EFFECTS OF PRODUCT VARIETY IN TECHNOLOGY SELECTION DECISION FOR CELLULAR MANUFACTURING SYSTEM DESIGN

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By

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July, 2003

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ABSTRACT

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In today's world, customers expect product variety. However, non-uniform products complicate the manufacturing processes significantly. In this study, we solved the cellular manufacturing system design and the technology selection problems simultaneously while taking the changing market dynamics into consideration. Cellular manufacturing system design problem aims the identification of existing part families and machine groups, while the technology selection decision determines the appropriate technology for the facility.

In order to integrate the market characteristics in our model, we proposed a new cost function. Further, we modified a well known similarity measure in order to handle the operational capability of available technology. This new coefficient is employed at the identification of part families. The technology selection decision is based on the individual properties of parts, namely the production volume, variability of the demand, and the design stability of the part. Integration of the product variety at the design stage leads us to the use of flexible machining systems and dedicated manufacturing systems at the same facility. In the thesis, our hybrid technology approach is presented via a multi-objective mathematical model. A filtered-beam based local search heuristic is proposed to solve the problem efficiently.

Keywords: Cellular manufacturing systems, technology selection, product variety.

ÖZET

HÜCRESEL ÜRETİM SİSTEMLERİNDE ÜRÜN ÇEŞİTLİLİĞİNİN TEKNOLOJİ SEÇİMİ KARARINA ETKİLERİ

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Günümüz pazarlarında ürün çeşitliliğinin sunulması bir zorunluluk haline gelmiştir. Ancak ürün çeşitliliği, üretim aşamasında birçok zorluğu da beraberinde getirir. Bu çalışmada, değişen pazar gereklilikleri göz önüne alınmış ve hücresel üretim sistemleri tasarımı ile aynı anda teknoloji seçimi kararı da verilmiştir. Hücresel üretim sistemleri tasarımı, sistem içinde varolan parça ailelerinin tanımlanması ve uygun makine gruplarının belirlenmesi esasına dayanır. Diğer yandan, teknoloji seçimi kararı da tasarımı yapılan tesiste kullanılacak teknolojiye karar verir.

Pazar özelliklerini modelimize katabilmek için yeni bir amaç fonksiyonu önerilmiştir. Buna ek olarak, varolan makinelerin operasyonel yeteneklerinin modelde kullanılmasına olanak sağlamak amacıyla çok bilinen bir benzerlik katsayısı yeniden düzenlenmiştir. Benzerlik katsayıları parça ailelerinin belirlenmesinde kullanılmaktadır. Ayrıca, modelde teknoloji seçimi kararı, parçaların bireysel özelliklerine dayandırılmıştır. Ürün çeşitliliğinin tasarım aşamasında göz önüne alınması, sonuçta esnek ve adanmış teknolojilerin bir üretim tesisinde aynı anda kullanılması gerekliliğini ortaya çıkarmıştır. Bu tezde, önerilen melez teknoloji yaklaşımı, çok amaçlı bir matematiksel modelle açıklanmaktadır. Ayrıca, söz konusu problemin olurlu çözümünün bulunabilmesi için filtrelenmiş ışın yerel tarama yöntemine dayalı bir algoritma önerilmiştir.

Anahtar sözcükler: Hücresel üretim sistemleri, teknoloji secimi, ürün çeşitliliği.

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Chapter 1

Introduction

Business world of the 21^{st} century witnesses an expanding global competition with increased variety of products and low demand. A company that wants to stay in this market should develop new manufacturing strategies. Old manufacturing technologies fail to meet the increasing demand for customized production.

In today's world, market is no longer satisfied with uniform products. While the customers are expecting product variety, it makes the manufacturing processes considerably difficult. Known manufacturing systems become inadequate to perform high variety production with low costs. Product variations in manufacturing brings high investment in equipment, high tooling costs, complex scheduling and loading, lengthy setup time and costs, excessive scrap, and high quality control costs.

Another effect of today's competitive environment is the change in product life cycles. The manufacturer has not spacious time for product design and production plan. The new product should rapidly be introduced to the market. Moreover, the total lifetime of the product has significantly decreased. In such an environment, the manufacturer cannot invest in dedicated lines at the whole facility. Because, the product design is likely to change before the dedicated facility has been paid for. This has implications for integration of flexible manufacturing systems. Flexible technology can be used both for existing designs and for future re-designs of the products.

Around the world, batch manufacturing is the dominant manufacturing activity. In literature, it is said to account for 60 to 80 percent of all manufacturing activities. However, batch manufacturing industries cannot compete in today's global market. Planning, controlling and scheduling must be simplified, material handling and setup times should be reduced, WIP inventories should be decreased, and quality must be improved. Furthermore, integration of design and manufacturing has to be achieved in order to provide customization to the market.

Group technology *(GT)* provides a gateway to achieve all these. Implementation of GT in manufacturing environment is the *Cellular Manufacturing System.* Machines are grouped into cells to produce a group of parts having similar design attributes or manufacturing requirements.

In today's world, technology selection is a more important issue. While designing a manufacturing system, the designer has considerably many alternatives. Especially flexible technology offers a variety of machines, some of which are highly capable in terms of operations. Today, there exist *'Done in one'* machines, which may process a part in just one setup.

Cellular manufacturing system design *(CMSD)* problem is very complex in nature. It is being studied for the last three decades. However, today, grouping of parts and machine selection problems are even harder because of the strong competition and product variety encountered in the new millennium. In literature, different approaches are proposed to solve CMSD problem. Methods used to identify machine-part families can be divided in three groups:

- Visual Inspection Method
- Part Characteristic Based Systems
- Production Process Based Systems

All these methods in the literature study part geometries, functions of parts, part routings, machine clustering requirements, layout types, etc. However, as far as we know, no study exists which takes product life cycle and technology selection attributes into account while determining the part families and machine groups simultaneously. In Chapter 2, the existing approaches to solve the cell formation problem and technology selection literature are reviewed. The advantages and disadvantages of the existing literature are discussed.

In our study, we will analyze single product attributes leading to variety. Processing times and available machine capacities will be taken into consideration while identifying part families and machine groups. Furthermore, a technology selection scheme is proposed for machine group formation. The problem is stated with the underlying assumptions in Chapter 3. In the same chapter, a mixed integer programming model is proposed to solve the problem. The model has five minimization objective functions regarding dissimilarity in part families, product variety costs, throughput time, machine investment, maintenance and labor costs and finally intercellular movement. This multi-objective mathematical model has machine capacity, utilization and cell size constraints.

The proposed mathematical programming model cannot be solved in a reasonable computation time because of the numerous binary and integer variables and quadratic functions used in the model. Consequently, a local search heuristic is proposed to solve the problem. The proposed algorithm has two main stages. In the first stage of the algorithm, a known fuzzy analysis is implemented by using an adapted similarity measure. According to the results of this fuzzy analysis, initial part families and machine groups are identified by the algorithm. In the second stage, filtered beam search principles are employed to improve the initial solution.

The proposed algorithm is discussed in detail in Chapter 4. The efficiency of the algorithm is tested by a number of randomly generated problems. In Chapter 5, the experimental results are analyzed and the findings about the algorithm are summarized. In the last chapter, the discussion about the study and some future research directions are provided.

Chapter 2

Literature Review

The cellular manufacturing and technology selection problems are studied separately in literature. That is, cellular manufacturing system models presume the technology is given and in most cases it is taken to be dedicated. Further, there exists no study incorporating market information to the CMSD problem. On the other hand, technology selection literature deals with a given partition of products. However, in real life applications, the manufacturer should make technology selection and cell formation decisions simultaneously while taking the changing market dynamics into consideration.

In this chapter, we review the literature on cellular manufacturing systems, and technology selection. CMSD problem is discussed in §2.1, in §2.2 technology selection literature is briefly introduced, and in §2.3 the motivations of our study is provided.

2.1 Cellular Manufacturing Systems

The nature of production processes can be classified in three groups: intermittent, continuous and repetitive [67]. If the demand is occurring at intervals and the jobs are different from each other, production is said to be intermittent. It is best to implement a standard machine layout, which is known as *job shop layout*. Nevertheless, the time part spends for waiting, travelling and setup is significantly high with such a layout. The time lost yields to low productivity.

On the other hand, if the production is in large scale and for a few part types, production has a continuous character. With such a layout, it is best to have the machines arranged in a sequence. This type of layout is known as *flow shop layout*. The cost of production is lowest in this case. However, to change the layout for production of a different part is a serious problem with such a layout. When the demand is repetitive, the best production system is the *batch production*. Around the world, batch manufacturing is the dominant manufacturing activity. It accounts for 60 to 80 percent of all manufacturing activities [66]. However, batch manufacturing industries cannot compete in today's global market.

The concept of *Group Technology (GT)* has risen to reduce WIP inventories, setups, material handling distances, and batch sizes. It was originally introduced as a single-machine concept in Russia by Mitrofanov in 1966 [49]. It is further extended to be a manufacturing principle which identifies related or similar parts and processes to take the advantages of the similarities that exist during all stages of design and production.

GT has several significant benefits. Material handling time is minimal since the part is completely processed within the cell. Furthermore, since the cell consist of the required machines, parts move from one machine to other completing the production much faster. In CMS, throughput time depends on the operation with the maximum processing time, whereas in batch production systems, it depends on the total processing time of the whole part. Setup time is also reduced by the similar part groupings. The development in technology further contributes to the reduction in setup. Moreover, improvement in quality by the immediate feedback and increase in job satisfaction of labor forming teams in the cell are other benefits of the GT. The CMSD problem is therefore basically focused on the identification of appropriate family membership of parts and formation of machine groups accordingly.

2.1.1 Machine Group - Part Family Identification Methods in Literature

In this subsection, visual inspection method, part characteristic based systems, and production process based systems are discussed briefly.

2.1.1.1 Visual Inspection Method

This method is mainly based on experience. The inspector analyzes the parts, and according to their geometric similarities, families are determined [51]. Although Burbidge reports in 1971 that the method can be used to distinguish up to 2000 parts, today it is rarely used in practice [12].

2.1.1.2 Part Characteristic Based Systems

Part characteristic based systems are also known as part coding and classification analysis *(PCA)*. PCA based methods group similar parts or separate dissimilar parts based on predetermined attributes. The code used to identify the part is a string of characters possessing information about the part. The PCA methods use parts coding schemes, which act as an instrument for the efficient recording, sorting, and retrieval of information [33].

There are three types of codes: Monocodes, Polycodes, and Mixed Codes. In literature, there exists various coding systems, e.g. BRISCH BIRN, CODE, MICLASS, OPITZ, KC-1. Extensive discussion on coding systems can be found in Hyer and Wemmerlov [33]. Generally coding schemes emphasized the identification of part families based on similarity in function, shape, etc. However, parts having similar shapes may have totally different manufacturing requirements. Hence, without any information about the machine groups, CMSD problem still has no clear answer.

2.1.1.3 Production Process Based Systems

Production oriented approaches solve part family formation problem based on the similarity in processing requirements of parts. These systems have the greatest amount of research attention. Most production oriented systems use routing of parts to determine the relationship between parts and the machines [51].

These approaches utilize either a simultaneous or sequential algorithms to define machine groups and part families. The sequential procedure determines the part families (or machine groups) first, followed by machine selection (or part allocation). The simultaneous procedure solves the machine group - part family formation problems concurrently [67].

2.1.2 Assumption Domain and Model Characteristics

Evaluation of design decisions can be categorized as relating to either system structure or system operation. During a design process, both structure and operation should be considered. In their review paper, Wemmerlov and Hyer [82] listed typical considerations related to system structure as equipment and tooling investment, equipment relocation cost, floor space requirements, manufacturing flexibility, extend to which parts are completed in the cell and existence of inter and intra movements of operators and material. The assumptions used to identify the problem characteristics might be either general or specific to each model. In literature, nine assumption domains are reported [51]:

1. Layout Type
2. Setup Time
3. Machine Clustering Requirements
4. Nature of Demand
5. Planning Horizon

6. Batch Size

7. Production Flow Policy

8. Number of machines per Machine type

9. Operation times

Few studies work on unknown demand. Harhalakis et al. [30] and Seifoddini [60] studied random product demand in cell formation problem. The authors mainly focused on the robustness over a certain range of demand variation ignoring any other market characteristic such as the age of the product or the number of design changes that the part underwent. Further, the authors use part-machine incidence matrix, assuming the only available technology is the dedicated technology.

The model of Fine and Freund [21] provides a firm the flexibility to respond to future demand variations, but at the expense of the increased cost of investment. The authors perform a stochastic study on the system performance.

Many of the CMSD problem models assume the demand for each part is known, and constant over the planning horizon [51]. This assumption is far from reality. Part demand nature changes with part's position in the life cycle. Product life cycle concept provides an appealing and readily understandable analysis for considering future growth opportunities and drawbacks. In contrary to the stability assumption, as time passes mean demand increases slowly at first and has high deviation at this introduction phase. At the growth phase, mean demand increases more quickly and deviation decreases. When the product is at maturity and saturation phases, it enjoys high and stable demand. Finally, demand decreases at the decline phase and before the demand falls to zero, the product fades away from the market.

As far as we know, characteristics of the market has not been studied in the CMSD literature. Many parts have evolving designs to satisfy the changing demands of customers [76]. Without considering the age and the frequency of design changes of the part, researchers carried out their calculations.

To have more realistic models, researchers should incorporate the product life cycle concept instead of taking the demand constant and stable over time. Especially in 21^{st} century, while the life cycles are getting shorter, designs are changing frequently and demand is subject to a significant decrease, constant demand assumption is no longer valid.

Arrangement of machines in a GT cell is generally assumed to be circular layout. Most of the models deal with the part family - machine group formation and do not consider actual manufacturing process activities such as scheduling and lot sizing. Setup times are also generally not included in the models.

When machine clusters are mutually separable, a machine can belong to one and only one cell. Some models assume machines could belong to more than one group. This may lead to a decrease in system efficiency by duplication. King et. al. and Kusiak et al. eliminate exceptional parts by subcontracting or forcing them to belong to one of the existing groups as discussed in [38] [41] and [42]. It is generally assumed that the model have m machines with one or more copies per machine type. The processing time of each machine for each operation on each part is also assumed to be known or varies probabilistically.

Based on these assumptions, in the literature models are built with the following attributes:

- Decision variables represent actions or policy decisions concerning the system:
  - Number of machines of a given type to be assigned to a given cell
  - Number of parts or machines assigned to any given cell
  - Number of operations or tool copies per part per group
  - Batch size
- Objectives: Several objectives of GT are cited in the literature. Ballakur and Steudel list eight such objectives [8]:
  - Min intercellular travels

  - Min intracellular travels
  - Min setup time or Max machine scheduling flexibility
  - Max similarity (Min dissimilarity)
  - Min total production cost
  - Min exceptional element costs (subcontracting, duplication)
  - Min machine idle time
  - Max machine utilization

- Constraints:
  - Number of groups (cells or part families)
  - Number of parts per group
  - Number of machines per group
  - Machine capacity
  - Each part, machine or both belongs to one part family or machine group
  - Annual operating budget
  - Tool or processing requirement of parts

In their review paper, Shambu, Suresh and Pegels [65] provide a taxonomy that summarizes operational issues and impact of cellular manufacturing. The authors figured out that some issues remain unclear in the literature such as: consideration of product mix, demand rates, uncertainties, etc., investigation of the performance of the entire shop floor rather than a single machine or a single cell, and providing help to industry in making informed decisions on when to implement CM and to what extend.

Wemmerlov and Hyer [83] stressed on the managerial aspects of cellular manufacturing applications. The authors raised questions on the operations strategy and social aspects of cellular manufacturing systems. Grouping efficiency is defined as the evaluative measure of the machine-part groups in cellular manufacturing systems. The measures are reported both descriptively and quantitatively.

Sarker and Khan [57] presented a comparison of existing grouping efficiency measures and proposed a new weighted grouping efficiency measure.

2.1.3 Algorithms in Literature

In this subsection, array based algorithms, similarity coefficient based clustering, mathematical programming, graph theoretic, and other approaches are discussed briefly.

2.1.3.1 Array Based Algorithms

CMSD solution identifies part families and machine groups for a production facility. Each part family processed within a machine group with minimum interaction with other groups. In literature, the processing requirements of parts on machines is obtained from the routing cards. This information is represented in a matrix called *the part-machine incidence matrix* with 0 or 1 entries. A 1 in row i and column m shows that part i requires machine m for an operation.

This kind of representation has serious drawbacks. If a part requires more than one operation on a machine, this cannot be identified in the part machine matrix using a 0-1 representation. On the other hand, with the available technology, there are flexible machines which can handle several operations with the same setup. A 0-1 matrix cannot represent any information about the flexible technology. In today's world, an analyst should not disregard the flexible technology. In literature, there are several matrix manipulation algorithms [67]. After rearranging rows and columns of the matrix, part families and machine groups are identified:

BEA - Bond Energy Algorithm: McCormick, Schweitzer and White developed BEA to identify natural groups that exist in complex data arrays [48]. This is a quadratic assignment based cluster analytic model. The authors define the bond strength between any two adjacent elements in a machine-part incidence matrix as their product, and the bond energy as the sum of the bond strengths. The objective is therefore to maximize the bond energy of the 0-1 matrix, resulting in a block diagonal matrix. It is reported that the final ordering is dependent on the initial row or column selected to initiate the process.

ROC - Rank Order Clustering: King [36] and [37] developed the ROC algorithm, which is the better known of the array-based clustering algorithms. Each row (column) in the part-machine matrix is read as a binary word. The procedure converts these binary words for each row (column) into decimal equivalents. The algorithm successively rearranges the rows (columns) in order of descending values until there is no change. However, even in well structured matrices it is not certain ROC will identify the block diagonal structure, and it possess computational difficulties.

ROC 2: King and Nakornchai [38] extended the basic ROC model to improve its computational efficiency. The new algorithm simultaneously sorts several rows and columns, thus enabling it to solve problems of much larger dimensions.

MODROC - Modified ROC: Chandrasekaran and Rajagapolan [15] identified the fact that ROC has a tendency to collect all the 1's in the top left corner. By removing this block of columns from the matrix and performing ROC again, MODROC collects another set of 1s in the top left corner. This process will identify mutually exclusive part families but may contain overlapping machines.

DCA - Direct Clustering Algorithm: Chan and Milner [14] proposed the DCA, which rearranges the rows with the left-most positive cells (i.e. 1s) to the top and the columns with the top-most positive cells to the left of the matrix. Wemmerlov [81] provided a correction to the original algorithm to get consistent results. This procedure, again, may not necessarily always produce diagonal solutions, even if one exists.

CIA - Cluster Identification Algorithm: Kusiak and Chow [41] and [42] present cluster identification and cost analysis algorithms to solve the machine-part grouping problem. In CIA, the machine-part incidence matrix is transformed into machine-part clusters using a form of cutting algorithm. It is not designed to decompose a matrix to a near-block diagonal form, but simply to identify disconnected blocks if there are any.

Modified CIA: In CIA, each element of the matrix is scanned twice. Boctor [11] proposed a new method where each element of the matrix is scanned only once.

2.1.3.2 Similarity Coefficient Based Clustering

Clustering is a mathematical method that is used to identify similar objects in a set. It is also used in the context of part-machine grouping. The methods of cluster analysis follow a set of steps [55]:

- Collect a data matrix, columns and rows of which stand for objects and attributes (parts and machines).
- Using the data matrix, compute the values of a resemblance matrix coefficient to measure the similarity.
- Use a clustering technique to process the values of the resemblance coefficient.

Although the basic steps are constant, there is a wide range in the definition of the resemblance matrix and the choice of clustering method. The similarity and distance measures using binary part-machine incidence matrix have the same disadvantages of not possessing part specific information like the array-based methods. Similarity coefficient concept is first introduced by McAuley [47]. The proposed similarity coefficient is also known as the Jaccard's similarity coefficient and it is widely accepted and used in the literature.

Seifoddini and Djassemi [61] modified Jaccard's coefficient by adding production volume data. Tam [74] integrated the operation sequence information in the calculation of the similarity coefficient. Nair and Narendran [50] proposed a weighted machine sequence similarity coefficient to cluster machines. Gupta and Seifoddini [27] presented a similarity coefficient using production volume, routing sequence and unit operation time. Akturk and Balkose [2] solved the part-machine grouping problem using a multi-objective cluster analysis. The authors suggested a new dissimilarity measure based on design and manufacturing attributes and operation sequences.

Recently, Yin and Yasuda [84] proposed a new similarity coefficient to cope with cell formation problems that consider alternative process routings, operation sequences, operation times and production volumes of parts simultaneously.

In two articles Shafer and Rogers [63], [64] reviewed the different similarity and distance measures used in cellular manufacturing. Manufacturing features other than the information provided in the part-machine matrix such as part volume, part sequence, tool requirements, setup features, etc. can be considered while computing the similarity measure. Some of the known clustering algorithms in literature are as follows:

SLC - Single Linkage Clustering: McAuley [47] is the first to apply single linkage clustering to cluster machines. The data matrix to be cluster-analyzed is the part-machine incidence matrix. A similarity coefficient is first defined between two machines in terms of number of parts that visit each machine. Once the similarity coefficients have been determined for machine pairs, SLC algorithm evaluates the similarity between two machine groups.

CLC - Complete Linkage Clustering: The algorithm remains the same with SLC except at the step of similar machine choice. It combines two clusters at minimum similarity level rather than at maximum level as in SLC.

ALC - Average Linkage Clustering: SLC and CLC are clustering based on extreme values. Instead, it may be of interest to cluster by considering the average of all links within a cluster. SLC produces compacted trees; CLC extended trees; and ALC trees are intermediate between these extremes.

Seifoddini [59] presented a comparative study of the two similarity coefficient based algorithms: SLC and ALC. The authors found that although SLC was relatively easy to apply, it might cause the chaining problem and produce more exceptional parts. ALC overcomes these drawbacks at a cost of more computation time.

LCC - Linear Cell Clustering: Wei and Kern proposed LCC [79], [80]. It clusters machines based on the use of a commonality score which defines the similarity between two machines. However, the worst case computational complexity of the algorithm is not linear as the name suggests.

2.1.3.3 Mathematical Programming and Graph Theoretic Approaches

The algorithmic procedures mentioned up to this point are heuristics. In literature, there also exist mathematical models which can provide optimal solutions. The heuristics are also utilized as a starting point towards an optimal solution.

One of the first approaches to forming part families using mathematical programming was by Kusiak [40]. The objective of *p-median model* is to find f part families optimally, such that the distance between parts in each family is minimized with respect to the median of the family. The drawback of the model is that it only identifies part families. Srinivasan, Narendran and Mahadevan [72] proposed an assignment model for the part families and machine grouping problem. The authors provided a sequential procedure to identify machine groups followed by identification of part families.

Selvan and Balasubramanian [62] present an integer programming formulation for grouping components based on their operation sequences. The objective of the model is to minimize the sum of material handling and machine idle costs subject to each component belonging to only one group, the one that minimizes the objective.

Some authors in literature integrate the production planning problem into cellular manufacturing systems. Akturk and Wilson [5] proposes a hierarchical cell loading approach to the hierarchical production planning problem simplified with CM shop configuration. Song and Hitomi [71] integrated production planning and layout decisions in CMSD problem at the same time. The authors define the flexibility as the optimal integration of production planning and cellular layout in a cellular manufacturing system. Schaller, Erenguc and Vakharia [58] proposed a mathematical approach for integrating the cell design and production planning decisions.

Choobineh [17] adopts a modified Jaccard similarity measure that uses operations sequences and proposes an integer programming formulation approach. Gunasingh and Lashkari [25], [26] propose two 0-1 integer programming formulations based on tooling requirements of the components (parts) in each family, available tooling on the machines, and processing times. Vakharia, Askin and Sen [75] present a 0-1 integer programming formulation with the objective of minimizing the total cost of machines required and intercell material handling costs, subject to each part being completely processed in each cell, machines required per cell, and number of cells visited by each part. Kandiller [34] used utilization levels, workload balances, exceptional elements and intercellular densities to compare the efficiency of some well known cell formation methods.

The clustering algorithms and p-median model minimize the distance of parts to the family median. Nevertheless, the parts within a family interact with each other. Kusiak, Vanelli and Kumar [44] proposed a quadratic programming model for this purpose. Kusiak and Chow [43] represented the machine-part incidence matrix as a graph formulation. The authors showed three types of graph depending on the representation of nodes and edges: bipartite graph, transition graph or boundary graph. Dahel and Smith [18] constructed a 0-1 integer programming formulation to design flexibility into cellular manufacturing systems. The flexibility concept the authors indicate in their paper is the intercell routing flexibility. This kind of flexibility reduces the proportion of parts being fully processed in one cell. Askin, Selim and Vakharia [7] studied demand flexibility simultaneously with routing flexibility.

Akturk and Turkcan [4] proposed a integer programming model to solve CMSD and layout problems simultaneously using a holonistic approach to maximize profit of individual cells. The authors also considered operation sequences, alternative routings, production volumes and processing times in their study. A more detailed discussion about mathematical formulations and graph theoretical approaches can be found in Offodile et al. [51] and Singh [67].

2.1.3.4 Other Approaches

Utilization of mathematical formulations brought the chance to integrate more information on the CMSD problem such as part volume, processing times, operation sequences, available machine capacity, etc. However, since the scope of the problem is broad, most of the proposed models cannot be solved optimally in a reasonable computation time.

On the other hand, the heuristics presented in former sections, although yielding an approximate solution in a reasonable computation time, are sensitive to the initial solution and groupability of the part-machine matrix. Hence, novel methods have emerged recently: Simulated annealing, genetic algorithms, neural networks, tabu search and beam search [67].

Simulated Annealing (SA) is inspired from the physical sciences. The design of SA is based on three key concepts [23]: The *temperature* controls the probability that a cost increasing solution will be accepted (in a min problem). The *equilibrium point* concept determines the point where no further improvement is expected in the objective with additional sampling. The *annealing schedule* defines the set of temperatures to be used and how many interchanges to consider before reducing the temperature. Adil, Rajamani and Strong have implemented SA to the grouping problem [1].

Tabu Search (TS) is in many ways similar to SA [53]: they both move from one schedule to another with the next solution being possibly worse than the one before. The basic difference between TS and SA is the mechanism used for approving a candidate schedule. In TS, at any stage of the process, a tabu list of mutations, which the procedure is not allowed to perform, is kept. For more information on tabu search see Glover [24].

Genetic Algorithms (GA): Holland developed GA as a random search technique in 1992 [31]. It was originally inspired by an analogy with the process of natural evolution. The design of GA is based on six key concepts: *representation, initialization, evaluation function, reproduction, crossover,* and *mutation* [28].

Neural Networks models mimic the way biological brain neurons generate intelligent decisions. Biological brains are superior at problems involving massive amount of uncertain data. Thus, neural network models are potential tools to solve the cell formation problems.

Beam Search: Enumerative branch and bound methods are currently the most widely used methods for obtaining optimal solutions to NP-hard problems. Beam search is a derivation of the branch and bound algorithm [53]. It eliminates some of the branches in an intelligent way. The number of nodes reserved for further evaluation is the *beam width* of the search. For these nodes, a simple evaluation procedure is applied, and some more non-promising nodes are fathomed. The number of nodes selected for a thorough evaluation is the *filter width*. After the final evaluation, a set of promising nodes are selected for next iteration. The size of this set is equal to the beam width. Ow and Morton [52] provide a thorough analysis of a filtered beam search methodology for different scheduling problems.

2.2 Technology Selection Literature

Increased product variety, low unit costs and lead times, high levels of product quality are necessary conditions to survive in today's markets. Large number of product variety, customized and instable product designs, increased international competition, the need to reduce manufacturing lead time, all require the development of manufacturing technologies [66]. The development of computer integrated manufacturing systems addresses some of these problems. Nevertheless, flexible manufacturing systems have high investment costs increasing the unit manufacturing costs. On the other hand, there still exists parts that have a standardized design and high production volume.

In literature, there are number of studies that analyze the trade off between flexible technology and dedicated technology. Singhal et al. [68] define the benefits of flexible technologies as the ability to respond quickly to changes in design and demand, lower direct manufacturing costs, improved quality, economies of scope, flexibility in scheduling. Basnet and Mize [10] provide a critical review on scheduling and control of flexible manufacturing systems. Sambasivarao and Deshmukh [56] classify and review the issues regarding the selection and implementation of advanced manufacturing technologies.

Flexibility is the key concept used in the design of modern automated manufacturing systems. Every manufacturing system is flexible to a certain degree. Barad and Nof [9] review CIM flexibility measures and provides a framework for analysis and applicability assessment. Gupta and Goyal [29] provide a comprehensive review of the literature on flexibility. Since there are various types of factors that affect the system performance, there exists various types of flexibility:

- Machine Flexibility
- Routing Flexibility
- Process Flexibility
- Product Flexibility
- Production Flexibility
- Expansion Flexibility
- Volume Flexibility
- Operation Flexibility

Flexible Manufacturing Systems *(FMS's)* provide most of the above flexibilities to a facility. FMS's also provide the ability to rapidly introduce new products to the market. This accelerates the implementation of flexible technology. Hutchinson and Holland [32] compared dedicated and flexible technologies. The authors simulated the effects of technology selection on manufacturing performance. Flexible technology is more preferable as the rate of new product introduction increases and as the average volume per part decreases. Fine and Li [22] studied optimality of automated manufacturing at some stages of the product and process life cycles. Li and Tirupati [46] constructed a mathematical program for selecting the optimal mix of dedicated and flexible technologies and timing of capacity additions to satisfy the deterministic demand over a finite planning horizon. Burstein [13]provided a convex programming model which incorporates production and technology selection decisions.

Some authors in the literature implement the multidimensional aspect of flexibility. Falkner and Benhajla [20] suggest to use the multi-attribute decision methods. Stam and Kuula [73] and Kuula and Stam [45] utilized multiple criteria optimization for FMS selection decisions. On the other hand, productivity, quality and flexibility are critical measures of manufacturing performance for justifying the investment in computer integrated manufacturing systems. Son and Park [69], [70] study the economic measure of these three critical performance measures in advanced manufacturing systems.

There exist studies which considered technology selection problem simultaneously with the facility location and capacity acquisition problems. Detailed information on this subject can be found in Verter and Dincer [78], Verter and Dasci [77], and Dasci and Verter [19]. Rajagopalan [54] and Li and Tirupati [46] studied technology selection problem integrated in the capacity expansion decision models.

Recently, Krishnan and Bhattacharya [39] studied the problem of technology selection and commitment under uncertainty. The authors formulate a mathematical model to compare a proven technology versus a prospective technology. The analysis shows the appropriateness of the different flexible design approaches.

2.3 Motivations of the Study

It is evident from the previous sections that the cellular manufacturing and technology selection problems are studied separately in literature. That is, cellular manufacturing system models presume the technology is given and in most cases it is taken to be dedicated. Further, there exists no study incorporating market information to the CMSD problem. On the other hand, technology selection literature deals with a given partition of products. However, in real life applications, the manufacturer should make technology selection and cell formation decisions simultaneously while taking the changing market dynamics into consideration.

