitelik seti temel alınarak sınıflandırılmasıdır. Bazı niteliklerin belirsiz olması sebebiyle bu niteliklerin ölçümlerinde uzman bilgisine ihtiyaç duyulmaktadır. Bu nitelikleri başarıyla ölçmek için bu çalışmada "Analitik Hiyerarşi Yöntemi" kullanılmıştır. Son olarak nitelikleri ölçülmüş projeler çeşitli sınıflandırma yöntemleri kullanılarak sınıflandırılmışlardır.

Anahtar sözcükler: Analitik Hiyerarşi Yöntemi, proje sınıflandırma, nicelendirme, çoklu kriter sınıflandırma, proje değerlendirme.

Acknowledgement

I am very grateful firstly to my thesis advisor, Prof. Dr. Ülkü Gürler for her patience, encouragement, guidance in my thesis work. I am indebted to other respected members of the thesis committee, Asst. Prof. Dr. Oya Ekin Karaşan and Asst. Prof. Dr. Yavuz Günalay for accepting to read and review this thesis and for their suggestions. Thanks to their invaluable contribution, I have managed to complete my thesis work.

I would like to express my sincere thanks and gratitude to my manager at Aselsan, Ms. Elif Baktır. It would be impossible for me to complete my thesis work without her understanding, support, and motivation.

I would like to thank my colleagues, Onur Kabul, Aybeniz Yiğit, and Aykut Özsoy at Aselsan for their support, friendship, suggestions during all Aselsan time. I also wish to express my thanks to MST (Microwave and System Technologies), one of the three divisions of Aselsan.

I would like to take this opportunity to thank Meray Y. İlhan for her help, support and love, especially during this thesis completion process.

I would also express my gratitude to mom, dad, my brother Hüseyin for their understanding, support, and most importantly for their love.

Contents

1 Introduction **1**

1.1 Motivation 1

1.2 Organization of the Thesis 2

2 Related Work **3**

2.1 Project Classification 3

2.2 Analytic Hierarchy Process (AHP) 4

2.2.1 Theoretical Structure of AHP 5

2.2.2 AHP Methodology 6

2.2.3 Pairwise Comparisons 7

2.2.4 Group Process 9

2.2.5 Deriving Relative Weights 11

2.2.6 Checking Consistency of the Results 11

2.2.7 Synthesis of Priorities 14

2.2.8 A Numerical Example 14

2.2.9 Criticisms of AHP and a Variant of AHP 19
2.3 Multicriteria Clustering 20
2.3.1 AHP Based Clustering 20
2.3.2 VAHP Based Clustering 24
2.3.3 An extension of the k-means algorithm 24
2.3.4 General Clustering Algorithms 25

3 Model and The Analysis 28

3.1 Defining The Projects In Terms of Features 29
3.1.1 Features 29
3.1.2 AHP Model 33
3.2 Clustering The Projects 37

4 Application of Project Clustering 39

4.1 Application of AHP 42
4.1.1 Evaluation of Features 42
4.1.2 Evaluation of Projects 43
4.2 Application of VAHP 44
4.3 Application of Clustering Algorithms 46
4.3.1 AHP Based Clustering 47
4.3.2 VAHP Based Clustering 54
4.3.3 Extension of K-means Algorithm for Multicriteria Clustering 56

4.4 Investigation of Project Clusters 56

5 Conclusions and Future Work 59

A Input Data: Pairwise Comparisons 61

B ProTer 79

List of Figures

2.1 Combined Judgments 10

2.2 Hierarchical Structure of Numerical Example 16

2.3 A Matrix for Criterion ‘Hardware Expandability’ 16

2.4 A Matrix and Priority Vector for Objective 16

2.5 A Matrix and Priority Vector for Criterion ‘Hardware Maintainability’ 17

2.6 A Matrix and Priority Vector for Criterion ‘Financing Available’ 17

2.7 A Matrix and Priority Vector for Criterion ‘User Friendly’ 17

2.8 Decision Matrix and Solution of Numerical Example 18

2.9 Hierarchy of the AHP Model in Ben-Arieh and Triantaphyllou’s Work 23

3.1 Hierarchical Structure 35

4.1 ProTer 41

4.2 Matrix-Based Clustering: Use of Hierarchical Clustering Algorithm 49

4.3 Matrix-Based Clustering: Silhouette Graph for $k = 5$ 50

4.4 Aggregate-value Clustering: Use of Hierarchical Clustering Algorithm 52
4.5 Aggregate-value Clustering: Silhouette Graph for $k = 4$ 53
4.6 VAHP Based Clustering: Use of Hierarchical Clustering Algorithm 55
4.7 VAHP Based Clustering: Silhouette Graph for $k = 5$ 55

A.1 Evaluation of Features by Pairwise Comparisons: Matrix A of Expert 1 61
A.2 Evaluation of Features by Pairwise Comparisons: Matrix A of Expert 2 62
A.3 Evaluation of Features by Pairwise Comparisons: Matrix A of Expert 3 62
A.4 Evaluation of Features by Pairwise Comparisons: Matrix A of Expert 4 62
A.5 Evaluation of Projects by Pairwise Comparisons With Respect to "Technological Uncertainty": Matrix A of Expert 1 63
A.6 Evaluation of Projects by Pairwise Comparisons With Respect to "Technological Uncertainty": Matrix A of Expert 2 64
A.7 Evaluation of Projects by Pairwise Comparisons With Respect to "Technological Uncertainty": Matrix A of Expert 3 65
A.8 Evaluation of Projects by Pairwise Comparisons With Respect to "Technological Uncertainty": Matrix A of Expert 4 66
A.9 Evaluation of Projects by Pairwise Comparisons With Respect to "Platform Type": Matrix A of Expert 1 67

A.10 Evaluation of Projects by Pairwise Comparisons With Respect to "Platform Type": Matrix A of Expert 2 68

A.11 Evaluation of Projects by Pairwise Comparisons With Respect to "Platform Type": Matrix A of Expert 3 69

A.12 Evaluation of Projects by Pairwise Comparisons With Respect to "Platform Type": Matrix A of Expert 4 70

A.13 Evaluation of Projects by Pairwise Comparisons With Respect to "Work and Test Environment": Matrix A of Expert 1 71

A.14 Evaluation of Projects by Pairwise Comparisons With Respect to "Work and Test Environment": Matrix A of Expert 2 72

A.15 Evaluation of Projects by Pairwise Comparisons With Respect to "Work and Test Environment": Matrix A of Expert 3 73

A.16 Evaluation of Projects by Pairwise Comparisons With Respect to "Work and Test Environment": Matrix A of Expert 4 74

A.17 Evaluation of Projects by Pairwise Comparisons With Respect to "System Scope": Matrix A of Expert 1 75

A.18 Evaluation of Projects by Pairwise Comparisons With Respect to "System Scope": Matrix A of Expert 2 76

A.19 Evaluation of Projects by Pairwise Comparisons With Respect to "System Scope": Matrix A of Expert 3 77

A.20 Evaluation of Projects by Pairwise Comparisons With Respect to "System Scope": Matrix A of Expert 4 78

B.1 ProTer's Parts 80

List of Tables

2.1 Scale of Relative Importance 8

2.2 Random Indices (RI)[13] 13

2.3 Membership Table 22

3.1 Output Table 36

4.1 Features' Weights 43

4.2 Projects' Weights for Subjective Features 44

4.3 Projects' Weights for Objective Feature 45

4.4 Output Table of AHP 45

4.5 Output Table of VAHP 46

4.6 Matrix-Based Clustering: Use of K-means Clustering Algorithm ($k = 7$) 49

4.7 Aggregate-value Clustering: Input Data 51

4.8 Aggregate-value Clustering: Use of K-means Clustering Algorithm ($k = 4$) 52

4.9 VAHP Based Clustering: Use of K-means Clustering Algorithm ($k = 5$) 54

4.10 Projects' Rankings 56

4.11 Extension of K-means Algorithm for Multicriteria Clustering ($k = 5$) 57

Chapter 1

Introduction

Due to imprecision or uncertainty that is inherent in the design process, the management of research and development projects is very challenging. With the growing complexity of the design process and need for different specializations, it is getting even more complicated. Especially, in an organization where tens of high-tech, military R&D projects are carried out concurrently, management should be supported with the state-of-the-art decision making and operations research methods.

1.1 Motivation

Aselsan A.Ş. is one of the biggest companies in the military electronics sector in Turkey. MST (Microwave and System Technologies) is one of the three divisions of Aselsan. In MST, there is a project-based structure, and it is one of the examples of organizations where tens of high-tech military R&D projects are carried out concurrently. As it is the case in MST, the projects may have completely different characteristics. For example, in terms of design type, some projects require breakthrough design, whereas some others require just redesign or continuous design of previously designed systems. Hence, the ways that each of the projects are managed and treated, may/should differ.

In this thesis, our aim is to form meaningful project clusters based on project characteristics. In other words, the objective is to identify project groups with common characteristics so that they can be managed in similar manners. This work can be thought as the first step of determining key issues for administrative processes of projects having same characteristics.

The features of projects have been determined beforehand for the case of MST, and the question of what should be the features of R&D projects, is not addressed in this thesis. The key point in this work is defining the projects in terms of features, and measuring the amount of each of the features that each project possesses. Some features are fuzzy, some others are not fuzzy, but their effects to project management are fuzzy. Therefore, expert knowledge is the main source. For this reason, Analytic Hierarchy Process (AHP) is used to define the projects in terms of features.

1.2 Organization of the Thesis

The thesis is organized as follows: Chapter 2 summarizes the studies in the literature on project classification, Analytic Hierarchy Process and multicriteria clustering. In Chapter 3, the model, constructed for the clustering of projects, is explained. Chapter 4 includes the application part of the study. Finally, Chapter 5 concludes the thesis.

Chapter 2

Related Work

2.1 Project Classification

A frequently used process by which products are created is the practice of project management. In fact, projects have become one of the most common forms of temporary organizations, and they are set for achieving a wide variety of organizational goals. Yet, ironically, as an organizational concept, project management is relatively new and probably not well understood. Most research literature on the management of projects is relatively inchoate and still suffers from a scanty theoretical basis and a lack of concepts [18].

One of the major barriers in understanding the nature of projects has been the little distinction made between the project type and its strategic and managerial problems. According to Pinto and Covin(1989) [12] "The prevailing tendency among the majority of academics has been to characterize all projects as fundamentally similar, and the implicit view of many academics could be represented by the axiom, a project is a project." However, works related to some distinctions among projects, based on levels change, exists (Blake (1978) [4], Hauptman (1986) [7], Whellwright and Clark(1992) [21], Shenhar(2001) [18]). Shenhar [18] noted that none of the typologies mentioned in the literature has developed into a standard, fully accepted theoretical framework. Moreover the proposed typologies are too general for MST's projects such that majority of the projects come together in the same grouping. In addition, there are some special factors of Aselsan, that must be taken into account in project classification. For this reason we have decided that we can not use the proposed typologies for the case of Aselsan.

2.2 Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process was developed by Thomas L. Saaty in 1970's. AHP provides a flexible and easily understood way to analyze and decompose the decision problem. It is a multi-criteria decision making methodology that allows subjective as well as objective factors to be considered in the evaluation process. In its general form, it is a framework for performing both deductive and inductive thinking. AHP was designed as a scaling procedure for measuring priorities in a hierarchical goal structure. It requires pairwise comparison judgments of criteria in terms of relative importance. These judgments can be expressed verbally and enable the decision-maker to incorporate subjectivity, experience and knowledge in an intuitive and natural way.

AHP's power has been validated in empirical use, extended by research, and expanded by new theoretical insights as reported in a series of annual international symposia on AHP. AHP has been widely used as a powerful multiple-criteria decision making tool. It has been applied to solve highly complex decision problems in planning and resource allocation as well as conflict resolutions. Zahedi [23] and Vargas [22] give comprehensive surveys of the method and its applications. In later applications, AHP was found to be a powerful tool for selecting projects and proposals, overcoming the limitations of other multiple-criteria decision making techniques ([8],[11],[5]).

In this study, AHP is chosen to evaluate projects in terms of predetermined features. In other words, for defining projects in terms of features, we use AHP.

Decision applications of the AHP are carried out in two phases: hierarchic design and evaluation. AHP requires the decision maker to first represent the problem within a hierarchical structure. The purpose of constructing the hierarchy is to evaluate and prioritize the influence of the criteria on the alternatives to attain or satisfy overall objectives. To set the problem in a hierarchical structure, the decision maker should identify his/her main purpose in solving a problem. In the most elementary form, a hierarchy is structured from the top level (objectives), through intermediate levels (criteria on which subsequent levels depend) to the lowest level (which is usually a list of alternatives). The evaluation phase is based on the concept of paired comparisons. The elements in a level of the hierarchy are compared in relative terms as to their importance or contribution to a given criterion that occupies the level immediately above the elements being compared. Two elements in the same level are pairwise compared only if they are connected to at least one common criterion in the level immediately above them. The structure of hierarchy designs the pairwise comparisons ([13],[15]).

The main motivation of the pairwise comparison approach is based on the fact that humans have serious difficulties evaluating many entities simultaneously. However, humans can perform rather well when they are asked to evaluate only two entities at a time.

2.2.1 Theoretical Structure of AHP

The axioms of the theory are as follows:

Axiom 1: (Reciprocal Comparison). The decision maker must be able to make comparisons and state the strength of his preferences. The intensity of these preferences must satisfy the reciprocal condition: If A is x times more preferred than B, then B is $1/x$ times more preferred than A.

Axiom 2: (Homogeneity). The preferences are represented by means of a bounded scale.

Axiom 3: (Independence). When expressing preference, criteria are assumed independent of the properties of the alternatives.

Axiom 4: (Expectations). For the purpose of making a decision, the hierarchic structure is assumed to be complete. That is, all the decision maker's intuition must be represented in terms of criteria and alternatives in the structure.

The first axiom says that if a decision maker is able to say something is five times more important than something else, then he should agree that the reciprocal property holds. The relaxation of Axiom 1 indicates that the question used to elicit the judgments or paired comparisons is not clearly or correctly stated. Axiom 2 says that infinite preferences are not allowed. If Axiom 2 is not satisfied, then the elements being compared are not homogeneous and clusters may need to be formed. Axiom 3 implies that the weights of the criteria must be independent of the alternatives considered. The weights of criteria can not be different for different alternatives. If Axiom 4 is not satisfied, then the decision maker does not use all the criteria and/or all the alternatives necessary to meet his reasonable expectations and hence the decision is incomplete.([17],[14])

2.2.2 AHP Methodology

1. The overall goal (objective) is identified, and the issue is clearly defined.

2. After finding the objective, the criteria used to satisfy the overall goal are identified. Then the sub-criteria under each criterion must be identified so that a suitable solution or alternative may be specified. The hierarchical structure is constructed.

3. Pairwise comparisons are constructed; elements of the problem are paired (with respect to their common relative impact on a property) and then compared.