PCA based methods use design similarity between part without considering the manufacturing requirements. Array based methods employ binary part-machine incidence matrices which possess no information about production volume, design stability, or operation sequences. Heuristics and mathematical formulations are best suited to CMSD problem. Important criteria, such as production volume, processing times, load-unload times, machine investment and maintenance costs, available machine capacities can be handled simultaneously.

In general, it is assumed that the market is stable with highly standardized products with high and stable demand patterns. However, in 21^{st} century, this is an unrealistic assumption. The market is no longer stable. Designs are evolving, production volumes are decreasing and life cycles are getting shorter. Thus, in order to design an efficient manufacturing system, analysts should incorporate this information effectively in the models.

In this study, our aim is to consider all important manufacturing system design factors such as properties of market and available technologies. Further, the model that we constructed takes important manufacturing issues such as production volume, throughput times, utilization levels, machine investment, maintenance and labor costs into consideration while identification of part families and machine groups is accomplished. In the following chapter, the problem is defined with the underlying assumptions and a mathematical programming model is proposed.

Chapter 3

Problem Statement

Cellular manufacturing system design (CMSD) is primarily concerned with the formation of part families and machine groups leading to appropriate *manufacturing cells* in order to achieve the benefits of group technology. In literature, various CMSD algorithms are proposed. In Chapter 2, these approaches to solve the problem are reviewed and it is emphasized that none of these procedures has taken market information and selection of available technology into consideration.

In literature, it is generally assumed that the market is stable with highly standardized products with high and stable demand patterns. However, in 21^{st} century, this is an unrealistic assumption. The market is no longer stable. Designs are evolving, production volumes are decreasing and life cycles are getting shorter. Thus, in order to design an efficient manufacturing system, we propose a new model to incorporate this information effectively.

Technology selection problem deals with selecting the best alternative among available technologies while designing a manufacturing system. Since the product life cycles have been shortening in today's market, productivity, flexibility, service time, quality and reliability as well as costs have become the major considerations for survival in the market. Thus, firms should adopt the automated manufacturing technologies in order to keep their competitiveness. In our study, we provide a model that make use of the automated technologies while keeping the dedicated technologies as an alternative.

In this study, an integrated approach is proposed to solve the cell design and technology selection problems simultaneously during the design of an advanced manufacturing system. We proposed a new cost function to integrate the market characteristics in our model. Further, we modified a well known similarity measure in order to handle the operational capability of available technology.

In section §3.1, the problem definition and our contributions to the definition are presented. In §3.2, a mathematical model is proposed to form manufacturing cells utilizing appropriate technology for management of product variety. In the last section §3.3, we present the concluding remarks about the problem.

3.1 Problem Definition and Assumptions

The aim is to solve the cellular manufacturing design problem and the technology selection problem simultaneously which, in general, is the case in real life problems. In today's world, a firm should benefit from the available computer integrated technology while sustaining the use of economies of scale inherited in dedicated manufacturing.

In this multi-objective study, a modification of a well known similarity measure is utilized in order to form part families, and the technology selection decision is based on the individual properties of parts, namely the production volume, variability of the demand, and the design stability of the part. The market information is quantified via a newly introduced cost function in the model.

First, basic assumptions of the model are presented. In the second subsection, some definitions related to the model are given, in the third subsection new cost function and new similarity coefficient are introduced, and finally the model is presented.

3.1.1 Basic Assumptions of The Model

- There are N parts with different demand variabilities, design patterns and production volumes.

- There are O operations required for the processing of the parts that can be handled by some of the FM flexible machine types, or DM dedicated machine types.

- Available technologies are flexible manufacturing systems, and dedicated machines.

- Each machine can perform a number of operations which are known a priori.

- The operations required in production of each part are known. Operation sequences of the parts are not taken into consideration in this model.

- Annual demand, age of the part, and the number of design changes up to date are known a priori. These attributes play an important role in calculation of the new product variety cost function and determination of the appropriate technology.

- The processing time of each operation of each part on each machine is predetermined. Processing times of parts are important not only because they are utilized in determining the number of machines required of each type, but also because they form a basis for the technology decision via determining the throughput times. The processing times of flexible machines are longer compared to that of dedicated machines. In terms of only the processing times, dedicated technology is preferable.

- Load and unload times are assumed to be equal for each part-machine pair and known a priori, but on the average load/unload times for flexible machines are taken to be longer than that of dedicated machines. However, each part should be loaded and unloaded on a dedicated machine for one operation while the flexible machines can handle a number of operations with a single load/unload. The load-unload times are utilized for calculation of the throughput times together with the processing times and provides a trade off to decide whether to process all the operations on a flexible machine or to process each operation on separate dedicated machines.

- The machine investment, maintenance, and labor costs are assumed to be known. They form a monetary basis for the decisions.

Under these assumptions, the following decisions need to be made:

- Part families
- Machine groups with appropriate technology
- Part assignments to cells
- Operation assignments of each part to machines
- Number of each machine type in each cell

3.1.2 Basic Definitions Used in The Model

Machine Capability Matrix (MCM)

It is a $0-1$ matrix presenting the operational capabilities of the machines. Rows of MCM are reserved for the machine types, where columns are reserved for operation types.

$$\mathbf{MCM} = \begin{pmatrix} MCM_{11} & MCM_{12} & \dots \\ MCM_{21} & MCM_{22} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$MCM_{mo} = \begin{cases} 1 & \text{if machine type } m \text{ can perform operation } o \\ 0 & \text{otherwise} \end{cases}$$

MCM has two basic blocks. Upper rows represent the values of dedicated machines. Thus, this block forms a unit matrix. Each row has only one positive value, since each dedicated machine is defined by a specific operation. Lower rows represent the values of flexible machines. In these rows, we observe more number of 1's. As the number of 1's in a machine's row increases, we say the machine gets more flexible, since the machine flexibility can be measured by the number of operations that can be handled by that machine. A representative MCM looks like the following:

$$\mathbf{MCM} = \begin{pmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \\ 1 & 1 & 0 & \dots \\ 0 & 1 & 1 & \dots \\ 1 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Part Requirement Matrix (PRM)

It is a 0-1 matrix presenting the processing requirements of the parts. Rows of PRM are reserved for the parts, where columns are reserved for operation types, as it is in the MCM.

$$\mathbf{PRM} = \begin{pmatrix} PRM_{11} & PRM_{12} & \dots \\ PRM_{21} & PRM_{22} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$PRM_{io} = \begin{cases} 1 & \text{if part } i \text{ requires operation } o \\ 0 & \text{otherwise} \end{cases}$$

3.1.3 Contributions

In order to integrate the market characteristics in our model, we proposed a new cost function. This is the first study to assign costs for design instabilities and demand variations. We minimize these costs resulting from the offered variety in today's markets.

Further, we modified a well known similarity measure in order to handle the operational capability of available technology. During the design of a cellular manufacturing system, it is generally treated that the only available technology is the dedicated technology. However, computer integrated manufacturing technologies are provided for the use of manufacturers. In order to make use of this computer numerically controlled machines, we propose a new similarity measure during the design of cellular manufacturing systems.

3.1.3.1 Product Variety Cost Function

By the term *product variety*, we imply the fluctuations in three characteristics of a product in a production environment:

. Production Volume

. Demand Pattern

. Design Stability

Following notation associated with the product variety costs is used in the thesis:

c_{id}	:	cost of assigning part i to a dedicated cell
c_{if}	:	cost of assigning part i to the flexible cell
a_{vol}	:	production volume coefficient of the part
a_{σ}	:	demand variation coefficient of the part
a_{des}	:	design stability coefficient of the part

Production volume of a part can differ from part to part. A part can have a high production volume whereas another, but operationally similar part can have a very small production volume. If these two parts are assigned in the same

Volume	a_{vol}	**Effect in** c_{id}	**Effect in** c_{if}
High	1	a_{vol}	4 - a_{vol}
Medium	2	a_{vol}	4 - a_{vol}
Low	3	a_{vol}	4 - a_{vol}

Table 3.1: Volume Costs in Product Variety Cost Function

cell, the frequent and interrupting set-up requirements can become a burden contradicting that set-up should have been an advantage of cellular manufacturing. To eliminate such a set-up problem, we should assign low-volume parts to FMS cell, whereas the high-volume parts to the cells composed of dedicated machines, namely dedicated cells.

We propose a costing scheme in Table 3.1 which is based on the volume characteristics of the parts. If the part is a low volume part, cost of assigning this part to a dedicated cell is 3, if it is a medium volume part, cost is 2, and for a high volume part, cost is only 1. Cost order is reversed for a flexible cell, which is 1 for low volume parts in flexible cells, 2 for medium volume parts, and 3 for high volume parts processed in a flexible cell. As a result of this volume cost, low volume parts tend to be processed in flexible cells, and high volume parts tend to be processed in dedicated cells.

Demand pattern of the part is a more important attribute of the part. Even we can determine the expected demand for the part, we should also care about the variation of this value. The traditional product life cycle curve is provided in Figure 3.1. It is true that in the early stages of the typical life cycle (introduction phase), variations in demand are high. However, the demand has much less variation during the periods of its half life-time (saturation phase). If the life of a part is divided in 6 equal periods, deviation is high in period 1, medium in period 2 and 5, and low in period 3 and 4. Generally, at the end of fifth period, the production of the part is ceased. Thus, it is not wrong to assume that there exists no product of period 6.

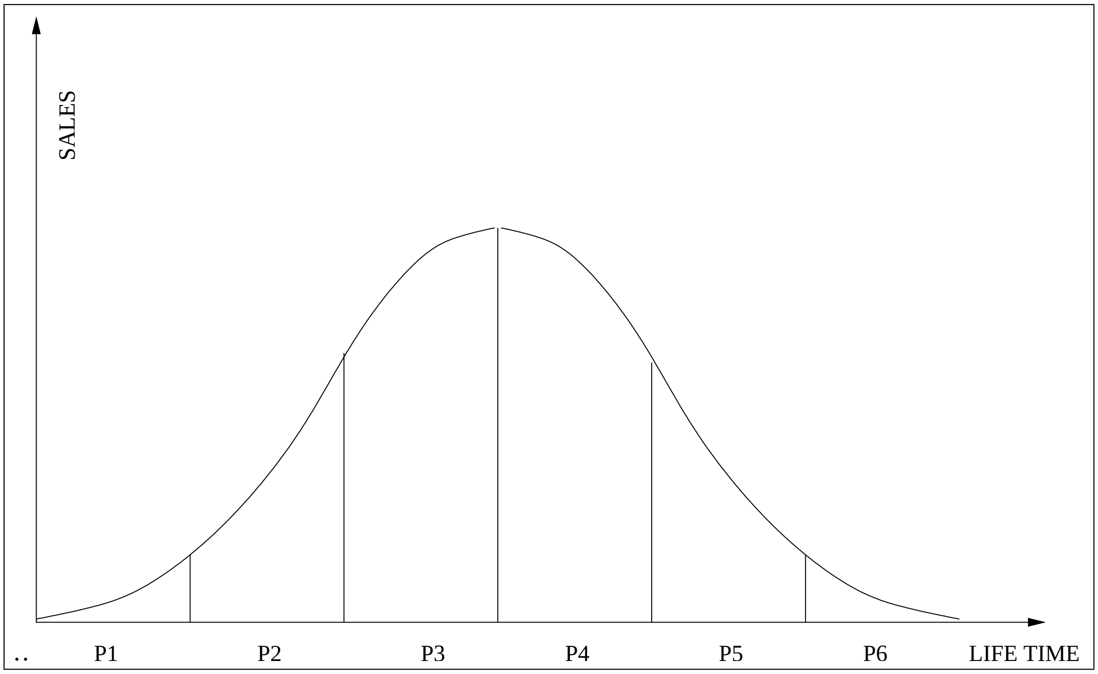

Figure 3.1: Traditional Product Life Cycle

Ages of parts differ, resulting various demand patterns at the production floor. Suppose a part is in its half-time periods of life, and another part is newly introduced in the market. While the first part is enjoying a stable demand pattern, the second one is subject to peaks and digressions. It is known for the first part how much to be produced pretty surely, whereas the demand for second part can change at any time. It may increase or diminish by time. Thus, under a risk of fading, to allocate resources for the second part may not end up with satisfactory results. To deal with this situation, we should benefit from the flexibility of flexible manufacturing cells, and put an influence on the high-variation parts to be assigned to FMS cells.

The proposed costing scheme is given in Table 3.2. The importance of the variation cost is emphasized by assigning the square power of the coefficients as the cost values. The coefficients are based on the life cycle positions of the parts. If the part is in its half life time periods, namely the 3^{rd} and 4^{th} periods, coefficient is 1 and cost of assigning this part to a dedicated cell is only 1, if it is a newly

Position in the life-cycle	a_σ	**Effect in** c_{id}	**Effect in** c_{if}
P3,P4	1	a_σ^2	$(4 - a_\sigma)^2$
P2,P5	2	a_σ^2	$(4 - a_\sigma)^2$
P1	3	a_σ^2	$(4 - a_\sigma)^2$

Table 3.2: Demand Variation Costs in Product Variety Cost Function

introduced part, i.e in its 1^{st} life period, the coefficient becomes 3 and cost gets as high as 9. On the other hand, a part which is in its 2^{nd} and 5^{th} life periods, is still subject to a variation in demand not as high as a newly introduced part, but not as small as a saturated part. It has a coefficient in the middle region, and cost is calculated to be 4.

Cost order is reversed for a flexible cell, which is 9 for saturated parts, 4 for medium volume parts, and 1 for newly introduced parts to be processed in a flexible cell. As a result of this costing scheme, parts that have high variations in demand tend to be processed in flexible cells, and parts that have stable demand patterns tend to be processed in dedicated cells.

Design stability is the most important attribute of the part. In today's markets, more customized designs need to be made in order to catch up with the competition. Many parts have evolving designs to satisfy the changing demands of customers. However, some parts still have stable design patterns.

When a design is said to be stable, the operations required are exactly defined for the part. When it is evolving, new operations may be added or some may be discarded from the routing. Thus, to design a dedicated manufacturing cell for an evolving part is a total jeopardy. We should assign an evolving part to a dedicated cell if and only if we have no other alternative.

The proposed cost structure is provided in Table 3.3. The significance and superiority of the design stability is emphasized by assigning the triple power of the coefficients as the cost values. The coefficients are based on the average number of design changes per life time unit of the product. If the part has underwent a high number of design changes in relatively short amount of time,

Number of Design Changes / **Part Age**	a_{des}	**Effect in** c_{id}	**Effect in** c_{if}
Low	1	a_{des}^3	$(4 - a_{des})^3$
Medium	2	a_{des}^3	$(4 - a_{des})^3$
High	3	a_{des}^3	$(4 - a_{des})^3$

Table 3.3: Design Stability Costs in Product Variety Cost Function

average becomes high and coefficient is 3. Associated cost value of assigning this unstable part to a dedicated cell is calculated to be as high as 27. At the other end, if the part has a very stable design, i.e. average number of changes is low, coefficient becomes 1 and cost of processing this stable part in a dedicated cell is as low as 1. Similarly, a part with medium range number of design changes has a coefficient in the middle region, and cost is calculated to be 8.

Cost order is reversed for a flexible cell, which is 1 for unstable parts, 8 for medium volume parts, and 27 for parts having stable design patterns to be processed in a flexible cell. As a result of this costing scheme, parts that have unstable design patterns tend to be processed in flexible cells, and parts that have stable designs tend to be processed in dedicated cells.

After identification of the values of coefficients and associated cost values of parts, final product variety cost function values are calculated. Having assigned different weights to the attributes, we make use of the simplicity and power of additivity in our proposed cost function.

$$
\begin{aligned}
c_{id} &= a_{vol} + a_{\sigma}^2 + a_{des}^3 \\
c_{if} &= (4 - a_{vol}) + (4 - a_{\sigma})^2 + (4 - a_{des})^3
\end{aligned}
$$

c_{id} and c_{if} are complementary costs. The more we prefer to assign a part to the flexible cell, the less we prefer to assign that part to a dedicated cell, and vice versa. The cost function have several important missions to be used in the solution procedure. It is basically used as a surrogate objective function of the model. Further, it provides us a strong basis for the selection of technology for each cell.

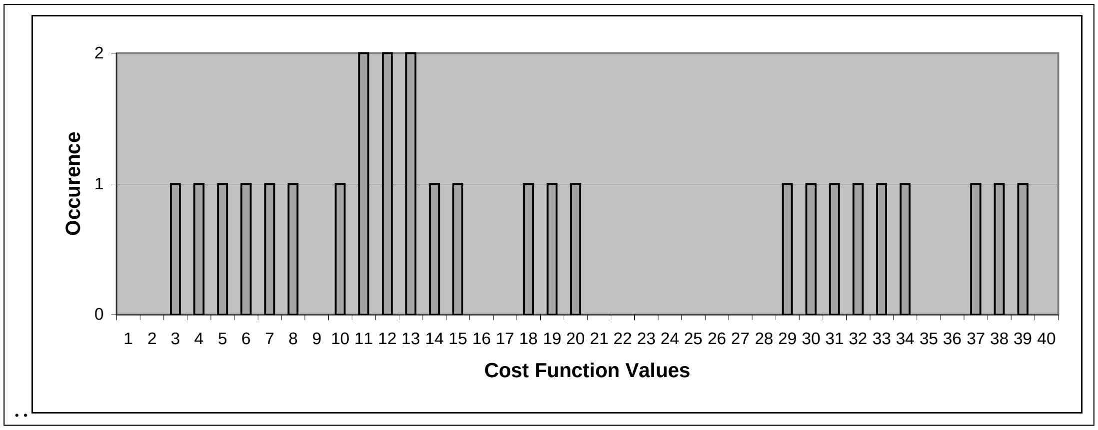

Figure 3.2: Possible Product Variety Cost Function Values

The range of the new cost function values begins from 3 and goes up to 39. All the possible observations of the cost function are calculated and plotted on the histogram presented in Figure 3.2. The same graph is true for both function values, c_{id} and c_{if}.

The observed values provide us very important information. Let the graph is plotted by the values of c_{id} costs. The parts providing the lowest costs are most probable members of families being processed on dedicated cells, and the ones with highest costs are selected for being processed on flexible cells. However, when it comes to choose from the middle region of the values, the analyst cannot decide for a threshold value clearly. It is observed that there exists a big jump at level 20. However, as it can be seen on the graph, some values occur more than once. When the number of occurrences is taken into consideration, the smaller jump on the value 15 is more meaningful to represent a threshold value to choose between dedicated and flexible technology. This value is utilized at two very critical points in the algorithm, which are discussed in detail in the next chapter.

3.1.3.2 Modified Jaccard Similarity Coefficient

The first similarity coefficient defined in the literature was between two machines in terms of the number of parts that visit each machine. The resulting image is a two-by-two matrix, where four types of matches are possible:

$$\begin{array}{c|cc} & 1 & 0 \\ \hline 1 & a & b \\ 0 & c & d \end{array}$$

In this matrix, a is the number of parts visiting both machines, b is the number of parts visiting only the first machine, c is that of second machine, and d is the number of parts not visiting any of the two machines. Jaccard coefficient (JC) is the most often used coefficient in the similarity context. It is not only a powerful coefficient but also very simple and effective as follows:

$$JC_{mn} = \frac{a}{a+b+c} \qquad 0 \leq JC_{mn} \leq 1$$

JC_{mn} shows the similarity between two machines, m and n, by calculating the ratio of common parts, to the total number of parts processed on these two machines. The main assumptions lying under this coefficient is that a specific operation can be handled by a specific machine, and whenever a part requires that operation, it has to visit that machine. However, with the available technology, an operation can be handled by several different types of machines. Thus, the assumption on which the coefficient is based has changed in today's world.

With the change of one-to-one assignment of machine operation pairs, similarity context should also be adapted to the technological advancements. As a first step for this adaptation, as we cannot relate a part to a machine directly, we should utilize operations as an indicator of similarity.

Operational similarity can be applied in part-similarity context, where previously a indicates the number of machines that both parts have operation and Jaccard is calculated as the similarity between parts i and j (JC_{ij}). With the new definition (JC'_{ij}), we may adapt a' just as the number of common operations in both parts' routings, b' as the number of operations required only by part one, and c' as that of part two.

$$JC'_{ij} = \frac{a'}{a' + b' + c'} \qquad 0 \leq JC'_{ij} \leq 1$$

Even after this adaptation, Jaccard still has imperfections. In this new formulation, we still do not have the machine flexibility information. When this information is integrated in the coefficient, then the modification can be accomplished fully.

The proposed model solves the calculation of similarity coefficient problem in two stages. A representative example is provided after the presentation of formal steps of the coefficient. In the first stage, a hypothetical manufacturing cell is designed to produce only the two parts. This cell is forced to have the minimum size without any other considerations, since we specifically need the numbers. As a second stage, we calculate the coefficient.

1. Design of a hypothetical manufacturing cell to find the minimum number of machines required to produce two parts in the same cell.

2. After the cell design, the following similarity table is formed between the two parts:

	1	0
1	k	l
0	m	n

In this matrix, k is the number of machines where both parts have an operation, l represents the number of machines which are required only by first part, and m is that of second part. Then the Modified Jaccard Similarity Coefficient (MJC_{ij}) follows:

$$MJC_{ij} = \frac{k}{k + l + m} \qquad 0 \leq MJC_{ij} \leq 1$$

As we have found the minimum possible cell size in the first stage, we have also found the greatest possible k, the number of common machines in the same cell. Hence, the greatest the modified similarity between the parts, the more number of machines in common between the parts, leading to a part family produced in a manufacturing cell composed of the common machines.

In the solution procedure, we minimize the dissimilarity between the parts, and attain homogeneity among the part families. Since we use dissimilarity coefficients in the model, a further calculation is needed. The Jaccard coefficient is defined from 0 to 1, and related dissimilarity is the complement of the coefficient value. Same range and complementarity applies to Modified Jaccard. Thus, the Dissimilarity Coefficient (DMJ_{ij}) is:

$$DMJ_{ij} = (1 - MJC_{ij}) \qquad 0 \leq DMJ_{ij} \leq 1$$

Example *Let the available Machine Capability Matrix (MCM) and Part Requirement Matrix values of parts i and j are as follows:*

MCM	op1	op2	op3
m1	1	0	0
m2	0	1	0
m3	0	0	1
fm1	0	1	1

PRM	op1	op2	op3
i	1	0	1
j	1	1	0

Jaccard coefficient finds the number of common operations between parts from the PRM and calculates the similarity as follows:

$$JC_{ij} = \frac{op1}{op1 + op2 + op3} = \frac{1}{3}$$

On the other hand, The Modified Jaccard make use of the available flexible technology and calculates the coefficient as follows:

Step 1: Construction of a hypothetical cell to produce only parts i and j. In the minimum best possible cell configuration, we have machines m1 and fm1.

Step 2: Both parts require m1 for operation 1 and operation 2 of i and operation 3 of j is handled on the same machine fm1. Then we calculate the similarity coefficient:

$$MJC_{ij} = \frac{m1 + fm1}{m1 + fm1} = \frac{2}{2} = 1$$

When we have taken the available technology into account, we may handle similarities much effectively. Ignoring the capability of machines, we may not end up with satisfactory results.

In the model, we utilize the proposed coefficient at two critical points. The coefficients are used basically as one of the objective functions. Further, these coefficients are input data for the fuzzy analysis.

However, in the model and solution approach, for the sake of technology selection, we limit the data of MCM. According to the selected technology of the cells and part families, we either use the flexible machines to calculate the dissimilarity or the dedicated machines. Initially since there exist no families before the fuzzy analysis, the input dissimilarities are calculated based on the average product variety cost (c_{id}'s) of the two parts. If the average cost of parts i and j is lower than the threshold value, the dedicated block of the MCM is available for the calculation (DMJ^d_{ij}). When only the dedicated machines are available for calculation, the proposed coefficient gives the same result as the classical Jaccard. Otherwise, if the average cost is high, flexible block of MCM is used to calculate the dissimilarity of parts (DMJ^f_{ij}).

At the end of design, we utilize the coefficients to calculate the dissimilarity objective function. At this stage, since the technology is known for the cell of the two parts, the appropriate dissimilarity coefficient is used for calculations.

3.2 Mathematical Model

After defining basic propositions, concepts and assumptions, we provide the mathematical representation of the problem. We not only utilize the classical assumptions of cellular manufacturing, but also propose a technology selection and a product variety management scheme.

In the proposed mathematical model, we make technology selection decision, while determining the appropriate machine groups for part families. Further, we identify part families not only according to their operational similarities but also according to their marketing positions. Management of all these attributes complicates the problem significantly.

We provide the notation of the problem in subsection 3.2.1, the decision variables in subsection 3.2.2, the objective functions are described in subsection 3.2.3, and the constraints of the model are presented in subsection 3.2.4.

3.2.1 Parameters of the Model

The notation that will be used through the thesis is presented as follows:

N	:	number of parts
FM	:	number of flexible machine types
DM	:	number of dedicated machine types
O	:	number of operations
K	:	maximum number of cells
MCM_{mo}	:	equals to 1 if machine m is capable of performing operation o, and 0 otherwise
PRM_{io}	:	equals to 1 if part i requires operation o, and 0 otherwise
c_{id}	:	cost of assigning part i to a dedicated cell
c_{if}	:	cost of assigning part i to the FMS cell
DMJ_{ij}^{d}	:	dissimilarity of parts i and j in a dedicated cell
DMJ_{ij}^{f}	:	dissimilarity of parts i and j in a flexible cell

$Ptime_{imo}$	:	processing time of operation o of part i on machine m
$Ltime_{im}$	:	load-unload time of part i on machine m
Inv_m	:	annual investment cost of machine m
$Maint_m$	:	annual maintenance cost of machine m
$Labor_d$	:	cost of one labor in a dedicated cell
$TotL_{dk}$	:	total cost of labor in dedicated cell k
$Labor_f$	:	cost of labor operating the FMS cell
$TotL_{fk}$	:	total cost of labor in FMS cell k
OR	:	operator ratio
SR	:	supplementary labor ratio
D_i	:	annual demand of part i
$TCap$	:	theoretical capacity of machines
α_m	:	upper utilization limit of machine m
β_m	:	lower utilization limit of machine m
UBK	:	upper bound on the cell size
M	:	a very large constant
$Util_{mk}$	:	utilization of machine type m in cell k
$Excess_{mk}$	:	excess capacity of machine type m in cell k
$Normf_1$	:	normalized value of objective 1
$GMinf_1$	:	global minimum value of objective 1
$LMaxf_1$	:	local maximum value of objective 1

3.2.2 Decision Variables of the Model

Under the assumptions and definitions presented in the previous section, we use the following decision variables:

x_{ik}	:	0-1 binary variable which is equal to 1 if part i is assigned to cell k, and 0 otherwise
y_{mk}	:	number of machine type m assigned in cell k
z_{imok}	:	0-1 binary variable which is equal to 1 if operation o of part i is performed by machine m in cell k, and 0 otherwise

L_{imk}	:	0-1 binary variable which is equal to 1 if part i is loaded on machine m in cell k, and 0 otherwise
$Open_{fk}$	:	0-1 binary variable which is equal to 1 if cell k is opened with flexible technology, and 0 otherwise
$Open_{dk}$	:	0-1 binary variable which is equal to 1 if cell k is opened with dedicated technology, and 0 otherwise
IM_{ik}	:	0-1 binary variable which is equal to 1 if part i makes an intercellular movement to cell k, and 0 otherwise

3.2.3 Objectives of The Model

The model is structured to be multi-objective. We minimize dissimilarity between parts, minimize product variety costs, minimize throughput time, minimize machine investment, maintenance and labor costs, and minimize intercellular movements of parts. We handle every aspect of the cellular manufacturing system design problem with new aspects integrated via use of multi objective criteria. All these objectives points to a different direction in the solution space, some of them totally contradicting directions. However, our aim is to find a good compromise solution in order to satisfy all objectives. Descriptions of each of the objectives are as follows:

Minimize Dissimilarity of Parts

$$\min f_1 = \sum_{i,j,f} x_{if} \cdot x_{jf} \cdot DMJ_{ij}^{f} \;+\; \sum_{i,j,d} x_{id} \cdot x_{jd} \cdot DMJ_{ij}^{d} \tag{3.1}$$

The function minimizes the value of dissimilarity if both parts are assigned in the same cell. The first term of the right-hand side of the equation is minimizing operational similarity in cells with flexible technologies, and the second term minimizes that in the other, namely dedicated cells.

We proposed a new similarity measure to handle the available technology that is used in the design. Since the similarity is based on operational similarity of parts, and operational capability of machines are different in the flexible machines and in the dedicated machines, coefficients of similarity comes out to be different for flexible cells and for dedicated cells as it is stated in the previous section. When considered alone, minimization objective 3.1 results in part families that have the best operational similarities among the member parts.

Minimize Product Variety Costs

$$\min f_2 = \sum_i x_{if} \cdot c_{if} \;\; + \;\; \sum_i x_{id} \cdot c_{id} \tag{3.2}$$

Product variety costs are minimized in this objective function. According to the definition provided in the previous section, parts have different product characteristics resulting in variety. However, management of variety is not possible with the available literature. In order to handle product variety, in our model, we proposed a new cost function. When considered alone, minimization objective 3.2 results in two groups of parts, that are processed either in flexible or dedicated cells. Each part is preferred to be placed in which the associated variety cost term is smaller.

However, when the first two objectives 3.1 and 3.2 act together in the same problem, the identification of part families can much effectively be performed via taking operational similarities and product characteristics into account at the same time.

Minimize Throughput Time

$$\min f_3 = \sum_{i,m,o,k} z_{imok} \cdot Ptime_{imo} \;\; + \;\; \sum_{i,m,k} L_{imk} \cdot 2 \cdot Ltime_{im} \tag{3.3}$$

Each operation of the part is assigned to a machine in a cell where processing time of the part's operation on that machine is known. Whenever a part has operation on a machine, then it has to be loaded and unloaded on the machine. Total time of production of a part is the sum of all processing times and load-unload times.

The processing times of flexible machines are longer compared to that of dedicated machines. In terms of only the processing times, dedicated technology is preferable. Further, on the average, load/unload times for flexible machines are longer than that of dedicated machines. However, each part should be loaded and unloaded on a dedicated machine for one operation while the flexible machines can handle a number of operations with a single load/unload.

The load-unload times are utilized for calculation of the throughput times together with the processing times and provides a trade off to decide whether to process all the operations on a flexible machine or to process each operation on separate dedicated machines.

The minimization objective 3.3 is significant because it has two contradicting parts regarding the technology selection at the same time. In the flexible cells processing times will be longer favoring dedicated cells, whereas in dedicated cells load-unload times will become a burden favoring flexible cells.

Minimize Monetary Costs

$$\min f_4 = \sum_{m,k} y_{mk} \cdot (Inv_m + Maint_m) + \sum_{k} Open_{fk} \cdot Labor_f + \sum_{m,k} Open_{dk} \cdot y_{mk} \cdot Labor_d \tag{3.4}$$

Monetary objective has two contradicting parts, machine investment, maintenance costs and labor costs. Machine costs are calculated annually for each machine type. Flexible machine costs are significantly higher compared to the dedicated machine costs.

Labor costs are different for the flexible and dedicated cells. A flexible operator of an FMS cell is paid more than a dedicated machine operator. However, for each flexible cell, 1 flexible operator is hired. On the other hand, each dedicated machine needs an operator in a dedicated cell. The number of operators in a dedicated cell is equal to the number of machines in that cell.

In the minimizing objective 3.4, the critical measures for the technology selection decision are analyzed. The two contradicting parts of the function, the investment costs and labor costs, form a strong basis for technology selection decision.

Minimize Intercellular Movement of Parts

$$\min f_5 = \sum_{i,k} IM_{ik} \tag{3.5}$$

In cellular manufacturing, one of the most critical objectives is the minimization of intercellular movements. Ideally, cellular manufacturing aims to have completely independent cells at the production floor. The decision variable IM_{ik} determines the movements of parts out of their own cells. The mechanism of the variable is presented in the next subsection.

The minimizing objective 3.5 acts towards other objectives in order not to allow exceptional parts. For example, for the sake of monetary objective 3.4, the decision to delete a machine from the system may be beneficial. However, when the machine is deleted, some of the parts will need to be processed in other cells contradicting the objective 3.5. Thus, we need to preserve this objective in order to comply with the rules of cellular manufacturing.

3.2.4 Constraints of The Model

In this subsection, we present the constraints of the proposed mathematical model. The following constraints are basically the classical cellular manufacturing constraints, and further there exist the supplementary constraints that provide descriptions for the decision variables.

Part and machine allocation constraints:

$$\sum_{k=1}^{K} x_{ik} = 1 \quad \forall i = 1, .., N \tag{3.6}$$

Each part is assigned to exactly one cell, that is, it is a member of only one family in the system. This constraint aims to form independent cells.

$$y_{mf} \;=\; 0 \qquad for \;\; m = 1, .., DM \tag{3.7}$$

$$y_{md} \;=\; 0 \qquad for \;\; m = 1, .., FM \tag{3.8}$$

None of the dedicated machines can be assigned in the FMS cell, in order not to destroy the total computer integration in the cell. Similarly, dedicated cells are totally composed of dedicated machines. This is a supplementary constraint that assures the technology selection decision.

$$M \cdot Open_{fk} \;\geq\; \sum_{m} x_{ik} \;\; \geq Open_{fk} \;\; \forall k \tag{3.9}$$

This constraint controls the opening of the flexible cell. At least one part should be assigned to the FMS cell to open and operate the cell. Open decision is an important decision for the whole system. However, the direct affect of this decision is observed at the labor costs. Same reasoning applies for the dedicated cells:

$$M \cdot Open_{dk} \;\geq\; \sum_{m} x_{ik} \;\; \geq Open_{dk} \;\; \forall k \tag{3.10}$$

Operational Allocation of Parts

$$z_{imok} \;\leq\; PRM_{io} \cdot MCM_{mo} \cdot y_{mk} \qquad \forall i, m, o, k \tag{3.11}$$

To be able to assign an operation of a part to a machine in a cell, three conditions should hold:

1 Operation should be necessary for the part. ($PRM_{io} = 1$)

2 At least one machine should exist in that cell. ($y_{mk} \geq 1$)

3 That machine should be capable of that operation. ($MCM_{mo} = 1$)

Unless these conditions hold, an operation cannot be assigned to a machine in a cell ($z_{imok} = 0$).Each necessary operation of a part should be handled by a machine in any one of the cells.

$$\sum_{m,k} z_{imok} \;=\; PRM_{io} \quad \forall i, o \tag{3.12}$$

Capacity and Utilization of Machines

$$TCap \cdot \beta_m \cdot y_{mk} \;\leq\; \sum_{i,o} z_{imok} \cdot Ptime_{imo} \cdot D_i \leq\; TCap \cdot \alpha_m \cdot y_{mk} \quad \forall m, k \tag{3.13}$$

With this constraint, we aim to incorporate the processing times and machine utilizations in our model. There exist other studies in literature that take machine utilizations into account. However, since we make technology selection simultaneously with part family-machine group formation, a different approach is constructed for this constraint.

The utilization levels of flexible and dedicated machines are different. Upper utilization level of dedicated machines is lower because of the longer setup requirements of the dedicated machines. Although we do not deal with setup times directly, we still consider this difference between technologies under utilization constraints. Furthermore, due to the high investment costs of flexible machines, companies might prefer increasing the lower utilization limits of these machines.

Central part of the equation 3.13 calculates the required total processing times of each cell and machine type. This amount should be neither smaller than a pre-specified utilization level of machines nor larger than the available capacity of the machines. The constraint determines the necessary number of machines for each machine type in each cell while controlling the utilization levels.

Cell Size Constraint

$$\sum_{m} y_{mk} \;\leq\; UBK \quad \forall k = 1, .., K \tag{3.14}$$

Total number of machines in a cell should be less than some upper bound. This is one of the classical cellular manufacturing constraints. If the number of machines in the cell increases, the system becomes more like a job shop production system. Thus, influencing a limit on the cell size is a necessity in a cellular manufacturing mathematical model.

Load-Unload Constraints

$$M \cdot L_{imk} \;\geq\; \sum_{o} z_{imok} \;\geq\; L_{imk} \quad \forall i, m, k \tag{3.15}$$

This constraint controls the loading of a part on a machine. At least one operation should be assigned to a machine in order a part to be loaded and unloaded on the machine. Load decision is important for the calculation of total production time of the part.

Intercellular Movement Constraints

$$M \cdot IM_{ik} \;\geq\; (\sum_{m,o} z_{imok}) \;\cdot\; (1 - x_{ik}) \;\geq\; IM_{ik} \quad \forall i, k \tag{3.16}$$

This constraint controls the intercellular movement of a part. At least one operation ($\sum_{m,o} z_{imok} > 0$) should be processed in a cell ($x_{ik} = 0$) other than the cell of which the part is a member. Then, an intercellular movement is said to be made by the part ($IM_{ik} = 1$). Movement decision is important for the calculation of the objective function 3.5. Intercellular movements should be minimized in order to design an efficient cellular manufacturing system.

3.3 Summary

In the market of 21^{st} century, product designs are evolving, production volumes are decreasing and product life cycles are getting shorter. Further, automated technologies offer a considerable number of different machine types which have different capabilities in terms of operations, setup requirements and utilizations.

On the other hand, there still exist stable design products with high and stable demand patterns. To allocate flexible resources for the production of these parts may not provide satisfactory results. Thus, we should make use of flexible technologies for management of product variety, while not losing the advantages of dedicated manufacturing via a hybrid technology application through a cellular manufacturing system design.

In our study, an integrated approach is proposed to solve the cell design and technology selection problems simultaneously during the design of an advanced manufacturing system. We proposed a new cost function to integrate the market characteristics in our model. Further, we modified a well known similarity measure in order to handle the operational capability of available technology.

This is the first model in literature that studies product variety management in cellular manufacturing systems via a hybrid choice of technology. The decisions of product variety management, part family - machine group formation, and technology selection are made simultaneously in the proposed model. In the next chapter, we propose a heuristic algorithm in order to solve the proposed model efficiently in a reasonable computation time.

Chapter 4

Solution Approach

In the previous chapter, a mathematical model is proposed to solve cell formation and technology selection problems simultaneously. It is difficult to obtain an optimal solution to this problem in a reasonable computation time. The model has a large number of binary and integer variables, and non-linear constraints, given in Table 4.1. Further the objectives f_1 and f_4 have quadratic functions of the decision variables. In order to solve this problem in an acceptable computation time, a local search heuristic is proposed below.

The proposed algorithm has two main stages. In the first stage, an initial feasible solution to the problem is constructed. The first stage of the algorithm can be analyzed in two phases: The major concern of the first phase is the minimization of variety costs and maximization of the similarity between parts. At the end of this phase, completely independent cells are formed. However, independent cells may not be feasible in terms of machine utilization and size constraints. This is overcome at the second phase by allowing parts to make intercellular movements. At the second stage initial solution is improved iteratively via a filtered beam based search heuristic.

In §4.1, the proposed algorithm is outlined and in §4.2 and §4.3 stages of the algorithm are explained in detail.

Effect	**Source**	**Quantity**
Binary Variable	x_{ik}	N · K
Binary Variable	z_{imok}	N · M · O · K
Binary Variable	L_{imk}	N · M · K
Binary Variable	$Open_{fk}$	K
Binary Variable	$Open_{dk}$	K
Binary Variable	IM_{ik}	N · K
Integer Variable	y_{mk}	M · K
Non-linear Constraint	Eq. 3.16	N · K

Table 4.1: Problem Complicating Items

4.1 Outline of The Algorithm

The basic steps of the algorithm are provided in this section. The details of each of the steps and representative examples are presented in the following sections. The brief outline and introductory information about the stages are as follows:

1 Initialization Stage

At this stage of the algorithm, a good initial solution is found under consideration of variety costs and operational similarity between parts. A fuzzy clustering technique is utilized to analyze the groups in part data. After the selection of technology for each cluster, machine groups are formed and feasibility is attained. Outline of the first stage is as follows:

1.1 Data Generation

1.2 Similarity Coefficient Calculation

1.3 Fuzzy Analysis

1.4 Part Family Formation

1.5 Machine Group Formation

1.6 Feasibility Check

1.6.1 Machine Utilization Feasibility

1.6.2 Cell Size Feasibility

2 Search Stage

At this stage of the algorithm, the objectives that have been ignored at the first stage are inserted back in the model. The initial solution aims to find a good solution in terms of similarity, product variety costs, and number of exceptional parts. However, the machine costs are at a very high level in this solution. This objective has been ignored at the first stage. The second stage searches for a better solution in terms of monetary costs, while not deviating much from the other objectives. Filtered beam search technique is applied. Outline of the second stage is as follows:

2.1 **while not** *stopping criteria met* **do**

  2.1.1 For Each Parent Solution

    2.1.1.1 Candidate Machine Selection

    2.1.1.2 Candidate Part Identification

    2.1.1.3 Alternative Solution Generation

  2.1.2 Evaluate Alternatives

  2.1.3 Go to Step 2.1

2.2 Return the final solution

4.2 Stage I - Finding an Initial Solution

At this stage of the algorithm, a good initial solution is found under consideration of variety costs and operational similarity between parts. Thus, the initial solution is very good in terms of product variety and similarity objectives, however, not as good in terms of monetary objectives. Since the investment and labor costs are ignored and similarity is maximized at this step, intercellular movements of parts are also at its best possible level. Details of the stages are as follows:

1.1 Data Generation

Part data is gathered from the literature. For the machine data, we preferred to use real data in order to be consistent with the real manufacturing environment. Details of the data generation procedures are explained in the experimental design chapter.

1.2 Similarity Coefficient Calculation

Modified Jaccard similarities are calculated for each pair of parts. As a first step for the partitioning, initial similarities are calculated according to the average dedicated variety cost of two parts. As described in the previous chapter, for the sake of technology selection, we limit the data of MCM in calculation of similarities.

Initially since there exist no families and technologies, the dissimilarities are calculated based on the average product variety cost (c_{id}'s) of the two parts. If the average cost of parts i and j is lower than the threshold value, explained in section 3.1, the dedicated block of the MCM is available for the calculation (DMJ_{ij}^{d}). When only the dedicated machines are available for calculation, the proposed coefficient gives the same result as the classical Jaccard. Otherwise, if the average cost is high, flexible block of MCM is used to calculate the dissimilarity of parts (DMJ_{ij}^{f}). High average shows a tendency to be placed in a flexible cell. Hence, if the parts are more likely to meet in a flexible cell, we calculate a flexibility based coefficient and vice versa.

Example *Let the threshold value be 15 and PRM and MCM data are given as follows:*

MCM	op1	op2	op3
m1	1	0	0
m2	0	1	0
m3	0	0	1
fm1	1	0	1
fm2	0	1	1

PRM	op1	op2	op3	Variety Costs (c_{id})
p1	1	1	1	15
p2	1	1	0	5
p3	0	1	0	33

$$Average\ Cost_{p1p2} = \frac{15+5}{2} = 10 < 15\ Dedicated\ machines\ are\ available.$$

$$DMJ^{d}_{12} = 1 - \frac{2}{3} = \frac{1}{3}$$

$$Average\ Cost_{p1p3} = \frac{15+33}{2} = 24 > 15\ Flexible\ machines\ are\ available.$$

$$DMJ^{f}_{13} = 1 - \frac{1}{2} = \frac{1}{2}$$

1.3 Fuzzy Analysis

As a third step, we perform a fuzzy analysis that uses dissimilarity coefficients and yields a list of membership coefficients for the parts. Fuzzy clustering is a generalization of partitioning. In a partition, each part of the set is assigned to only one cluster. On the other hand, a fuzzy clustering method allows for some uncertainty in the data leading to a number of choices for the part to be assigned. Fuzzy analysis uses dissimilarity coefficients and provides a list of membership coefficients for the parts.

Fuzzy clustering technique is utilized to analyze the groups in part data. We have adapted the fuzzy algorithm of Kaufmann and Rousseeuw [35]. Steps of the algorithm are provided in the Appendix A. The main advantage of the fuzzy clustering over hard clustering is that it yields much more detailed information on the structure of the data. The fuzziness principle is very appealing because it allows a description of some of the uncertainties that often retained in real data.

In the CMSD problem, fuzzy algorithm eases the alternative solution generation process. A part has membership value for each of the clusters. We have a chance to choose any cluster having acceptable values. Further, it is a quantifiable basis where to move the candidate part in the next iteration.

Example *Let the output of a fuzzy analysis performed on the dissimilarity matrix of a CMSD problem is as follows, where c1, c2, c3 represent the possible clusters of the system:*

Membership	c1	c2	c3
p1	0,53	0,23	0,24
p2	0,33	0,32	0,35
p3	0,29	0,69	0,02
⋮	⋮	⋮	⋮

> *In this output, we read that it is 53% beneficial for the part 1 to be in cluster 1, 23% beneficial to be in cluster 2 and 24% beneficial to be in cluster 3 in terms of operational similarities. The decision maker have the chance to choose between these alternatives. While choosing cluster 2 for part 3 is an obvious alternative, for part 2 all clusters are candidates to be assigned in. We do not lose much from the similarity objective if we change the membership of part 2 from cluster 3 to cluster 1.*

Fuzzy analysis output leaves room for the decision maker to form different part families. The level of satisfaction in terms of similarities can be decided via the alternatives provided in the fuzzy analysis output, namely the membership matrix.

We make use of membership matrix frequently in our solution approach. From the initial part families to the last iteration of the search process one of the most critical tools used in the algorithm is the fuzzy membership matrix. The fuzzy output provide us valuable information, such that we know how much we deviate from the optimal similarity when we change the cluster of a part. Hence, we generate solution alternatives more effectively via use of fuzzy membership matrix.

1.4 Part Family Formation

After completing the initial calculations, the algorithm constructs the initial solution. The first step of finding the initial solution is the identification of the initial part families. In order to have maximum similarity, at this stage, each part is assigned to the cluster where it has the greatest membership value.

Example *Take the fuzzy membership matrix provided in the example of the Step* 1.3*. For the sake of optimality in similarities, we assign part 1 to cluster 1 ($x_{11} = 1$), part 2 to cluster 3 ($x_{23} = 1$) and part 3 to cluster 2 ($x_{32} = 1$).*

At this point in the algorithm, the initial part families are identified. These clusters not only identifies the part families but also are selected to form cells by the algorithm. The opening of cells are decided at this point in the algorithm. The clusters that have member parts are opened, and if a cluster is not selected by any of the parts, it is not considered any more in the algorithm.

Once the opening decision is taken for a cluster, it is time to decide the technology of the cell. We based our technology selection decision on the average product variety costs of the parts. If the member parts of a cell tend to have unstable market characteristics, i.e. high variety costs (c_{id}'s), it is good to open a cell with flexible technology to process these parts. However, if the general tendency of the member parts is stability in terms of design and demand, i.e. low variety costs (c_{id}'s), the algorithm prefers to open a cell with dedicated technology to process these parts.

Technology selection decision of each cell is based on the average product variety costs of the member parts. If the average is larger than the threshold value, the technology for the cell is selected to be the flexible technology. Otherwise, it is a dedicated cell. For each cluster, the technology that minimizes the variety costs is selected. Following is a representative example that clarifies the steps of technology selection procedure.

Example *Let the threshold value be 15 and the product variety costs and fuzzy membership matrix values of a problem be as follows:*

Membership	c1	c2	c3	c_{id}		Chosen Cluster
p1	0,50	0,20	0,30	20	$\Rightarrow$	1
p2	0,33	0,32	0,35	19	$\Rightarrow$	3
p3	0,20	0,60	0,20	7	$\Rightarrow$	2
p4	0,25	0,30	0,45	5	$\Rightarrow$	3
p5	0,25	0,25	0,50	8	$\Rightarrow$	3
p6	0,75	0,15	0,10	11	$\Rightarrow$	1
p7	0,80	0,15	0,05	4	$\Rightarrow$	1
p8	0,15	0,70	0,15	34	$\Rightarrow$	2
p9	0,25	0,40	0,35	20	$\Rightarrow$	2
p10	0,05	0,90	0,05	37	$\Rightarrow$	2
p11	0,30	0,55	0,15	32	$\Rightarrow$	2

Each part is assigned to the cluster where it has the greatest membership value. In such a configuration, the algorithm opens all the cells since each cluster has members. To select the technology of each cell, the average variety costs are calculated:

$$Average\ Cost\ of\ Cluster\ 1 = \frac{20 + 11 + 4}{3} = 11,66 < 15$$

Technology of cluster 1 is selected to be the dedicated technology.

$$Average\ Cost\ of\ Cluster\ 2 = \frac{7 + 34 + 20 + 37 + 32}{5} = 26 > 15$$

Technology of cluster 2 is selected to be the flexible technology.

$$Average\ Cost\ of\ Cluster\ 3 = \frac{19 + 5 + 8}{3} = 10,66 < 15$$

Technology of cluster 3 is selected to be the dedicated technology.

1.5 Machine Group Formation

After the selection of technology for each cluster, machine groups are formed. As an initial configuration of the machines, any necessary machine is placed in the cell by the algorithm. The important issue at this step is the technology of the machines, no dedicated machine exists in flexible cells and no flexible machine exists in dedicated cells. Initial operational allocation of parts is also performed simultaneously.

Example *If part i in cell k ($x_{ik} = 1$) requires operation o ($PRM_{io} = 1$), and no machine that is capable of the operation exists in the cell, the first machine in the list capable of operation o ($MCM_{mo} = 1$) is assigned to the cell ($y_{mk} = 1$) and operation is allocated to the machine ($z_{imok} = 1$). If there exists a machine ($MCM_{mo} = 1$ and $y_{mk} = 1$), the operation is directly allocated to the machine ($z_{imok} = 1$). After allocations, according to the utilization levels of machine types, appropriate number of machines are calculated for the cell k.*

After assignment of operations to machines, utilizations of machine types are calculated in order to find the necessary number of machines in the cell. The required number of machines is calculated according to the upper utilization levels of each machine type.

$$Util_{mk} = \sum_{i,o}(z_{imok} \cdot Ptime_{imo} \cdot D_i)$$

$$y_{mk} = \lceil \frac{Util_{mk}}{TCap \cdot \alpha_m} \rceil$$

1.6 Feasibility Check

At this point of the algorithm, initial part families and machine groups are identified. However, this configuration may not be feasible. Because neither utilization levels nor size of the cells are taken into consideration. Since the cells are designed to be independent, some machines may exist with very low utilizations in the cells, and sizes of the cells might be larger than the acceptable level.

1.6.1 Machine Utilization Feasibility

First, utilization levels of the machines are checked. The left side of the equation 4.1 is true for each machine, since the greater integer value is assigned in the previous step. However, the right side of the equation should be checked for feasibility.

$$TCap \cdot \alpha_m \cdot y_{mk} \geq Util_{mk} \geq TCap \cdot \beta_m \cdot y_{mk} \qquad \forall m, k \tag{4.1}$$

In a cell, if the utilization level of a machine type turns out to be low, first the operations performed on the machine are determined. If the remaining capacity of the system is sufficient to perform these operations, one machine is deleted from the system. If the excess capacity is not enough to delete the machine, then the problem is infeasible in terms of utilization constraints or the lower limit of the utilization is high. In such a situation, the analyst should consult the decision maker to change the suggested utilization levels.

If the machine is a flexible one, after it is deleted, parts having operation on the machine are forwarded to other machines in the same cell. If none of the machines are capable of the required operation, algorithm allows for the intercellular movement of parts.

Example *Let a flexible cell k is composed of 2 flexible machines MCM of which are given and let machine 1 be a low utilization machine. Let parts i and j be the only two parts having operation on this machine. PRM values of two parts are also given.*

MCM	op1	op2	op3	op4
m1	1	1	0	0
m2	1	0	1	1

PRM	op1	op2	op3	op4
i	1	1	0	1
j	0	1	1	0

Let the initial operational allocations of the parts (z_{imok}) are as follows:

$z_{i11k} = 1$ $z_{i12k} = 1$ $z_{i24k} = 1$

$z_{j12k} = 1$ $z_{j23k} = 1$

Since machine 1 is not utilized at a satisfactory level, it is deleted from cell k. Then, the operations handled on this machine should be re-allocated ($z'_{i11k} = 0$ $z'_{i12k} = 0$ $z'_{j12k} = 0$).

Since there exist another machine in the cell that can handle operation 1, part i can be directed to machine 2 for operation 1 ($z'_{i21k} = 1$).

However, the only machine that can handle operation 2 is deleted from the cell. Thus, parts i and j make an intercellular movement to cell c where another machine 1 exists with available capacity ($z'_{i12c} = 1$ $z'_{j12c} = 1$).

If the machine is a dedicated one, after it is deleted, parts having operation on the machine are forwarded to other cells since no machine in a dedicated cell may perform more than one type of operation.

1.6.2 Cell Size Feasibility

After the utilization feasibility is attained, cell size constraints are checked according to the cell size constraint 3.14 provided in the Problem Statement chapter. If the design is feasible in terms of sizes, we go on with the search algorithm. Otherwise, in order to maintain size feasibility, algorithm first considers the deletion alternative. The least utilized machine is a candidate for deletion from the cell where the constraint is violated. Delete procedure is as described in Step 1.6.1. If still the constraint is not satisfied, the algorithm moves a required number of the least utilized machines to another cell. Since it produces a number of intercellular movements, move is not a preferable action, and generally it is not needed at all.

4.3 Stage II - Local Search Heuristic

At the second stage of the algorithm, the objectives that have been ignored at the first stage, namely the monetary objectives and throughput times are inserted back in the model. The initial solution is constructed in order to find high similarity, low product variety costs, and low number of exceptional parts. However, the machine costs are at a high level in this solution. The second stage searches for a better solution in terms of monetary costs, while not deviating much from the other objectives.

Filtered beam search technique is the tool employed for the local search. In this study, we adapted the classical filtered beam to our problem. The filtered beam local search is an efficient branch and bound technique. It uses heuristics to figure out a number of promising paths. Other alternatives are fathomed and not evaluated ever again. It has two distinguishing parameters, *beam width (b)* and *filter width (f)*. Akturk and Kilic [3] employ a third parameter, namely *child* *width (c)*. This parameter determines the number of children allowed for each parent (beam) limiting the number of beams that originate from the same parent.

In our study, beam width represents the number of parent solutions that generate alternative solutions. Filtering mechanism has 2 steps in the algorithm. Before fully generating the alternative designs, the algorithm prunes the paths via filters located at two steps. Child width mechanism is also employed in the algorithm in order to perform the search in a wider solution space. Details of the steps are as follows:

2.1 while not *stopping criteria met* do

Iterative procedure goes on until the stopping criteria is met. The algorithm stops either no other candidate machines or parts can be identified in the search space or a pre-specified number of iterations is reached. The details of one search iteration are provided below:

2.1.1 For Each Parent Solution

Each iteration begins with the generating solutions for each parent solution. The number of parents is the beam width parameter of the algorithm. The aim of newly generated alternative solutions is decreasing the number of machines that exist more than necessary in the whole system. Generating alternatives is an important step in our study. Phases of this step are detailed below:

2.1.1.1 Candidate Machine Selection

First step of the alternative generation phase is the identification of candidate machines. We should decrease the number of machines which have been assigned greedily in the initial solution. Initially, whenever an operation is needed in a cell, the associated machine is made available in the cell. In some cells, we assigned machines with very low utilizations. If we consider deleting these machines from the cells, we might improve our solution in terms of monetary costs.

The critical question is the identification of the candidate machines. We use utilization levels accumulated for each machine type in a cell. Thus, number of machines of each type determines the utilization levels ($Util_{mk}$). To decide whether the utilization is high or low from the $Util_{mk}$ data is misleading. The best way of finding the low utilization machines is to identify the highest excess capacity machine types in a cell. Whatever the number of machines in the cell, excess capacity data provide information about the utilization level of one machine in the cell. We assume at this point that, for each type of machine in a cell, the next machine is bought only if the previously bought machines are fully utilized.

$$Excess_{mk} = (TCap \cdot \alpha_m \cdot y_{mk}) - Util_{mk}$$

First filtering mechanism is employed at this step. The highest excess capacity **f1** number of machines are selected for further analysis. These low utilization machines are selected to be deleted from the system. Other machines with excess capacities are not considered since the least utilized machines provide the most promising paths around the solution space.

Example *Let there exist 5 machines (m1, m2, m3, m4, m5) in different cells (k1-dedicated, k2-dedicated, k3-flexible) which have excess capacities. Let f1 be 3. Let* $TCap = 119.808$ *min./year and* α_m *be 80% for dedicated machines and 95% for flexible machines.*

$0 \leq Excess_{mk} \leq 95.846$ *for dedicated machines*

$0 \leq Excess_{mk} \leq 113.817$ *for flexible machines*

$$\begin{aligned}
Excess_{m1k1} &= 75.482 \\
Excess_{m2k1} &= 54.237 \\
Excess_{m3k2} &= 15.156 \\
Excess_{m4k2} &= 80.872 \\
Excess_{m5k3} &= 45.278
\end{aligned}$$

The algorithm chooses 3 highest excess capacity machines regardless of their technologies, m4 of k2, m1 of k1 and m2 of *k1 to design new solutions by deleting in the next steps.*

2.1.1.2 Candidate Part Identification

Parts having operation on the candidate machines are candidates to change clusters. We consider changing the cluster of the part, because the machine might only be needed by this part which might initially be placed in a wrong cluster. When we change the cluster of the part, and delete the machine from the cell, objectives might improve further without any loss. We increase the number of alternatives by considering different part-cluster allocations. Second filtering mechanism is employed at this step: **f2** number of part-cluster pairs which have the greatest fuzzy membership coefficients are selected for further evaluation. In the new design, part will be the member of this new cluster. Highest coefficient parts lead to better solutions since the loss in the objective function values occurs the least with relatively high similarity coefficients.

Example *Let's assume that there exist 3 parts (p1,p2,p3) having operation on a candidate machine. These parts are candidates to change clusters. Fuzzy membership matrix is given below. Let f2 = 3, and all the cells be open (If a cell is not open, it is not considered at all).*

Membership	k1	k2	k3
p1	0,50	0,15	0,35
p2	0,32	0,35	0,33
p3	0,25	0,30	0,45

.

$x_{11} = 1 \;\; x_{22} = 1 \;\; x_{33} = 1$

The algorithm finds the part-cluster pairs that give the greatest coefficients and different than the current assignments. Since the filter parameter is equal to 3, only 3 of new pairs are selected for further evaluation: (p1-k3)(p2-k3)(p2-k1)

$x'_{13} = 1 \;\; x'_{22} = 1 \;\; x'_{33} = 1$

$x''_{23} = 1 \;\; x''_{11} = 1 \;\; x''_{33} = 1$

$x'''_{21} = 1 \;\; x'''_{11} = 1 \;\; x'''_{33} = 1$

2.1.1.3 Alternative Solution Generation

At this point in the algorithm, we have identified the candidate machines and associated parts to change clusters. At this step, we generate new alternative solutions based on the candidate machines and parts. For each candidate machine, following alternatives are generated:

- Without any part transfers, all the operations previously performed on the machine are forwarded to other machines. If the machine is in a dedicated cell, then the parts make intercellular movement to have the operation previously processed by the deleted machine. Otherwise, if the machine is in a flexible cell, the algorithm searches for each operation processed previously on the deleted machine to find an available capable machine in the same cell. If no machine in the same cell might perform the operation, the part is directed to other cells to have the operation. (A single alternative design is created.)
- Candidate parts are transferred to their candidate cells (one for each alternative) and remaining operations are forwarded to other machines. (f2 alternative designs are created.)
- For each new design, the number of intercellular movements are calculated, and if any part travels more than a pre-defined number of movements, new and revised designs considering the problematic parts are constructed. (Less than (f2+1) alternative designs are created.)

At each iteration, for each b parent solutions, the algorithm identifies f1 candidate machines and f2 candidate parts. Then, the number of alternatives (Alt) at each iteration depends on the parameters of the algorithm.

$$b \cdot f1 \cdot (f2 + 1) \leq Alt \leq b \cdot f1 \cdot 2 \cdot (f2 + 1)$$

2.1.2 Evaluate Alternatives

Since the model is a multi-objective model, for each alternative we have 5 different objectives. The three objectives, dissimilarity, product cost variety and number of exceptional parts, are aimed to be minimized in the initial solution. At the second stage, for the sake of monetary objectives, the algorithm gives up from these three objectives, results get worse, but the investment and labor costs are significantly reduced.

In all of the search algorithms, the analyst first generates, then evaluates the alternatives according to some fitness value of the alternatives. When the model has a single objective, it is easy to compare the alternative solutions. However, when there exist more than one objective, evaluation procedure gets complicated. In simple words, the analyst should compare apples to oranges and decide the best. There exist solution approaches in the literature for unification of multi objective criteria. However, in any case, the analyst loses information. Thus, to keep the algorithm simple and effective, we preferred a 0-1 normalization procedure applied among the alternatives of each iteration.

At the end of each iteration, objective function values (f_1, f_2, f_3, f_4, and f_5) of each alternative are calculated according to the Equations 3.1, 3.2, 3.3, 3.4, 3.5 provided in the Problem Statement Chapter. Each objective function is normalized compared to the same objective functions of the remaining alternatives of the iteration.

$$Normf_1 = \frac{f_1 - GMinf_1}{LMaxf_1 - GMinf_1} \tag{4.2}$$

Normalization of objective 1 is achieved through equation 4.2. The same equation applies for all of the objectives except the fifth objective, the intercellular movement function, local maximum value should be changed to global maximum value. Because of the power of 0 value, which is the global minimum value in general for f_5, we observed that this function results in very dominant and misleading results. For normalization other than f_5, we prefer to use the minimum value attained at that point in the algorithm as the global minimum value, and the maximum value achieved just in that iteration as the local maximum value. Since our aim is minimization, we should compare the alternatives relative to the best (global min) value. On the other hand, we should not lose information by taking the upper bound (max) value unnecessarily high. Thus, we prefer to use just the maximum value of that iteration in the calculations.