4. Weights of the decision elements are estimated by using the eigenvalue method. Consistency of judgments is checked.

5. Working downward through the hierarchy, hierarchical composition is used

to combine the weight vectors and arrive at global and local relative contributions (priorities) of each element.[17]

2.2.3 Pairwise Comparisons

In AHP, once the hierarchy has been constructed, the decision maker begins the prioritization procedure to determine the relative importance of the elements on each level of the hierarchy. Elements of a problem on each level are paired (with respect to their common relative impact on a property) and then compared. The comparisons are made in the following form: How important is element 1 when compared to element 2 with respect to a specific element in the level immediately higher? If two elements are not connected to a common element in the level immediately higher, they are not pairwise compared. If two elements are connected to more than one common element in the level immediately higher, these two elements are pairwise compared for each common element in the level immediately higher. The hierarchy determines the pairwise comparisons. Therefore, special attention must be given to the form of the hierarchy.

For each common element in the level immediately higher, starting from the top of the hierarchy and going down, the pairwise comparisons are reduced in the square matrix form, A given in equation (2.1). Breaking a complex system into a set of pairwise comparisons is the major characteristic of AHP.

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & & & & & \\ . & & & & & \\ . & & & & & \\ a_{n1} & . & . & . & . & a_{nn} \end{pmatrix} \tag{2.1}$$

A is an $n \times n$ matrix in which n is the number of elements being compared. Entries of A, a_{ij}'s are the judgments or the relative scale of alternative i to alternative j with respect to a common element. They have the following characteristics:

$$a_{ij} = 1/a_{ji} \text{ for } \forall i, j \tag{2.2}$$

Table 2.1: Scale of Relative Importance

Intensity or Relative Importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Moderate importance of one over another	Experience and judgment slightly favor one activity over another
5	Essential or strong importance	Experience and judgment strongly favor one activity over another
7	Very strong importance	An activity is strongly favored and its dominance demonstrated in practice
9	Extremely important	The evidence favor one activity over another is of the highest order of affirmation
2,4,6,8	Intermediate values between the two adjacent judgments	When comparison is needed
Reciprocals of above non-zero numbers		If the activity i has one of the above none-zero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared to i

To fill the matrix of A, Saaty [13] proposed the use of a one-to-nine scale to express the decision maker's preferences and intensity of that preference for one element over another. Table 2.1 contains the recommended scale from 1-9, which is used to assign a judgment in comparing pairs of elements at each level of the hierarchy against a criterion in the next highest level. For example, if $a_{12} = 5$, this means that the first alternative is strongly favored over the second alternative based on experience and judgment.

An obvious case of a consistent matrix is one in which the comparisons are based on exact measurements; that is, the weights $w_1, w_2, w_3, ..., w_n$ are already known. Then a_{ij} can be written as follows:

$$a_{ij} = w_i/wj \tag{2.3}$$

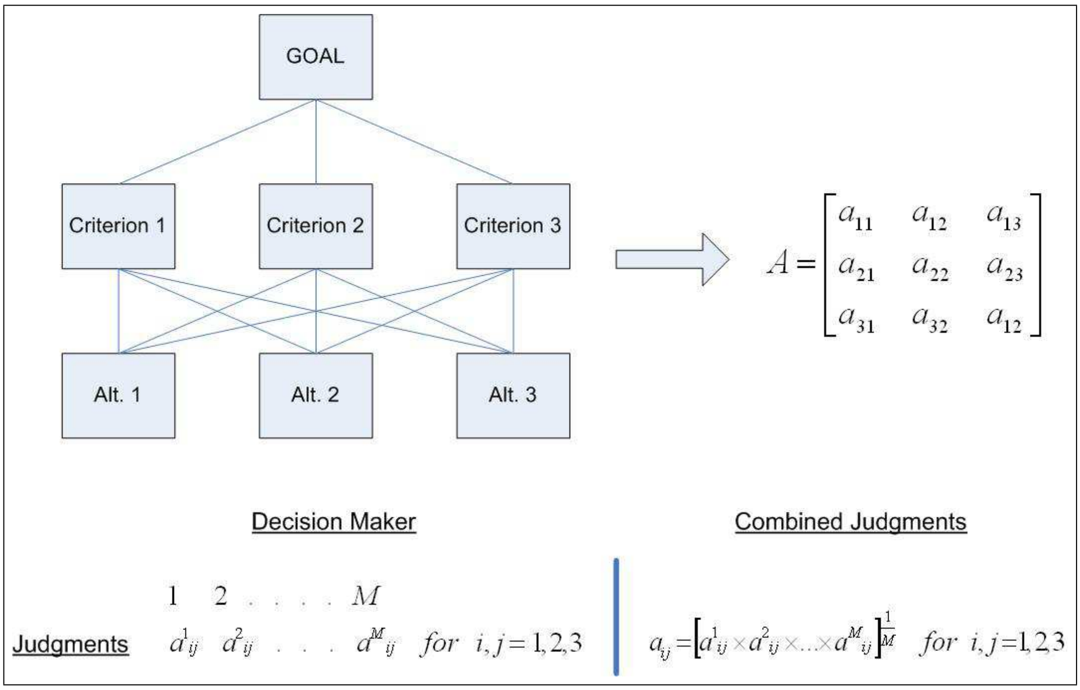
where w_i is the relative weight of alternative i.

2.2.4 Group Process

Higher complexity and need for different specializations necessitate the participation of many individuals in the decision making process. AHP allows each decision maker to specify a value and then combine the individual judgments as follows: Use the geometric mean of the individual judgments to obtain the group judgment for each pairwise comparison (see Figure 2.1). Aczel and Saaty [1] showed that the geometric mean is the uniquely appropriate rule for combining judgments in the AHP because it preserves the reciprocal property in the combined pairwise comparison matrix.

In Figure 2.1, a simple hierarchy is given, and matrix A which may be formed by the evaluation of criterions or evaluation of alternatives with respect to a criteria is shown. M is the number of decision makers and each of them evaluate pairwise comparisons individually, so, for the same set of pairwise comparisons, there are M different A matrices. The combined judgments are obtained as shown in Figure 2.1, by taking geometric mean of each element of A matrices. M different A matrices are transformed to one A matrix which is formed by combined judgments. [6]

Figure 2.1: Combined Judgments



2.2.5 Deriving Relative Weights

The next step is to estimate the relative weights of the decision elements by using the eigenvalue method. The mathematical basis for determining the weights has been determined by Saaty [13] based on matrix theory. The procedure is called an eigenvector approach, which takes advantage of characteristics of a special type of matrix called a reciprocal matrix.

The entries a_{ij} are defined by equation 2.2 and according to 2.3 the *consistent* pairwise comparison matrix, A in 2.1, can be represented in the form shown in 2.4

$$A = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} & . & . & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} & . & . & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \frac{w_3}{w_3} & . & . & \frac{w_3}{w_n} \\ . & & & & & . \\ . & & & & & . \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & . & . & \frac{w_n}{w_n} \end{pmatrix} \tag{2.4}$$

The objective is to find eigenvector w corresponding to maximum eigenvalue λ_{max} which is the relative weights of the objects:

$$w = (w_1, w_2, w_3, ..., w_n) \tag{2.5}$$

If the pairwise comparison matrix is not consistent as stated above, the weights of the objects obtained by using eigenvalue method may not be valid. For this reason we should check the consistency of the matrix A.

2.2.6 Checking Consistency of the Results

In decision-making, it is important to know how good the consistency is. Consistency in this case means that the decision procedure is producing coherent judgments in specifying the pairwise comparison of the criteria or alternatives.

The cardinal consistency rule is:

$$a_{ij}a_{jk} = a_{ik} \text{ for } i, j, k = 1, ...n. \tag{2.6}$$

When A is consistent, and

$$a_{ij} = \frac{w_i}{w_j} \Rightarrow w_i = a_{ij}w_j \text{ for } i, j = 1, ...n. \tag{2.7}$$

$$\begin{aligned}
Aw &= \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & & & & & \\ . & & & & & \\ . & & & & & \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{pmatrix} = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} & . & . & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} & . & . & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \frac{w_3}{w_3} & . & . & \frac{w_3}{w_n} \\ . & & & & & . \\ . & & & & & . \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & . & . & \frac{w_n}{w_n} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{pmatrix} \\
Aw &= \begin{pmatrix} w_1 + w_1 + ... + w_1 \\ w_2 + w_2 + ... + w_2 \\ . \\ . \\ . \\ w_n + w_n + ... + w_n \end{pmatrix} = \begin{pmatrix} nw_1 \\ nw_2 \\ . \\ . \\ . \\ nw_n \end{pmatrix} \\
Aw &= nw \qquad (2.8)
\end{aligned}$$

In matrix theory, equation (2.8) is satisfied only if w is an eigenvector of A with eigenvalue of n.

All the rows in the represented matrix are constant multiplies of the first row. From linear algebra all the eigenvalues λ_i, i=1,...n are zero except one. Since A is a reciprocal matrix and all the entries are positive, all the eigenvalues of A are non-negative. Therefore λ_i which is greater than zero can be called λ_{max}.

$$\sum_{i=1}^{n} \lambda_i = Trace(A) = n \tag{2.9}$$

Table 2.2: Random Indices (RI)[13]

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49	1.51	1.48	1.56	1.57	1.59

The trace of a matrix is a summation of the diagonal entries. Since all the diagonal elements of A are one, the trace of A is n.

Since all the eigenvalues λ_i are zero except λ_{max},

$$\sum_{i=1}^{n} \lambda_i = \lambda_{max} \tag{2.10}$$

This implies that $\lambda_{max} = n$ and λ_{max} can be used as an approximation for n.

An index is needed to measure the consistency of weights. The following index, the consistency index (CI), was suggested by Saaty [13]:

$$\text{Consistency Index, } CI = \frac{\lambda_{max} - n}{n - 1} \tag{2.11}$$

This is an index to assess how much the consistency of pairwise comparisons differs from the perfect consistency. The numerator signifies the deviation of maximum eigenvalue (λ_{max}) from perfect consistency, which is n. The denominator is needed to compute an average of each pairwise comparison from perfectly consistent judgment. A value of one subtracted from the order of matrix n, because one of the pairwise comparisons is a self-comparison, and there should be no inconsistency involved in self-comparison.

The consistency check of pairwise comparison is done by comparing the computed consistency index with the average consistency index of randomly generated reciprocal matrices using one-to-nine scale . Table 2.2 shows the random indices (RI) for matrices of order 1 through 15. RI values are taken from Saaty [13].

AHP measures the overall consistency of judgments by means of a consistency ratio (CR). The consistency ratio is obtained by dividing the computed consistency index by the random index.

$$CR = \frac{CI}{RI} \tag{2.12}$$

Saaty [13] stated that a consistency ratio of 0.10 or less can be considered acceptable; otherwise the judgments should be improved.

2.2.7 Synthesis of Priorities

After finding normalized eigenvectors (sum up to 1) that corresponds to λ_{max}'s for each evaluation, and verifying that the pairwise comparisons are acceptable in terms of consistency criteria, the last step is the synthesis of priorities. Priorities are synthesized from the top level down by multiplying local priorities by the priority of their corresponding criterion in the level above, and adding them for each element in a level according to the criteria it affects. (The second level elements are each multiplied by unity, the weight of the single top level goal.) This gives the composite or global priority of that element which is then used to weigh the local priorities of elements in the level below compared by it as criterion, and so on to the bottom level. [13]

2.2.8 A Numerical Example

To clarify AHP methodology, it is appropriate to investigate a numerical example. We take an example from Triantaphyllou and Mann's [20] work. Suppose that the best computer system is tried to be chosen among three alternative configurations (configuration A, configuration B, configuration C). We have four criteria which are 'hardware expandability', 'hardware maintainability', 'financing available', and 'user friendly'. The hierarchical structure can be seen from Figure 2.2. For this hierarchical structure, the evaluation process has five main parts:

1. Evaluation of criteria with respect to the objective
2. Evaluation of alternative configurations with respect to criterion 'hardware expandability'
3. Evaluation of alternative configurations with respect to criterion 'hardware maintainability'

4. Evaluation of alternative configurations with respect to criterion ‘financing available’

5. Evaluation of alternative configurations with respect to criterion ‘user friendly’

In case there is only one decision-maker, for each evaluation part we form an A matrix as in Figure 2.3. For each A matrix we estimate the relative weights/priorities by using the eigenvalue method, and then we check the consistency. In Figures 2.4, 2.5, 2.6, 2.7 you can see A matrices for other evaluation parts with priority vectors and consistency ratios (CR). The priority vectors are used to form the entries of the decision matrix. The decision matrix and the resulted final priorities are in Figure 2.8. The final priority for each alternative configuration is obtained by multiplying the columns of the decision matrix (each column corresponds to a criterion) with the corresponding criterion’s weight, then by taking summation of the elements for each row (each row corresponds to an alternative), and finally, by normalizing the resulting vector such that summation of the vector is equal to one.

Figure 2.2: Hierarchical Structure of Numerical Example

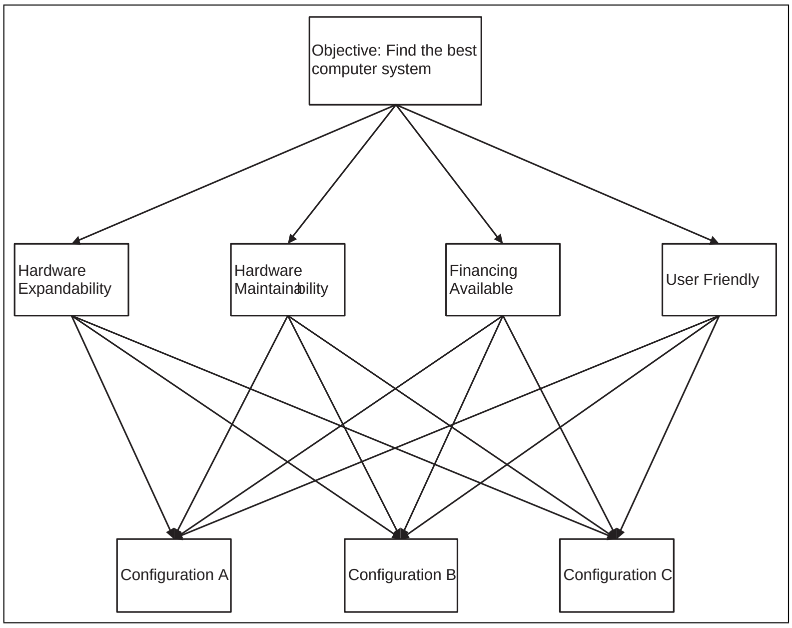


Figure 2.3: A Matrix for Criterion 'Hardware Expandability'

C_1: Hardware Expandability	A	B	C
A	1	6	8
B	1/6	1	4
C	1/8	1/4	1

Figure 2.4: A Matrix and Priority Vector for Objective

The four Criteria	C_1	C_2	C_3	C_4	Priority Vector
C_1	1	5	3	7	0.553
C_2	1/5	1	1/3	5	0.131
C_3	1/3	3	1	6	0.271
C_4	1/7	1/5	1/6	1	0.045

$\lambda_{max} = 4.252$, CI $= 0.084$, and CR $= 0.093$.