The fitness value of each alternative is simply the summation of 5 normalized objective function values. Best solution is preserved as the incumbent solution, and best alternatives of the iteration are chosen to be the parents of new iteration. The child mechanism is employed at this step. Child width limits the number of new parents originating from the same old parent. Total number of new parents is the beam width.

Example *Let the number of alternatives generated at an iteration is 5. Let b is 3, and the objective function values are given as the following.*

Alternatives	f_1	f_2	f_3	f_4	f_5
Initial	175	415	315.565	978.247	0
Current Best	180	457	375.672	672.169	0
Alt 1	182	415	289.614	742.245	2
Alt 2	197	567	197.723	691.837	3
Alt 3	248	502	298.521	572.893	2
Alt 4	177	499	214.345	619.361	1
Alt 5	314	467	155.983	580.347	0
G Min	175	402	155.983	565.741	0
Max	314	567	375.672	978.247	8

.

With the given objective function values, the following normalized objective function values are calculated.

	$Normf_1$	$Normf_2$	$Normf_3$	$Normf_4$	$Normf_5$	Fitness
Initial	0,00	0,08	0,72	1,00	0,00	1,80
C.B.	0,04	0,33	1,00	0,26	0,00	1,63
Alt1	0,05	0,08	0,61	0,43	0,25	1,42
Alt2	0,16	1,00	0,19	0,31	0,38	2,04
Alt3	0,52	0,61	0,65	0,02	0,25	2,05
Alt4	0,01	0,59	0,27	0,13	0,13	1,13
Alt5	1,00	0,39	0,00	0,04	0,00	1,43

.

As it is seen from the fitness values of each of the alternatives that Alternative 4 is the best value attained at that point. The incumbent solution, which is represented as the current best, is changed to Alternative 4 at the next iteration. Secondly, the algorithm chooses the parents of the new iteration. They are the best 3 alternatives of this iteration, namely the alternatives 4, 1 and 5.

2.1.3 Go to Step 2.1

Iterative procedure goes on until the stopping criteria is met. The algorithm stops either no other candidate machines or parts can be identified in the search space or a pre-specified number of iterations is reached.

2.2 Return the final solution

At the end of the search, the algorithm terminates. Finally, the initial solution, best solution, and global minimum values are reported to the analyst.

4.4 Summary

In order to solve the proposed CMSD problem in an acceptable computation time, a local search heuristic is proposed in this chapter. The algorithm has two main stages. In the first stage, an initial feasible solution to the problem is constructed. The first stage of the algorithm can be analyzed in two phases: The major concern of the first phase is the minimization of variety costs and maximization of the similarity between parts. At the end of this phase, completely independent cells are formed. However, independent cells may not be feasible in terms of machine utilization and size constraints. This is overcome at the second phase by allowing parts to make intercellular movements. At the second stage initial solution is improved iteratively via a filtered beam based search heuristic.

The efficiency of the algorithm is tested by using a set of randomly generated problems. In the next chapter, the experimental design of the proposed algorithm will be discussed.

Chapter 5

Experimental Design

The algorithm proposed in Chapter 4 is coded in C language. The code is compiled with Gnu C 5.0 compiler and the problem is solved on a 12x400 MHz UltraSparc Station under Solaris 2.7. In this chapter, we present the parameters of the experimental design, and the factors that have effects on system performance are analyzed. The experimental setting is explained in §5.1 and computational results are discussed in §5.2. In §5.3, chapter is concluded with a summary.

5.1 Experimental Setting

Parameters of the model can be analyzed in two main categories: (i) Factors that effect the system performance and (ii) Parameters that are fixed to a predetermined value.

Five factors that provide different system properties with their different levels are shown in Table 5.1. Factor A determines the number of high-low-medium volume parts in the entire production area. Factor B controls the ratio of the production amount of a high volume part to a low volume part. Factor C determines the characteristics of the marketing environment. If the market demands more customized products, namely the design stability of the parts is low, and

Factors	**Definition**	**Level 0**	**Level 1**	**Level 2**
A	Highest Number of Parts	High Vol.	Low Vol.	Medium Vol.
B	($D_{HighVol.Part}$ / $D_{LowVol.Part}$)	Low Ratio	High Ratio	-
C	Stability of Environment	Stable	Volatile	-
D	Flexibility	Low	High	-
E	($Labor_f$ / $Labor_d$)	Low Ratio	High Ratio	-

Table 5.1: Experimental Design Factors

variation of the demand is high, the factor is equal to 1. Factor D controls the operational capability of the flexible machines. Factor E determines the relation between the labor costs of flexible cells and dedicated cells.

Remaining parameters of the model are the number of parts, part requirement matrix, ages of the parts, number of cells, number of machines, variety cost threshold value, total annual demand, cell size limit, processing times, load-unload times, theoretical capacities, utilization limits and investment costs of machines. Detailed description of experimental factors and parameters are provided in this section.

5.1.1 Factors A and B

Parts can be classified into 3 categories: High volume parts, medium volume parts and low volume parts. At the production floor, the most important data are the number of parts to be produced and the amount of production volume for each part. Factors *A* and *B* together determine the volume data for each part and

Level	**High Vol**	**Med.Vol**	**Low Vol**	**Total**
0	0,35	0,50	0,15	1,00
1	0,15	0,50	0,35	1,00
2	0,15	0,70	0,15	1,00

Table 5.2: Factor A

Level	High / Low	Med / Low	Low
0	8	4	1
1	27	9	1

Table 5.3: Factor B

the amount of production. Table 5.2 shows the probability of a part to become a high, medium or low volume part. For example, during the experimentation, if the setting is favoring high volume parts (Level is 0), 35% of the parts have a high production volume, 50% of the parts have a medium production volume, and 15% of the parts have a low production volume.

In Table 5.3, ratios of mean production amount of high and medium volume parts to mean production amount of low volume parts is presented. If the factorial combination favors high volume production (Level is 1), mean production amount of a high volume part is determined to be 27 times of the mean production amount of a low volume part, and the ratio for a medium volume part is equal to 9.

In order to receive consistent results, we have fixed the total demand. For each run, the algorithm partitions the constant demand to each part type according to the factorial combination. The ratio of the average production amounts to the entire demand comes out to be as it is shown in Table 5.4. Calculation of these values are explained on an example:

Example *Take the factorial setting for factors A and B as 0 and 1 respectively, favoring the high volume environment.*

According to the 0 setting of factor A, 35 of 100 parts have high volume, 50

Setting	00	01	10	11	20	21
Ratio for Low. Avg.	0,202	0,071	0,282	0,112	0,241	0,095
Ratio for Med. Avg.	0,808	0,639	1,128	1,008	0,964	0,855
Ratio for High Avg.	1,616	1,917	2,256	3,024	1,928	2,565

Table 5.4: Average Demand Ratios

of 100 have medium and 15 of 100 have low production volume.

According to the 1 setting of factor B, high amount means 27 times of low amount, and medium amount equals 9 times of low amount.

$$Mean\ \ Low\ \ Volume\ \ =\ \ \frac{Total\ \ Demand\ \cdot\ \frac{15\ \cdot\ 1}{35\cdot 27\ +\ 50\cdot 9\ +\ 15\cdot 1}}{N\ \cdot\ \frac{15}{100}} \tag{5.1}$$

The numerator of the Equation 5.1 gives the concentration of the low volume parts in the entire system. The denominator is the number of low volume parts in the system.

$$Mean\ \ Low\ \ Volume\ \ =\ \ \frac{Total\ \ Demand}{N}\ \ \cdot\ \ 0,071$$

$$Mean\ \ Med\ \ Volume\ \ =\ \ \frac{Total\ \ Demand}{N}\ \ \cdot\ \ 0,639$$

$$Mean\ \ High\ \ Volume\ \ =\ \ \frac{Total\ \ Demand}{N}\ \ \cdot\ \ 1,917$$

After identification of the average values of production amounts, the algorithm assigns volume data for each part randomly around the averages. The range of the random assignment is determined to be ±10% in order to preserve the clear distinctions between part volumes. At the end of the demand calculation process, it is verified to maintain the constant demand in total.

5.1.2 Factor C

In order to adapt the evolving market dynamics of the world, we incorporate a key marketing concept, namely the product life cycle concept, in our model. There exist different life cycle curves in the marketing literature. However, for

Demand	P1	P2	P3	P4	P5
High	0,00	0,15	0,35	0,35	0,15
Medium	0,10	0,30	0,15	0,15	0,30
Low	0,50	0,25	0,00	0,00	0,25

Table 5.5: Demand - Age Ratios

the sake of generality, we assume the traditional product life cycle curve as shown in Figure 3.1. The associated parameters used in the design are provided in Table 5.5. In the table, the probability of a part whose demand is known, to have a position in each period of the life cycle is presented.

Example *If a part a has low demand, according to the traditional product life cycle curve, it is in its first period of life with probability 0.50, in its second life period with probability 0.25, and in its fifth life period with probability 0.25.*

In order to be in accordance with the traditional life cycle curve, we assumed that no part might have a high demand in its initial period of lifetime, and no part might have low demand if it is in its maturity phase of life, namely the third and fourth life periods.

In the life cycle curve, we assume, each part have independent positions, i.e. the ages of parts are independent from each other. On the other hand, for each part, there exist direct relation between age, demand and design variability of the part. The amount of this relation is affected by the market characteristics. The market might demand design changes either frequently, that is unstable, or seldom, that is stable. This triple relation is presented in Table 5.6 under the effect of market characteristics. The table shows the probability of a part to have a design feature under the volume and age information. In the stable environment setting, the stability is favored, while in the unstable environment setting, it is quite likely that the designs will be unstable as well.

Example *For example, if a part has a high demand, and it is in its second life*

		Stable Environment 0			Unstable Environment 1		
Volume	**Life Cycle Position**	Stable Design	Moderate Design	Volatile Design	Stable Design	Moderate Design	Volatile Design
High	**2**	0,50	0,30	0,20	0,20	0,30	0,50
High	**3**	0,60	0,30	0,10	0,40	0,30	0,30
High	**4**	0,60	0,30	0,10	0,40	0,30	0,30
High	**5**	0,50	0,30	0,20	0,20	0,30	0,50
Medium	**1**	0,45	0,30	0,25	0,25	0,30	0,45
Medium	**2**	0,50	0,30	0,20	0,20	0,30	0,50
Medium	**3**	0,55	0,30	0,15	0,35	0,30	0,35
Medium	**4**	0,55	0,30	0,15	0,35	0,30	0,15
Medium	**5**	0,50	0,30	0,20	0,20	0,30	0,50
Low	**1**	0,30	0,30	0,40	0,10	0,30	0,60
Low	**2**	0,35	0,30	0,35	0,15	0,30	0,55
Low	**5**	0,35	0,30	0,35	0,15	0,30	0,55

Table 5.6: Factor C

period, the probability of having a stable design is 0,50 in a stable market, whereas 0,20 in an unstable market.

5.1.3 Factor D

Machine flexibility is another factor that affects the system performance. We understand the flexibility as the operational capability of the machines. With this definition, machines of dedicated technology are the least capable machines of the system. On the other hand, we decide the capability of flexible technology machines randomly. If the level of factor D is 1, randomness favors more flexibility. Otherwise, machines are designed to be less capable.

The expected level of flexibility is determined in accordance with the part-operation matrix. We assumed that the requirement frequency of an operation shows the expected capability of a machine on that operation. In other words, as the requirement of an operation increases, the probability of a flexible machine to be capable of that operation increases. After calculation of expected probabilities,

PRM	Operations						
Parts	1	2	3	4	5	6	7
1	1	1	1				
2	1	1					
3		1	1				
4				1	1	1	1
5					1	1	1
6				1	1	1	
7					1	1	1
8				1	1		1
9				1		1	1
10				1			1

Table 5.7: An example PRM

the two levels of flexibility are determined according to these probabilities. A representative example is presented below:

Example *Take the part requirement matrix in Table 5.7 as the input data to the algorithm. According to this part requirement matrix, we calculate the flexibility probability expectation matrix with the following formula:*

$$Probability = \frac{Number\ of\ 1's\ in\ the\ column}{Total\ number\ of\ parts}$$

MCM Probability	Operations						
Machines	1	2	3	4	5	6	7
1	0,20	0,30	0,20	0,50	0,50	0,50	0,60
2	0,20	0,30	0,20	0,50	0,50	0,50	0,60
3	0,20	0,30	0,20	0,50	0,50	0,50	0,60
4	0,20	0,30	0,20	0,50	0,50	0,50	0,60
5	0,20	0,30	0,20	0,50	0,50	0,50	0,60

Each figure in the table shows the probability of the machine to be capable of the operation, i.e. the probability of machine capability matrix (MCM) entry to be 1. Thus,

Level	Range
0 Less Flexible	-20%
1 More Flexible	+20%

Table 5.8: Factor D

$$\begin{aligned} MCM_{51} &= \textit{1 with expected probability } 0,20 \\ &= \textit{0 with expected probability } 0,80 \end{aligned}$$

Let's assume that the design is constructed in order to favor more flexibility. Let the range parameter be 20% *and our random number generator gives* $0,50$. *Then, the effective probabilities to produce 1's in the* MCM_{51} *increases by factor* $(1 + (0,20 \cdot 0,50))$. *Thus,*

$$\begin{aligned} MCM_{51} &= \textit{1 with effective probability } 0,22 \ \textit{(}0,20 \cdot 1,10\textit{)} \\ &= \textit{0 with effective probability } 0,78 \end{aligned}$$

After construction of the machine capability matrix with respect to these probabilities, a control mechanism checks the matrix whether a meaningful matrix is produced or not. If there exists operations not handled by the flexible machines, the algorithm finds 2 flexible machines randomly and assigns the MCM_{mo} entry to be 1. If there exist flexible machines with operational capability less than 2, the algorithm randomly finds an operation and assigns the MCM_{mo} entry to be 1. The two levels of flexibility are determined according to the values presented in Table 5.8.

5.1.4 Factor E

Labor cost is the last factor that we analyze the effect on the system performance. In terms of financial concern, labor is a significant attribute. Further, it requires different characteristics for a dedicated system and an advanced manufacturing system. Labor costs get higher when computers are integrated in the system. Not only the operator is paid more, but also there exists supplementary labor who is responsible from the software, maintenance, etc. On the other hand, while one

Level	Operator Ratio	Supplementary Ratio
0 Low Level	2	0,20
1 High Level	5	0,50

Table 5.9: Factor E

operator is enough for the entire flexible cell, each machine requires an operator in the dedicated cell. This brings a trade off and needs to be analyzed.We re-define the flexible labor cost as a factor of dedicated labor cost.

$$TotL_{fk} = (Labor_d \cdot OR) + \sum_{m=1}^{FM}(Labor_d \cdot SR \cdot y_{mk}) \qquad \forall\, k \tag{5.2}$$

$$TotL_{dk} = \sum_{m=1}^{DM}(Labor_d \cdot y_{mk}) \qquad \forall\, k \tag{5.3}$$

Equations 5.2 and 5.3 determine the relation between labor cost of flexible cells and dedicated cells. The flexible labor cost is always greater than the dedicated labor, however if the factor is at level 0, the difference is low and when it is 1, the difference is high. Further, the size of the cells also affect the final labor cost.

5.1.5 Parameters of The Experimental Design

Remaining parameters of the model are the number of parts, part requirement matrix, number of cells, number of machines, variety cost threshold value, total annual demand, cell size limit, processing times, load-unload times, theoretical capacities, utilization limits and investment costs of machines.

5.1.5.1 Part Requirement Matrix

Cellular manufacturing success depends on the groupability of the part data. There should exist virtual part groups in order to apply cellular manufacturing.

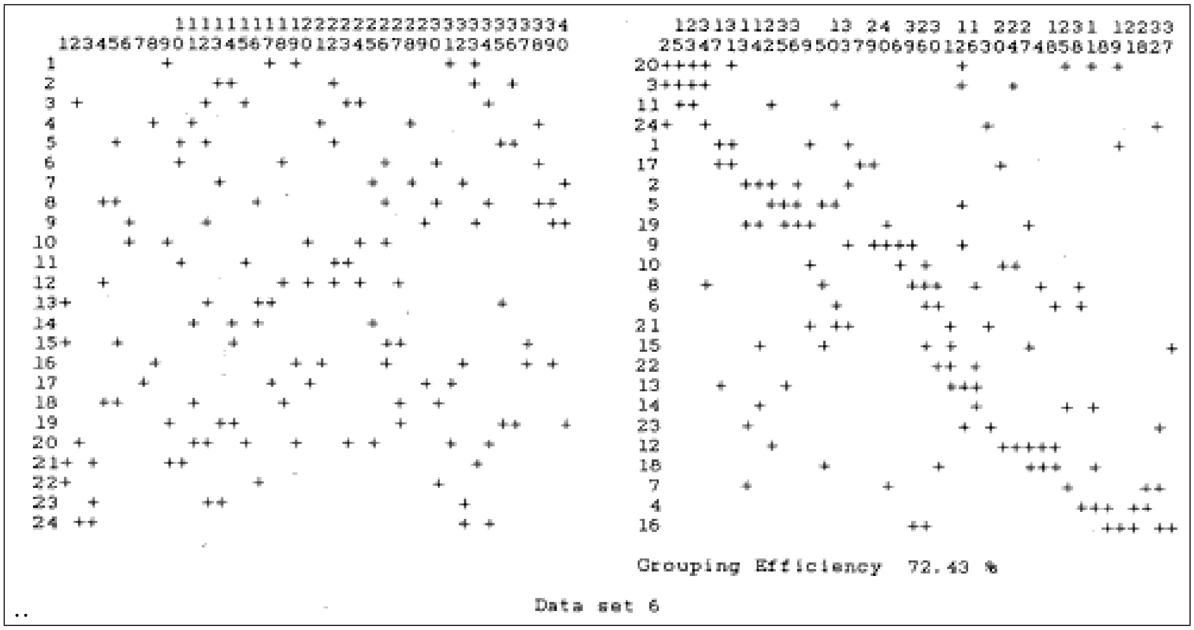

Figure 5.1: Machine-Part Incidence Matrix

Totally independent parts cannot form part families. Thus, in the experimental design, random data should not be used.

We have employed the part-machine incidence matrices of Chandrasekharan and Rajagopalan [16]. Authors have performed a detailed analysis of the machine-part incidence matrices and identified the major factors that defines the groupability of data sets. They have provided many data sets ranging from perfectly groupable ones to very ill structured ones. We have implemented the data set 6 in our algorithm since it is neither perfectly groupable nor badly structured. The data is presented in figure 5.1. The data set has an average grouping efficiency of 72,4% according to the measures presented by Chandrasekharan and Rajagopalan. It is one of the most representative data provided in their study.

The data include 24 machines and 40 parts. Each machine in the matrix is directly referred to as an operation. The machine-part incidence matrix is utilized as the part requirement matrix of our algorithm. Derived from this data, number of parts in the system is 40, number of operations is 24, and number of dedicated machines is also 24. The number of available flexible machines is taken to be 12.

5.1.5.2 Machine Investment Data

Machine investment cost of the machines is another key concept in the proposed algorithm. In order to be as close as possible to the real environment, we have employed the real cost data. Table 5.10 shows the machines and average investment costs of these machines.

Machine costs are directly related with the capabilities of the machines. There are different measures of capability in machine selection literature. Arslan, Catay and Budak [6] provide a decision support system for machine tool selection which guides the decision maker in the machine selection process.

We had very valuable data from Mazak Corporation. Mazak is one of the largest machine tool manufacturers in the world, with manufacturing facilities located in the United States, the United Kingdom, Singapore in addition to those in Japan. Information technology is thoroughly integrated in their flexible machine tools and systems. Thus, Mazak has been an effective source of information for our study. We analyzed the properties of numerous types of flexible machines produced by Mazak, and selected some benchmark machine types that are the most representative ones to be employed in our study. Machining center properties of the selected types are provided in Table 5.11. We have selected the machines with nearly same size specifications while with different operational capabilities, since the investment cost of machines change with sizes of the machines as well as operational capabilities of the machines. Thus, we tried to be as fair as possible in terms of motor speeds, chuck sizes, etc.

Machine	**Capability**	**Average Value**
Universal Turning Machine	1 operation	Euro 15.000
Mazak Super Quick Turn 200 Series	2 operations	Euro 100.000
Mazak Super Quick Turn 200M Series	3 operations	Euro 130.000
Mazak Super Quick Turn 200MY Series	4 operations	Euro 150.000
Mazak Integrex 200Y Series	5 operations	Euro 175.000
Mazak Variaxis 500 Series	$\geq$ 6 operations	Euro 200.000

Table 5.10: Machine Investment Costs

	SQT 200	**SQT 200M**	**SQT 200MY**	**Integrex 200Y**	**Variaxis 500**
Chuck Size	8”	8”	8”	8”	-
Max.Diameter	350 mm	300 mm	300 mm	540 mm	-
Spindle speed	5000 rpm	5000 rpm	5000 rpm	5000 rpm	12000 rpm
RTS speed	-	4500 rpm	4500 rpm	10000 rpm	-
Axes	X,Z	X,Z,C	X,Z,C,Y	X,Z,C,Y,B	X,Z,C,Y,A

Table 5.11: Machining Center Specifications

SQT 200 is one of the least capable flexible machines. It has very similar machining properties to universal dedicated turning machines. However, the computer integrated system of SQT 200 provides much accurate turning of small diameter workpieces up to heavy duty cutting of large diameter workpieces while controlling the process with a 3D simulation of the produced part. However, computer integration results in very high investment difference compared to its dedicated alternative.

SQT 200M and SQT 200MY are multi-tasking CNC turning centers that can turn and mill a workpiece in a single machine setup. The number of axes in a machining center significantly affects the operational capability of these machines. Thus, with more axes, SQT 200MY is a more capable machine than SQT 200M, while SQT 200M is a more flexible machine than SQT 200.

Integrex Series is the most widely used multi-tasking machine tool in the world. Turning, milling, grinding, contouring with the C-axis, off-center machining with Y-axis, milling angled surfaces with B-axis, and heat treatment with laser are some of the operational capabilities of Integrex Series machining centers. Integrex 200Y series is a representative machine type for our 5-operations case with its 5 axes (X,Z,C,Y,B) available for different operations.

Variaxis Series is a multi-face and simultaneously controlled 5-axis double-column machining center. It is one of the most flexible machines in the world. With its tilting rotary table, it is able to finish complex workpieces in just a single machine setup. While completing all machining in a single process, the workpiece accuracy is superior to multiple machine processing. We have selected Variaxis 500 to represent the flexible machines that are very capable in terms of operations.

According to the data, lifetimes of machines are assumed to be the same (15 years) for all types of flexible machines and 10 years for the dedicated machines. Under consideration of lifetimes of the machines, annual machine investment costs are selected randomly in a range of ±5% of costs provided in Table 5.10, based on the capabilities of the machines. Maintenance costs of machines are simply taken to be 10% of investment costs, since it is directly proportional to the investment made.

5.1.5.3 Other Parameters

Number of machines in each cell has an initial upper bound (UBK) of 15 machines. This initial limit tends to be halved at the end of the search procedure. On the other hand, number of machines directly depends on the total annual demand parameter. In the final design, total annual demand ($\sum_i D_i$) is taken to be 50,000 units and number of cells is assumed to be 4. Theoretical capacity of machines is calculated as follows:

$$\begin{aligned} TCap &= 48\, min/hour \;\cdot\; 8\, hours/day \;\cdot\; 6\, days/week \;\cdot\; 52\, weeks/year \\ &= 119{,}808\, min/year \end{aligned} \tag{5.4}$$

Because of setups and maintenance stops, the theoretical capacities can never be achieved. Especially for dedicated machines upper bound of utilization is less than it is for flexible machines. Upper level of utilization (α_m) for dedicated machines is taken to be 80%, and for flexible machines it is 95%. The lower utilization bound (β_m) is determined to be the same for both types of machines as 5%.

The processing time of a part on a machine is selected randomly from an interval. The interval is different for dedicated and flexible machines. Although the processing times of dedicated machines are shorter, when the load-unload times are taken into consideration, the total throughput times might be longer. Processing times of parts on dedicated machines are selected randomly from the uniform interval $U[5, 10]$ whereas that of flexible machines are selected from the uniform interval $U[5, 15]$.

Load-unload times of parts on dedicated machines are selected randomly from the uniform interval $U[1, 2]$ whereas that of flexible machines are selected from the uniform interval $U[1.5, 2.5]$. Although when single load time is taken it seems longer, a part generally receives more than one operation on a flexible machine with only one load-unload.

The last and one of the most important parameters is the variety cost threshold parameter. As shown previously in Figure 3.2, product variety costs of parts take values from 3 to 39. It is observed that there exists a big jump at level 20. However, when the number of occurrences is taken into consideration, little jump on the value 15 is more meaningful to represent a threshold value to chose between dedicated and flexible technology. This value is utilized at two very critical points in the algorithm: at the initial similarity calculation of parts and in the technology choice of cells. Results of the experimental design are discussed in the next section.

5.2 Experimental Results

The experimentation is performed by $(3 \cdot 2^4)$ full factorial designs with the factors detailed in the previous section. Factor A has 3 levels and the others have 2 levels each, resulting in 48 full factorial designs. With 5 different random number seeds, 240 randomly generated problems are solved by the proposed algorithm and a challenger algorithm. The results of the proposed algorithm are presented in Appendices B, C, and D, and the results of the challenger algorithm are presented in Appendices E, F, and G.

It is clearly expressed in the previous sections that the cellular manufacturing and technology selection problems are studied separately in literature. That is, cellular manufacturing system models presume the technology is given and in most cases it is taken to be dedicated. Further, there exists no study incorporating market information to the CMSD problem. Since there does not exist hybrid technology approaches in the literature handling product variety, and most of the existing cellular manufacturing literature assumes dedicated technology, this is the challenging algorithm in our case. We have re-constructed the proposed algorithm such that the available technology is the dedicated technology for the whole system as it is assumed generally by the researchers.

For both of the algorithms, each problem is run with various beam widths. We employed two different beam widths: 3 and 6. With such a construction, each problem has been run 4 times, resulting in 960 runs. Significant improvements are observed after the experimentation.

We have five objective functions to be minimized, namely the dissimilarity, product variety costs, throughput times, monetary costs and intercellular movements (f_1, f_2, f_3, f_4, and f_5). Each objective function and best achieved minimums are recorded for each run. In order to compare these results, we need to normalize them. Normalization is achieved through the results of runs of same factorial combination and seeding. We have six values for each combination: initial solutions of the two challengers and final results of the 4 runs. The normalization equation is exactly same as the Equation 4.2 of alternative evaluation phase of the solution approach, which is given below again:

$$Normf_1 = \frac{f_1 - GMinf_1}{LMaxf_1 - GMinf_1}$$

Normalization of objective 1 is achieved through equation 4.2. The global minimum values achieved during the experimentations for each of the objective function values are provided in the Appendix H. Like the case of alternative evaluation step, the same equation applies for all of the objectives except the fifth objective, the intercellular movement function. At this final point, we changed the local maximum value of f_5 to its theoretical maximum value $(N \cdot (K-1))$, maximum attainable intercellular movements. Because of the power of 0 value, which is the global minimum value in general for f_5, we observed that the function 4.2 results in misleading results. For the simplicity of calculations, the algorithm records the results of objective 5 after normalization, since global minimum is 0 and theoretical maximum is a known value. For normalization other than f_5, we prefer to use the minimum value attained during the runs, and the maximum value observed among 6 results as the local maximum value.

Example *Let six values of the objective* 5 *of a factor combination are:* $0, 0, 0, 0, 0$, *and* 3, *and normalized total value of the other functions* ($\sum_{h=1}^{4} Normf_h$) *is* 2.10 *for the run with* 5^{th} *objective value* 3. *Let's assume that for another factor combination, the six values of the objective* 5 *are:* $0, 0, 0, 0, 0$ *and* 24, *and normalized total value of the other functions is again* 2.10 *for the run with* 5^{th} *objective value* 24. *These two different experimentations will give the same final objective value of* 3.10. *However, the decision maker has to differentiate between* 3 *and* 24 *intercellular movements. Thus, without changing the normalization function, we have replaced the local maximum value with the theoretical maximum for the fifth objective.*

We utilized the summation of normalized objective function values as the performance measures. The relative difference between the two challenger algorithms shows the performance of the algorithms for different factorial combinations. The computation of relative difference is achieved in two ways:

$$\text{Measure 1} = \frac{\text{Dedicated}-\text{Hybrid}}{\text{Dedicated}}$$

$$\text{Measure 2} = \frac{\text{Dedicated}-\text{Hybrid}}{\text{Hybrid}}$$

Both measures compute the difference between results of the proposed hybrid algorithm and challenger dedicated algorithm. In measure 1, we rate this difference compared to the value of the challenger. This measure should be used when the value of the difference is negative, i.e. challenger proves better. In measure 2, we rate the difference according to the value of the proposed algorithm. This

Factor		Measure 1			Measure 2		
Type	Level	Min	Avg	Max	Min	Avg	Max
A	0	-64,8	20,8	62,4	-39,3	45,4	165,9
A	1	-42,4	34,2	59,5	-29,8	68,7	146,7
A	2	-64,5	19,3	61,7	-39,2	46,7	160,9
B	0	-64,7	25,4	60,4	-39,3	54,4	152,8
B	1	-64,5	24,2	62,4	-39,2	52,8	165,9
C	0	-64,7	10,6	59,3	-39,3	27,9	145,6
C	1	-40,9	38,9	62,4	-29,1	79,3	165,9
D	0	-64,5	18,4	62,4	-39,2	45,5	165,9
D	1	-64,7	31,1	60,4	-39,3	61,7	152,8
E	0	-57,1	27,3	62,4	-36,3	60,2	165,9
E	1	-64,7	22,2	54,7	-39,3	47,1	120,6

Table 5.12: Improvements for Each Factor

measure should be used when the value of the difference is positive, i.e. hybrid approach proves better.

The overall improvement that the hybrid algorithm achieves is 24.8% according to the first measure, and 53.6% according to the second measure. Table 5.12 presents the average values for each factor level. At each row, the value shows the average of half of the entire experimentation where the given factor is at the same level.

Factor A determines the number of parts in the system. At level 0, high volume parts, at level 1, low volume parts, and at level 2, medium volume parts are prevailing. As expected, when the general tendency is low volume, hybrid technologies prove much better.

Factor B controls the ratio between average production amount of low and high volume parts. This factor, on its own, does not provide an effect on the system. Improvement values are very close to the overall values.