Figure 2.5: A Matrix and Priority Vector for Criterion ‘Hardware Maintainability’

C_2: Hardware Maintainability	A	B	C	Priority Vector
A	1	7	1/5	0.233
B	1/7	1	1/8	0.055
C	5	8	1	0.713

λ_{max} = 3.247, CI = 0.124, and CR = 0.213.

Figure 2.6: A Matrix and Priority Vector for Criterion ‘Financing Available’

C_3: Financing Available	A	B	C	Priority Vector
A	1	8	6	0.745
B	1/8	1	1/4	0.065
C	1/6	4	1	0.181

λ_{max} = 3.130, CI = 0.068, and CR = 0.117.

Figure 2.7: A Matrix and Priority Vector for Criterion ‘User Friendly’

C_4: User Friendly	A	B	C	Priority Vector
A	1	5	4	0.674
B	1/5	1	1/3	0.101
C	1/4	3	1	0.226

λ_{max} = 3.086, CI = 0.043, and CR = 0.074.

Figure 2.8: Decision Matrix and Solution of Numerical Example

	Criterion				
	C_1	C_2	C_3	C_4	
Alt.	(0.553	0.131	0.271	0.045)	**Final Priority**
A	0.754	0.233	0.745	0.674	0.680
B	0.181	0.055	0.065	0.101	0.130
C	0.065	0.713	0.181	0.226	0.190

2.2.9 Criticisms of AHP and a Variant of AHP

The AHP and its use of pairwise comparisons has inspired the creation of many other decision-making methods. Beside, its wide acceptance, it also created some considerable criticism. Belton and Gear(1983) [2] observed that the AHP may reverse the ranking of the alternatives when an alternative identical to one of the already existing alternatives is introduced (well-known rank reversal problem). In order to overcome this deficiency, Belton and Gear proposed that each column of the AHP decision matrix to be divided by the maximum entry of that column. Thus, they introduced a variant of the original AHP, called the revised-AHP. Later, Saaty [16] accepted the previous variant of the AHP and now it is called the Ideal Mode AHP. The following guidelines were developed by Millet and Saaty [10] to reflect the core differences in translating performance measures to preference measures of alternatives. The original AHP should be used when the decision maker is concerned with the extent to which each alternative dominates all other alternatives under the criterion. The Ideal Mode AHP should be used when the decision maker is concerned with how well each alternative performs relative to a fixed benchmark. For example, consider selecting a car:

Two different decision makers may approach the same problem from two different points of view even if the criteria and standards are the same. The one who is interested in “getting a well performing car” should use the Ideal Mode. The one who is interested in “getting a car that stands out” among the alternatives should use the original AHP. [17]

Another main drawback of AHP, is the high number of pairwise comparisons, especially for large hierarchies. Assigning a numerical value for each pairwise comparison is also not easy. For this reason, for large number of pairwise comparisons, considerable amount of effort is needed.

2.3 Multicriteria Clustering

Projects are evaluated in terms of features by using a multicriteria decision making methodology, AHP. The next step is the clustering of projects.

The two mostly used technique for grouping objects with similar properties are: classification and clustering. Generally both names are used interchangeably but some important differences exist between them. Classification techniques use supervised learning; which means that the objects are assigned to pre-defined classes. On the contrary, clustering is an unsupervised technique that finds potential groups in data such that the objects within a cluster are more similar to each other than to objects in other clusters [19].

In literature, the research related to grouping objects with respect to multiple criteria are mainly focused on the assignment of actions to pre-defined classes; in other words multicriteria classification field [19]. Since the model studied in this thesis requires a multicriteria clustering method, only clustering part of the field is given.

In the context of AHP, so far we have found three different applications in literature, related to multicriteria clustering field. Two of them can be considered as extensions of AHP method. The third one is a kind of generic method that can be applied by using the results of any multicriteria decision methodology.

2.3.1 AHP Based Clustering

Ben-Arieh and Triantaphyllou [3] used the AHP in the group technology application. The problem discussed in the paper is grouping N different part types based on a predetermined set of features. In the proposed methodology, parts' data about different features are expressed in terms of membership values. That is, for each part, the membership value of a given feature in the part is determined.

The hierarchy of the model can be seen at Figure 2.9. As it can be seen from the hierarchy, the features are considered as criteria, and the parts are considered as the alternatives. Membership values are the weights of the parts for each feature. In this work, the method used for determining membership values is based on Ideal Mode AHP.

Three types of features are identified in the paper:

1. Quantitative features. Such features represent properties of parts that can be expressed numerically.
2. Qualitative(fuzzy) features. These features describe the part attributes in fuzzy terms such as 'large, medium, small' or other terms agreed upon by the system users.
3. Quantitative features with subjective meaning. Features of this type have numerical values which do not quantitatively represent the actual meaning of these features in the relevant environment. Therefore these features are also fuzzy.

The method presented in the paper treat the fuzzy features and quantitative features differently. For the fuzzy features, the membership values are determined by using Ideal Mode AHP. The membership values for the quantitative features are determined by normalizing the values such that the maximum one is equal to one. The membership matrix can be seen in Table 2.3 where w_{f_k} represents the weight of Feature k (where $k = 1....m$), and $w_{p_{sr}}$ represents the membership value of Feature r (where $r = 1....m$) in the Part s (where $s = 1....n$).

According to Ben-Arieh and Triantaphyllou [3], once the features that are used for the part grouping are described in terms of their membership values, and their relative importance, it is possible to cluster the parts into groups. Two different ways are proposed to conduct grouping process:

1. Matrix-based clustering: By multiplying the membership values of the features by the features' weight (importance), a new matrix is generated. In this matrix, each part is represented as an m dimensional point in Euclidean

Table 2.3: Membership Table

Feature	Feature Weight	Part 1	Part 2	.	.	.	Part n
1	w_{f_1}	$w_{p_{11}}$	$w_{p_{21}}$	.	.	.	$w_{p_{n1}}$
2	w_{f_2}	$w_{p_{12}}$	$w_{p_{22}}$	.	.	.	$w_{p_{n2}}$
.	.	.	.				.
.	.	.	.				.
.	.	.	.				.
m	w_{f_m}	$w_{p_{1m}}$	$w_{p_{2m}}$	.	.	.	$w_{p_{nm}}$

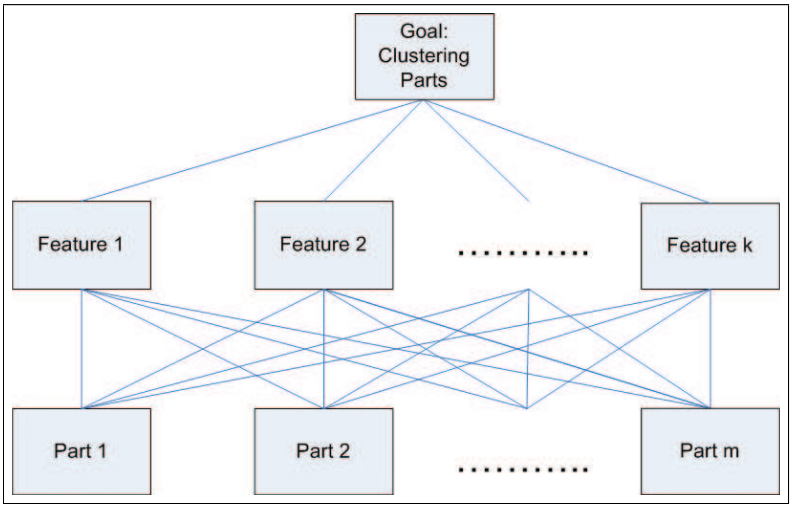
space(each feature is a different dimension). Using this matrix, grouping is accomplished by using any clustering method.

2. Aggregate-value clustering: Each part represented by an aggregate value which is

$$v_i = \sum_{j=1}^{m} w_{f_j} w_{p_{ij}} \text{ for } i = 1, ...n. \tag{2.13}$$

Then parts are clustered based on these single values.

Figure 2.9: Hierarchy of the AHP Model in Ben-Arieh and Triantaphyllou's Work



2.3.2 VAHP Based Clustering

Zahir [25] has developed a Euclidean version of the AHP in a vector space (VAHP). Due to the fact that AHP uses summation normalization, using such numerical data in a subsequent clustering procedure that uses Euclidean distance as the similarity measure may be problematic. However, the VAHP based approach allows obtaining representations of objects that satisfy Euclidean normalization and thus is consistent with the use of Euclidean distance in the clustering technique. [26]

The VAHP has the same decision hierarchy as the conventional AHP and uses the same eigenvector method. The decision space is assumed to be a linear vector space spanned by the number of objects to decide from and each eigenvector is defined with an Euclidean norm. However, here Zahir [25] introduces the preference operator P obtained by taking the square roots of each element of A, where A is the corresponding preference matrix of the conventional AHP. If A is consistent, P is also consistent and $\lambda_{max} = n$ for both. If the eigenvector w of A, such that $\sum w_i = 1$, the eigenvector v of P is normalized such that $v^T v = 1$ or $\sum_{i=1}^{n} v_i^2 = 1$. According to 'Eigenvalue Power Law' theorem [15] only when A satisfies the generalized consistency condition, we have $v_i = \sqrt{w_i}$ or $w_i = v_i^2$. This preserves the validation successes of conventional AHP. Thus, as we interpret w_i as the relative priority for A, v_i^2 is taken as the relative priority for P.

To sum up, the VAHP uses the same structure, the same decision hierarchy, the same eigenvector method, and it makes it possible to develop a meaningful grouping or clustering technique based on Euclidean distance.

2.3.3 An extension of the k-means algorithm

Smet and Guzman [19] proposed an extension of the well-known k-means algorithm to the multicriteria framework. This extension relies on the definition of a multicriteria distance based on the preference structure defined by the decision maker. With the proposed multicriteria distance two alternatives will be similar if they are preferred, indifferent, and incomparable to more or less the same actions.

The method proposed by Smet and Guzman [19] can be applied by using the results of any multicriteria decision making method. It just needs the preference structure as the input to partition the set of alternatives into classes that are meaningful from the multicriteria perspective.

2.3.4 General Clustering Algorithms

2.3.4.1 K-Means Clustering Algorithm

The k-means clustering algorithm is one of the simplest and the most commonly used algorithms. It employs squared error similarity criteria, which is widely used criterion function in clustering. It starts with predefined number (k) of initial set of clusters and at each iteration, patterns/objects, that are tried to be clustered, are reassigned to the nearest cluster based on the distance based similarity measure, this process is repeated until a converge criterion is met such as no reassignment of any pattern to a new cluster or predefined error value. [9]

In detail, the algorithm of the method is as follows:

There are n input patterns and patterns are denoted by P_1, P_2, ., P_n. The pattern P_i (ith pattern) consists of a tuple of describing features where features are denoted by f_{i1}, f_{i2}, . , f_{id}. A dimension represents each feature, where d is the number of dimensions of the value space. The second input of the algorithm is the predefined number of clusters, denoted by k. The number of the clusters cannot be changed during the execution of the algorithm. Let C_1, C_2,., C_k be the clusters, and each cluster is represented by its centroid. Let c_1, c_2, . , c_k be the centroids of the clusters.

First, the initial cluster centroids are formed randomly. The distances between pattern P_i and all clusters are calculated and pattern P_i is assigned to the closest cluster C_d. This process is repeated for all patterns and all patterns are assigned to a unique cluster. At the end of the iteration all centroids $(c_1,c_2, \;.\;\;, c_k)$ are updated. In the next iteration, distance calculations between patterns and clusters are repeated with the updated centroids. The algorithm will iterate until predefined number of iteration is reached or no pattern is moved to a different cluster. At the end of the algorithm, quality of the clustering is measured by the error function:

$$E = \sum_{d=1}^{k} \sum_{P_i \in C_d} \|P_i - C_d\|^2 \tag{2.14}$$

Moreover, we need a measure of how good the clusters, so that we can choose the right value of k. The silhouette value for each object is a measure of how similar that object is to objects in its own cluster compared to objects in other clusters, and ranges from -1 to +1. It is defined as

$$\begin{aligned} s(P_i) &= \frac{b(P_i) - a(P_i)}{\max(a(P_i), b(P_i))} \qquad (2.15) \\ \text{where} & \\ a(P_i) &= \text{average dissimilarity of } P_i \text{ to all other objects of } A \\ d(P_i, C) &= \text{average dissimilarity of } P_i \text{ to all objects of } C \\ b(P_i) &= \min_{C \neq A} d(P_i, C) \end{aligned}$$

where A is the first assigned cluster of object P_i, and C is any cluster different from A. As the $s(P_i)$ value comes close to 1, it means that object i is at the right cluster. The average of the $s(P_i)$ for $i = 1, 2, ..., n$ which is called the average silhouette width for the entire data set can be used for the selection of a 'best' value of k, by choosing that k for which the average silhouette width for the entire data set is as high as possible. [9]

The objective of the k-means clustering algorithm is to select the best clustering with k groups.

2.3.4.2 Hierarchical Clustering Algorithm

Hierarchical algorithms do not construct a single partition with k clusters, but they deal with all values of k in the same run. That is, both the partition with $k = 1$ and the partition with $k = n$ are part of the output. The output is in the form of *dendrogram*, where nested partitions and similarity levels at partitions change are presented. [9]

There are two basic approaches in hierarchical clustering:

- Agglomerative (starts when all objects are apart, the case where there are n clusters, and merges two clusters at each step until only one is left.)
- Divisive (starts with when all objects are together, the case where there is one cluster, and splits up clusters at each step, until there are n clusters.)

Due to its advantages and easy to implement speciality, agglomerative hierarchical clustering algorithms are used frequently. It starts when all objects are apart and in all succeeding steps, the two closest clusters are merged.

Hierarchical methods suffer from the defect that they can never repair what was done in previous steps. Indeed, once an agglomerative algorithm has joined two objects, they can not be separated anymore. Also, a cluster that has been split up by a divisive algorithm can not be reunited. [9]

The objective of hierarchical clustering algorithms is not finding the best clusterings but to describe the objects in a totally different way.

Chapter 3

Model and The Analysis

Based on the studies related to project management in literature, we observed that "a project is a project" is the dominating idea. However the idea that different projects must be managed in different ways, comes out from the real-life applications at MST, one of the three divisions of Aselsan, as a need. What we need is basically a framework to distinguish projects that have common characteristics that are meaningful in terms of project management concept. Then practical guidelines on how to manage projects in different ways can be established. However creating such a framework which has substantial importance for the company, Aselsan, requires great effort.

Our strategy for creating the framework can be summarized as follows:

1. Firstly, we look at the clustering of projects for which data is available. The natural groupings among the projects hopefully come out.
   (a) The features that can reveal the differences between the projects in terms of project management concept are determined.
   (b) The projects for which data is available, and there is no problem to be evaluated for the clustering work are determined.
   (c) After obtaining the related data of the projects, projects are clustered by using appropriate methods.

2. Secondly, the project groupings are examined to see whether they are meaningful and they can be used for creating the framework.
3. By benefiting the proposed frameworks in the literature and the project groupings that come out in previous steps, the framework is created.

In this thesis we focused on (1c), "After obtaining the related data of the projects, projects are clustered by using appropriate methods."

The problem discussed in this study is grouping N different projects based on a predetermined set of features. It is a multicriteria clustering problem. Basically, the problem has two parts:

Defining the projects in terms of features: The amount of each of the features that each project possesses is measured by using a multicriteria decision method. In other words, if we think each feature as a fuzzy set, we are trying to find membership values of projects for each fuzzy set.

Clustering the projects: The projects are clustered based on the representing vectors that are formed with the amount of each of the features that each project possesses.

3.1 Defining The Projects In Terms of Features

The choice of method for defining the projects in terms of features, closely related with the features characteristics. For this reason, we first look at the features.

3.1.1 Features

There are five predetermined features. Each of them is explained below.

3.1.1.1 Technological Uncertainty

In general, technological uncertainty is associated with the degree of using new (to the company) versus mature technology within the product or process. Since most projects employ a mixture of technologies, our interpretation is based on the share of new technology within the product. In addition; as a project progresses, the related technological uncertainty of project tends to change; therefore, technological uncertainty of a project implies the related uncertainty at the time of project initiation.

3.1.1.2 Platform Type

Platform type defines the working environment of product(s). Platform types can be grouped under two main titles. One of them is commercial platforms, and the other one is military platforms. In general form, platform types are:

- Commercial Platforms
  - Stationary Ground
  - Mobile Wheel Drive Ground
  - Airborne
  - Naval
- Military Platforms
  - Stationary Ground
  - Mobile Wheel Drive Ground
  - Mobile Tracked Ground
  - Airborne
  - Naval

Platform types are directly related with the standards and specifications that must be obeyed. Therefore it is a crucial parameter for a project.

3.1.1.3 Work and Test Environment

Efficiency is strongly dependent on the work and test environment. Due to structural and operational characteristics of systems, the work and test environment has to be sometimes a military area in battlefield conditions, sometimes a construction facility which does not belong to Aselsan for a considerable time interval. For this reason, work and test environment is the feature that should be incorporated into the evaluation of projects.

3.1.1.4 System Scope

Products are composed of components and systems of subsystems. Every product has its own hierarchy with different design and managerial implications. Projects can be classified as follows in terms of their hierarchies in its most general form :

1. Assembly project: dealing with either a single component or with a complete assembly (collection of components and modules combined into a single unit).
2. System project: a collection of interactive elements functioning together within a single product.
3. Array project: dispersed collection of systems that function together to achieve a common purpose.

3.1.1.5 Amount of Resource (Labor)

Human resources are extremely important for an organization like MST where tens of R&D projects are carried out concurrently. In addition, the amount of human resources (man*hour) needed for a project is an appropriate indicator about the size of the project.

We can classify the features by using the Ben-Arieh and Triantaphyllou's [3] classification.

- Quantitative features:
  - Amount of resource (labor): The numerical data of amount of resource (labor) is available for all projects.

- Qualitative fuzzy features:
  - Technological Uncertainty: There is no numerical data related to technological uncertainty of projects. Technological uncertainty of the projects are described by the terms low, medium, high, super high.
  - Platform Type: There is no numerical data related to platform types of projects. Although each of the projects' platform type is known, its effects to project management should be quantified.
  - Work and Test Environment: There is no numerical data related to work and test environment of projects.
  - System Scope: There is no numerical data related to system scope of projects. The general classification of projects as assembly, system and array is known.

As it can be seen above, four features out of five are characterized as 'qualitative fuzzy feature'. These features can be considered as subjective feature, because for these features, projects' related attributes may be evaluated differently by different experts. The other one, amount of resource (labor), is characterized as 'quantitative feature' or objective feature. For this reason, the choice of method for defining the projects in terms of features should permit the evaluation of both objective features and subjective features.

One of the most appropriate method to quantify subjective features is using expert knowledge, so the method should aim to get expert knowledge as good as possible. Moreover, the method should permit a group of people to realize the evaluation together and/or one by one, since subjective evaluations may change from person to person and the best way to reach more accurate results is to get data from a group of people who are experts in the related area.

To sum up, we need a multicriteria decision making method, which supports subjective and objective evaluations, gets expert knowledge in an effective way, to determine the membership value of each feature in the projects. For this reason, Analytic Hierarchy Process is chosen since it is a powerful multicriteria decision making method which meets the requirements of the problem.

3.1.2 AHP Model

The model is constructed in the way that AHP methodology proposes. For this reason, the model is explained based on AHP methodology:

Overall goal (objective): The objective is to evaluate the projects in terms of project management concept.

Hierarchical structure: Hierarchical structure can be seen in Figure 3.1. The hierarchy starts with the objective at the top level. The second level is formed by the features determined previously. The third level, which is the final one, is formed by the projects.

Pairwise comparisons: The elements in a level of the hierarchy are compared in relative terms with respect to a given criterion that occupies the level immediately above the elements being compared. In this model, we can distinguish two main groups of pairwise comparisons. One group is the features' pairwise comparisons with respect to objective. The other group consists of pairwise comparisons of projects with respect to each feature. For the features' pairwise comparisons, the question that is tried to answer for each comparison is like "Which one of the Feature A and Feature B is more important/effective than the other one with respect to project management concept and how much relatively?". For projects' comparisons, the questions should be identified for each feature:

- For Technological Uncertainty: Which one of the Project A and Project B has a higher degree of technological uncertainty and by how much relatively?

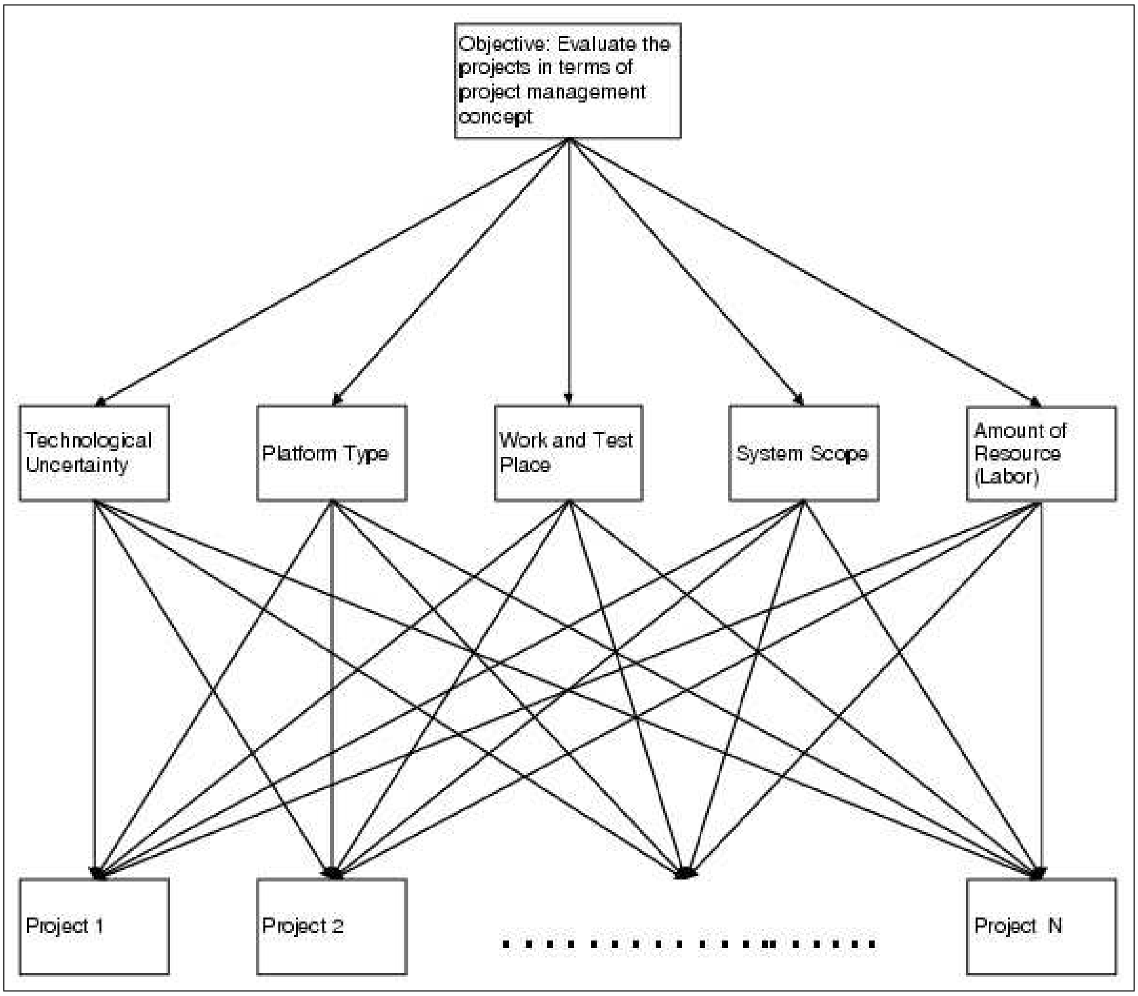
- For Platform Type: Which one of the Project A's platform type and Project B's platform type has a stronger effect in the way to complicate the management activities and by how much relatively?
- For Work and Test Environment: Which one of the Project A's work and test environment and Project B's work and test environment has a stronger effect in the way to complicate the management activities and by how much relatively?
- For System Scope: Which one of the Project A's system scope and Project B's system scope has a stronger effect in the way to complicate the management activities and by how much relatively?
- For Amount of Resource (Labor): Which one of the Project A's amount of resource (labor) and Project B's amount of resource (labor) is larger and how much relatively?

Estimation of Weights: Weights of the elements are estimated by using the eigenvalue method. Consistency of judgments is checked.

The AHP model that is described above has to conform to axioms of AHP. Below, the model is checked whether it conforms to the axioms or not.

- Axiom 1 (Reciprocal Comparison): Since the questions used to elicit the judgments or paired comparisons are clearly, correctly stated and asking for a reciprocal relation. This axiom is conformed.
- Axiom 2 (Homogeneity): The elements of the model that are evaluated with pairwise comparisons are comparable and they do not differ by too much in the property being compared. Both features' set and projects' set are homogeneous. The model conforms to Axiom 2.
- Axiom 3 (Independence): In expressing preferences, the features' importance are independent of the properties of projects. Features are evaluated with respect to their importance/effectiveness to project management activities in general, and the model conforms Axiom 3.

Figure 3.1: Hierarchical Structure



- Axiom 4 (Expectations): For the purpose of evaluating projects in terms of project management activities, the hierarchy is assumed to be complete. Axiom 4 is conformed.

In literature, as it is mentioned, there are two versions of AHP. One of them is the original AHP which is called "Distributive Mode" and the other one is called "Ideal Mode". By following the guidelines developed by Saaty and Millet [10], the one, which fits best to the case, must be chosen. The guideline states that the original AHP, Distributive Mode AHP, should be used when the decision

Table 3.1: Output Table

Feature	Feature Weight	Project 1	Project 2	.	.	Project m
1- Technological Uncertainty	w_{f_1}	$w_{p_{11}}$	$w_{p_{21}}$	.	.	$w_{p_{m1}}$
2- Platform Type	w_{f_2}	$w_{p_{12}}$	$w_{p_{22}}$	.	.	$w_{p_{m2}}$
3- Work and Test Environment	w_{f_3}	$w_{p_{13}}$	$w_{p_{23}}$			$w_{p_{m3}}$
4- System Scope	w_{f_4}	$w_{p_{14}}$	$w_{p_{24}}$			$w_{p_{m4}}$
5- Amount of Resource (Labor)	w_{f_5}	$w_{p_{15}}$	$w_{p_{25}}$	.	.	$w_{p_{m5}}$

maker is concerned with the extent to which each alternative dominates all other alternatives under the criterion. On the other hand, Ideal Mode AHP should be used when the decision maker is concerned with how well each alternative performs relative to a fixed benchmark. In our case, main objective is to construct a general framework for projects, so the aim here is not to find the dominating project. The aim is to define projects in terms of features, and to evaluate projects relative to a fixed benchmark. For this reason, Ideal Mode AHP seems to fit best to the needs of the problem.

The output of the model explained above constitutes the features weights, and weights/membership values of projects in terms of each one of the features. In a table format, the output is like Table 3.1, where w_{f_a} defines the weight of feature numbered 'a', $w_{p_{ab}}$ defines the weight of project numbered 'b' with respect to the feature numbered 'a' for $a = 1, 2....5$ and $b = 1, 2...m$.

Ben-Arieh and Triantaphyllou [3] proposed a different way to evaluate alternatives with respect to objective features. Since quantitative data is available for objective features, the alternatives do not need to be evaluated with pairwise comparisons. By just normalizing the quantitative data such that the largest number in the vector is equal to one, the membership values of projects in terms of an objective feature are obtained. Therefore, smaller number of pairwise comparisons have to be evaluated and experts can focus on the evaluations where expert knowledge is the inevitable source. In our model, amount of resource (labor) is the only objective feature, and it is appropriate to obtain membership values of projects in terms of this feature by using the method proposed in Ben-Arieh and Triantaphyllou [3] work. In the output table 3.1, only $w_{p_{15}}$ $w_{p_{25}}$ . . . $w_{p_{m5}}$ values are obtained by normalizing the available quantitative data.

3.2 Clustering The Projects

Once the projects are defined in terms of features, and the features' weights are determined, it is possible to cluster the projects into groups. However, due to the multicriteria nature of the problem, there are different views related to distance concept.

As it is mentioned, there are three applications related to multi-criteria clustering. The crucial differences between the applications result from different distance concepts. Ben-Arieh and Triantaphyllou [3] assumed that the matrix formed by multiplication of the features' weights with weight/membership vectors of projects can be used to form representative vectors of projects. With these vectors, each project can be represented in Euclidean space where each feature is a different dimension. Hence, the distance between two projects can be calculated as the Euclidean distance of two points (representing vectors of two projects).

On the other hand, Zahir [26] states that in order to obtain a meaningful pattern discovery, the underlying similarity measure cannot be independent of the type of normalization imposed on the data. In addition, since the AHP uses summation normalization, using such numerical data in a subsequent clustering procedure that uses Euclidean distance as the similarity measure may be problematic. For this reason, Zahir [25] developed a Euclidean version of the AHP in a vector space (VAHP). The VAHP based approach allows obtaining representations of objects that satisfy Euclidean normalization and thus is consistent with the use of Euclidean distance in clustering technique.