Factor C is the most effective factor in the entire system. The stability of the environment is a crucial attribute which should never be discarded. It is

	Initial			Beam Width = 3			Beam Width = 6		
Algorithm	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Dedicated	518,3	518,3	518,3	436,4	448,7	478,3	435,0	447,0	518,3
Hybrid	6,8	161,2	491,2	1,5	123,0	443,4	1,5	121,0	444,6

Table 5.13: Dis-Similarity Objective (f_1)

seen from the results that when the market gets more volatile, the need for flexible manufacturing systems significantly rises. The improvement we get by implementing hybrid strategies instead of dedicated technology is as high as 80% on the average.

Factor D controls the flexibility level of computer numerically controlled machines in the system. This factor also provided meaningful comparable results. When the machines get more capable in terms of operations, to justify the high investment in these machines gets easier.

Factor E is another factor that we analyzed during the experimentations. It also has an effect on the system performance. As expected, when the difference between flexible and dedicated labor costs decreases, in other words, to hire a flexible machine operator becomes cheaper, the algorithm tends to choose flexible systems as expected.

In terms of values of objective functions, we provide the data of the total 240 runs. Although, all these 240 runs are of different problems, the average values are comparable with each other. Because, the same 240 problems are solved for dedicated and hybrid algorithms. On the other hand, maximum and minimum values are presented for information.

The results for dissimilarity objective shown in Table 5.13 are significant for two different reasons. It is observed that the dissimilarity of dedicated technology is considerably higher, or in other words, the similarity achieved in hybrid approach is higher. This difference is because of the new similarity measure employed in the study. The study shows the necessity of integration of operational flexibility of machines in the similarity context.

	Initial			Beam Width = 3			Beam Width = 6		
Algorithm	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Dedicated	414,0	707,9	995,0	414,0	707,9	995,0	414,0	707,9	995,0
Hybrid	414,0	600,1	839,0	372,0	587,6	839,0	370,0	588,0	839,0

Table 5.14: Product Variety Cost Objective (f_2)

The other point that should be noted at this point is the improvement we have achieved after the search. Although we designed the search algorithm against the similarity objective, we have found a better solution after the search stage. The algorithm achieves this by means of fuzzy analysis. Not all the time hard clustering works, but it is better to perform search with the help of fuzzy membership coefficients in cellular manufacturing system design problems.

The results for product variety cost objective are shown in Table 5.14. We have the same results in three phases of the dedicated technology, since all the parts are in dedicated cells all the time with same product variety costs. However, since the flexible and dedicated variety costs are different, the results of hybrid application varies.

It is observed that the risks are significantly higher when we use dedicated technology. The study shows the necessity of hybrid technology implementation at the production floor to hedge against the market fluctuations. Another conclusion we draw from the result is that the search has worked well. Although the proposed search algorithm does not work in favor of product variety costs, it aims to preserve the initial results. It is observed that, on the average, the costs do not change, even there is a slight decrease.

The third objective is the throughput time objective. The associated results are provided in Table 5.15. The initial stage of the algorithm considers mainly the similarity and product variety costs. Yet, the intercellular movement objective is indirectly minimized at this stage. The second stage of the proposed algorithm considers mainly the monetary objectives and aims to find a better solution in terms of machine investments. On the other hand, throughput time objective

	Initial			Beam Width = 3			Beam Width = 6		
	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Ded.	160902	219163	266576	399027	470915	528848	190964	466468	528848
Hyb.	225233	500127	751864	345757	545828	663791	471284	543840	663791

Table 5.15: Throughput Time Objective (f_3)

is used as a secondary objective in the search algorithm. It affects the results indirectly at the evaluation step, together with the other objectives.

The other noteworthy objective of this study is the machine investment, maintenance and labor costs objective (f_4). The averages of monetary cost objective results are presented in Table 5.16. Costs of hybrid approach are significantly higher, because of the flexible machines integrated in the system. This is an expected result. Yet, in the study, the important point that should be noted is: when the overall averages are analyzed, we observe that the justification of high investment of flexible systems is achieved easily.

The last objective is the intercellular movement objective (f_5). The average results are provided in Table 5.17. Even at the initial stage, we observe up to 20 intercellular movements out of 120 ($N \cdot K$) theoretical limit. The first reason of this relatively high number of intercellular movements is the groupability efficiency of the part data. Since we have employed a relatively less efficient part matrix, the results are high as expected. The grouping of the data according to Jaccard similarity coefficient is provided in Figure 5.1. The important point in these results is the decrease of intercellular movement of parts in hybrid approach. Since we make use of the operational capabilities of flexible machines, if

	Initial			Beam Width = 3			Beam Width = 6		
	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Ded.	326715	492928	653021	221557	267569	291480	221557	268737	326715
Hyb.	450230	681907	976300	279163	473421	734839	279163	468692	680833

Table 5.16: Monetary Objective (f_4)

	Initial			Beam Width = 3			Beam Width = 6		
Algorithm	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Dedicated	3	10,8	22	30	39,7	47	14	39,7	49
Hybrid	0	2,77	20	0	16,9	53	0	17,2	47

Table 5.17: Intercellular Movement Objective (f_5)

a machine is deleted from the cell, the part can most probably be processed in the same cell by another machine. However, in a dedicated cell, the part should definitely visit another cell to have the same operation. This increases the exceptional parts in the system, decreasing the efficiency of the CMSD. In other words, integration of flexible systems decreases the exceptional parts in the system, increasing the efficiency of the CMSD. On the other hand, another effect that cause high number of intercellular movements might be the tight utilization limits. In order to have a feasible solution initially, and justify the investment in flexible machines at the search stage, we let the parts make intercellular movements. The search algorithm works against the fifth objective. The number of parts making intercellular movements increases at the second stage of the proposed algorithm.

In Table 5.18, improvements of hybrid approach against the dedicated approach for several selected factorial combinations are presented. Setting (101–) constructs a system in an unstable marketing environment which has more number of low volume parts and with high average demand ratio for low volume parts as provided in Table 5.4. Hybrid technology selection for cellular manufacturing

Factors	Measure 1			Measure 2		
(ABCDE)	Min	Avg	Max	Min	Avg	Max
(101- -)	35,5	50,1	59,5	55,1	103,8	146,7
(101-0)	39,1	52,9	59,5	64,1	116,1	146,7
(- -110)	33,2	48,5	60,4	49,7	98,9	152,8
(1-110)	44,8	51,4	57,9	81,3	107,9	137,9

Table 5.18: Improvements for Selected Factor Combinations - I

Factors	Measure 1			Measure 2		
(ABCDE)	Min	Avg	Max	Min	Avg	Max
(210- -)	-64,5	0,66	46,5	-39,2	14,2	87,1
(010- -)	-50,2	10,1	40,5	-33,4	23,8	68,1
(- -001)	-64,5	2,6	52,4	-39,2	18,5	110,3
(-1001)	-64,5	-7,9	38,4	-39,2	4,5	62,4
(21001)	-64,5	-21,9	38,0	-39,2	-7,3	61,3
(11001)	-42,4	-13,6	38,4	-29,8	-3,2	62,4

Table 5.19: Improvements for Selected Factor Combinations - II

systems prove more than 100% better than the fully dedicated cellular manufacturing systems in such an environment. Further, if the flexible labor cost is low, i.e. in the setting (101-0), average improvement reaches up to 116%.

In setting (–110), algorithm constructs a system in an unstable environment which has highly capable machines and low costs of flexible labor. This kind of construction also proves very well. Improvement is more than 95% regardless of the volumes of the parts. Even in setting (01110), where (01—) provides a high volume dominant production environment, the average improvement is still around 95%. Moreover, when the environment is set to favor low volume parts (1-110), average improvement achieved becomes 108%.

Not all the time the proposed hybrid algorithm provides improvement. There are cases where the investment in flexible technology cannot be justified. These cases generally occur in stable environments. Some selected factorial combinations where dedicated technology should be selected are given in Table 5.19.

When the setting is (210–), the system provides one of the highest demand ratio for high volume parts, as presented in Table 5.4, in a stable environment. (010–) setting also favors high volume production in a stable environment. In these settings, the algorithm proves that challenger works better. When further factors are added to the environment, namely the less capability of the flexible machines and higher cost of flexible labor, the averages decrease further to below 20% favoring the dedicated technology.

	Initial			Beam Width = 3			Beam Width = 6		
Algorithm	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Dedicated	5,8	6,3	6,9	45,6	104,1	169,1	72,5	199,2	330,2
Hybrid	5,7	6,3	7,5	9,0	93,6	325,5	14,9	189,0	619,5

Table 5.20: CPU Times (seconds)

In fact, these results are in accordance with the reality. Dedicated systems are still very powerful when the market is stable and production volumes are high. Further, in literature analysts have always assumed such a setting and carried out their calculations. However, when the stability assumption is relaxed, things change in favor of flexible manufacturing systems. Besides the power of stability, we understand from the results that the proposed algorithm has a tendency to implement flexible technologies. Because, if the problem could be solved optimally, the proposed model would also have implemented the dedicated technology for all the cells like in the case of challenger, which is a possible configuration in hybrid applications.

Another significant attribute of the algorithm is the beam width selection. As the beam width gets larger, the solution space that is inspected also enlarges. This brings a higher cost of computation time. Table 5.20 shows the final average CPU times of the algorithms, while the Appendix I provides each individual CPU seconds of the experimentation.

Both algorithms provide 20% improvement from the initial solution at the search phase. This improvement is worth some seconds of computation. However, the quality of the final solution of larger beam width is only in 1% neighborhood of the smaller beam width. We believe this much improvement is not worth twice much computation time.

The reason of not improving further with the increasing beam width should be because of the algorithm converge fast. It provides the best available solution rapidly and more computation just causes movements around the solution.

5.3 Summary

In this chapter, the experimental design of the proposed algorithm is presented. First the factors that affect the system performance and the parameters of the design are given. Then the results attained from the experimentation is discussed. Further, it is compared with the challenger algorithm. Outcomes of the experimentation can be summarized as follows:

- The study shows the significance of hybrid technology implementation at the production floor to hedge against the market fluctuations. As the design stability decreases and demand variation increases, flexible systems become crucial necessity to survive in the market.
- The study shows the necessity of integration of operational capability of machines in the similarity context.
- Integration of flexible systems decreases the exceptional parts in the system, increasing the efficiency of the CMSD.
- As the available technology provides more capable machines, system prefers flexible machines more likely.
- If the difference between dedicated labor costs and flexible labor costs are low, flexible manufacturing systems are more advantageous.
- As the demand for each part decreases, hybrid manufacturing systems becomes favorable.
- It is better to perform search with the help of fuzzy analysis in cellular manufacturing system design problems.
- Finally, on the average, hybrid algorithm proves to work quite well, provides better solutions in less computation times.

Chapter 6

Conclusions and Future Work

In today's world, market is no longer satisfied with uniform products. While the customers are expecting product variety, it makes the manufacturing processes considerably difficult. Known manufacturing systems become inadequate to perform high variety production with low costs. In such an environment, the manufacturer cannot invest in dedicated lines at the whole facility. Because, the product design is likely to change before the dedicated facility has been paid for. This has implications for integration of flexible manufacturing systems. Flexible technology can be used both for existing designs and for future re-designs of the products.

GT has several significant benefits. Material handling time is minimal since the part is completely processed within the cell. Furthermore, since the cell consist of the required machines, parts move from one machine to other completing the production much faster. Setup time is also reduced by the similar part groupings. The development in technology further contributes to the reduction in setup. Moreover, improvement in quality by the immediate feedback and increase in job satisfaction of labor forming teams in the cell are other benefits of the GT.

It is presented that the cellular manufacturing and technology selection problems are studied separately in literature. That is, cellular manufacturing system models assume the available technology is the dedicated technology. Further, there exists no study incorporating market information to the CMSD problem. We proposed a model that makes technology selection and cell formation decisions simultaneously while taking the changing market dynamics into consideration.

6.1 Results

In our multi-objective study, we modified a well known similarity measure in order to handle the operational capability of available technology. This new coefficient is utilized in order to form part families, and the technology selection decision is based on the individual properties of parts, namely the production volume, variability of the demand, and the design stability of the part.

In order to integrate the market characteristics in our model, we proposed a new cost function. This cost function measures the unquantifiable properties of the parts. This is the first study to assign costs for design stabilities and demand variations. We minimize these discarded costs resulting from the offered variety in today's markets.

We proposed a mathematical representation for the problem. In the model, we make technology selection decision, while determining the appropriate machine groups for part families. Further, we identify part families not only according to their operational similarities but also according to their marketing positions. The model is structured to be multi-objective. We minimize dissimilarity between parts, minimize product variety costs, minimize throughput time, minimize machine investment, maintenance and labor costs, and minimize intercellular movements of parts. We handle every aspect of the cellular manufacturing system design problem with new aspects integrated via use of multi objective criteria. Management of all these attributes complicates the problem significantly.

In order to solve this problem in an acceptable computation time, a local search heuristic is proposed. The proposed algorithm has two main stages. In the first stage, an initial feasible solution to the problem is constructed. The major concern of the first stage is the minimization of variety costs and maximization of the similarity between parts. At the second stage initial solution is improved iteratively via a filtered beam based search heuristic.

The study shows the significance of hybrid technology implementation at the production floor to hedge against the market fluctuations. As the design stability decreases and demand variation increases, flexible systems become crucial necessity to survive in the market. We showed the necessity of integration of operational capability of machines in the similarity context. Integration of flexible systems decreases the exceptional parts in the system, increasing the efficiency of the CMSD. As the available technology provides more capable machines, system prefers flexible machines much likely. If the difference between dedicated labor costs and flexible labor costs are low, flexible manufacturing systems are more advantageous. As the demand for each part decreases, hybrid manufacturing systems becomes favorable. It is better to perform search with the help of fuzzy analysis in cellular manufacturing system design problems. Finally, on the average, hybrid algorithm proves to work quite well, provides better solutions in less computation times.

6.2 Future Research Directions

Some future research directions can be summarized as follows:

- The proposed model handles the mean and variation of demand in a deterministic nature. Since one of the most important concerns of the proposed model is the market fluctuation, stochastic models can be considered.
- The model can be enhanced further to design a virtual cellular manufacturing system that changes dynamically with the demand changes.
- We used the traditional life cycle in the model. Other life cycle patterns, such as style, fad and fashion life cycle patterns can be used to test the efficiency of the proposed algorithm in different market settings.

- In this study, we emphasize the design problem, although the impact of operational issues, such as cell loading and scheduling can be analyzed. The proposed model do not take into account neither the operation sequences of parts nor the layout of the facility. Integration of these attributes in the model can be a challenging study.

- Other local search heuristics, such as simulated annealing and genetic algorithms can be used for comparison purposes.

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VITA

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Appendix A

Fuzzy Analysis Algorithm

Fuzzy clustering is a generalization of partitioning. In a partition, each object of the data set is assigned to one and only one cluster. Therefore, partitioning methods are sometimes said to produce a hard clustering, because they make a clear-cut decision for each object. On the other hand, a fuzzy clustering method allows for some ambiguity in the data, which often occurs. Kaufmann and Rousseeuw use the following notation for the fuzzy analysis algorithm they propose in their book [35]:

C	:	minimization objective function
k	:	number of clusters
n	:	number of objects
u_{iv}	:	fuzzy membership coefficient of object i in cluster v
$d_{i,j}$	:	given dissimilarities between objects i and j

$$\min C = \sum_{v=1}^{k} \frac{\sum_{i,j=1}^{n} (u_{iv})^2 \cdot (u_{jv})^2 \cdot d_{i,j}}{2 \cdot \sum_{j=1}^{n} (u_{jv})^2} \tag{A.1}$$

The algorithm that the authors propose attempts to minimize the objective function C in Equation A.1. It contains the dissimilarities ($d_{i,j}$) and the membership coefficients that we are trying to find. The sum in the numerator ranges over all pairs of objects (i, j). Since (j, i) also occurs, the sum is divided by 2. The outer sum is over all clusters, so the objective is comprehensive. The proposed algorithm works iteratively and stops when the objective function converges. The membership functions are subject to the constraints A.2 and A.3 expressing that memberships cannot be negative and that each object has a constant total membership, distributed over the different clusters; by convention this total membership is normalized to 1:

$$u_{iv} \geq 0 \qquad for \;\; i = 1, ..., n; \;\; v = 1, ..., k \tag{A.2}$$

$$\sum_{v} u_{iv} = 1 \qquad for \;\; i = 1, ..., n \tag{A.3}$$

The authors provide a characterization of the local optima of Equation A.1 from the Lagrange equations. Via use of Lagrange multipliers, they write down the corresponding Kuhn and Tucker conditions taking into account the objective function (A.1) and the constraints (A.2, A.3). The interested reader on the mathematical proof of the algorithm is referred to Kaufmann and Rousseeuw [35]. We provide the steps of the iterative algorithm, derived from the proposed model. Having some initial values for all u_{iv}, the algorithm computes new and better membership coefficients at each iteration, until the objective function converges. The proposed iterative algorithm has the following form: (Note that superscripts stand for the number of iteration step.)

1 Initialize the membership functions as u^0_{iv} for all $i = 1, ..., n$ and all $v = 1, ..., k$, taking into account constraints A.2 and A.3.Calculate the objective function C^0 by A.1.

In our algorithm, we have $k = 4$, and the following arbitrary initial membership coefficients are provided as initial feasible values in terms of the constraints:

$u^0_{i1} = 0.10$ $u^0_{i2} = 0.20$

$u^0_{i3} = 0.30$ $u^0_{i4} = 0.40$

2 Compute for each $i = 1, ..., n$ the following quantities:

2.1 Compute for each $v = 1, ..., k$:

$$a_{iv}^{m} = \frac{Num1 - (Num2 + Num3 + Num4 + Num5)}{Den}$$

$$Num1 = 2 \cdot (\sum_{j=1}^{i-1} (u_{jv}^{m+1})^2 \cdot d_{ij} + \sum_{j=i}^{n} (u_{jv}^{m})^2 \cdot d_{ij})$$

$$Num2 = \sum_{j=1}^{i-1} \sum_{h=1}^{i-1} (u_{jv}^{m+1})^2 \cdot (u_{hv}^{m+1})^2 \cdot d_{ij}$$

$$Num3 = \sum_{j=1}^{i-1} \sum_{h=i}^{n} (u_{jv}^{m+1})^2 \cdot (u_{hv}^{m})^2 \cdot d_{ij}$$

$$Num4 = \sum_{j=i}^{n} \sum_{h=1}^{i-1} (u_{jv}^{m})^2 \cdot (u_{hv}^{m+1})^2 \cdot d_{ij}$$

$$Num5 = \sum_{j=i}^{n} \sum_{h=i}^{n} (u_{jv}^{m})^2 \cdot (u_{hv}^{m})^2 \cdot d_{ij}$$

$$Den = \sum_{j=1}^{i-1} (u_{jv}^{m+1})^2 + \sum_{j=i}^{n} (u_{jv}^{m})^2$$

2.2 Compute for each $v = 1, ..., k$:

$$A_v = \frac{1/a_{iv}^{m}}{\sum_w (1/a_{iw}^{m})}$$

2.2.1 if $A_v \leq 0 \Rightarrow V1 = V1 \bigcup \{u\}$

2.2.2 if $A_v > 0 \Rightarrow V2 = V2 \bigcup \{u\}$

2.3 Put for all $v \in V1$

$$u_{iv}^{m+1} = 0$$

2.4 Compute for all $v \in V2$

$$u_{iv}^{m+1} = \frac{1/a_{iv}^{m}}{\sum_{w \in V2} (1/a_{iw}^{m})}$$

2.5 Put $V1 = V2 = \emptyset$ and restart from Step 2.1 with the next i.

3 Calculate the new objective function value C^{m+1} by equation A.1. If $(C^m/C^{m+1} - 1) < \epsilon$, then go to Step 2; otherwise stop.

Appendix B

Initial Stage Results of Hybrid Algorithm

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
1	0 0 0 0 0	370,7	414	282356,4	652427	17
1	0 0 0 0 1	370,7	414	282356,4	652427	17
1	0 0 0 1 0	491,2	488	244560,2	593420	9
1	0 0 0 1 1	491,2	488	244560,2	593420	9
1	0 0 1 0 0	28,5	769	604115,9	554268	0
1	0 0 1 0 1	28,5	769	604115,9	776268	0
1	0 0 1 1 0	175,7	650	460168,7	601135	0
1	0 0 1 1 1	175,7	650	460168,7	766135	0
1	0 1 0 0 0	347,4	499	301098	582531	20
1	0 1 0 0 1	347,4	499	301098	582531	20
1	0 1 0 1 0	491,2	488	242269,2	558425	13
1	0 1 0 1 1	491,2	488	242269,2	558425	13
1	0 1 1 0 0	56,7	722	615538,1	497956	0
1	0 1 1 0 1	56,7	722	615538,1	683956	0
1	0 1 1 1 0	175,7	650	457382,1	577820	0
1	0 1 1 1 1	175,7	650	457382,1	742820	0

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
1	1 0 0 0 0	482,2	602	252454,4	511920	7
1	1 0 0 0 1	482,2	602	252454,4	562920	7
1	1 0 0 1 0	238,4	505	371781,5	677457	5
1	1 0 0 1 1	238,4	505	371781,5	731457	5
1	1 0 1 0 0	38,4	626	586298	476012	0
1	1 0 1 0 1	38,4	626	586298	656012	0
1	1 0 1 1 0	18,6	644	643700,3	621702	0
1	1 0 1 1 1	18,6	644	643700,3	855702	0
1	1 1 0 0 0	191,8	546	482091,3	708125	2
1	1 1 0 0 1	191,8	546	482091,3	825125	2
1	1 1 0 1 0	238,4	505	375214,3	660206	5
1	1 1 0 1 1	238,4	505	375214,3	711206	5
1	1 1 1 0 0	38	689	678085,1	487881	0
1	1 1 1 0 1	38	689	678085,1	670881	0
1	1 1 1 1 0	18,6	644	643422,8	636983	1
1	1 1 1 1 1	18,6	644	643422,8	873983	1
1	2 0 0 0 0	307,7	434	347133,1	727078	4
1	2 0 0 0 1	307,7	434	347133,1	784078	4
1	2 0 0 1 0	441,7	452	326944,7	803618	4
1	2 0 0 1 1	441,7	452	326944,7	866618	4
1	2 0 1 0 0	41,4	707	583007,4	475769	0
1	2 0 1 0 1	41,4	707	583007,4	655769	0
1	2 0 1 1 0	17,2	671	603519,3	608304	0
1	2 0 1 1 1	17,2	671	603519,3	836304	0
1	2 1 0 0 0	234,1	593	358943,9	654430	5
1	2 1 0 0 1	234,1	593	358943,9	714430	5
1	2 1 0 1 0	441,7	452	329212,1	803618	4
1	2 1 0 1 1	441,7	452	329212,1	866618	4
1	2 1 1 0 0	59,9	655	655310	488508	0
1	2 1 1 0 1	59,9	655	655310	671508	0
1	2 1 1 1 0	17,2	671	608024,3	625587	0
1	2 1 1 1 1	17,2	671	608024,3	856587	0

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
3	0 0 0 0 0	327,8	561	355831,6	507320	7
3	0 0 0 0 1	327,8	561	355831,6	561320	7
3	0 0 0 1 0	238,8	471	370400,7	692148	9
3	0 0 0 1 1	238,8	471	370400,7	752148	9
3	0 0 1 0 0	16,7	684	637846,1	561947	0
3	0 0 1 0 1	16,7	684	637846,1	783947	0
3	0 0 1 1 0	57,5	577	517557,8	647015	0
3	0 0 1 1 1	57,5	577	517557,8	824015	0
3	0 1 0 0 0	327,8	561	359250,5	507320	7
3	0 1 0 0 1	327,8	561	359250,5	561320	7
3	0 1 0 1 0	448	485	275920,9	533930	14
3	0 1 0 1 1	448	485	275920,9	578930	14
3	0 1 1 0 0	16,7	684	643404	574602	0
3	0 1 1 0 1	16,7	684	643404	799602	0
3	0 1 1 1 0	254,8	535	446318,9	584839	0
3	0 1 1 1 1	254,8	535	446318,9	734839	0
3	1 0 0 0 0	19,6	839	615353,4	521787	0
3	1 0 0 0 1	19,6	839	615353,4	734787	0
3	1 0 0 1 0	378,3	508	253967,9	468346	13
3	1 0 0 1 1	378,3	508	253967,9	507346	13
3	1 0 1 0 0	16,6	619	617426,2	556191	0
3	1 0 1 0 1	16,6	619	617426,2	778191	0
3	1 0 1 1 0	7,8	608	629204,4	683194	0
3	1 0 1 1 1	7,8	608	629204,4	923194	0
3	1 1 0 0 0	19,6	839	610361,4	521787	0
3	1 1 0 0 1	19,6	839	610361,4	734787	0
3	1 1 0 1 0	378,3	508	251817,5	450230	19
3	1 1 0 1 1	378,3	508	251817,5	492230	19
3	1 1 1 0 0	16,6	619	612119,4	553977	0
3	1 1 1 0 1	16,6	619	612119,4	775977	0
3	1 1 1 1 0	7,8	608	616276,7	632901	1
3	1 1 1 1 1	7,8	608	616276,7	863901	1

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
3	2 0 0 0 0	208	709	457521,9	482295	0
3	2 0 0 0 1	208	709	457521,9	590295	0
3	2 0 0 1 0	207,5	517	429637,7	726467	2
3	2 0 0 1 1	207,5	517	429637,7	837467	2
3	2 0 1 0 0	16,6	672	634275,4	549502	0
3	2 0 1 0 1	16,6	672	634275,4	768502	0
3	2 0 1 1 0	71,3	589	538820,7	685530	0
3	2 0 1 1 1	71,3	589	538820,7	868530	0
3	2 1 0 0 0	208	709	462904,1	465067	0
3	2 1 0 0 1	208	709	462904,1	570067	0
3	2 1 0 1 0	461	506	265603,4	615556	10
3	2 1 0 1 1	461	506	265603,4	660556	10
3	2 1 1 0 0	16,6	672	634794,1	575607	0
3	2 1 1 0 1	16,6	672	634794,1	800607	0
3	2 1 1 1 0	340,8	489	345280	741928	1
3	2 1 1 1 1	340,8	489	345280	852928	1
4	0 0 0 0 0	205,7	673	497895,9	554833	0
4	0 0 0 0 1	205,7	673	497895,9	680833	0
4	0 0 0 1 0	411,4	613	310441,8	540244	5
4	0 0 0 1 1	411,4	613	310441,8	600244	5
4	0 0 1 0 0	11,7	704	709814	596493	0
4	0 0 1 0 1	11,7	704	709814	830493	0
4	0 0 1 1 0	123,9	592	495913,7	649446	0
4	0 0 1 1 1	123,9	592	495913,7	823446	0
4	0 1 0 0 0	205,7	673	486013,2	485034	0
4	0 1 0 0 1	205,7	673	486013,2	611034	0
4	0 1 0 1 0	420,4	469	291095	604900	13
4	0 1 0 1 1	420,4	469	291095	664900	13
4	0 1 1 0 0	11,7	704	720621,1	596493	0
4	0 1 1 0 1	11,7	704	720621,1	830493	0
4	0 1 1 1 0	15,2	656	597206,4	581659	0
4	0 1 1 1 1	15,2	656	597206,4	806659	0

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
4	1 0 0 0 0	18	754	673241,4	632126	0
4	1 0 0 0 1	18	754	673241,4	872126	0
4	1 0 0 1 0	77,6	660	507287	602174	2
4	1 0 0 1 1	77,6	660	507287	722174	2
4	1 0 1 0 0	9,8	565	719700,8	637029	0
4	1 0 1 0 1	9,8	565	719700,8	877029	0
4	1 0 1 1 0	72,1	459	501884,5	592952	0
4	1 0 1 1 1	72,1	459	501884,5	763952	0
4	1 1 0 0 0	18	754	689770,1	656592	0
4	1 1 0 0 1	18	754	689770,1	902592	0
4	1 1 0 1 0	77,6	660	490095,9	602174	2
4	1 1 0 1 1	77,6	660	490095,9	722174	2
4	1 1 1 0 0	9,8	565	751863,6	656741	0
4	1 1 1 0 1	9,8	565	751863,6	902741	0
4	1 1 1 1 0	72,1	459	474607,6	592952	0
4	1 1 1 1 1	72,1	459	474607,6	763952	0
4	2 0 0 0 0	207,6	505	500732,8	770571	2
4	2 0 0 0 1	207,6	505	500732,8	893571	2
4	2 0 0 1 0	175,1	705	412682,4	664207	1
4	2 0 0 1 1	175,1	705	412682,4	781207	1
4	2 0 1 0 0	21,7	606	651547,3	606737	0
4	2 0 1 0 1	21,7	606	651547,3	786737	0
4	2 0 1 1 0	72,4	547	530653,8	616218	0
4	2 0 1 1 1	72,4	547	530653,8	787218	0
4	2 1 0 0 0	205,7	667	543366,1	543171	0
4	2 1 0 0 1	205,7	667	543366,1	666171	0
4	2 1 0 1 0	175,1	705	391013,7	640854	2
4	2 1 0 1 1	175,1	705	391013,7	757854	2
4	2 1 1 0 0	31,1	619	700066,2	631485	0
4	2 1 1 0 1	31,1	619	700066,2	823485	0
4	2 1 1 1 0	72,4	547	511856,8	599571	0
4	2 1 1 1 1	72,4	547	511856,8	767571	0
4	2 1 1 1 1	72,4	547	511856,8	767571	0

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
5	0 0 0 0 0	368,4	438	309385,3	624473	10
5	0 0 0 0 1	368,4	438	309385,3	678473	10
5	0 0 0 1 0	255,5	665	423310,7	514986	0
5	0 0 0 1 1	255,5	665	423310,7	631986	0
5	0 0 1 0 0	164	667	504816,4	607235	0
5	0 0 1 0 1	164	667	504816,4	772235	0
5	0 0 1 1 0	16,7	688	669531,6	584883	0
5	0 0 1 1 1	16,7	688	669531,6	809883	0
5	0 1 0 0 0	368,4	438	307290,4	531419	17
5	0 1 0 0 1	368,4	438	307290,4	585419	17
5	0 1 0 1 0	270,6	698	435622,7	602455	2
5	0 1 0 1 1	270,6	698	435622,7	716455	2
5	0 1 1 0 0	164	667	485979,9	572344	0
5	0 1 1 0 1	164	667	485979,9	737344	0
5	0 1 1 1 0	22,8	616	714622,9	689802	0
5	0 1 1 1 1	22,8	616	714622,9	932802	0
5	1 0 0 0 0	347,4	480	345756,7	566783	13
5	1 0 0 0 1	347,4	480	345756,7	623783	13
5	1 0 0 1 0	55,3	662	605920,1	649359	0
5	1 0 0 1 1	55,3	662	605920,1	823359	0
5	1 0 1 0 0	32,8	751	581192,8	609813	0
5	1 0 1 0 1	32,8	751	581192,8	843813	0
5	1 0 1 1 0	6,8	568	706261,8	682947	0
5	1 0 1 1 1	6,8	568	706261,8	925947	0
5	1 1 0 0 0	174,8	761	449672,1	485488	0
5	1 1 0 0 1	174,8	761	449672,1	641488	0
5	1 1 0 1 0	55,3	662	616597,3	649359	0
5	1 1 0 1 1	55,3	662	616597,3	823359	0
5	1 1 1 0 0	36,1	658	558115,8	564029	0
5	1 1 1 0 1	36,1	658	558115,8	789029	0
5	1 1 1 1 0	6,8	568	724066	693528	0
5	1 1 1 1 1	6,8	568	724066	939528	0
5	1 1 1 1 1	6,8	568	724066,0	939528	0

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
5	2 0 0 0 0	343	548	385354,8	607117	6
5	2 0 0 0 1	343	548	385354,8	718117	6
5	2 0 0 1 0	66,7	739	605655,5	661003	0
5	2 0 0 1 1	66,7	739	605655,5	835003	0
5	2 0 1 0 0	32,8	772	597804,9	612988	0
5	2 0 1 0 1	32,8	772	597804,9	846988	0
5	2 0 1 1 0	14,7	595	693000,3	724300	0
5	2 0 1 1 1	14,7	595	693000,3	976300	0
5	2 1 0 0 0	343	548	414952,2	619101	7
5	2 1 0 0 1	343	548	414952,2	733101	7
5	2 1 0 1 0	179,6	607	506980,6	756480	7
5	2 1 0 1 1	179,6	607	506980,6	825480	7
5	2 1 1 0 0	32,8	772	583888	605924	1
5	2 1 1 0 1	32,8	772	583888	839924	1
5	2 1 1 1 0	13,3	635	652521,6	628049	0
5	2 1 1 1 1	13,3	635	652521,6	859049	0
6	0 0 0 0 0	294,3	511	347694,5	675267	8
6	0 0 0 0 1	294,3	511	347694,5	735267	8
6	0 0 0 1 0	486,3	503	237696,1	654283	5
6	0 0 0 1 1	486,3	503	237696,1	702283	5
6	0 0 1 0 0	76,1	634	550870,3	597619	0
6	0 0 1 0 1	76,1	634	550870,3	771619	0
6	0 0 1 1 0	127,7	480	513311	692779	0
6	0 0 1 1 1	127,7	480	513311	857779	0
6	0 1 0 0 0	294,3	511	338363,9	558681	15
6	0 1 0 0 1	294,3	511	338363,9	618681	15
6	0 1 0 1 0	486,3	503	225233	572805	11
6	0 1 0 1 1	486,3	503	225233	620805	11
6	0 1 1 0 0	76,1	634	548536,7	540618	0
6	0 1 1 0 1	76,1	634	548536,7	717618	0
6	0 1 1 1 0	127,7	480	481481,3	665138	0
6	0 1 1 1 1	127,7	480	481481,3	827138	0
6	1 0 0 0 0	112,1	687	533056,4	615485	0
6	1 0 0 0 0	112,1	687	533056,4	615485	0,00

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
6	1 0 0 0 1	112,1	687	533056,4	795485	0
6	1 0 0 1 0	218,9	421	379724,5	633197	2
6	1 0 0 1 1	218,9	421	379724,5	732197	2
6	1 0 1 0 0	76,1	621	573688,6	576111	0
6	1 0 1 0 1	76,1	621	573688,6	750111	0
6	1 0 1 1 0	30,7	495	612555,9	607867	0
6	1 0 1 1 1	30,7	495	612555,9	781867	0
6	1 1 0 0 0	112,1	687	558094,9	588591	0
6	1 1 0 0 1	112,1	687	558094,9	762591	0
6	1 1 0 1 0	218,9	421	358055,1	604525	2
6	1 1 0 1 1	218,9	421	358055,1	697525	2
6	1 1 1 0 0	76,1	621	610821,9	603429	0
6	1 1 1 0 1	76,1	621	610821,9	783429	0
6	1 1 1 1 0	30,7	495	631886,8	621099	0
6	1 1 1 1 1	30,7	495	631886,8	798099	0
6	2 0 0 0 0	294,3	532	375784	714827	6
6	2 0 0 0 1	294,3	532	375784	777827	6
6	2 0 0 1 0	182,3	702	466682,6	688145	3
6	2 0 0 1 1	182,3	702	466682,6	808145	3
6	2 0 1 0 0	76,1	636	572139,5	609246	0
6	2 0 1 0 1	76,1	636	572139,5	783246	0
6	2 0 1 1 0	138,2	429	487073,9	644836	1
6	2 0 1 1 1	138,2	429	487073,9	755836	1
6	2 1 0 0 0	294,3	532	382996,1	703149	7
6	2 1 0 0 1	294,3	532	382996,1	766149	7
6	2 1 0 1 0	182,3	702	468290,7	688145	3
6	2 1 0 1 1	182,3	702	468290,7	808145	3
6	2 1 1 0 0	76,1	636	601557,8	625501	0
6	2 1 1 0 1	76,1	636	601557,8	802501	0
6	2 1 1 1 0	138,2	429	487441,5	644836	1
6	2 1 1 1 1	138,2	429	487441,5	755836	1

Appendix C

Final Results of Hybrid Alg. with b=3

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	0 0 0 0 0	319,8	414	494807,9	279637	53
1	0 0 0 0 1	319,8	414	494807,9	279637	53
1	0 0 0 1 0	443,4	488	484505,2	279163	34
1	0 0 0 1 1	443,4	488	484505,2	279163	34
1	0 0 1 0 0	17,2	769	533387,9	337232	21
1	0 0 1 0 1	17,2	769	533387,9	514232	21
1	0 0 1 1 0	149,2	648	511189,1	472199	8
1	0 0 1 1 1	149,2	648	511189,1	610199	8
1	0 1 0 0 0	323,4	499	479802,1	279632	49
1	0 1 0 0 1	323,4	499	479802,1	279632	49
1	0 1 0 1 0	442,7	488	479516,3	279163	37
1	0 1 0 1 1	442,7	488	479516,3	279163	37
1	0 1 1 0 0	35,1	722	550309,1	334378	11
1	0 1 1 0 1	35,1	722	550309,1	484378	11
1	0 1 1 1 0	199,4	578	493883,7	432528	14
1	0 1 1 1 1	199,4	578	493883,7	567528	14
1	1 0 0 0 0	351,1	546	527141,8	372296	26

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	1 0 0 0 1	351,1	546	527141,8	423296	26
1	1 0 0 1 0	134	479	511448,3	375285	21
1	1 0 0 1 1	134	479	511448,3	429285	21
1	1 0 1 0 0	28,6	626	520351,1	333442	7
1	1 0 1 0 1	28,6	626	520351,1	483442	7
1	1 0 1 1 0	9,3	644	535977,3	388842	8
1	1 0 1 1 1	8,9	644	525858,3	577904	5
1	1 1 0 0 0	139,4	520	561537,1	454802	13
1	1 1 0 0 1	122,1	578	572190,5	545056	10
1	1 1 0 1 0	142,7	479	497978,4	346345	28
1	1 1 0 1 1	142,7	479	497978,4	397345	28
1	1 1 1 0 0	30,8	689	597242,9	346701	6
1	1 1 1 0 1	30,8	689	597242,9	499701	6
1	1 1 1 1 0	6,8	644	551759,5	365963	16
1	1 1 1 1 1	6,8	644	551759,5	545963	16
1	2 0 0 0 0	223,7	406	566695,1	377759	18
1	2 0 0 0 1	223,7	406	566695,1	434759	18
1	2 0 0 1 0	193,3	380	548276,1	454552	23
1	2 0 0 1 1	193,3	380	548276,1	517552	23
1	2 0 1 0 0	33,9	707	538456	337485	11
1	2 0 1 0 1	33,9	707	538456	487485	11
1	2 0 1 1 0	7,8	671	534573,1	386780	13
1	2 0 1 1 1	8,5	671	532538,1	569780	12
1	2 1 0 0 0	183,7	557	515119,5	339940	29
1	2 1 0 0 1	183,7	557	515119,5	399940	29
1	2 1 0 1 0	176,4	372	554238,3	431245	26
1	2 1 0 1 1	176,4	372	554238,3	494245	26
1	2 1 1 0 0	43,1	655	539488,3	339421	7
1	2 1 1 0 1	43,1	655	539488,3	489421	7
1	2 1 1 1 0	7,8	671	534190,1	396649	8
1	2 1 1 1 1	7,8	671	534190,1	582649	8
3	0 0 0 0 0	231,1	509	523090,7	379227	20
3	0 0 0 0 1	231,1	509	523090,7	433227	20

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
3	0 0 0 1 0	140,2	477	528689,3	423284	20
3	0 0 0 1 1	140,2	477	528689,3	483284	20
3	0 0 1 0 0	8,8	684	567823,5	373060	12
3	0 0 1 0 1	9	684	570714,4	541348	19
3	0 0 1 1 0	44,9	567	492950,1	455371	9
3	0 0 1 1 1	44,9	567	492950,1	593371	9
3	0 1 0 0 0	245,8	543	513773	379227	18
3	0 1 0 0 1	245,8	543	513773	433227	18
3	0 1 0 1 0	377,8	485	492511,8	347028	37
3	0 1 0 1 1	377,8	485	492511,8	392028	37
3	0 1 1 0 0	8,5	684	559626,3	358949	12
3	0 1 1 0 1	10,2	684	556312,7	540166	10
3	0 1 1 1 0	254,8	535	446318,9	584839	0
3	0 1 1 1 1	254,8	535	446318,9	734839	0
3	1 0 0 0 0	10,7	839	558891,5	376472	9
3	1 0 0 0 1	10,7	839	558919,3	559472	9
3	1 0 0 1 0	337,9	508	496596,3	339855	40
3	1 0 0 1 1	337,9	508	496596,3	378855	40
3	1 0 1 0 0	12,1	619	551060,1	360406	13
3	1 0 1 0 1	12,1	619	551060,1	540406	13
3	1 0 1 1 0	4,7	608	561629,1	427439	4
3	1 0 1 1 1	4,5	608	545427,7	584458	13
3	1 1 0 0 0	7,7	839	548396,5	360406	12
3	1 1 0 0 1	7,7	839	548396,5	540406	12
3	1 1 0 1 0	339,4	508	503433,3	356903	45
3	1 1 0 1 1	339,4	508	503433,3	398903	45
3	1 1 1 0 0	9,8	619	545843,6	348969	22
3	1 1 1 0 1	9,8	619	545843,6	525969	22
3	1 1 1 1 0	3	608	558467,3	398352	15
3	1 1 1 1 1	3	608	558467,3	581352	15
3	2 0 0 0 0	158,5	751	543764,4	449000	10
3	2 0 0 0 1	158,5	751	543764,4	551000	10
3	2 0 0 1 0	77	563	538382,2	472364	8
3	2 0 0 1 1	78,5	563	539264,5	562702	9

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
3	2 0 1 0 0	10,6	672	561189,8	366936	11
3	2 0 1 0 1	10,6	672	561189,8	546936	11
3	2 0 1 1 0	102,8	475	524661,1	476959	14
3	2 0 1 1 1	102,8	475	524661,1	617959	14
3	2 1 0 0 0	159	751	561238,8	449001	13
3	2 1 0 0 1	159	751	561238,8	551001	13
3	2 1 0 1 0	376,8	506	534277,8	370372	40
3	2 1 0 1 1	376,8	506	534277,8	415372	40
3	2 1 1 0 0	10,2	672	561367,9	379590	6
3	2 1 1 0 1	10,2	672	561367,9	562590	6
3	2 1 1 1 0	172	375	516158,3	444408	18
3	2 1 1 1 1	172	375	516158,3	537408	18
4	0 0 0 0 0	205,7	673	497895,9	554833	0
4	0 0 0 0 1	205,7	673	497895,9	680833	0
4	0 0 0 1 0	306,5	613	520357,6	435260	19
4	0 0 0 1 1	306,5	613	520357,6	495260	19
4	0 0 1 0 0	7,2	704	620065,9	365482	13
4	0 0 1 0 1	7,2	704	621044,3	533653	15
4	0 0 1 1 0	104,2	616	507963,2	472369	10
4	0 0 1 1 1	104,2	616	507963,2	610369	10
4	0 1 0 0 0	181,9	699	623005,6	420747	20
4	0 1 0 0 1	181,9	699	623005,6	531747	20
4	0 1 0 1 0	251,2	433	493925,3	441719	29
4	0 1 0 1 1	251,2	433	493925,3	501719	29
4	0 1 1 0 0	7,1	704	581592,9	344511	17
4	0 1 1 0 1	7,1	704	584501,8	536340	15
4	0 1 1 1 0	7	656	509828,6	368935	15
4	0 1 1 1 1	7	656	509828,6	548935	15
4	1 0 0 0 0	9,7	754	604738,8	356340	12
4	1 0 0 0 1	9,7	754	604738,8	536340	12
4	1 0 0 1 0	66,5	622	490780,4	447796	8
4	1 0 0 1 1	66,5	622	490780,4	549796	8

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
4	1 0 1 0 0	6,9	565	619627,7	371247	10
4	1 0 1 0 1	5,4	565	624983,1	535622	13
4	1 0 1 1 0	70,8	459	489687,2	437503	9
4	1 0 1 1 1	70,8	459	489687,2	575503	9
4	1 1 0 0 0	9,4	754	619803,8	345078	15
4	1 1 0 0 1	9,4	754	619803,8	522078	15
4	1 1 0 1 0	73,8	564	491715,3	420848	12
4	1 1 0 1 1	73,8	564	491715,3	522848	12
4	1 1 1 0 0	6,5	565	663791,2	374462	15
4	1 1 1 0 1	6,5	565	663791,2	557462	15
4	1 1 1 1 0	71,3	459	474103,8	424468	11
4	1 1 1 1 1	71,3	459	474103,8	559468	11
4	2 0 0 0 0	135,7	443	605292,4	482719	16
4	2 0 0 0 1	135,7	443	606223,8	576129	20
4	2 0 0 1 0	134,6	647	517465,6	500822	13
4	2 0 0 1 1	134,6	647	517465,6	605822	13
4	2 0 1 0 0	21,7	606	611455,3	461716	8
4	2 0 1 0 1	21,7	606	611455,3	611716	8
4	2 0 1 1 0	72,6	547	523025,9	486685	2
4	2 0 1 1 1	72,6	547	523025,9	630685	2
4	2 1 0 0 0	205,4	667	639307,2	499827	11
4	2 1 0 0 1	205,4	667	639307,2	613827	11
4	2 1 0 1 0	140,6	601	507678,1	484076	17
4	2 1 0 1 1	140,6	601	502604,9	586076	15
4	2 1 1 0 0	29,3	619	635126,3	410460	16
4	2 1 1 0 1	29,3	619	635126,3	554460	16
4	2 1 1 1 0	70	547	512938,1	473650	7
4	2 1 1 1 1	70	547	512938,1	614650	7
5	0 0 0 0 0	253,5	402	522080,9	356746	31
5	0 0 0 0 1	253,5	402	522080,9	410746	31
5	0 0 0 1 0	229	701	610492,2	456768	12
5	0 0 0 1 1	229	701	610492,2	564768	12

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
5	0 0 1 0 0	141,1	691	522394,8	475971	18
5	0 0 1 0 1	141,1	691	522394,8	613971	18
5	0 0 1 1 0	8,6	688	617617,6	384001	10
5	0 0 1 1 1	8,6	688	617617,6	567001	10
5	0 1 0 0 0	268	482	535325,8	368327	38
5	0 1 0 0 1	268	482	535325,8	422327	38
5	0 1 0 1 0	145,9	652	621628,4	495008	18
5	0 1 0 1 1	145,9	652	621628,4	606008	18
5	0 1 1 0 0	140,1	691	500689,4	428136	21
5	0 1 1 0 1	140,1	691	500689,4	563136	21
5	0 1 1 1 0	7,1	616	581708,9	402308	11
5	0 1 1 1 1	7,1	616	581708,9	588308	11
5	1 0 0 0 0	347,4	480	345756,7	566783	13
5	1 0 0 0 1	347,4	480	345756,7	623783	13
5	1 0 0 1 0	81,5	628	576243,5	481198	11
5	1 0 0 1 1	81,5	628	576243,5	622198	11
5	1 0 1 0 0	20,7	751	500040,8	359399	14
5	1 0 1 0 1	22,5	751	476909,5	525216	21
5	1 0 1 1 0	1,5	568	602981,4	429124	12
5	1 0 1 1 1	1,5	568	602981,4	621124	12
5	1 1 0 0 0	130	667	503903,5	373548	16
5	1 1 0 0 1	130	667	503903,5	505548	16
5	1 1 0 1 0	53,5	662	589047,1	496659	7
5	1 1 0 1 1	53,5	662	589047,1	640659	7
5	1 1 1 0 0	27,5	658	489162,6	346091	16
5	1 1 1 0 1	27,5	658	489162,6	523091	16
5	1 1 1 1 0	2,2	568	620377,2	394544	15
5	1 1 1 1 1	2,2	568	620377,2	580544	15
5	2 0 0 0 0	216,9	578	550932,3	445368	25
5	2 0 0 0 1	216,9	578	550932,3	550368	25
5	2 0 0 1 0	67	739	558717,5	491357	12
5	2 0 0 1 1	67	739	558717,5	632357	12
5	2 0 1 0 0	20,1	772	510632,2	363477	12

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
5	2 0 1 0 1	20,1	772	510632,2	543477	12
5	2 0 1 1 0	5,5	595	569521,1	411905	5
5	2 0 1 1 1	5,5	595	569521,1	600905	5
5	2 1 0 0 0	181,1	610	551690,3	448493	28
5	2 1 0 0 1	181,1	610	551690,3	550493	28
5	2 1 0 1 0	119	561	621539,8	431137	20
5	2 1 0 1 1	119	561	621539,8	500137	20
5	2 1 1 0 0	20,9	772	475354,7	347416	17
5	2 1 1 0 1	19,4	772	513619,2	533372	23
5	2 1 1 1 0	8,4	635	571870,8	389241	13
5	2 1 1 1 1	8,4	635	571870,8	572241	13
6	0 0 0 0 0	187,3	473	558180,8	407582	28
6	0 0 0 0 1	187,3	473	558180,8	467582	28
6	0 0 0 1 0	414,6	503	529850,9	386327	41
6	0 0 0 1 1	414,6	503	529850,9	434327	41
6	0 0 1 0 0	62,4	606	546528,2	461268	12
6	0 0 1 0 1	62,4	606	546528,2	605268	12
6	0 0 1 1 0	128,3	480	559945,6	532671	7
6	0 0 1 1 1	128,3	480	559945,6	664671	7
6	0 1 0 0 0	244,3	487	570893,2	395996	32
6	0 1 0 0 1	244,3	487	570893,2	455996	32
6	0 1 0 1 0	420,2	503	528154,1	386387	40
6	0 1 0 1 1	420,2	503	528154,1	434387	40
6	0 1 1 0 0	46,5	640	551244,6	404804	15
6	0 1 1 0 1	46,5	640	551244,6	551804	15
6	0 1 1 1 0	127,6	480	566010,5	507765	8
6	0 1 1 1 1	127,6	480	566010,5	636765	8
6	1 0 0 0 0	112,8	639	556993,6	424303	21
6	1 0 0 0 1	112,8	639	556993,6	562303	21
6	1 0 0 1 0	151,9	481	562567,5	441393	14
6	1 0 0 1 1	163,7	455	575379,6	537393	18

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
6	1 0 1 0 0	63,4	555	561183,6	454006	12
6	1 0 1 0 1	63,4	555	561183,6	601006	12
6	1 0 1 1 0	31,6	457	583042,3	449632	6
6	1 0 1 1 1	31,6	457	583042,3	590632	6
6	1 1 0 0 0	95,2	673	537942,6	443617	24
6	1 1 0 0 1	95,2	673	535950,9	584617	22
6	1 1 0 1 0	166,2	455	566726,2	429815	18
6	1 1 0 1 1	166,2	445	562678,1	503787	20
6	1 1 1 0 0	60,6	565	597613	465990	16
6	1 1 1 0 1	60,6	565	597613	615990	16
6	1 1 1 1 0	22,8	493	565332,4	462335	8
6	1 1 1 1 1	22,8	493	565332,4	606335	8
6	2 0 0 0 0	204,1	514	587604,9	412091	22
6	2 0 0 0 1	204,1	514	587604,9	475091	22
6	2 0 0 1 0	125,9	640	554988,6	431128	12
6	2 0 0 1 1	124	638	548767,2	536175	17
6	2 0 1 0 0	65	608	542303	472895	10
6	2 0 1 0 1	65	608	542303	616895	10
6	2 0 1 1 0	97,7	383	608413,3	462892	11
6	2 0 1 1 1	97,7	383	608413,3	564892	11
6	2 1 0 0 0	203,7	506	598913,9	423687	26
6	2 1 0 0 1	203,7	506	598913,9	486687	26
6	2 1 0 1 0	104,4	692	558574,2	424701	13
6	2 1 0 1 1	104,4	692	558574,2	535701	13
6	2 1 1 0 0	91,7	546	529308,1	463566	13
6	2 1 1 0 1	91,7	546	529308,1	604566	13
6	2 1 1 1 0	98,1	385	612394,6	462821	11
6	2 1 1 1 1	98,1	385	612394,6	564821	11

Appendix D

Final Results of Hybrid Alg. with b=6

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	0 0 0 0 0	312,52	414	494807,9	279637	45
1	0 0 0 0 1	312,52	414	494807,9	279637	45
1	0 0 0 1 0	439,67	488	484505,2	279163	40
1	0 0 0 1 1	439,67	488	484505,2	279163	40
1	0 0 1 0 0	15,67	769	527814,8	365322	9
1	0 0 1 0 1	15,73	769	534776	514232	17
1	0 0 1 1 0	199,39	578	505245,9	472199	9
1	0 0 1 1 1	199,39	578	505245,9	610199	9
1	0 1 0 0 0	321,27	499	479802,1	279632	47
1	0 1 0 0 1	321,27	499	479802,1	279632	47
1	0 1 0 1 0	444,65	488	479516,3	279163	36
1	0 1 0 1 1	444,65	488	479516,3	279163	36
1	0 1 1 0 0	35,07	722	550309,1	334378	11
1	0 1 1 0 1	35,07	722	550309,1	484378	11
1	0 1 1 1 0	199,39	578	493883,7	432528	14
1	0 1 1 1 1	199,39	578	493883,7	567528	14

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	1 0 0 0 0	326,55	582	532512,4	383981	26
1	1 0 0 0 1	326,55	582	532512,4	434981	26
1	1 0 0 1 0	136,68	479	502311,6	375271	22
1	1 0 0 1 1	136,68	479	502311,6	429271	22
1	1 0 1 0 0	27,8	626	520173,2	333442	7
1	1 0 1 0 1	27,8	626	520173,2	483442	7
1	1 0 1 1 0	8,9	644	525858,3	394904	5
1	1 0 1 1 1	8,9	644	525858,3	577904	5
1	1 1 0 0 0	103,47	636	560998,5	408135	15
1	1 1 0 0 1	94,83	662	558865,4	510135	14
1	1 1 0 1 0	142,66	479	497978,4	346345	28
1	1 1 0 1 1	142,66	479	497978,4	397345	28
1	1 1 1 0 0	29,13	689	597326,6	346701	7
1	1 1 1 0 1	29,13	689	597326,6	499701	7
1	1 1 1 1 0	7,07	644	521116,4	416052	8
1	1 1 1 1 1	7,07	644	521116,4	605052	8
1	2 0 0 0 0	223,74	406	566695,1	377759	18
1	2 0 0 0 1	223,74	406	566695,1	434759	18
1	2 0 0 1 0	170	370	558206	431117	23
1	2 0 0 1 1	170	370	558206	494117	23
1	2 0 1 0 0	31,77	707	541838,3	337485	13
1	2 0 1 0 1	31,77	707	541838,3	487485	13
1	2 0 1 1 0	5,17	671	525607,5	375560	13
1	2 0 1 1 1	5,17	671	525607,5	555560	13
1	2 1 0 0 0	187,24	557	504174,9	374890	24
1	2 1 0 0 1	187,24	557	504174,9	434890	24
1	2 1 0 1 0	172,18	370	546984,8	419539	25
1	2 1 0 1 1	172,18	370	546984,8	482539	25
1	2 1 1 0 0	39,27	655	538078,1	339421	7
1	2 1 1 0 1	39,27	655	538078,1	489421	7
1	2 1 1 1 0	7,83	671	527124,9	396649	7
1	2 1 1 1 1	7,83	671	527124,9	582649	7

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
3	0 0 0 0 0	210,11	569	506197,7	379227	17
3	0 0 0 0 1	210,11	569	506197,7	433227	17
3	0 0 0 1 0	156,4	495	521003	399964	22
3	0 0 0 1 1	156,4	495	521003	459964	22
3	0 0 1 0 0	9	684	570714,4	361348	19
3	0 0 1 0 1	9	684	570714,4	541348	19
3	0 0 1 1 0	44,91	567	492950,1	455371	9
3	0 0 1 1 1	44,91	567	492950,1	593371	9
3	0 1 0 0 0	231,14	509	519473,2	379227	17
3	0 1 0 0 1	231,14	509	519473,2	433227	17
3	0 1 0 1 0	377,84	485	486255,5	347028	38
3	0 1 0 1 1	377,84	485	486255,5	392028	38
3	0 1 1 0 0	9,5	684	558329,6	371607	9
3	0 1 1 0 1	9,5	684	558329,6	554607	9
3	0 1 1 1 0	253,37	535	525605,8	509608	12
3	0 1 1 1 1	253,37	535	525605,8	644608	12
3	1 0 0 0 0	9	839	546824,5	360406	12
3	1 0 0 0 1	9	839	546824,5	540406	12
3	1 0 0 1 0	336,59	508	499770,5	339855	41
3	1 0 0 1 1	336,59	508	499770,5	378855	41
3	1 0 1 0 0	8,17	619	559774,9	347554	14
3	1 0 1 0 1	9,17	619	535645,4	540406	18
3	1 0 1 1 0	4,5	608	545427,7	401458	13
3	1 0 1 1 1	4,5	608	545427,7	584458	13
3	1 1 0 0 0	8,33	839	540687,5	360406	9
3	1 1 0 0 1	8,33	839	540687,5	540406	9
3	1 1 0 1 0	338,58	508	477468,8	356830	47
3	1 1 0 1 1	338,58	508	477468,8	398830	47
3	1 1 1 0 0	10	619	554483,9	373060	10
3	1 1 1 0 1	10,5	619	530192,9	524554	12
3	1 1 1 1 0	3	608	543431,5	395503	16
3	1 1 1 1 1	3	608	543431,5	578503	16

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
3	2 0 0 0 0	158,52	751	543764,4	449000	10
3	2 0 0 0 1	158,52	751	543764,4	551000	10
3	2 0 0 1 0	130,06	513	532493,1	492489	11
3	2 0 0 1 1	67,97	563	541969,6	562586	10
3	2 0 1 0 0	10,57	672	561189,8	366936	11
3	2 0 1 0 1	10,57	672	561189,8	546936	11
3	2 0 1 1 0	103,26	475	524127,8	476959	15
3	2 0 1 1 1	103,26	475	524127,8	617959	15
3	2 1 0 0 0	159,02	751	561238,8	449001	13
3	2 1 0 0 1	159,02	751	561238,8	551001	13
3	2 1 0 1 0	376,75	506	534277,8	370372	40
3	2 1 0 1 1	376,75	506	534277,8	415372	40
3	2 1 1 0 0	10,57	672	545667,7	366141	7
3	2 1 1 0 1	10,17	672	561367,9	562590	6
3	2 1 1 1 0	172,04	375	516158,3	444408	18
3	2 1 1 1 1	172,04	375	516158,3	537408	18
4	0 0 0 0 0	205,7	673	497895,9	554833	0
4	0 0 0 0 1	205,7	673	497895,9	680833	0
4	0 0 0 1 0	291	613	520357,6	435260	19
4	0 0 0 1 1	291	613	520357,6	495260	19
4	0 0 1 0 0	5	704	593582,1	340395	16
4	0 0 1 0 1	5	704	593582,1	517395	16
4	0 0 1 1 0	102,91	616	503193,7	456990	15
4	0 0 1 1 1	102,91	616	503193,7	591990	15
4	0 1 0 0 0	182,03	699	623899,6	408917	18
4	0 1 0 0 1	182,03	699	623899,6	516917	18
4	0 1 0 1 0	265,84	397	501245,8	441719	30
4	0 1 0 1 1	265,84	397	501245,8	501719	30
4	0 1 1 0 0	5	704	579014	319370	19
4	0 1 1 0 1	5	704	579014	490370	19
4	0 1 1 1 0	3,5	656	497944,8	365240	11
4	0 1 1 1 1	5,5	656	480122,5	545240	10

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
4	1 0 0 0 0	7,87	754	601216,6	359047	15
4	1 0 0 0 1	7,87	754	601216,6	539047	15
4	1 0 0 1 0	55,53	622	495357,2	412762	8
4	1 0 0 1 1	66,53	622	490780,4	549796	8
4	1 0 1 0 0	5,4	565	617209,9	344414	20
4	1 0 1 0 1	5,4	565	617209,9	521414	20
4	1 0 1 1 0	69,5	459	490588,7	437503	9
4	1 0 1 1 1	70,83	459	489687,2	575503	9
4	1 1 0 0 0	8,8	754	604404,4	348391	15
4	1 1 0 0 1	11,4	754	606800,4	551169	9
4	1 1 0 1 0	63,34	588	490675,1	420848	11
4	1 1 0 1 1	86,84	538	494451	522848	14
4	1 1 1 0 0	5,17	565	639786,2	343841	17
4	1 1 1 0 1	6,5	565	663791,2	557462	15
4	1 1 1 1 0	71,33	459	474103,8	424468	11
4	1 1 1 1 1	71,33	459	474103,8	559468	11
4	2 0 0 0 0	135,67	443	606223,8	468129	20
4	2 0 0 0 1	135,67	443	606223,8	576129	20
4	2 0 0 1 0	130,5	637	502266,6	495773	12
4	2 0 0 1 1	134,6	647	517465,6	605822	13
4	2 0 1 0 0	21,71	606	611455,3	461716	8
4	2 0 1 0 1	21,71	606	611455,3	611716	8
4	2 0 1 1 0	70	547	524765,1	470039	7
4	2 0 1 1 1	70	547	524765,1	611039	7
4	2 1 0 0 0	205,44	667	639307,2	499827	11
4	2 1 0 0 1	205,44	667	639307,2	613827	11
4	2 1 0 1 0	140,6	601	502604,9	484076	15
4	2 1 0 1 1	138,2	629	503204,1	576481	13
4	2 1 1 0 0	27,68	619	607016,8	425012	12
4	2 1 1 0 1	27,68	619	607016,8	572012	12
4	2 1 1 1 0	70,5	547	512589,7	457004	7
4	2 1 1 1 1	70,5	547	512589,7	595004	7