In Smet and Guzman's work [19], a multicriteria distance, based on the preference structure defined by the decision maker, is proposed. This multicriteria distance is based on the idea that two alternatives will be similar if they are preferred and indifferent to more or less the same features. In preference modelling, usually the following relations are considered:

$$\begin{cases} a_i P a_j & \text{if } a_i \text{ is preferred to } a_j, \\ a_i I a_j & \text{if } a_i \text{ is indifferent to } a_j, \\ a_i J a_j & \text{if } a_i \text{ is incomparable to } a_j. \end{cases}$$

The multicriteria distance definition proposed by Smet and Guzman [19] is:

$$d(a_i, a_j) = 1 - \frac{\sum_{k=1}^{4} |P_k(a_i) \cap P_k(a_j)|}{n} \tag{3.1}$$

where

- $P_1(a_i) = \{a_j \in A | a_i J a_j\}$,
- $P_2(a_i) = \{a_j \in A | a_j P a_i\}$,
- $P_3(a_i) = \{a_j \in A | a_i I a_j\}$,
- $P_4(a_i) = \{a_j \in A | a_i P a_j\}$,

In AHP, since there is no incomparable relation, the J relation remains empty and the comparison between pairs of actions are restricted to P and I relations.

In this thesis, all the distance concepts mentioned above are applied in project clustering application by using well-known clustering algorithms: k-means and hierarchical clustering.

Chapter 4

Application of Project Clustering

In this section, we present the details of the application performed in Aselsan. We look at the clustering of projects, and we expect that the natural groupings among the projects come out. Based on the model, the projects are evaluated with respect to predefined features by using AHP, and the representing vectors are formed for each project. Then, by using well-known clustering algorithms, k-means and hierarchical clustering, the projects are clustered.

For this study, fourteen projects are chosen to be evaluated. The set of fourteen projects is assumed to represent the full set of MST's applications. Since Aselsan is in military electronic sector, and most of its projects have high secrecy, we can not give the names of the projects in this study. However, we will mention the characteristics of the projects while the project clusters are being investigated.

Experts, as being the main source of projects' data, are the most important actors of the model. The projects are evaluated by experts with respect to all features except 'Amount of Resource (Labor)' feature which was identified as objective feature in the model. Moreover, project clusters coming out of the model are investigated by the experts to see whether they are meaningful or not. For this reason, two issues gain importance:

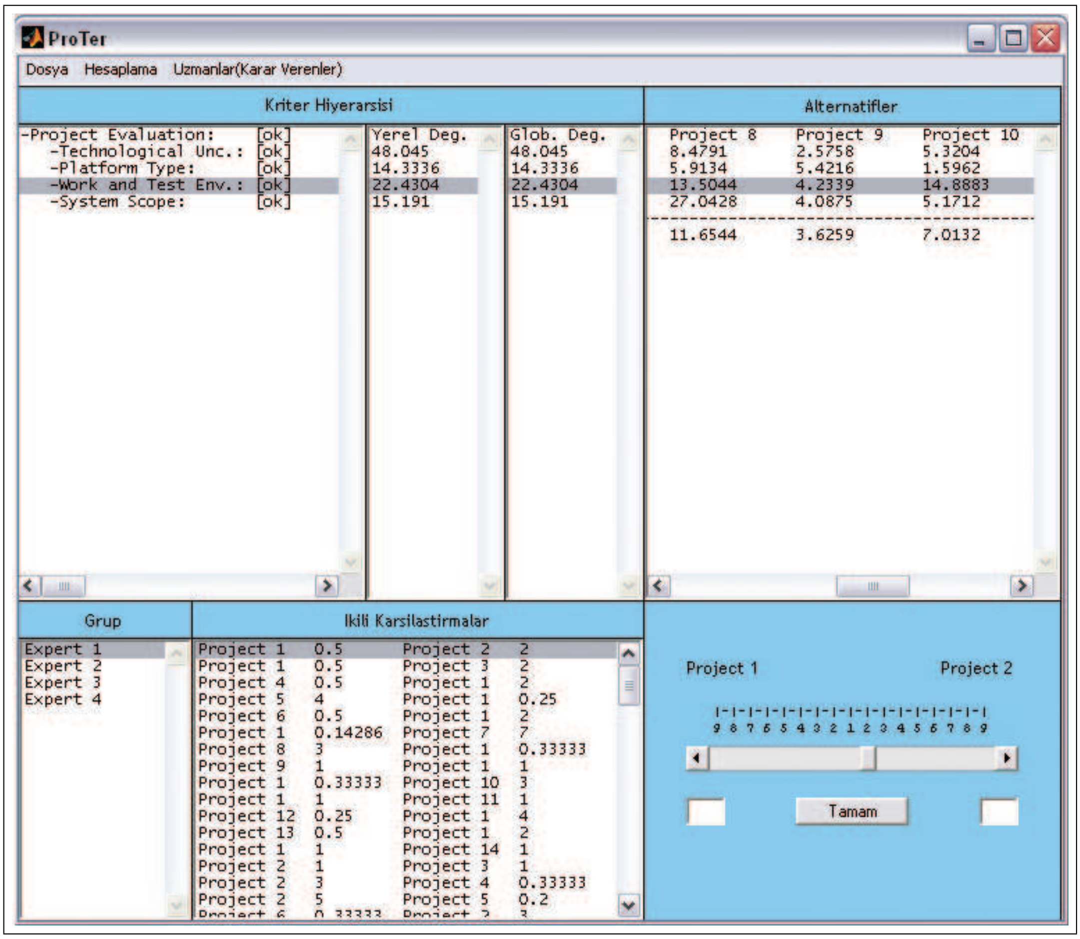
- Selection of experts
- Common understanding of the concepts used in the model

The vital characteristic for experts is to have enough knowledge about the set of fourteen projects such that for each subjective feature, pairwise comparisons of projects can be realized consciously. Since some of the projects out of the set of fourteen projects started at the beginning of 1990s, the number of possible experts possessing the vital characteristic is small. Among the possible candidates, four managers of MST kindly accept to provide data about the projects and to be part of the study.

Lack of common understanding of the concepts causes high group inconsistency and getting low quality of data which harms the quality of output, project clusters. For this reason, before initializing the process, we have worked together with the experts to give a common meaning to each concept in the model. The projects have also been discussed.

Expert Choice is the most well-known AHP software. In fact, Expert Choice is a software which has accelerated the wide-spread use of AHP. However, Expert Choice software is unavailable at Aselsan and the trial version of it restricts the number of alternatives to eight. For this reason, we create a software called '*ProTer*'. ProTer is a stand-alone AHP software that is capable of group decision making. The model's AHP part is conducted by using ProTer (Figure 4.1). You can find more information related to ProTer in Appendix.

Figure 4.1: ProTer



4.1 Application of AHP

In application of AHP, there are two main parts:

- Evaluation of Features
- Evaluation of Projects

All four experts perform both evaluation of features and evaluation of projects.

4.1.1 Evaluation of Features

In this part of the study, all the features including both the subjective features

- Technological Uncertainty
- Platform Type
- Work and Test Environment
- System Scope,

and the objective feature

- Amount of Resource (Labor)

are evaluated with pairwise comparisons by the experts. Since there are 5 features, each expert evaluate 10 pairwise comparisons. (Number of Pairwise Comparisons $= \frac{n\times(n-1)}{2}$, where n is the number of elements being evaluated.) You can find input values in Appendix.

After all the pairwise comparisons are evaluated by all the experts, the weights of the features can be calculated by using the AHP methodology. The results are as in Table 4.1.

Table 4.1: Features' Weights

Feature	Weight of the Feature
Technological Uncertainty	0.41
Platform Type	0.12
Work and Test Environment	0.20
System Scope	0.14
Amount of Resource(Labor)	0.13

As it can be seen from Table 4.1, 'Technological Uncertainty' feature is the dominant feature such that its weight nearly equals to two times of the weight of the second important feature of all, 'Work and Test Environment'.

The final weights of features are approved by the experts.

4.1.2 Evaluation of Projects

In this part of the study, all fourteen projects are evaluated pairwise by the experts in terms of the subjective features. For the objective feature, since we have the quantitative data, there is no need for the projects to be evaluated with pairwise comparisons.

Each expert, for each of the feature (subjective ones), has to evaluate 91 (Number of Pairwise Comparisons = $\frac{14\times(14-1)}{2}$) pairwise comparisons which is quite a high number. Totally, experts evaluated 364 (91×4) pairwise comparisons. (You can find input values in Appendix.) High number of pairwise comparisons is one of the weak points of AHP.

After all the pairwise comparisons are evaluated by all experts, the weights of the projects for each subjective feature can be calculated by using the eigenvalue method. The results are as in Table 4.2.

As it is seen at Table 4.2, for each feature, the weight vectors are normalized such that the max value is equal to one. This type of normalization is the result

Table 4.2: Projects' Weights for Subjective Features

	Technological Uncertainty	Platform Type	Work and Test Environment	System Scope
Project 1	0.0555	0.0777	0.2287	0.1010
Project 2	0.0830	1.0000	0.4400	0.3473
Project 3	0.0709	0.9356	0.4045	0.1098
Project 4	0.1978	0.1922	0.1635	0.2611
Project 5	0.1741	0.1514	0.0685	0.1235
Project 6	0.0946	0.1544	0.1574	0.0954
Project 7	0.8074	0.9730	1.0000	0.6161
Project 8	0.3646	0.2906	0.6380	1.0000
Project 9	0.1109	0.2665	0.1999	0.1513
Project 10	0.2287	0.0787	0.7037	0.1912
Project 11	0.4170	0.2856	0.2519	0.2862
Project 12	0.0649	0.1598	0.0647	0.0947
Project 13	0.6298	0.1514	0.1167	0.1513
Project 14	1.0000	0.1996	0.2878	0.1697

of the usage of Ideal Mode AHP.

For the objective feature, 'Amount of Resource (Labor)', the vector is formed by normalizing the related quantitative data of projects such that the max value is equal to one. The weight vector for the 'Amount of Resource (Labor)' feature is shown at Table 4.3.

The final weights of the projects for each feature are approved by the experts.

The output table of AHP application can be seen at Table 4.4.

4.2 Application of VAHP

As it has been mentioned previously, VAHP is a variation of AHP. VAHP uses the same input, same hierarchical model, same structure. For this reason, the input entered for the AHP model by the experts can be used without any change for VAHP model. By modifying ProTer, we manage to run VAHP algorithm for

Table 4.3: Projects' Weights for Objective Feature

	Amount of Resource (Labor)
Project 1	0.02
Project 2	0.14
Project 3	0.08
Project 4	0.07
Project 5	0.05
Project 6	0.01
Project 7	1.00
Project 8	0.42
Project 9	0.09
Project 10	0.31
Project 11	0.47
Project 12	0.16
Project 13	0.14
Project 14	0.34

Table 4.4: Output Table of AHP

	Technological Uncertainty	Platform Type	Work and Test Environment	System Scope	Amount of Resource (Labor)
Features' Weight	0.41	0.12	0.20	0.14	0.13
Project 1	0.0555	0.0777	0.2287	0.1010	0.02
Project 2	0.0830	1.0000	0.4400	0.3473	0.14
Project 3	0.0709	0.9356	0.4045	0.1098	0.08
Project 4	0.1978	0.1922	0.1635	0.2611	0.07
Project 5	0.1741	0.1514	0.0685	0.1235	0.05
Project 6	0.0946	0.1544	0.1574	0.0954	0.01
Project 7	0.8074	0.9730	1.0000	0.6161	1.00
Project 8	0.3646	0.2906	0.6380	1.0000	0.42
Project 9	0.1109	0.2665	0.1999	0.1513	0.09
Project 10	0.2287	0.0787	0.7037	0.1912	0.31
Project 11	0.4170	0.2856	0.2519	0.2862	0.47
Project 12	0.0649	0.1598	0.0647	0.0947	0.16
Project 13	0.6298	0.1514	0.1167	0.1513	0.14
Project 14	1.0000	0.1996	0.2878	0.1697	0.34

Table 4.5: Output Table of VAHP

	Technological Uncertainty	Platform Type	Work and Test Environment	System Scope	Amount of Resource (Labor)
Features' Weight	0.6411	0.3451	0.4515	0.3733	0.3559
Project 1	0.2357	0.2783	0.4808	0.3192	0.13
Project 2	0.2915	1.0000	0.6664	0.5870	0.38
Project 3	0.2692	0.9678	0.6387	0.3324	0.29
Project 4	0.4496	0.4378	0.4043	0.5094	0.26
Project 5	0.4210	0.3897	0.2619	0.3517	0.22
Project 6	0.3113	0.3934	0.3982	0.3101	0.09
Project 7	0.9065	0.9871	1.0000	0.7807	1.00
Project 8	0.6095	0.5394	0.8022	1.0000	0.65
Project 9	0.3364	0.5165	0.4496	0.3895	0.30
Project 10	0.4826	0.2797	0.8419	0.4383	0.56
Project 11	0.6532	0.5343	0.5041	0.5334	0.68
Project 12	0.2560	0.3996	0.2551	0.3090	0.40
Project 13	0.7995	0.3899	0.3432	0.3903	0.38
Project 14	1.0000	0.4453	0.5382	0.4128	0.58

the same input. The output table of the VAHP is at Table 4.5.

There is one point that should be identified related to 'Amount of Resource (Labor)' feature. For AHP, we reach the weight vector of projects for 'Amount of Resource (Labor)' feature by normalizing the quantitative data such that the maximum element is equal to one. However, for VAHP, we form the weight vector of projects for 'Amount of Resource (Labor)' feature by employing the proposed normalization method of VAHP to related quantitative data.

4.3 Application of Clustering Algorithms

Once the projects are described in terms of weight vectors which are five dimensional (each dimension represents each one of the features) and the features' relative importance are determined, we can continue with the clustering part of the study.

4.3.1 AHP Based Clustering

By using the AHP results as the input, Ben-Arieh and Triantaphyllou [3] proposed two different ways to conduct clustering process:

1. Matrix-based clustering: By multiplying the membership values of the features by the features' weight (importance), a new matrix is generated. In this matrix, each part (project) is represented as an m dimensional point in Euclidean space (each feature is a different dimension). Using this matrix, grouping is accomplished by using any clustering method.

2. Aggregate-value clustering: Each part (project) represented by an aggregate value which is

$$v_i = \sum_{j=1}^{m} w_{f_j} w_{p_{ij}} \text{ for } i = 1, ...n. \tag{4.1}$$

   Then parts (projects) are clustered based on these single values.

4.3.1.1 Matrix-based Clustering

By multiplying the membership values of the features by the features' weight (importance), we form a new matrix which is input of the clustering algorithms. We employed two well-known clustering algorithms: k-means and agglomerative hierarchical clustering.

The dendrogram coming out of the application of hierarchical clustering algorithm can be seen at Figure 4.2. In the dendrogram, the numbers on the X axis represent the project numberings, and the numbers on the Y axis represent the distances between clusters. By determining a limit distance, we can define the clusters. Limit distance is the maximum distance between two elements in the same cluster. For example, if we set distance limit to 20, there will be three clusters. It is like drawing a horizontal line that intersects the Y axis at 20. The clusters formed just below the line are the clusters that we are looking for.

In order to apply k-means clustering algorithm, we should determine the possible k values. As it is mentioned, there is a measure called the average silhouette width for the entire data set. The higher the value of the measure is, the more appropriate the value of k is.

Also, there are both an intuitive lower limit and an upper limit of k such that

$$k \geq 3 \text{ and } k \leq 5, \tag{4.2}$$

since for fourteen projects, two clusters can not reveal enough details. In addition, with six or more clusters, the clustering study is getting meaningless. For this reason, we should find the k value in between 3 and 5 for which the average silhouette width for the entire data set is the highest.

The desired k value comes out as 5 and the resulting silhouette values are at Figure 4.3. The clusters are at Table 4.6.

Table 4.6: Matrix-Based Clustering: Use of K-means Clustering Algorithm ($k = 7$)

Cluster Number	Project
4	Project 1
3	Project 2
3	Project 3
4	Project 4
4	Project 5
4	Project 6
1	Project 7
5	Project 8
4	Project 9
5	Project 10
5	Project 11
4	Project 12
2	Project 13
2	Project 14

Figure 4.2: Matrix-Based Clustering: Use of Hierarchical Clustering Algorithm

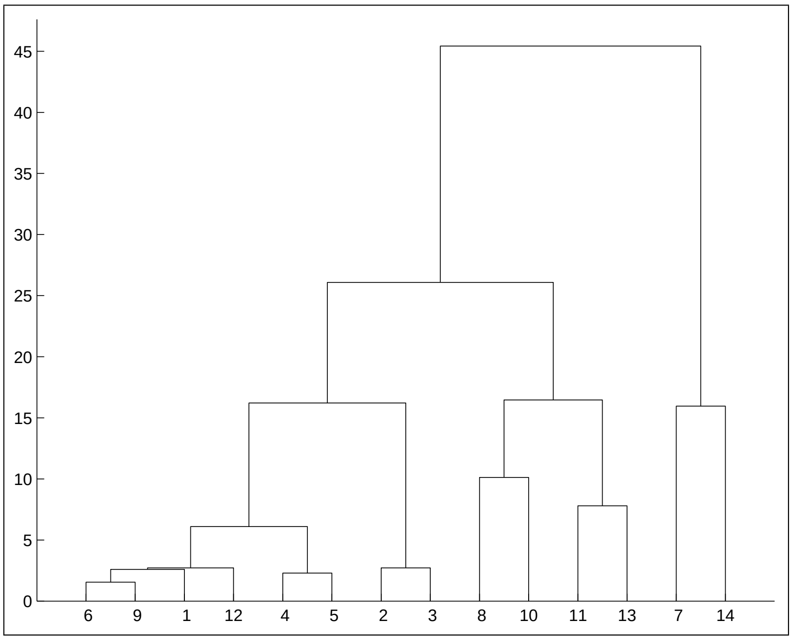


Figure 4.3: Matrix-Based Clustering: Silhouette Graph for $k = 5$

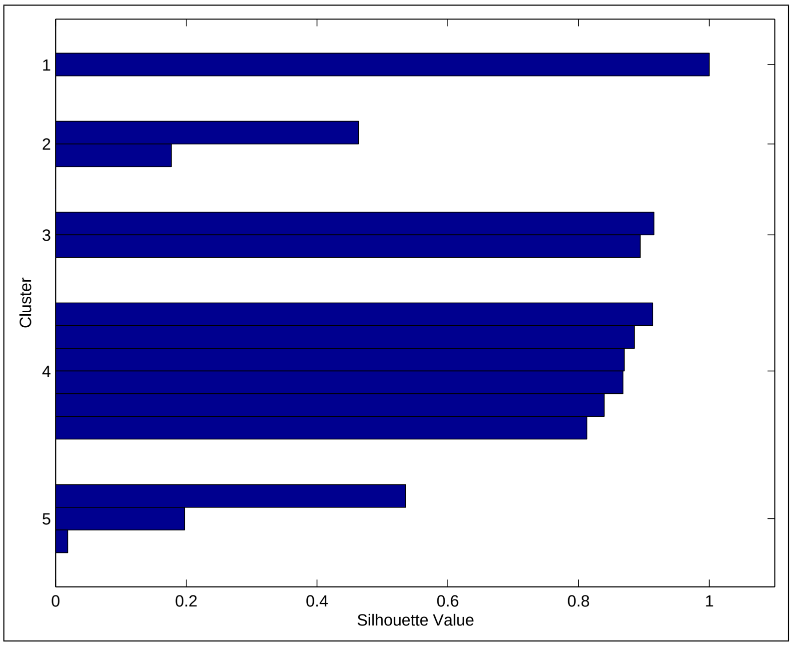


Table 4.7: Aggregate-value Clustering: Input Data

Project	Aggregate Value
Project 1	2.3791
Project 2	7.6195
Project 3	6.2760
Project 4	4.5268
Project 5	3.1752
Project 6	2.6716
Project 7	19.2448
Project 8	11.6544
Project 9	3.6259
Project 10	7.0132
Project 11	7.7718
Project 12	1.8887
Project 13	8.5061
Project 14	13.6468

4.3.1.2 Aggregate-value clustering

Each project is represented by an aggregate value, which is the ultimate output of AHP. The aggregate values can be seen at Table 4.7.

The dendrogram coming out of the application of hierarchical clustering algorithm can be seen at Figure 4.4.

The desired k value for k-means clustering algorithm comes out as 4 for aggregate-value clustering, and the resulting silhouette values are at Figure 4.5. The clusters are at Table 4.8.

Table 4.8: Aggregate-value Clustering: Use of K-means Clustering Algorithm ($k = 4$)

Cluster Number	Project
4	Project 1
3	Project 2
3	Project 3
4	Project 4
4	Project 5
4	Project 6
2	Project 7
1	Project 8
4	Project 9
3	Project 10
3	Project 11
4	Project 12
3	Project 13
1	Project 14

Figure 4.4: Aggregate-value Clustering: Use of Hierarchical Clustering Algorithm

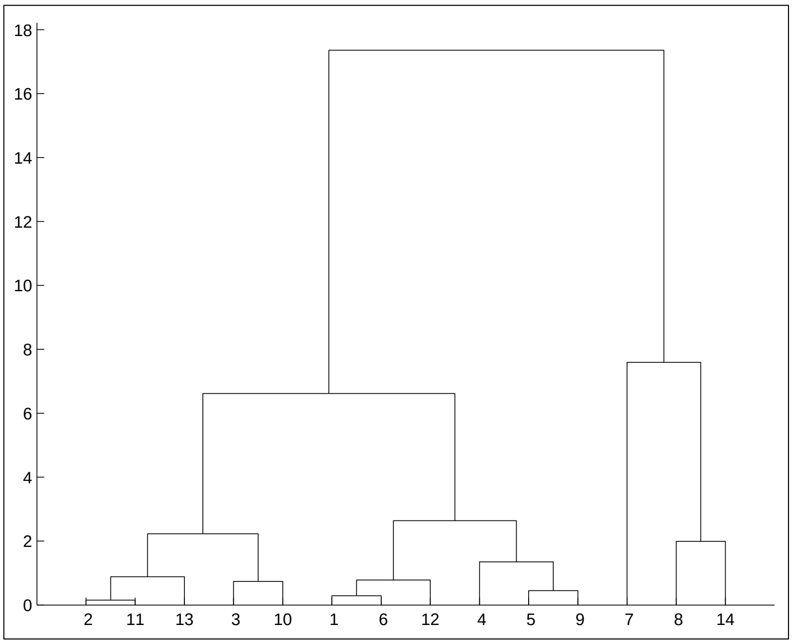


Figure 4.5: Aggregate-value Clustering: Silhouette Graph for $k = 4$

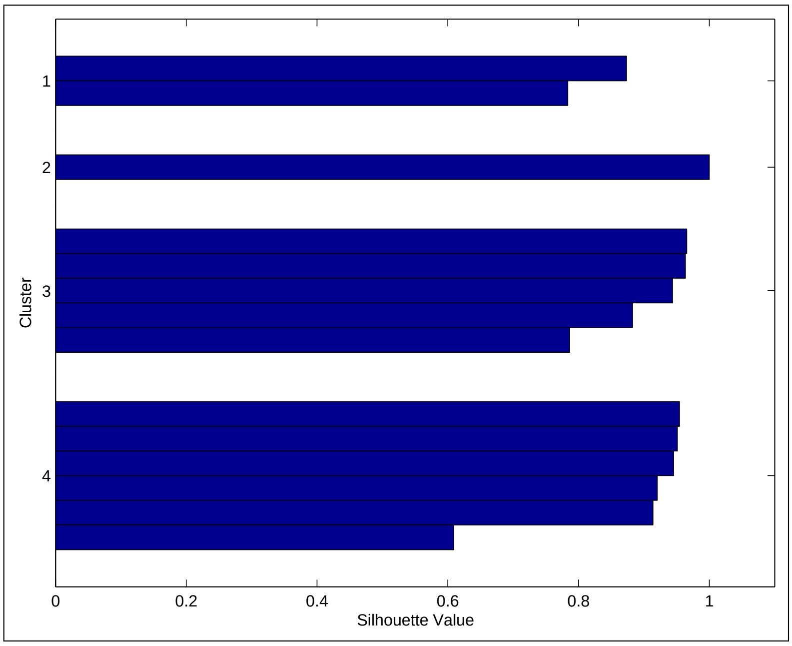


Table 4.9: VAHP Based Clustering: Use of K-means Clustering Algorithm ($k = 5$)

Cluster Number	Project
2	Project 1
3	Project 2
3	Project 3
2	Project 4
2	Project 5
2	Project 6
1	Project 7
5	Project 8
2	Project 9
5	Project 10
5	Project 11
2	Project 12
4	Project 13
4	Project 14

4.3.2 VAHP Based Clustering

By using the VAHP results as the input, the clustering process is conducted. By multiplying the membership values of the features by the features' weight (importance), we form a new matrix which is input of the clustering algorithms. We employed two well-known clustering algorithms: k-means and agglomerative hierarchical clustering.

The dendrogram coming out of the application of hierarchical clustering algorithm can be seen at Figure 4.6. The desired k value for k-means clustering algorithm comes out as 5 for VAHP based clustering, and the resulting silhouette values are at Figure 4.7. The clusters are at Table 4.9.

Figure 4.6: VAHP Based Clustering: Use of Hierarchical Clustering Algorithm

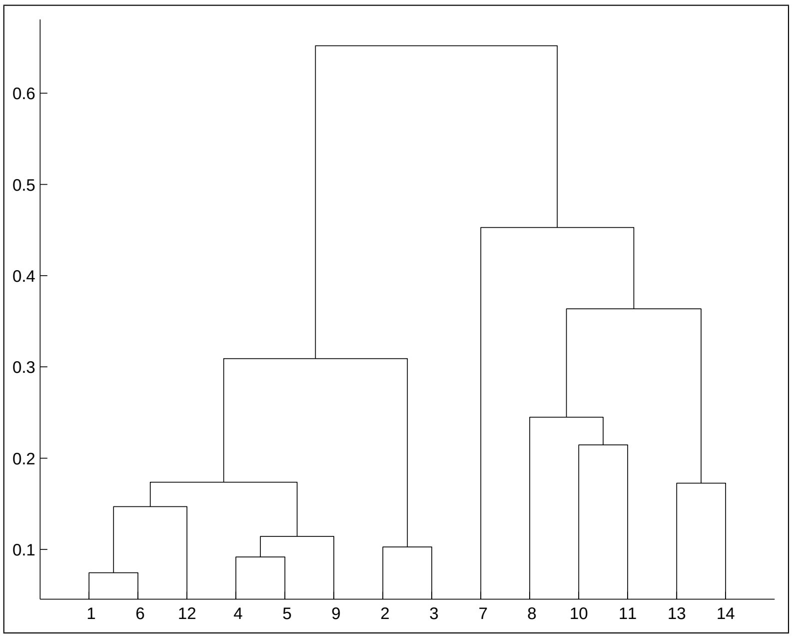


Figure 4.7: VAHP Based Clustering: Silhouette Graph for $k = 5$

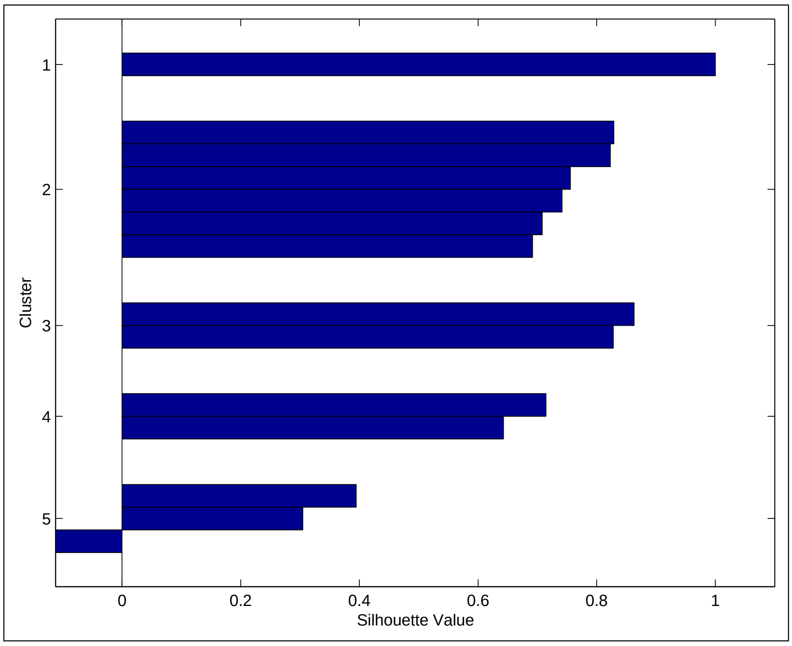


Table 4.10: Projects' Rankings

Rankings	Projects
13	Project 1
7	Project 2
8	Project 3
9	Project 4
11	Project 5
12	Project 6
1	Project 7
3	Project 8
10	Project 9
6	Project 10
4	Project 11
14	Project 12
5	Project 13
2	Project 14

4.3.3 Extension of K-means Algorithm for Multicriteria Clustering

As mentioned earlier, Smet and Guzman [19] proposed an extension to k-means clustering algorithm to the multicriteria framework by defining a multicriteria distance. The pairwise distances between the projects are calculated based on the AHP's resulting ranking of projects. The rankings of projects can be seen at Table 4.10.

The desired k value for k-means clustering algorithm comes out as 5 and the clusters are at Table 4.11.