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
5	0 0 0 0 0	253,51	402	522080,9	356746	31
5	0 0 0 0 1	253,51	402	522080,9	410746	31
5	0 0 0 1 0	229,01	701	610492,2	456768	12
5	0 0 0 1 1	229,01	701	610492,2	564768	12
5	0 0 1 0 0	141,1	691	522394,8	475971	18
5	0 0 1 0 1	141,1	691	522394,8	613971	18
5	0 0 1 1 0	8,9	688	601914,6	389584	12
5	0 0 1 1 1	8,9	688	601914,6	572584	12
5	0 1 0 0 0	271,91	422	536888,7	379976	41
5	0 1 0 0 1	271,91	422	536888,7	433976	41
5	0 1 0 1 0	130,66	662	608734	483418	17
5	0 1 0 1 1	130,66	662	608734	594418	17
5	0 1 1 0 0	140,1	691	500689,4	428136	21
5	0 1 1 0 1	140,1	691	500689,4	563136	21
5	0 1 1 1 0	4,33	616	579821,5	388789	11
5	0 1 1 1 1	4,33	616	579821,5	571789	11
5	1 0 0 0 0	230	526	528028,5	380351	29
5	1 0 0 0 1	230	526	528028,5	437351	29
5	1 0 0 1 0	55,58	662	575936,4	505505	12
5	1 0 0 1 1	55,58	662	575936,4	652505	12
5	1 0 1 0 0	20,47	751	487999,7	330567	17
5	1 0 1 0 1	20,47	751	487999,7	504567	17
5	1 0 1 1 0	1,5	568	602981,4	429124	12
5	1 0 1 1 1	1,83	568	596109,8	606783	11
5	1 1 0 0 0	130	667	503903,5	373548	16
5	1 1 0 0 1	130	667	503903,5	505548	16
5	1 1 0 1 0	53,08	662	590471,1	494164	7
5	1 1 0 1 1	67,91	636	593190,7	638164	9
5	1 1 1 0 0	26,37	658	471283,7	313503	25
5	1 1 1 0 1	26,37	658	471283,7	484503	25
5	1 1 1 1 0	2,67	568	600713,9	394544	15
5	1 1 1 1 1	2,67	568	600713,9	580544	15

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
5	2 0 0 0 0	199,08	602	532499,6	432903	28
5	2 0 0 0 1	199,08	602	532499,6	534903	28
5	2 0 0 1 0	66,59	739	556331,3	491357	10
5	2 0 0 1 1	66,59	739	556331,3	632357	10
5	2 0 1 0 0	20,1	772	510632,2	363477	12
5	2 0 1 0 1	20,1	772	510632,2	543477	12
5	2 0 1 1 0	6	595	562433,9	402632	6
5	2 0 1 1 1	5,5	595	565520,6	588632	7
5	2 1 0 0 0	197,1	584	549252,3	448493	27
5	2 1 0 0 1	197,1	584	549252,3	550493	27
5	2 1 0 1 0	116,92	561	618956,1	431137	20
5	2 1 0 1 1	116,92	561	618956,1	500137	20
5	2 1 1 0 0	21	772	474240,8	375459	9
5	2 1 1 0 1	21	772	474240,8	558459	9
5	2 1 1 1 0	8,23	635	556393,6	376275	15
5	2 1 1 1 1	7,9	635	576021,9	572099	8
6	0 0 0 0 0	182,23	499	535521,9	384368	29
6	0 0 0 0 1	182,23	499	535521,9	444368	29
6	0 0 0 1 0	414,57	503	529850,9	386327	40
6	0 0 0 1 1	414,57	503	529850,9	434327	40
6	0 0 1 0 0	76,19	570	543109,4	461268	14
6	0 0 1 0 1	76,19	570	543109,4	605268	14
6	0 0 1 1 0	128,27	480	559596,1	532671	7
6	0 0 1 1 1	128,27	480	559596,1	664671	7
6	0 1 0 0 0	230,41	485	558876,7	395996	32
6	0 1 0 0 1	230,41	485	558876,7	455996	32
6	0 1 0 1 0	418,55	503	544854,3	386387	41
6	0 1 0 1 1	418,55	503	544854,3	434387	41
6	0 1 1 0 0	46,46	640	551244,6	404804	15
6	0 1 1 0 1	46,46	640	551244,6	551804	15
6	0 1 1 1 0	126,6	480	550528,3	507765	10
6	0 1 1 1 1	128,27	480	563287,8	636765	9
6	1 0 0 0 0	112	647	529905,2	424303	21

Appendix E

Initial Stage Results of Dedicated Algorithm

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
1	0 0 0 0 0	518,3	414	201561,5	489143	14
1	0 0 0 0 1	518,3	414	201561,5	489143	14
1	0 0 0 1 0	518,3	488	222134,7	523529	11
1	0 0 0 1 1	518,3	488	222134,7	523529	11
1	0 0 1 0 0	518,3	621	201561,5	489143	14
1	0 0 1 0 1	518,3	621	201561,5	489143	14
1	0 0 1 1 0	518,3	748	222134,7	523529	11
1	0 0 1 1 1	518,3	748	222134,7	523529	11
1	0 1 0 0 0	518,3	499	185917,6	360999	18
1	0 1 0 0 1	518,3	499	185917,6	360999	18
1	0 1 0 1 0	518,3	488	210368,3	465255	10
1	0 1 0 1 1	518,3	488	210368,3	465255	10
1	0 1 1 0 0	518,3	718	185917,6	360999	18
1	0 1 1 0 1	518,3	718	185917,6	360999	18
1	0 1 1 1 0	518,3	748	210368,3	465255	10
1	0 1 1 1 1	518,3	748	210368,3	465255	10

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
1	1 0 0 0 0	518,3	582	216549,2	477447	12
1	1 0 0 0 1	518,3	582	216549,2	477447	12
1	1 0 0 1 0	518,3	579	225283,1	476901	10
1	1 0 0 1 1	518,3	579	225283,1	476901	10
1	1 0 1 0 0	518,3	794	216549,2	477447	12
1	1 0 1 0 1	518,3	794	216549,2	477447	12
1	1 0 1 1 0	518,3	858	225283,1	476901	10
1	1 0 1 1 1	518,3	858	225283,1	476901	10
1	1 1 0 0 0	518,3	550	234467,8	465752	12
1	1 1 0 0 1	518,3	550	234467,8	465752	12
1	1 1 0 1 0	518,3	579	236565	476901	10
1	1 1 0 1 1	518,3	579	236565	476901	10
1	1 1 1 0 0	518,3	729	234467,8	465752	12
1	1 1 1 0 1	518,3	729	234467,8	465752	12
1	1 1 1 1 0	518,3	858	236565	476901	10
1	1 1 1 1 1	518,3	858	236565	476901	10
1	2 0 0 0 0	518,3	498	224194,7	535735	9
1	2 0 0 0 1	518,3	498	224194,7	535735	9
1	2 0 0 1 0	518,3	504	235168	570156	5
1	2 0 0 1 1	518,3	504	235168	570156	5
1	2 0 1 0 0	518,3	703	224194,7	535735	9
1	2 0 1 0 1	518,3	703	224194,7	535735	9
1	2 0 1 1 0	518,3	809	235168	570156	5
1	2 0 1 1 1	518,3	809	235168	570156	5
1	2 1 0 0 0	518,3	527	211819,6	500810	11
1	2 1 0 0 1	518,3	527	211819,6	500810	11
1	2 1 0 1 0	518,3	504	243869,8	558530	6
1	2 1 0 1 1	518,3	504	243869,8	558530	6
1	2 1 1 0 0	518,3	753	211819,6	500810	11
1	2 1 1 0 1	518,3	753	211819,6	500810	11
1	2 1 1 1 0	518,3	809	243869,8	558530	6
1	2 1 1 1 1	518,3	809	243869,8	558530	6

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
3	0 0 0 0 0	518,3	597	216389,6	500824	9
3	0 0 0 0 1	518,3	597	216389,6	500824	9
3	0 0 0 1 0	518,3	549	213446,2	490173	12
3	0 0 0 1 1	518,3	549	213446,2	490173	12
3	0 0 1 0 0	518,3	824	216389,6	500824	9
3	0 0 1 0 1	518,3	824	216389,6	500824	9
3	0 0 1 1 0	518,3	809	213446,2	490173	12
3	0 0 1 1 1	518,3	809	213446,2	490173	12
3	0 1 0 0 0	518,3	597	222683,5	489158	10
3	0 1 0 0 1	518,3	597	222683,5	489158	10
3	0 1 0 1 0	518,3	581	225249,9	420192	17
3	0 1 0 1 1	518,3	581	225249,9	420192	17
3	0 1 1 0 0	518,3	824	222683,5	489158	10
3	0 1 1 0 1	518,3	824	222683,5	489158	10
3	0 1 1 1 0	518,3	737	225249,9	420192	17
3	0 1 1 1 1	518,3	737	225249,9	420192	17
3	1 0 0 0 0	518,3	679	233639,5	465828	12
3	1 0 0 0 1	518,3	679	233639,5	465828	12
3	1 0 0 1 0	518,3	658	210562,7	408505	13
3	1 0 0 1 1	518,3	658	210562,7	408505	13
3	1 0 1 0 0	518,3	875	233639,5	465828	12
3	1 0 1 0 1	518,3	875	233639,5	465828	12
3	1 0 1 1 0	518,3	892	210562,7	408505	13
3	1 0 1 1 1	518,3	892	210562,7	408505	13
3	1 1 0 0 0	518,3	679	240693,5	442582	12
3	1 1 0 0 1	518,3	679	240693,5	442582	12
3	1 1 0 1 0	518,3	658	208946,6	385187	17
3	1 1 0 1 1	518,3	658	208946,6	385187	17
3	1 1 1 0 0	518,3	875	240693,5	442582	12
3	1 1 1 0 1	518,3	875	240693,5	442582	12
3	1 1 1 1 0	518,3	892	208946,6	385187	17
3	1 1 1 1 1	518,3	892	208946,6	385187	17

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
3	2 0 0 0 0	518,3	621	222621,1	547491	6
3	2 0 0 0 1	518,3	621	222621,1	547491	6
3	2 0 0 1 0	518,3	599	210419,8	513467	9
3	2 0 0 1 1	518,3	599	210419,8	513467	9
3	2 0 1 0 0	518,3	848	222621,1	547491	6
3	2 0 1 0 1	518,3	848	222621,1	547491	6
3	2 0 1 1 0	518,3	833	210419,8	513467	9
3	2 0 1 1 1	518,3	833	210419,8	513467	9
3	2 1 0 0 0	518,3	621	233938,7	524212	8
3	2 1 0 0 1	518,3	621	233938,7	524212	8
3	2 1 0 1 0	518,3	604	218115,8	478585	13
3	2 1 0 1 1	518,3	604	218115,8	478585	13
3	2 1 1 0 0	518,3	848	233938,7	524212	8
3	2 1 1 0 1	518,3	848	233938,7	524212	8
3	2 1 1 1 0	518,3	779	218115,8	478585	13
3	2 1 1 1 1	518,3	779	218115,8	478585	13
4	0 0 0 0 0	518,3	629	210919,6	536394	7
4	0 0 0 0 1	518,3	629	210919,6	536394	7
4	0 0 0 1 0	518,3	535	205351,8	559729	11
4	0 0 0 1 1	518,3	535	205351,8	559729	11
4	0 0 1 0 0	518,3	842	210919,6	536394	7
4	0 0 1 0 1	518,3	842	210919,6	536394	7
4	0 0 1 1 0	518,3	814	205351,8	559729	11
4	0 0 1 1 1	518,3	814	205351,8	559729	11
4	0 1 0 0 0	518,3	629	202865	431453	16
4	0 1 0 0 1	518,3	629	202865	431453	16
4	0 1 0 1 0	518,3	513	192375,2	466448	15
4	0 1 0 1 1	518,3	513	192375,2	466448	15
4	0 1 1 0 0	518,3	842	202865	431453	16
4	0 1 1 0 1	518,3	842	202865	431453	16
4	0 1 1 1 0	518,3	856	192375,2	466448	15
4	0 1 1 1 1	518,3	856	192375,2	466448	15

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
4	1 0 0 0 0	518,3	794	200071,9	373382	17
4	1 0 0 0 1	518,3	794	200071,9	373382	17
4	1 0 0 1 0	518,3	716	219567,6	454856	13
4	1 0 0 1 1	518,3	716	219567,6	454856	13
4	1 0 1 0 0	518,3	995	200071,9	373382	17
4	1 0 1 0 1	518,3	995	200071,9	373382	17
4	1 0 1 1 0	518,3	945	219567,6	454856	13
4	1 0 1 1 1	518,3	945	219567,6	454856	13
4	1 1 0 0 0	518,3	794	190964,5	326715	14
4	1 1 0 0 1	518,3	794	190964,5	326715	14
4	1 1 0 1 0	518,3	716	206924,8	443197	14
4	1 1 0 1 1	518,3	716	206924,8	443197	14
4	1 1 1 0 0	518,3	995	190964,5	326715	14
4	1 1 1 0 1	518,3	995	190964,5	326715	14
4	1 1 1 1 0	518,3	945	206924,8	443197	14
4	1 1 1 1 1	518,3	945	206924,8	443197	14
4	2 0 0 0 0	518,3	663	222987,4	618025	4
4	2 0 0 0 1	518,3	663	222987,4	618025	4
4	2 0 0 1 0	518,3	569	237086	653021	3
4	2 0 0 1 1	518,3	569	237086	653021	3
4	2 0 1 0 0	518,3	902	222987,4	618025	4
4	2 0 1 0 1	518,3	902	222987,4	618025	4
4	2 0 1 1 0	518,3	855	237086	653021	3
4	2 0 1 1 1	518,3	855	237086	653021	3
4	2 1 0 0 0	518,3	647	223594,2	582991	5
4	2 1 0 0 1	518,3	647	223594,2	582991	5
4	2 1 0 1 0	518,3	569	229151,2	641421	4
4	2 1 0 1 1	518,3	569	229151,2	641421	4
4	2 1 1 0 0	518,3	879	223594,2	582991	5
4	2 1 1 0 1	518,3	879	223594,2	582991	5
4	2 1 1 1 0	518,3	855	229151,2	641421	4
4	2 1 1 1 1	518,3	855	229151,2	641421	4

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
5	0 0 0 0 0	518,3	488	221801	524315	9
5	0 0 0 0 1	518,3	488	221801	524315	9
5	0 0 0 1 0	518,3	583	250656,6	546200	11
5	0 0 0 1 1	518,3	583	250656,6	546200	11
5	0 0 1 0 0	518,3	679	221801	524315	9
5	0 0 1 0 1	518,3	679	221801	524315	9
5	0 0 1 1 0	518,3	774	250656,6	546200	11
5	0 0 1 1 1	518,3	774	250656,6	546200	11
5	0 1 0 0 0	518,3	488	213992,5	477706	13
5	0 1 0 0 1	518,3	488	213992,5	477706	13
5	0 1 0 1 0	518,3	584	266576,1	441623	9
5	0 1 0 1 1	518,3	584	266576,1	441623	9
5	0 1 1 0 0	518,3	679	213992,5	477706	13
5	0 1 1 0 1	518,3	679	213992,5	477706	13
5	0 1 1 1 0	518,3	858	266576,1	441623	9
5	0 1 1 1 1	518,3	858	266576,1	441623	9
5	1 0 0 0 0	518,3	566	184982,9	373007	16
5	1 0 0 0 1	518,3	566	184982,9	373007	16
5	1 0 0 1 0	518,3	664	244127,9	569479	7
5	1 0 0 1 1	518,3	664	244127,9	569479	7
5	1 0 1 0 0	518,3	769	184982,9	373007	16
5	1 0 1 0 1	518,3	769	184982,9	373007	16
5	1 0 1 1 0	518,3	936	244127,9	569479	7
5	1 0 1 1 1	518,3	936	244127,9	569479	7
5	1 1 0 0 0	518,3	623	160901,7	349740	16
5	1 1 0 0 1	518,3	623	160901,7	349740	16
5	1 1 0 1 0	518,3	664	247908,2	569479	7
5	1 1 0 1 1	518,3	664	247908,2	569479	7
5	1 1 1 0 0	518,3	874	160901,7	349740	16
5	1 1 1 0 1	518,3	874	160901,7	349740	16
5	1 1 1 1 0	518,3	936	247908,2	569479	7
5	1 1 1 1 1	518,3	936	247908,2	569479	7

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
5	2 0 0 0 0	518,3	526	217759,7	524245	10
5	2 0 0 0 1	518,3	526	217759,7	524245	10
5	2 0 0 1 0	518,3	637	231310,1	604329	6
5	2 0 0 1 1	518,3	637	231310,1	604329	6
5	2 0 1 0 0	518,3	748	217759,7	524245	10
5	2 0 1 0 1	518,3	748	217759,7	524245	10
5	2 0 1 1 0	518,3	885	231310,1	604329	6
5	2 0 1 1 1	518,3	885	231310,1	604329	6
5	2 1 0 0 0	518,3	526	204932,9	500939	10
5	2 1 0 0 1	518,3	526	204932,9	500939	10
5	2 1 0 1 0	518,3	573	226519,2	581075	8
5	2 1 0 1 1	518,3	573	226519,2	581075	8
5	2 1 1 0 0	518,3	748	204932,9	500939	10
5	2 1 1 0 1	518,3	748	204932,9	500939	10
5	2 1 1 1 0	518,3	859	226519,2	581075	8
5	2 1 1 1 1	518,3	859	226519,2	581075	8
6	0 0 0 0 0	518,3	505	219614,1	512023	14
6	0 0 0 0 1	518,3	505	219614,1	512023	14
6	0 0 0 1 0	518,3	537	223758,3	571256	5
6	0 0 0 1 1	518,3	537	223758,3	571256	5
6	0 0 1 0 0	518,3	768	219614,1	512023	14
6	0 0 1 0 1	518,3	768	219614,1	512023	14
6	0 0 1 1 0	518,3	842	223758,3	571256	5
6	0 0 1 1 1	518,3	842	223758,3	571256	5
6	0 1 0 0 0	518,3	505	214866,3	383834	22
6	0 1 0 0 1	518,3	505	214866,3	383834	22
6	0 1 0 1 0	518,3	537	220279	466298	12
6	0 1 0 1 1	518,3	537	220279	466298	12
6	0 1 1 0 0	518,3	768	214866,3	383834	22
6	0 1 1 0 1	518,3	768	214866,3	383834	22
6	0 1 1 1 0	518,3	842	220279	466298	12
6	0 1 1 1 1	518,3	842	220279	466298	12
6	1 0 0 0 0	518,3	617	218264,4	442215	14

Seed	(ABCDE)	I - F1	I - F2	I - F3	I - F4	I - F5
6	1 0 0 0 1	518,3	617	218264,4	442215	14
6	1 0 0 1 0	518,3	645	210119	419697	14
6	1 0 0 1 1	518,3	645	210119	419697	14
6	1 0 1 0 0	518,3	837	218264,4	442215	14
6	1 0 1 0 1	518,3	837	218264,4	442215	14
6	1 0 1 1 0	518,3	931	210119	419697	14
6	1 0 1 1 1	518,3	931	210119	419697	14
6	1 1 0 0 0	518,3	617	216777,2	395575	13
6	1 1 0 0 1	518,3	617	216777,2	395575	13
6	1 1 0 1 0	518,3	645	205047,1	408082	16
6	1 1 0 1 1	518,3	645	205047,1	408082	16
6	1 1 1 0 0	518,3	837	216777,2	395575	13
6	1 1 1 0 1	518,3	837	216777,2	395575	13
6	1 1 1 1 0	518,3	931	205047,1	408082	16
6	1 1 1 1 1	518,3	931	205047,1	408082	16
6	2 0 0 0 0	518,3	528	228665,5	570122	9
6	2 0 0 0 1	518,3	528	228665,5	570122	9
6	2 0 0 1 0	518,3	586	222672,9	582909	4
6	2 0 0 1 1	518,3	586	222672,9	582909	4
6	2 0 1 0 0	518,3	798	228665,5	570122	9
6	2 0 1 0 1	518,3	798	228665,5	570122	9
6	2 0 1 1 0	518,3	879	222672,9	582909	4
6	2 0 1 1 1	518,3	879	222672,9	582909	4
6	2 1 0 0 0	518,3	528	232462,5	523650	13
6	2 1 0 0 1	518,3	528	232462,5	523650	13
6	2 1 0 1 0	518,3	586	219296,1	582909	4
6	2 1 0 1 1	518,3	586	219296,1	582909	4
6	2 1 1 0 0	518,3	798	232462,5	523650	13
6	2 1 1 0 1	518,3	798	232462,5	523650	13
6	2 1 1 1 0	518,3	879	219296,1	582909	4
6	2 1 1 1 1	518,3	879	219296,1	582909	4

Appendix F

Final Results of Dedicated Alg. with b=3

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	0 0 0 0 0	445,6	414	473516,9	267966	40
1	0 0 0 0 1	445,6	414	473516,9	267966	40
1	0 0 0 1 0	447,3	488	484505,2	279151	46
1	0 0 0 1 1	447,3	488	484505,2	279151	46
1	0 0 1 0 0	445,6	621	473516,9	267966	40
1	0 0 1 0 1	445,6	621	473516,9	267966	40
1	0 0 1 1 0	447,3	748	484505,2	279151	46
1	0 0 1 1 1	447,3	748	484505,2	279151	46
1	0 1 0 0 0	458,5	499	417176,9	244675	42
1	0 1 0 0 1	458,5	499	417176,9	244675	42
1	0 1 0 1 0	451,4	488	412365,4	232557	40
1	0 1 0 1 1	451,4	488	412365,4	232557	40
1	0 1 1 0 0	458,5	718	417176,9	244675	42
1	0 1 1 0 1	458,5	718	417176,9	244675	42
1	0 1 1 1 0	451,4	748	412365,4	232557	40
1	0 1 1 1 1	451,4	748	412365,4	232557	40
1	1 0 0 0 0	444,5	582	471535,3	267989	38

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	1 0 0 0 1	444,5	582	471535,3	267989	38
1	1 0 0 1 0	449,3	579	445227,9	255836	39
1	1 0 0 1 1	449,3	579	445227,9	255836	39
1	1 0 1 0 0	444,5	794	471535,3	267989	38
1	1 0 1 0 1	444,5	794	471535,3	267989	38
1	1 0 1 1 0	449,3	858	445227,9	255836	39
1	1 0 1 1 1	449,3	858	445227,9	255836	39
1	1 1 0 0 0	450,9	550	469554,8	267890	32
1	1 1 0 0 1	450,9	550	469554,8	267890	32
1	1 1 0 1 0	447,9	579	444098,5	267451	38
1	1 1 0 1 1	447,9	579	444098,5	267451	38
1	1 1 1 0 0	450,9	729	469554,8	267890	32
1	1 1 1 0 1	450,9	729	469554,8	267890	32
1	1 1 1 1 0	447,9	858	444098,5	267451	38
1	1 1 1 1 1	447,9	858	444098,5	267451	38
1	2 0 0 0 0	450,7	498	484527,3	279646	38
1	2 0 0 0 1	450,7	498	484527,3	279646	38
1	2 0 0 1 0	441,5	504	487577,8	279151	34
1	2 0 0 1 1	441,5	504	487577,8	279151	34
1	2 0 1 0 0	450,7	703	484527,3	279646	38
1	2 0 1 0 1	450,7	703	484527,3	279646	38
1	2 0 1 1 0	441,5	809	487577,8	279151	34
1	2 0 1 1 1	441,5	809	487577,8	279151	34
1	2 1 0 0 0	447,5	527	485145,7	279646	37
1	2 1 0 0 1	447,5	527	485145,7	279646	37
1	2 1 0 1 0	445,3	504	484141,4	279151	36
1	2 1 0 1 1	445,3	504	484141,4	279151	36
1	2 1 1 0 0	447,5	753	485145,7	279646	37
1	2 1 1 0 1	447,5	753	485145,7	279646	37
1	2 1 1 1 0	445,3	809	484141,4	279151	36
1	2 1 1 1 1	445,3	809	484141,4	279151	36

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
3	0 0 0 0 0	447,8	597	449715,5	256254	43
3	0 0 0 0 1	447,8	597	449715,5	256254	43
3	0 0 0 1 0	445,7	549	445649,6	268494	43
3	0 0 0 1 1	445,7	549	445649,6	268494	43
3	0 0 1 0 0	447,8	824	449715,5	256254	43
3	0 0 1 0 1	447,8	824	449715,5	256254	43
3	0 0 1 1 0	445,7	809	445649,6	268494	43
3	0 0 1 1 1	445,7	809	445649,6	268494	43
3	0 1 0 0 0	444,6	597	451022,9	256254	44
3	0 1 0 0 1	444,6	597	451022,9	256254	44
3	0 1 0 1 0	447,5	581	448336,5	268494	43
3	0 1 0 1 1	447,5	581	448336,5	268494	43
3	0 1 1 0 0	444,6	824	451022,9	256254	44
3	0 1 1 0 1	444,6	824	451022,9	256254	44
3	0 1 1 1 0	447,5	737	448336,5	268494	43
3	0 1 1 1 1	447,5	737	448336,5	268494	43
3	1 0 0 0 0	444,7	679	476631,4	267834	43
3	1 0 0 0 1	444,7	679	476631,4	267834	43
3	1 0 0 1 0	447,3	658	453092,8	268494	37
3	1 0 0 1 1	447,3	658	453092,8	268494	37
3	1 0 1 0 0	444,7	875	476631,4	267834	43
3	1 0 1 0 1	444,7	875	476631,4	267834	43
3	1 0 1 1 0	447,3	892	453092,8	268494	37
3	1 0 1 1 1	447,3	892	453092,8	268494	37
3	1 1 0 0 0	441,7	679	464231,7	256254	44
3	1 1 0 0 1	441,7	679	464231,7	256254	44
3	1 1 0 1 0	447	658	423588,2	268470	39
3	1 1 0 1 1	447	658	423588,2	268470	39
3	1 1 1 0 0	441,7	875	464231,7	256254	44
3	1 1 1 0 1	441,7	875	464231,7	256254	44
3	1 1 1 1 0	447	892	423588,2	268470	39
3	1 1 1 1 1	447	892	423588,2	268470	39

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
3	2 0 0 0 0	448	621	456440,7	256254	37
3	2 0 0 0 1	448	621	456440,7	256254	37
3	2 0 0 1 0	449,6	599	450604,2	268494	39
3	2 0 0 1 1	449,6	599	450604,2	268494	39
3	2 0 1 0 0	448	848	456440,7	256254	37
3	2 0 1 0 1	448	848	456440,7	256254	37
3	2 0 1 1 0	449,6	833	450604,2	268494	39
3	2 0 1 1 1	449,6	833	450604,2	268494	39
3	2 1 0 0 0	446,5	621	463058,2	267911	45
3	2 1 0 0 1	446,5	621	463058,2	267911	45
3	2 1 0 1 0	450,4	604	454887,9	280176	41
3	2 1 0 1 1	450,4	604	454887,9	280176	41
3	2 1 1 0 0	446,5	848	463058,2	267911	45
3	2 1 1 0 1	446,5	848	463058,2	267911	45
3	2 1 1 1 0	450,4	779	454887,9	280176	41
3	2 1 1 1 1	450,4	779	454887,9	280176	41
4	0 0 0 0 0	460,9	629	486973,9	256528	32
4	0 0 0 0 1	460,9	629	486973,9	256528	32
4	0 0 0 1 0	442,9	535	473543,4	279997	46
4	0 0 0 1 1	442,9	535	473543,4	279997	46
4	0 0 1 0 0	460,9	842	486973,9	256528	32
4	0 0 1 0 1	460,9	842	486973,9	256528	32
4	0 0 1 1 0	442,9	814	473543,4	279997	46
4	0 0 1 1 1	442,9	814	473543,4	279997	46
4	0 1 0 0 0	469,5	629	491718,4	256528	46
4	0 1 0 0 1	469,5	629	491718,4	256528	46
4	0 1 0 1 0	447,6	513	459258,9	268338	44
4	0 1 0 1 1	447,6	513	459258,9	268338	44
4	0 1 1 0 0	469,5	842	491718,4	256528	46
4	0 1 1 0 1	469,5	842	491718,4	256528	46
4	0 1 1 1 0	447,6	856	459258,9	268338	44
4	0 1 1 1 1	447,6	856	459258,9	268338	44

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
4	1 0 0 0 0	467,5	794	512166,6	268252	47
4	1 0 0 0 1	467,5	794	512166,6	268252	47
4	1 0 0 1 0	458	716	468741,9	279997	40
4	1 0 0 1 1	458	716	468741,9	279997	40
4	1 0 1 0 0	467,5	995	512166,6	268252	47
4	1 0 1 0 1	467,5	995	512166,6	268252	47
4	1 0 1 1 0	458	945	468741,9	279997	40
4	1 0 1 1 1	458	945	468741,9	279997	40
4	1 1 0 0 0	464,3	794	457787,9	256628	39
4	1 1 0 0 1	464,3	794	457787,9	256628	39
4	1 1 0 1 0	446,2	716	439246	268338	43
4	1 1 0 1 1	446,2	716	439246	268338	43
4	1 1 1 0 0	464,3	995	457787,9	256628	39
4	1 1 1 0 1	464,3	995	457787,9	256628	39
4	1 1 1 1 0	446,2	945	439246	268338	43
4	1 1 1 1 1	446,2	945	439246	268338	43
4	2 0 0 0 0	453	663	528847,7	279976	35
4	2 0 0 0 1	453	663	528847,7	279976	35
4	2 0 0 1 0	446,2	569	487569,8	279997	38
4	2 0 0 1 1	446,2	569	487569,8	279997	38
4	2 0 1 0 0	453	902	528847,7	279976	35
4	2 0 1 0 1	453	902	528847,7	279976	35
4	2 0 1 1 0	446,2	855	487569,8	279997	38
4	2 0 1 1 1	446,2	855	487569,8	279997	38
4	2 1 0 0 0	478,3	647	517685,6	256528	30
4	2 1 0 0 1	478,3	647	517685,6	256528	30
4	2 1 0 1 0	438,4	569	478061	279997	36
4	2 1 0 1 1	438,4	569	478061	279997	36
4	2 1 1 0 0	478,3	879	517685,6	256528	30
4	2 1 1 0 1	478,3	879	517685,6	256528	30
4	2 1 1 1 0	438,4	855	478061	279997	36
4	2 1 1 1 1	438,4	855	478061	279997	36