4.4 Investigation of Project Clusters

If we look at the outputs of the different methods conducted to cluster the projects, we can see that the results of some methods are the same. These are

Table 4.11: Extension of K-means Algorithm for Multicriteria Clustering ($k = 5$)

Cluster Number	Project
1	Project 1
2	Project 2
3	Project 3
3	Project 4
5	Project 5
1	Project 6
4	Project 7
4	Project 8
5	Project 9
2	Project 10
4	Project 11
1	Project 12
4	Project 13
4	Project 14

- Matrix Based Clustering by using K-means Clustering Algorithm (Table 4.6)
- VAHP Based Clustering by using Hierarchical Clustering Algorithm (cut off point = 0.25), (Figure 4.6)
- VAHP Based Clustering by using K-means Clustering Algorithm (Table 4.9)

In fact, the clusterings coming out of these methods are prescribed by the experts as the most appropriate groupings of the projects among all the resulting project groupings. We should also note that the resulting clustering structure coming out of 'Matrix Based Clustering by using K-means Clustering Algorithm' is similar to the output of methods stated above.

This results are not so surprising, because the methods mentioned above are using detailed data of projects and features for clustering the projects. Each project is represented by five dimensional vector, each dimension corresponds to each feature, and these vectors are the input of clustering algorithms. The other methods are using aggregate data, which causes loss of information.

Moreover, we see that VAHP based approach gives the same results for hierarchical and k-means clustering algorithms. On the other hand, AHP based approach (matrix based clustering) gives different results for hierarchical and k-means clustering algorithms. This also supports the Zahir [26]'s claim that 'in order to obtain a meaningful pattern discovery, the underlying similarity measure can not be independent of the type of normalization imposed on the data'. According to the results of this application based study, VAHP based approach seems to be the best one.

The clustering of projects considered as the one representing the reality best, is in the following form:

Group 1: Project 1, Project 4, Project 5, Project 6, Project 9, and Project 12 are in Group 1. The common characteristic of these projects is that they are somehow simple projects for all of the features. These projects have a homogeneous structure such that each project possesses approximately the same amount of each feature characteristics.

Group 2: Project 2, and Project 3 are in Group 2. The common characteristic of the projects in this group is that they have quite a high degree of 'Platform Type' characteristics but a small degree of other features' characteristics.

Group 3: Project 7 is in Group 3. The cluster having only Project 7 represents the projects such that they are quite difficult projects in terms of all features.

Group 4: Project 8, Project 10, and Project 11 are in Group 4. The common characteristic of the projects in this group is that they are moderately difficult in terms of all the features.

Group 5: Project 13, and Project 14 are in Group 5. Group 5 shows a characteristic such that the projects that belong to this cluster have quite a high degree of 'Technological Uncertainty' characteristics but a small degree of other features' characteristics.

Chapter 5

Conclusions and Future Work

In this thesis, we try to solve a real life problem which is the clustering of R&D projects based on predetermined set of features. We are motivated to solve this problem, since formally differentiating the processes for different projects showing different characteristics comes out as a crucial need for MST, one of the three divisions of Aselsan.

Firstly, by using both AHP and VAHP, we evaluate the projects with respect to predefined features, and we obtain representing values and vectors for each projects. Then by employing several approaches existing in the literature we manage to cluster the projects. At the end of the study, we reach 5 project clusters, which shows different characteristics. The clusters are also approved by the experts and they think that these clusters can be used for constructing the general framework in order to classify the projects. From the results of this application based study, VAHP based approach comes out as the best one.

This study is constructed on two main fields: multicriteria decision making field and clustering field. In fact, the combination of these fields, multicriteria clustering area, is nearly untouchable. This study is important for bringing together the clustering applications of multicriteria decision making methodology (AHP) and applying the methods in a real life problem.

Since multicriteria clustering field is quite a new area, it is very appropriate for future works. In addition, the general accepted idea of ‘a project is a project’ in project management literature is being weakened, and the researches related to classifying projects is gaining importance.

Appendix A

Input Data: Pairwise Comparisons

Figure A.1: Evaluation of Features by Pairwise Comparisons: Matrix A of Expert 1

	Technological Unc.	Platform Type	Work and Test Env.	System Scope	Amount of Res.(Work)
Technological Unc.	1	0.25	0.5	0.33333	0.25
Platform Type		1	3	2	1
Work and Test Env.			1	0.5	0.5
System Scope				1	1
Amount of Res.(Work)					1

Figure A.2: Evaluation of Features by Pairwise Comparisons: Matrix A of Expert 2

	Technological Unc.	Platform Type	Work and Test Env.	System Scope	Amount of Res.(Work)
Technological Unc.	1	0.2	0.33333	0.25	0.33333
Platform Type		1	2	2	2
Work and Test Env.			1	0.5	0.5
System Scope				1	1
Amount of Res.(Work)					1

Figure A.3: Evaluation of Features by Pairwise Comparisons: Matrix A of Expert 3

	Technological Unc.	Platform Type	Work and Test Env.	System Scope	Amount of Res.(Work)
Technological Unc.	1	0.33333	0.25	0.25	0.33333
Platform Type		1	0.5	0.5	0.5
Work and Test Env.			1	1	0.5
System Scope				1	0.33333
Amount of Res.(Work)					1

Figure A.4: Evaluation of Features by Pairwise Comparisons: Matrix A of Expert 4

	Technological Unc.	Platform Type	Work and Test Env.	System Scope	Amount of Res.(Work)
Technological Unc.	1	0.5	1	0.5	0.5
Platform Type		1	2	1	2
Work and Test Env.			1	0.5	1
System Scope				1	1
Amount of Res.(Work)					1

Figure A.5: Evaluation of Projects by Pairwise Comparisons With Respect to "Technological Uncertainty": Matrix A of Expert 1

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	1	1	3	3	2	8	6	2	4	6	1	7	9
Project 2		1	1	3	3	1	8	6	1	3	6	1	7	9
Project 3			1	3	3	1	8	6	1	3	6	1	7	9
Project 4				1	1	0.5	5	2	0.5	1	0.5	0.5	4	6
Project 5					1	0.5	5	2	0.5	1	2	0.5	4	6
Project 6						1	8	4	1	2	4	1	7	9
Project 7							1	0.5	0.125	0.25	0.5	0.11111	1	2
Project 8								1	0.33333	0.5	1	0.14286	2	3
Project 9									1	2	4	1	6	8
Project 10										1	2	0.33333	4	6
Project 11											1	0.16667	2	4
Project 12												1	8	9
Project 13													1	2
Project 14														1

Figure A.6: Evaluation of Projects by Pairwise Comparisons With Respect to "Technological Uncertainty": Matrix A of Expert 2

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	2	2	4	4	2	9	7	2	4	7	2	8	9
Project 2		1	1	3	3	1	9	6	1	2	6	1	8	9
Project 3			1	3	3	1	9	6	2	3	6	1	8	9
Project 4				1	1	0.5	5	2	0.5	1	2	0.33333	4	6
Project 5					1	0.5	4	2	0.5	1	2	0.33333	4	5
Project 6						1	9	4	1	2	4	1	7	9
Project 7							1	0.5	0.125	0.25	0.5	0.11111	1	2
Project 8								1	0.25	0.5	1	0.16667	2	4
Project 9									1	2	4	0.5	6	9
Project 10										1	2	0.33333	4	6
Project 11											1	0.16667	2	3
Project 12												1	9	9
Project 13													1	2
Project 14														1

Figure A.7: Evaluation of Projects by Pairwise Comparisons With Respect to "Technological Uncertainty": Matrix A of Expert 3

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	2	2	5	5	2	9	8	3	5	8	2	9	9
Project 2		1	0.5	1	1	2	9	7	1	4	7	0.5	8	9
Project 3			1	3	3	2	9	5	2	4	6	1	8	9
Project 4				1	0.5	0.33333	6	3	0.5	2	4	0.2	3	6
Project 5					1	0.33333	5	3	0.5	3	4	0.2	6	8
Project 6						1	7	4	2	3	6	0.33333	4	8
Project 7							1	0.33333	0.25	0.2	0.33333	0.125	0.5	2
Project 8								1	0.25	0.5	2	0.16667	3	5
Project 9									1	3	5	0.25	6	8
Project 10										1	3	0.16667	4	6
Project 11											1	0.14286	2	3
Project 12												1	8	9
Project 13													1	3
Project 14														1

Figure A.8: Evaluation of Projects by Pairwise Comparisons With Respect to "Technological Uncertainty": Matrix A of Expert 4

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	3	2	4	6	3	9	7	5	6	6	2	7	9
Project 2		1	0.5	6	3	2	9	4	3	5	6	0.25	4	8
Project 3			1	4	4	2	9	5	4	6	7	0.33333	7	9
Project 4				1	0.33333	0.25	8	3	0.25	3	4	0.2	4	6
Project 5					1	0.33333	5	6	0.33333	3	5	0.25	6	5
Project 6						1	9	3	2	4	7	0.33333	6	7
Project 7							1	0.2	0.11111	0.16667	0.2	0.11111	0.33333	0.5
Project 8								1	0.25	0.5	2	0.16667	4	7
Project 9									1	4	5	0.25	5	6
Project 10										1	3	0.14286	5	7
Project 11											1	0.14286	2	4
Project 12												1	7	7
Project 13													1	3
Project 14														1

Figure A.9: Evaluation of Projects by Pairwise Comparisons With Respect to "Platform Type": Matrix A of Expert 1

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	9	9	2	2	2	9	3	2	1	5	9	2	2
Project 2		1	1	0.14286	0.16667	0.25	1	0.25	0.5	0.11111	0.25	0.14286	0.25	0.33333
Project 3			1	0.5	0.33333	0.33333	1	0.5	0.5	0.11111	0.5	0.14286	0.125	0.33333
Project 4				1	1	1	3	2	1	0.33333	2	1	1	1
Project 5					1	1	4	2	2	1	2	1	1	1
Project 6						1	4	2	2	0.5	2	1	1	1
Project 7							1	0.5	0.33333	0.11111	0.5	0.14286	0.25	0.33333
Project 8								1	1	0.2	0.5	0.5	0.5	1
Project 9									1	0.25	1	0.5	0.5	1
Project 10										1	5	2	2	3
Project 11											1	0.5	0.5	1
Project 12												1	1	1
Project 13													1	1
Project 14														1

Figure A.10: Evaluation of Projects by Pairwise Comparisons With Respect to "Platform Type": Matrix A of Expert 2

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	9	9	3	2	2	9	5	4	1	5	2	2	3
Project 2		1	1	0.16667	0.14286	0.14286	1	0.33333	0.33333	0.11111	0.25	0.125	0.125	0.16667
Project 3			1	0.14286	0.125	0.125	1	0.25	0.2	0.11111	0.25	1	0.125	0.14286
Project 4				1	1	1	7	1	1	0.33333	1	1	1	1
Project 5					1	1	8	2	2	0.5	2	1	1	1
Project 6						1	8	2	2	0.5	2	1	1	1
Project 7							1	0.25	0.2	0.11111	0.25	0.125	0.125	0.16667
Project 8								1	1	0.2	1	0.5	0.5	1
Project 9									1	0.25	1	0.5	0.5	1
Project 10										1	5	2	2	3
Project 11											1	0.5	0.5	1
Project 12												1	1	1
Project 13													1	1
Project 14														1

Figure A.11: Evaluation of Projects by Pairwise Comparisons With Respect to "Platform Type": Matrix A of Expert 3

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	9	9	4	2	2	9	5	4	1	5	2	2	3
Project 2		1	1	0.14286	0.125	0.125	1	0.25	0.2	0.11111	0.25	0.125	0.125	0.14286
Project 3			1	0.14286	0.125	0.125	1	0.25	0.2	0.11111	0.25	0.125	0.125	0.14286
Project 4				1	1	1	7	1	1	0.33333	1	1	1	1
Project 5					1	1	8	2	2	0.5	2	1	1	1
Project 6						1	8	2	2	0.5	2	1	1	1
Project 7							1	0.25	0.2	0.11111	0.25	0.125	0.125	0.125
Project 8								1	1	0.2	1	0.5	0.5	1
Project 9									1	0.25	1	0.5	0.5	1
Project 10										1	5	2	2	3
Project 11											1	0.5	0.5	1
Project 12												1	1	1
Project 13													1	1
Project 14														1

Figure A.12: Evaluation of Projects by Pairwise Comparisons With Respect to "Platform Type": Matrix A of Expert 4

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	9	9	3	2	2	9	5	4	1	5	2	2	3
Project 2		1	1	0.14286	0.125	0.125	1	0.25	0.2	0.11111	0.25	0.125	0.125	0.14286
Project 3			1	0.14286	0.125	0.125	1	0.25	0.2	0.11111	0.25	0.125	0.125	0.14286
Project 4				1	1	1	6	1	1	0.33333	1	1	1	1
Project 5					1	1	8	2	2	0.5	2	1	1	1
Project 6						1	8	2	2	0.5	2	1	1	1
Project 7							1	0.25	0.2	0.11111	0.25	0.125	0.125	0.2
Project 8								1	1	0.2	1	0.5	0.5	1
Project 9									1	0.25	1	0.5	0.5	1
Project 10										1	5	2	2	3
Project 11											1	0.5	0.5	1
Project 12												1	1	1
Project 13													1	1
Project 14														1

Figure A.13: Evaluation of Projects by Pairwise Comparisons With Respect to "Work and Test Environment": Matrix A of Expert 1

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	2	2	0.5	4	0.5	7	3	1	3	1	0.25	0.5	1
Project 2		1	1	0.33333	0.2	0.33333	0.33333	2	0.5	2	0.5	0.16667	0.25	0.5
Project 3			1	0.5	0.16667	0.33333	3	2	0.5	2	0.5	0.16667	0.25	1
Project 4				1	0.5	1	7	4	1	5	2	0.5	1	2
Project 5					1	2	9	9	3	9	4	1	2	4
Project 6						1	9	4	1	5	2	0.5	1	2
Project 7							1	0.5	0.2	0.5	0.25	0.11111	0.11111	0.33333
Project 8								1	0.33333	1	0.5	0.11111	0.16667	0.5
Project 9									1	4	1	0.33333	0.5	1
Project 10										1	0.33333	0.11111	0.16667	0.33333
Project 11											1	0.25	0.5	1
Project 12												1	1	4
Project 13													1	3
Project 14														1

Figure A.14: Evaluation of Projects by Pairwise Comparisons With Respect to "Work and Test Environment": Matrix A of Expert 2

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	2	2	0.5	0.2	0.5	5	3	1	3	1	0.2	0.5	2
Project 2		1	1	0.33333	0.16667	0.33333	3	2	0.5	2	0.5	0.14286	0.25	0.5
Project 3			1	0.33333	0.14286	0.33333	3	2	0.5	2	0.5	0.14286	0.25	0.5
Project 4				1	0.33333	1	8	4	1	5	2	0.5	1	3
Project 5					1	2	9	9	3	9	4	1	2	5
Project 6						1	8	4	1	5	2	0.33333	1	2
Project 7							1	0.5	0.16667	0.5	0.25	0.11111	0.11111	0.25
Project 8								1	0.33333	2	0.33333	0.11111	0.2	0.5
Project 9									1	4	1	0.33333	0.5	2
Project 10										1	0.33333	0.11111	0.125	0.33333
Project 11											1	0.25	0.5	1
Project 12												1	2	5
Project 13													1	3
Project 14														1

Figure A.15: Evaluation of Projects by Pairwise Comparisons With Respect to "Work and Test Environment": Matrix A of Expert 3

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	2	2	0.5	0.2	0.5	4	3	1	3	1	0.2	0.5	1
Project 2		1	1	0.33333	0.125	0.33333	2	2	0.5	2	0.5	0.125	0.25	1
Project 3			1	0.5	0.125	0.5	2	2	0.5	2	0.5	0.125	0.25	1
Project 4				1	0.33333	1	7	4	1	5	1	0.33333	1	0.25
Project 5					1	3	9	8	4	8	5	1	2	6
Project 6						1	7	4	1	5	1	0.33333	1	2
Project 7							1	0.5	0.16667	0.5	0.25	0.11111	0.14286	0.33333
Project 8								1	0.33333	1	0.5	0.11111	0.16667	0.25
Project 9									1	4	1	0.25	0.5	2
Project 10										1	0.33333	0.11111	0.14286	0.33333
Project 11											1	0.2	0.5	1
Project 12												1	2	6
Project 13													1	3
Project 14														1

Figure A.16: Evaluation of Projects by Pairwise Comparisons With Respect to "Work and Test Environment": Matrix A of Expert 4

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	2	2	1	0.2	1	4	2	1	3	1	0.2	0.5	1
Project 2		1	1	0.33333	0.125	0.33333	3	2	0.5	2	0.5	0.125	0.25	1
Project 3			1	0.33333	0.125	3	4	0.5	0.5	2	0.5	0.125	0.25	1
Project 4				1	0.33333	1	7	4	1	5	2	0.33333	1	0.5
Project 5					1	3	9	8	4	8	5	1	2	6
Project 6						1	7	4	1	5	2	0.33333	1	2
Project 7							1	0.5	0.2	0.5	0.25	0.11111	0.125	0.33333
Project 8								1	0.33333	0.16667	0.5	0.11111	0.16667	0.5
Project 9									1	5	1	0.25	0.5	2
Project 10										1	0.33333	0.11111	0.14286	0.5
Project 11											1	0.2	0.5	0.5
Project 12												1	2	6
Project 13													1	3
Project 14														1

Figure A.17: Evaluation of Projects by Pairwise Comparisons With Respect to "System Scope": Matrix A of Expert 1

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	4	1	2	1	1	6	9	2	2	3	1	1	2
Project 2		1	0.25	1	0.33333	0.25	1	3	2	0.5	1	0.33333	0.33333	0.5
Project 3			1	2	1	1	5	9	1	2	0.5	1	0.5	2
Project 4				1	0.5	0.33333	2	5	0.5	1	1	1	0.5	1
Project 5					1	1	5	9	1	2	3	1	1	1
Project 6						1	6	9	2	2	3	1	2	2
Project 7							1	2	0.25	0.33333	0.5	0.14286	0.25	0.33333
Project 8								1	0.11111	0.16667	0.25	0.11111	0.125	0.14286
Project 9									1	1	2	0.5	1	1
Project 10										1	1	0.5	1	1
Project 11											1	0.33333	0.5	0.5
Project 12												1	2	2
Project 13													1	1
Project 14														1

Figure A.18: Evaluation of Projects by Pairwise Comparisons With Respect to "System Scope": Matrix A of Expert 2

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	4	1	2	1	1	6	9	1	2	3	1	2	0.5
Project 2		1	0.25	1	0.33333	0.25	1	3	0.33333	0.5	1	0.2	0.33333	0.5
Project 3			1	2	1	1	5	9	1	2	5	1	2	2
Project 4				1	0.5	0.33333	2	5	0.5	1	1	0.33333	0.5	1
Project 5					1	1	5	9	1	2	0.5	1	1	1
Project 6						1	6	9	2	2	3	1	2	2
Project 7							1	2	0.25	0.33333	0.5	0.14286	0.25	0.33333
Project 8								1	0.11111	0.16667	0.25	0.11111	0.125	0.14286
Project 9									1	1	2	0.5	1	1
Project 10										1	1	0.5	1	1
Project 11											1	0.33333	0.5	0.5
Project 12												1	2	2
Project 13													1	1
Project 14														1

Figure A.19: Evaluation of Projects by Pairwise Comparisons With Respect to "System Scope": Matrix A of Expert 3

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	4	1	2	1	1	6	9	2	2	3	1	2	2
Project 2		1	0.25	1	0.5	0.25	1	1	0.33333	0.5	8	0.25	0.33333	0.5
Project 3			1	2	1	1	5	9	1	2	3	1	2	2
Project 4				1	0.5	0.5	2	5	0.5	1	1	0.33333	1	1
Project 5					1	1	5	9	1	2	2	1	1	1
Project 6						1	6	9	2	2	3	1	2	2
Project 7							1	0.25	0.25	0.33333	0.5	0.14286	0.25	0.33333
Project 8								1	0.11111	0.16667	0.25	0.11111	0.125	0.14286
Project 9									1	1	2	0.5	1	1
Project 10										1	1	0.5	1	1
Project 11											1	0.33333	0.5	0.5
Project 12												1	2	2
Project 13													1	1
Project 14														1

Figure A.20: Evaluation of Projects by Pairwise Comparisons With Respect to "System Scope": Matrix A of Expert 4

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10	Project 11	Project 12	Project 13	Project 14
Project 1	1	4	1	2	1	1	6	9	2	2	3	1	2	2
Project 2		1	0.33333	2	0.5	0.33333	1	4	0.5	0.5	1	0.33333	0.5	0.5
Project 3			1	2	1	1	5	9	1	2	3	1	1	2
Project 4				1	0.33333	0.5	0.5	4	0.33333	0.5	1	0.33333	0.5	1
Project 5					1	1	5	9	1	2	2	1	1	1
Project 6						1	6	9	2	2	3	1	2	2
Project 7							1	0.33333	0.2	0.2	0.33333	0.125	0.2	0.2
Project 8								1	0.11111	0.16667	0.25	0.11111	0.125	0.14286
Project 9									1	1	2	0.5	1	1
Project 10										1	1	0.5	1	1
Project 11											1	0.33333	0.5	0.5
Project 12												1	2	2
Project 13													1	1
Project 14														1

Appendix B

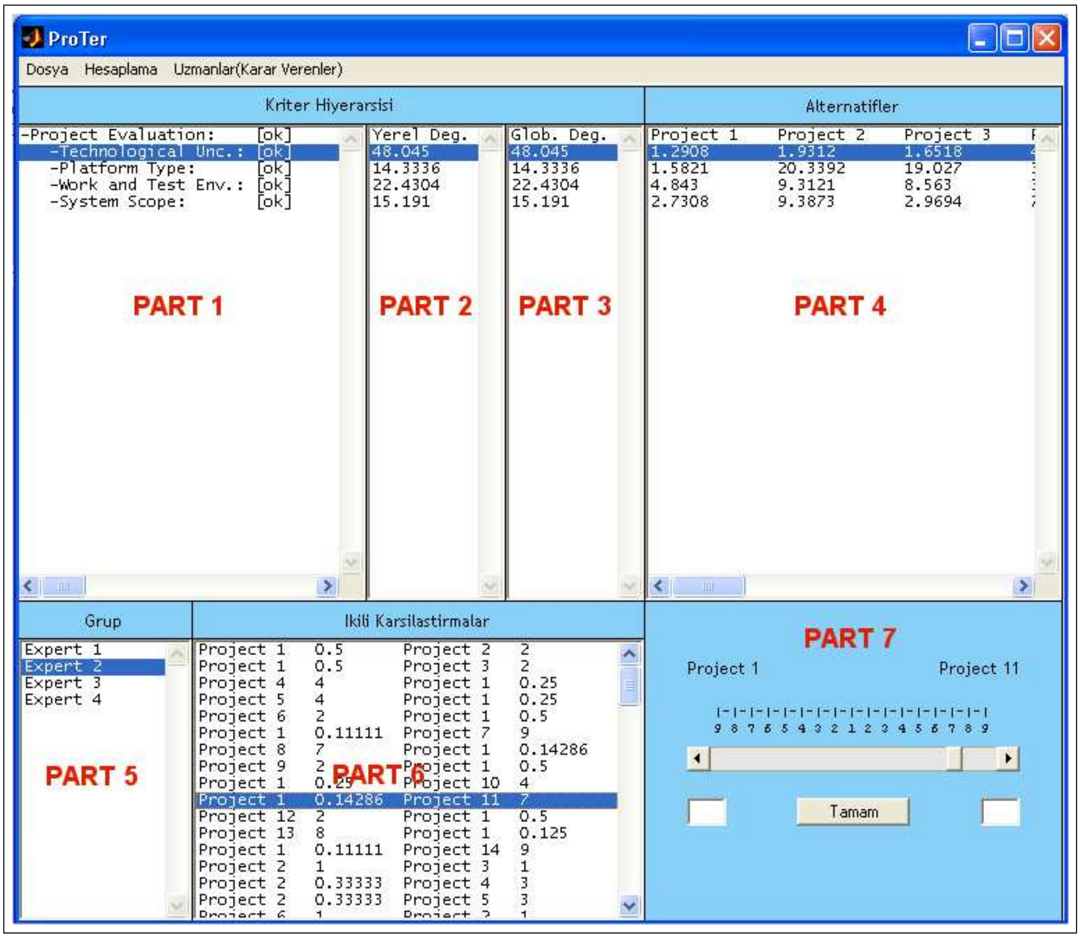
ProTer

ProTer is a stand-alone AHP software that is capable of group decision making. It is created as part of this thesis work. However, it is not created to meet only the thesis's model requirements. By using ProTer, it is possible to apply AHP method to any multicriteria decision problem. With its user interface, everyone that is familiar with AHP methodology, and has watched a short demonstration can use ProTer easily.

ProTer is created by using Matlab. ProTer makes it possible to save a work, and open it.

The user interface of ProTer has seven main parts, Figure B.1. Part 1 is the list-box, where hierarchy of criteria are entered into the program. The objective is also entered in Part 1, as the top of the hierarchy. Part 2 is the list-box where local weights (before synthesis of priorities) of criteria are shown. Part 3 is the list-box where global weights (after synthesis of priorities) of criteria are shown. Part 4 is the list-box where both alternatives and the alternatives' weights for each criterion are shown. Part 5 is the list-box where the group of experts are shown. Part 6 is the list-box where pairwise comparisons are shown. Part 7 is formed for evaluating each pairwise comparisons by either using sliding bar or text boxes.

Figure B.1: ProTer's Parts



Bibliography

[1] Aczel, J., and Saaty, T. L., (1983), "Procedures for Synthesizing Ratio Judgments", *Journal of Mathematical Psychology*, 27 93-102.

[2] Belton, V., and Gear, T., (1983), "On a Short-coming of Saaty's Method of Analytic Hierarchies", *Omega* 228-230.

[3] Ben-Arieh, D., and Triantaphyllou, E., (1992), "Quantifying data for group technology with fuzzy features", *International Journal of Production Research* 30(6) 1285-1299.

[4] Blake, S. B., (1978), "Managing for Responsive Research and Development", Freeman and Co., San Francisco, CA.

[5] Ferrari, P., (2003), "A method for choosing from among alternative transportation projects", *European Journal Of Operational Research* 150 194-203.

[6] Golden, B. L., Wasil, E. A., and Harker P. T., (1989) "The Analytic Hierarchy Process", Springer-Verlag, Berlin.

[7] Hauptman, O., (1986), "Influence of task type on the relationship between communication and performance: the case of software development", *R&D Management* 16(2) 127-139.

[8] Kahraman, C., Cebeci, U., Ruan, D., (2004) "Multi-attribute coparison of catering service companies using fuzzy AHP: The case of Turkey", *International Journal of Production Economics* 87 171-184.

[9] Kaufman, L., and Rousseeuw, P. J., (1989) “Finding Groups in Data: An Introduction to Cluster Analysis”, John Wiley&Sons, New York.

[10] Millet I., and Saaty, T. L., (2000), “On the Relativity of Relative Measures-Accomodating Both Rank Preservation and Rank Reversal In the AHP”, *European Journal of Operational Research* 121(1) 205-212.

[11] Ozdemir, M. S., Gasimov, R. N., (2004), “The analytic hierarchy process and multiobjective 0-1 faculty course assignment”, *European Journal of Operational Research* 157(2) 398-408.

[12] Pinto, J. K., and Covin, J. G., (1989) “Critical factors in project implementation: A comparison of construction and R&D projects”, *Technovation* 9 49-62.

[13] Saaty, T. L., (1980), “The Analytic Hierarchy Process”, McGraw-Hill, New York.

[14] Saaty, T. L., (1986), “Axiomatic Foundation of the Analytic Hierarchy Process”, *Management Science* 32(7) 841-855.

[15] Saaty, T. L., (1990), “How to make a decision: The Analytic Hierarchy Process”, *European Journal Of Operational Research* 48 9-26.

[16] Saaty, T. L., (1994), “Fundamentals of Decision Making and Priority Theory with the AHP”, RWS Publications, Pittsbugh, PA, U.S.A.

[17] Saaty, T. L., (2001), “The seven pillars of the analytic hierarchy process”, *Multiple Criteria Decision Making in the New Millennium Lecture Notes in Economics and Mathematical Systems* 507 15-37.

[18] Shenhar, A. J., (2001), “One Size Does Not Fit All Projects: Exploring Classical Contingency Domains”, *Management Science* 47(3) 394-414.

[19] Smet, Y. D., and Guzman, L. M., (2004), “Towards multicriteria clustering: An extension of the k-means algorithm ”, *European Journal of Operational Research* 158(2) 390-398.

[20] Triantaphyllou, E., Mann H. S., (1995), “Using The Analytical Hierarchy Process For Decision Making In Engineering Applications: Some Challenges”, *International Journal of Industrial Engineering: Applications and Practice* 2(1) 35-44.

[21] Wheelwright, S. C., and Clark, K. B., (1992), “Revolutionizing Product Development”, The Free Press, New York.

[22] Vargas, L. G., (1990), “An Overview of the Analytic Hierarchy Process and its applications”, *European Journal of Operational Research* 48 2-8.

[23] Zahedi, F., (1986), “The Analytic Hierarchy Process-A survey of the Method and Its Applications”, *Interfaces* 16 96-108.

[24] Zahir, S., (1999), “Clusters in a group: Decision making in the vector space formulation of the analytic hierarchy process”, *European Journal of Operational Research* 112 620-634.

[25] Zahir, S., (1999), “Geometry of decision making and the vector space formulation of the analytic hierarcy process”, *European Journal of Operational Research* 373-396.

[26] Zahir, S., (2002), “Using the Analytic Hierarchy Process for quantifying and classifying objects with multiple attributes”, *Infor* 40(2) 149-172.