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
5	0 0 0 0 0	442,9	488	467932,8	268023	41
5	0 0 0 0 1	442,9	488	467932,8	268023	41
5	0 0 0 1 0	442	583	505726,1	267357	39
5	0 0 0 1 1	442	583	505726,1	267357	39
5	0 0 1 0 0	442,9	679	467932,8	268023	41
5	0 0 1 0 1	442,9	679	467932,8	268023	41
5	0 0 1 1 0	442	774	505726,1	267357	39
5	0 0 1 1 1	442	774	505726,1	267357	39
5	0 1 0 0 0	445,9	488	459262,9	268023	43
5	0 1 0 0 1	445,9	488	459262,9	268023	43
5	0 1 0 1 0	472,4	584	486770,2	255711	40
5	0 1 0 1 1	472,4	584	486770,2	255711	40
5	0 1 1 0 0	445,9	679	459262,9	268023	43
5	0 1 1 0 1	445,9	679	459262,9	268023	43
5	0 1 1 1 0	472,4	858	486770,2	255711	40
5	0 1 1 1 1	472,4	858	486770,2	255711	40
5	1 0 0 0 0	447,2	566	422919,7	244824	36
5	1 0 0 0 1	447,2	566	422919,7	244824	36
5	1 0 0 1 0	444,6	664	488179,9	255711	34
5	1 0 0 1 1	444,6	664	488179,9	255711	34
5	1 0 1 0 0	447,2	769	422919,7	244824	36
5	1 0 1 0 1	447,2	769	422919,7	244824	36
5	1 0 1 1 0	444,6	936	488179,9	255711	34
5	1 0 1 1 1	444,6	936	488179,9	255711	34
5	1 1 0 0 0	445,9	623	399027	221557	37
5	1 1 0 0 1	445,9	623	399027	221557	37
5	1 1 0 1 0	445,5	664	495931,8	255711	40
5	1 1 0 1 1	445,5	664	495931,8	255711	40
5	1 1 1 0 0	445,9	874	399027	221557	37
5	1 1 1 0 1	445,9	874	399027	221557	37
5	1 1 1 1 0	445,5	936	495931,8	255711	40
5	1 1 1 1 1	445,5	936	495931,8	255711	40

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
5	2 0 0 0 0	437,6	526	468494,3	268023	39
5	2 0 0 0 1	437,6	526	468494,3	268023	39
5	2 0 0 1 0	437,4	637	501376,2	267357	32
5	2 0 0 1 1	437,4	637	501376,2	267357	32
5	2 0 1 0 0	437,6	748	468494,3	268023	39
5	2 0 1 0 1	437,6	748	468494,3	268023	39
5	2 0 1 1 0	437,4	885	501376,2	267357	32
5	2 0 1 1 1	437,4	885	501376,2	267357	32
5	2 1 0 0 0	447,3	526	460236,7	268023	44
5	2 1 0 0 1	447,3	526	460236,7	268023	44
5	2 1 0 1 0	436,4	573	495184,8	267357	37
5	2 1 0 1 1	436,4	573	495184,8	267357	37
5	2 1 1 0 0	447,3	748	460236,7	268023	44
5	2 1 1 0 1	447,3	748	460236,7	268023	44
5	2 1 1 1 0	436,4	859	495184,8	267357	37
5	2 1 1 1 1	436,4	859	495184,8	267357	37
6	0 0 0 0 0	446	505	477277,9	267610	44
6	0 0 0 0 1	446	505	477277,9	267610	44
6	0 0 0 1 0	442,6	537	482500,1	279793	38
6	0 0 0 1 1	442,6	537	482500,1	279793	38
6	0 0 1 0 0	446	768	477277,9	267610	44
6	0 0 1 0 1	446	768	477277,9	267610	44
6	0 0 1 1 0	442,6	842	482500,1	279793	38
6	0 0 1 1 1	442,6	842	482500,1	279793	38
6	0 1 0 0 0	452,6	505	475319,8	279231	44
6	0 1 0 0 1	452,6	505	475319,8	279231	44
6	0 1 0 1 0	465,7	537	483792,9	291480	36
6	0 1 0 1 1	465,7	537	483792,9	291480	36
6	0 1 1 0 0	452,6	768	475319,8	279231	44
6	0 1 1 0 1	452,6	768	475319,8	279231	44
6	0 1 1 1 0	465,7	842	483792,9	291480	36
6	0 1 1 1 1	465,7	842	483792,9	291480	36
6	1 0 0 0 0	446,4	617	485585,9	290837	39

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
6	1 0 0 0 1	446,4	617	485585,9	290837	39
6	1 0 0 1 0	444,4	645	497404,4	279793	41
6	1 0 0 1 1	444,4	645	497404,4	279793	41
6	1 0 1 0 0	446,4	837	485585,9	290837	39
6	1 0 1 0 1	446,4	837	485585,9	290837	39
6	1 0 1 1 0	444,4	931	497404,4	279793	41
6	1 0 1 1 1	444,4	931	497404,4	279793	41
6	1 1 0 0 0	446,5	617	451456,3	255875	34
6	1 1 0 0 1	446,5	617	451456,3	255875	34
6	1 1 0 1 0	447,7	645	505194,6	279793	43
6	1 1 0 1 1	447,7	645	505194,6	279793	43
6	1 1 1 0 0	446,5	837	451456,3	255875	34
6	1 1 1 0 1	446,5	837	451456,3	255875	34
6	1 1 1 1 0	447,7	931	505194,6	279793	43
6	1 1 1 1 1	447,7	931	505194,6	279793	43
6	2 0 0 0 0	441	528	483389,3	267610	44
6	2 0 0 0 1	441	528	483389,3	267610	44
6	2 0 0 1 0	440,9	586	485594,9	279793	38
6	2 0 0 1 1	440,9	586	485594,9	279793	38
6	2 0 1 0 0	441	798	483389,3	267610	44
6	2 0 1 0 1	441	798	483389,3	267610	44
6	2 0 1 1 0	440,9	879	485594,9	279793	38
6	2 0 1 1 1	440,9	879	485594,9	279793	38
6	2 1 0 0 0	447,4	528	486656,3	279237	44
6	2 1 0 0 1	447,4	528	486656,3	279237	44
6	2 1 0 1 0	440,9	586	491437,9	279793	38
6	2 1 0 1 1	440,9	586	491437,9	279793	38
6	2 1 1 0 0	447,4	798	486656,3	279237	44
6	2 1 1 0 1	447,4	798	486656,3	279237	44
6	2 1 1 1 0	440,9	879	491437,9	279793	38
6	2 1 1 1 1	440,9	879	491437,9	279793	38

Appendix G

Final Results of Dedicated Alg. with b=6

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	0 0 0 0 0	445,6	414	473516,9	267966	40
1	0 0 0 0 1	445,6	414	473516,9	267966	40
1	0 0 0 1 0	453	488	484505,2	279151	43
1	0 0 0 1 1	453	488	484505,2	279151	43
1	0 0 1 0 0	445,6	621	473516,9	267966	40
1	0 0 1 0 1	445,6	621	473516,9	267966	40
1	0 0 1 1 0	453	748	484505,2	279151	43
1	0 0 1 1 1	453	748	484505,2	279151	43
1	0 1 0 0 0	458,5	499	417176,9	244675	42
1	0 1 0 0 1	458,5	499	417176,9	244675	42
1	0 1 0 1 0	446,9	488	412365,4	232557	38
1	0 1 0 1 1	446,9	488	412365,4	232557	38
1	0 1 1 0 0	458,5	718	417176,9	244675	42

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	0 1 1 0 1	458,5	718	417176,9	244675	42
1	0 1 1 1 0	446,9	748	412365,4	232557	38
1	0 1 1 1 1	446,9	748	412365,4	232557	38
1	1 0 0 0 0	444,5	582	471535,3	267989	38
1	1 0 0 0 1	444,5	582	471535,3	267989	38
1	1 0 0 1 0	450,4	579	445228	255836	39
1	1 0 0 1 1	450,4	579	445228	255836	39
1	1 0 1 0 0	444,5	794	471535,3	267989	38
1	1 0 1 0 1	444,5	794	471535,3	267989	38
1	1 0 1 1 0	450,4	858	445228	255836	39
1	1 0 1 1 1	450,4	858	445228	255836	39
1	1 1 0 0 0	450,9	550	469554,8	267890	32
1	1 1 0 0 1	450,9	550	469554,8	267890	32
1	1 1 0 1 0	446,2	579	444098,4	267451	38
1	1 1 0 1 1	446,2	579	444098,4	267451	38
1	1 1 1 0 0	450,9	729	469554,8	267890	32
1	1 1 1 0 1	450,9	729	469554,8	267890	32
1	1 1 1 1 0	446,2	858	444098,4	267451	38
1	1 1 1 1 1	446,2	858	444098,4	267451	38
1	2 0 0 0 0	448,2	498	484527,3	279646	35
1	2 0 0 0 1	448,2	498	484527,3	279646	35
1	2 0 0 1 0	444,1	504	487577,8	279151	36
1	2 0 0 1 1	444,1	504	487577,8	279151	36
1	2 0 1 0 0	448,2	703	484527,3	279646	35
1	2 0 1 0 1	448,2	703	484527,3	279646	35
1	2 0 1 1 0	444,1	809	487577,8	279151	36
1	2 0 1 1 1	444,1	809	487577,8	279151	36
1	2 1 0 0 0	443	527	485145,7	279646	41
1	2 1 0 0 1	443	527	485145,7	279646	41
1	2 1 0 1 0	444,2	504	484141,4	279151	36
1	2 1 0 1 1	444,2	504	484141,4	279151	36
1	2 1 1 0 0	443	753	485145,7	279646	41
1	2 1 1 0 1	443	753	485145,7	279646	41
1	2 1 1 1 0	444,2	809	484141,4	279151	36

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
1	2 1 1 1 1	444,2	809	484141,4	279151	36
3	0 0 0 0 0	441	597	449715,5	256254	45
3	0 0 0 0 1	441	597	449715,5	256254	45
3	0 0 0 1 0	445	549	445649,6	268494	43
3	0 0 0 1 1	445	549	445649,6	268494	43
3	0 0 1 0 0	441	824	449715,5	256254	45
3	0 0 1 0 1	441	824	449715,5	256254	45
3	0 0 1 1 0	445	809	445649,6	268494	43
3	0 0 1 1 1	445	809	445649,6	268494	43
3	0 1 0 0 0	444,6	597	451022,9	256254	44
3	0 1 0 0 1	444,6	597	451022,9	256254	44
3	0 1 0 1 0	450,2	581	448336,5	268494	42
3	0 1 0 1 1	450,2	581	448336,5	268494	42
3	0 1 1 0 0	444,6	824	451022,9	256254	44
3	0 1 1 0 1	444,6	824	451022,9	256254	44
3	0 1 1 1 0	450,2	737	448336,5	268494	42
3	0 1 1 1 1	450,2	737	448336,5	268494	42
3	1 0 0 0 0	439,4	679	476631,4	267834	42
3	1 0 0 0 1	439,4	679	476631,4	267834	42
3	1 0 0 1 0	442,1	658	453092,9	268494	41
3	1 0 0 1 1	442,1	658	453092,9	268494	41
3	1 0 1 0 0	439,4	875	476631,4	267834	42
3	1 0 1 0 1	439,4	875	476631,4	267834	42
3	1 0 1 1 0	442,1	892	453092,9	268494	41
3	1 0 1 1 1	442,1	892	453092,9	268494	41
3	1 1 0 0 0	440	679	464231,7	256254	40
3	1 1 0 0 1	440	679	464231,7	256254	40
3	1 1 0 1 0	449,8	658	423588,2	268470	42
3	1 1 0 1 1	449,8	658	423588,2	268470	42
3	1 1 1 0 0	440	875	464231,7	256254	40
3	1 1 1 0 1	440	875	464231,7	256254	40
3	1 1 1 1 0	449,8	892	423588,2	268470	42
3	1 1 1 1 1	449,8	892	423588,2	268470	42
3	2 0 0 0 0	440,8	621	456440,7	256254	37

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
3	2 0 0 0 1	440,8	621	456440,7	256254	37
3	2 0 0 1 0	446,2	599	450604,2	268494	40
3	2 0 0 1 1	446,2	599	450604,2	268494	40
3	2 0 1 0 0	440,8	848	456440,7	256254	37
3	2 0 1 0 1	440,8	848	456440,7	256254	37
3	2 0 1 1 0	446,2	833	450604,2	268494	40
3	2 0 1 1 1	446,2	833	450604,2	268494	40
3	2 1 0 0 0	440,7	621	463058,1	267911	44
3	2 1 0 0 1	440,7	621	463058,1	267911	44
3	2 1 0 1 0	450,2	604	454887,9	280176	41
3	2 1 0 1 1	450,2	604	454887,9	280176	41
3	2 1 1 0 0	440,7	848	463058,1	267911	44
3	2 1 1 0 1	440,7	848	463058,1	267911	44
3	2 1 1 1 0	450,2	779	454887,9	280176	41
3	2 1 1 1 1	450,2	779	454887,9	280176	41
4	0 0 0 0 0	442,5	629	486973,9	256528	34
4	0 0 0 0 1	442,5	629	486973,9	256528	34
4	0 0 0 1 0	437,7	535	473543,4	279997	44
4	0 0 0 1 1	437,7	535	473543,4	279997	44
4	0 0 1 0 0	442,5	842	486973,9	256528	34
4	0 0 1 0 1	442,5	842	486973,9	256528	34
4	0 0 1 1 0	437,7	814	473543,4	279997	44
4	0 0 1 1 1	437,7	814	473543,4	279997	44
4	0 1 0 0 0	468,7	629	491718,4	256528	46
4	0 1 0 0 1	468,7	629	491718,4	256528	46
4	0 1 0 1 0	445,8	513	459258,9	268338	45
4	0 1 0 1 1	445,8	513	459258,9	268338	45
4	0 1 1 0 0	468,7	842	491718,4	256528	46
4	0 1 1 0 1	468,7	842	491718,4	256528	46
4	0 1 1 1 0	445,8	856	459258,9	268338	45
4	0 1 1 1 1	445,8	856	459258,9	268338	45
4	1 0 0 0 0	468,4	794	512166,6	268252	47
4	1 0 0 0 1	468,4	794	512166,6	268252	47
4	1 0 0 1 0	450,8	716	468742	279997	41

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
4	1 0 0 1 1	450,8	716	468742	279997	41
4	1 0 1 0 0	468,4	995	512166,6	268252	47
4	1 0 1 0 1	468,4	995	512166,6	268252	47
4	1 0 1 1 0	450,8	945	468742	279997	41
4	1 0 1 1 1	450,8	945	468742	279997	41
4	1 1 0 0 0	518,3	794	190964,5	326715	14
4	1 1 0 0 1	518,3	794	190964,5	326715	14
4	1 1 0 1 0	446,2	716	439246	268338	43
4	1 1 0 1 1	446,2	716	439246	268338	43
4	1 1 1 0 0	518,3	995	190964,5	326715	14
4	1 1 1 0 1	518,3	995	190964,5	326715	14
4	1 1 1 1 0	446,2	945	439246	268338	43
4	1 1 1 1 1	446,2	945	439246	268338	43
4	2 0 0 0 0	445,4	663	528847,7	279976	38
4	2 0 0 0 1	445,4	663	528847,7	279976	38
4	2 0 0 1 0	435,6	569	487569,8	279997	36
4	2 0 0 1 1	435,6	569	487569,8	279997	36
4	2 0 1 0 0	445,4	902	528847,7	279976	38
4	2 0 1 0 1	445,4	902	528847,7	279976	38
4	2 0 1 1 0	435,6	855	487569,8	279997	36
4	2 0 1 1 1	435,6	855	487569,8	279997	36
4	2 1 0 0 0	437,5	647	517685,6	256528	38
4	2 1 0 0 1	437,5	647	517685,6	256528	38
4	2 1 0 1 0	438,4	569	478061	279997	36
4	2 1 0 1 1	438,4	569	478061	279997	36
4	2 1 1 0 0	437,5	879	517685,6	256528	38
4	2 1 1 0 1	437,5	879	517685,6	256528	38
4	2 1 1 1 0	438,4	855	478061	279997	36
4	2 1 1 1 1	438,4	855	478061	279997	36
5	0 0 0 0 0	441,9	488	467932,7	268023	42
5	0 0 0 0 1	441,9	488	467932,7	268023	42
5	0 0 0 1 0	445,5	583	505726,1	267357	40
5	0 0 0 1 1	445,5	583	505726,1	267357	40
5	0 0 1 0 0	441,9	679	467932,7	268023	42

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
5	0 0 1 0 1	441,9	679	467932,7	268023	42
5	0 0 1 1 0	445,5	774	505726,1	267357	40
5	0 0 1 1 1	445,5	774	505726,1	267357	40
5	0 1 0 0 0	444,8	488	459262,9	268023	41
5	0 1 0 0 1	444,8	488	459262,9	268023	41
5	0 1 0 1 0	470,8	584	486770,2	255711	41
5	0 1 0 1 1	470,8	584	486770,2	255711	41
5	0 1 1 0 0	444,8	679	459262,9	268023	41
5	0 1 1 0 1	444,8	679	459262,9	268023	41
5	0 1 1 1 0	470,8	858	486770,2	255711	41
5	0 1 1 1 1	470,8	858	486770,2	255711	41
5	1 0 0 0 0	447,2	566	422919,7	244824	36
5	1 0 0 0 1	447,2	566	422919,7	244824	36
5	1 0 0 1 0	440,5	664	488179,9	255711	36
5	1 0 0 1 1	440,5	664	488179,9	255711	36
5	1 0 1 0 0	447,2	769	422919,7	244824	36
5	1 0 1 0 1	447,2	769	422919,7	244824	36
5	1 0 1 1 0	440,5	936	488179,9	255711	36
5	1 0 1 1 1	440,5	936	488179,9	255711	36
5	1 1 0 0 0	445,6	623	399027	221557	36
5	1 1 0 0 1	445,6	623	399027	221557	36
5	1 1 0 1 0	449	664	495931,7	255711	38
5	1 1 0 1 1	449	664	495931,7	255711	38
5	1 1 1 0 0	445,6	874	399027	221557	36
5	1 1 1 0 1	445,6	874	399027	221557	36
5	1 1 1 1 0	449	936	495931,7	255711	38
5	1 1 1 1 1	449	936	495931,7	255711	38
5	2 0 0 0 0	439,1	526	468494,3	268023	48
5	2 0 0 0 1	439,1	526	468494,3	268023	48
5	2 0 0 1 0	436,1	637	501376,2	267357	33
5	2 0 0 1 1	436,1	637	501376,2	267357	33
5	2 0 1 0 0	439,1	748	468494,3	268023	48
5	2 0 1 0 1	439,1	748	468494,3	268023	48
5	2 0 1 1 0	436,1	885	501376,2	267357	33

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
5	2 0 1 1 1	436,1	885	501376,2	267357	33
5	2 1 0 0 0	443,2	526	460236,7	268023	49
5	2 1 0 0 1	443,2	526	460236,7	268023	49
5	2 1 0 1 0	440,2	573	495184,8	267357	36
5	2 1 0 1 1	440,2	573	495184,8	267357	36
5	2 1 1 0 0	443,2	748	460236,7	268023	49
5	2 1 1 0 1	443,2	748	460236,7	268023	49
5	2 1 1 1 0	440,2	859	495184,8	267357	36
5	2 1 1 1 1	440,2	859	495184,8	267357	36
6	0 0 0 0 0	450,3	505	477277,9	267610	43
6	0 0 0 0 1	450,3	505	477277,9	267610	43
6	0 0 0 1 0	440,8	537	482500,1	279793	40
6	0 0 0 1 1	440,8	537	482500,1	279793	40
6	0 0 1 0 0	450,3	768	477277,9	267610	43
6	0 0 1 0 1	450,3	768	477277,9	267610	43
6	0 0 1 1 0	440,8	842	482500,1	279793	40
6	0 0 1 1 1	440,8	842	482500,1	279793	40
6	0 1 0 0 0	449,7	505	475319,8	279231	42
6	0 1 0 0 1	449,7	505	475319,8	279231	42
6	0 1 0 1 0	449,2	537	483792,9	291480	38
6	0 1 0 1 1	449,2	537	483792,9	291480	38
6	0 1 1 0 0	449,7	768	475319,8	279231	42
6	0 1 1 0 1	449,7	768	475319,8	279231	42
6	0 1 1 1 0	449,2	842	483792,9	291480	38
6	0 1 1 1 1	449,2	842	483792,9	291480	38
6	1 0 0 0 0	446,4	617	485585,9	290837	39
6	1 0 0 0 1	446,4	617	485585,9	290837	39
6	1 0 0 1 0	443,2	645	497404,4	279793	42
6	1 0 0 1 1	443,2	645	497404,4	279793	42
6	1 0 1 0 0	446,4	837	485585,9	290837	39
6	1 0 1 0 1	446,4	837	485585,9	290837	39
6	1 0 1 1 0	443,2	931	497404,4	279793	42
6	1 0 1 1 1	443,2	931	497404,4	279793	42
6	1 1 0 0 0	446,5	617	451456,3	255875	34

Seed	(ABCDE)	F - F1	F - F2	F - F3	F - F4	F - F5
6	1 1 0 0 1	446,5	617	451456,3	255875	34
6	1 1 0 1 0	446,4	645	505194,6	279793	45
6	1 1 0 1 1	446,4	645	505194,6	279793	45
6	1 1 1 0 0	446,5	837	451456,3	255875	34
6	1 1 1 0 1	446,5	837	451456,3	255875	34
6	1 1 1 1 0	446,4	931	505194,6	279793	45
6	1 1 1 1 1	446,4	931	505194,6	279793	45
6	2 0 0 0 0	435	528	483389,3	267610	46
6	2 0 0 0 1	435	528	483389,3	267610	46
6	2 0 0 1 0	440,5	586	485594,9	279793	40
6	2 0 0 1 1	440,5	586	485594,9	279793	40
6	2 0 1 0 0	435	798	483389,3	267610	46
6	2 0 1 0 1	435	798	483389,3	267610	46
6	2 0 1 1 0	440,5	879	485594,9	279793	40
6	2 0 1 1 1	440,5	879	485594,9	279793	40
6	2 1 0 0 0	446,7	528	486656,3	279237	41
6	2 1 0 0 1	446,7	528	486656,3	279237	41
6	2 1 0 1 0	441,2	586	491437,9	279793	37
6	2 1 0 1 1	441,2	586	491437,9	279793	37
6	2 1 1 0 0	446,7	798	486656,3	279237	41
6	2 1 1 0 1	446,7	798	486656,3	279237	41
6	2 1 1 1 0	441,2	879	491437,9	279793	37
6	2 1 1 1 1	441,2	879	491437,9	279793	37

Appendix H

Global Minimum Objective Values

Seed	(ABCDE)	MIN1	MIN2	MIN3	MIN4	MIN5
1	0 0 0 0 0	312,5	414	201561,5	267966	14
1	0 0 0 0 1	312,5	414	201561,5	267966	14
1	0 0 0 1 0	439,7	488	222134,7	279151	9
1	0 0 0 1 1	439,7	488	222134,7	279151	9
1	0 0 1 0 0	15,7	621	201561,5	267966	0
1	0 0 1 0 1	15,7	621	201561,5	267966	0
1	0 0 1 1 0	149,2	578	222134,7	279151	0
1	0 0 1 1 1	149,2	578	222134,7	279151	0
1	0 1 0 0 0	321,3	499	185917,6	244675	18
1	0 1 0 0 1	321,3	499	185917,6	244675	18
1	0 1 0 1 0	442,7	488	210368,3	232557	10
1	0 1 0 1 1	442,7	488	210368,3	232557	10
1	0 1 1 0 0	35,1	718	185917,6	244675	0

Seed	(ABCDE)	MIN1	MIN2	MIN3	MIN4	MIN5
1	0 1 1 0 1	35,1	718	185917,6	244675	0
1	0 1 1 1 0	175,7	578	210368,3	232557	0
1	0 1 1 1 1	175,7	578	210368,3	232557	0
1	1 0 0 0 0	326,6	546	216549,2	267989	7
1	1 0 0 0 1	326,6	546	216549,2	267989	7
1	1 0 0 1 0	134	479	225283,1	255836	5
1	1 0 0 1 1	134	479	225283,1	255836	5
1	1 0 1 0 0	27,8	626	216549,2	267989	0
1	1 0 1 0 1	27,8	626	216549,2	267989	0
1	1 0 1 1 0	8,9	644	225283,1	255836	0
1	1 0 1 1 1	8,9	644	225283,1	255836	0
1	1 1 0 0 0	103,5	520	234467,8	267890	2
1	1 1 0 0 1	94,8	546	234467,8	267890	2
1	1 1 0 1 0	142,7	479	236565	267451	5
1	1 1 0 1 1	142,7	479	236565	267451	5
1	1 1 1 0 0	29,1	689	234467,8	267890	0
1	1 1 1 0 1	29,1	689	234467,8	267890	0
1	1 1 1 1 0	6,8	644	236565	267451	1
1	1 1 1 1 1	6,8	644	236565	267451	1
1	2 0 0 0 0	223,7	406	224194,7	279646	4
1	2 0 0 0 1	223,7	406	224194,7	279646	4
1	2 0 0 1 0	170	370	235168	279151	4
1	2 0 0 1 1	170	370	235168	279151	4
1	2 0 1 0 0	31,8	703	224194,7	279646	0
1	2 0 1 0 1	31,8	703	224194,7	279646	0
1	2 0 1 1 0	5,2	671	235168	279151	0
1	2 0 1 1 1	5,2	671	235168	279151	0
1	2 1 0 0 0	183,7	527	211819,6	279646	5
1	2 1 0 0 1	183,7	527	211819,6	279646	5
1	2 1 0 1 0	172,2	370	243869,8	279151	4
1	2 1 0 1 1	172,2	370	243869,8	279151	4
1	2 1 1 0 0	39,3	655	211819,6	279646	0
1	2 1 1 0 1	39,3	655	211819,6	279646	0
1	2 1 1 1 0	7,8	671	243869,8	279151	0

Seed	(ABCDE)	MIN1	MIN2	MIN3	MIN4	MIN5
1	2 1 1 1 1	7,8	671	243869,8	279151	0
3	0 0 0 0 0	210,1	509	216389,6	256254	7
3	0 0 0 0 1	210,1	509	216389,6	256254	7
3	0 0 0 1 0	140,2	471	213446,2	268494	9
3	0 0 0 1 1	140,2	471	213446,2	268494	9
3	0 0 1 0 0	8,8	684	216389,6	256254	0
3	0 0 1 0 1	9	684	216389,6	256254	0
3	0 0 1 1 0	44,9	567	213446,2	268494	0
3	0 0 1 1 1	44,9	567	213446,2	268494	0
3	0 1 0 0 0	231,1	509	222683,5	256254	7
3	0 1 0 0 1	231,1	509	222683,5	256254	7
3	0 1 0 1 0	377,8	485	225249,9	268494	14
3	0 1 0 1 1	377,8	485	225249,9	268494	14
3	0 1 1 0 0	8,5	684	222683,5	256254	0
3	0 1 1 0 1	9,5	684	222683,5	256254	0
3	0 1 1 1 0	253,4	535	225249,9	268494	0
3	0 1 1 1 1	253,4	535	225249,9	268494	0
3	1 0 0 0 0	9	679	233639,5	267834	0
3	1 0 0 0 1	9	679	233639,5	267834	0
3	1 0 0 1 0	336,6	508	210562,7	268494	13
3	1 0 0 1 1	336,6	508	210562,7	268494	13
3	1 0 1 0 0	8,2	619	233639,5	267834	0
3	1 0 1 0 1	9,2	619	233639,5	267834	0
3	1 0 1 1 0	4,5	608	210562,7	268494	0
3	1 0 1 1 1	4,5	608	210562,7	268494	0
3	1 1 0 0 0	7,7	679	240693,5	256254	0
3	1 1 0 0 1	7,7	679	240693,5	256254	0
3	1 1 0 1 0	338,6	508	208946,6	268470	17
3	1 1 0 1 1	338,6	508	208946,6	268470	17
3	1 1 1 0 0	9,8	619	240693,5	256254	0
3	1 1 1 0 1	9,8	619	240693,5	256254	0
3	1 1 1 1 0	3	608	208946,6	268470	1
3	1 1 1 1 1	3	608	208946,6	268470	1
3	2 0 0 0 0	158,5	621	222621,1	256254	0

Seed	(ABCDE)	MIN1	MIN2	MIN3	MIN4	MIN5
3	2 0 0 0 1	158,5	621	222621,1	256254	0
3	2 0 0 1 0	77	513	210419,8	268494	2
3	2 0 0 1 1	68	517	210419,8	268494	2
3	2 0 1 0 0	10,6	672	222621,1	256254	0
3	2 0 1 0 1	10,6	672	222621,1	256254	0
3	2 0 1 1 0	71,3	475	210419,8	268494	0
3	2 0 1 1 1	71,3	475	210419,8	268494	0
3	2 1 0 0 0	159	621	233938,7	267911	0
3	2 1 0 0 1	159	621	233938,7	267911	0
3	2 1 0 1 0	376,8	506	218115,8	280176	10
3	2 1 0 1 1	376,8	506	218115,8	280176	10
3	2 1 1 0 0	10,2	672	233938,7	267911	0
3	2 1 1 0 1	10,2	672	233938,7	267911	0
3	2 1 1 1 0	172	375	218115,8	280176	1
3	2 1 1 1 1	172	375	218115,8	280176	1
4	0 0 0 0 0	205,7	629	210919,6	256528	0
4	0 0 0 0 1	205,7	629	210919,6	256528	0
4	0 0 0 1 0	291	535	205351,8	279997	5
4	0 0 0 1 1	291	535	205351,8	279997	5
4	0 0 1 0 0	5	704	210919,6	256528	0
4	0 0 1 0 1	5	704	210919,6	256528	0
4	0 0 1 1 0	102,9	592	205351,8	279997	0
4	0 0 1 1 1	102,9	592	205351,8	279997	0
4	0 1 0 0 0	181,9	629	202865	256528	0
4	0 1 0 0 1	181,9	629	202865	256528	0
4	0 1 0 1 0	251,2	397	192375,2	268338	13
4	0 1 0 1 1	251,2	397	192375,2	268338	13
4	0 1 1 0 0	5	704	202865	256528	0
4	0 1 1 0 1	5	704	202865	256528	0
4	0 1 1 1 0	3,5	656	192375,2	268338	0
4	0 1 1 1 1	5,5	656	192375,2	268338	0
4	1 0 0 0 0	7,9	754	200071,9	268252	0
4	1 0 0 0 1	7,9	754	200071,9	268252	0
4	1 0 0 1 0	55,5	622	219567,6	279997	2

Seed	(ABCDE)	MIN1	MIN2	MIN3	MIN4	MIN5
4	1 0 0 1 1	66,5	622	219567,6	279997	2
4	1 0 1 0 0	5,4	565	200071,9	268252	0
4	1 0 1 0 1	5,4	565	200071,9	268252	0
4	1 0 1 1 0	69,5	459	219567,6	279997	0
4	1 0 1 1 1	70,8	459	219567,6	279997	0
4	1 1 0 0 0	8,8	754	190964,5	256628	0
4	1 1 0 0 1	9,4	754	190964,5	256628	0
4	1 1 0 1 0	63,3	564	206924,8	268338	2
4	1 1 0 1 1	73,8	538	206924,8	268338	2
4	1 1 1 0 0	5,2	565	190964,5	256628	0
4	1 1 1 0 1	6,5	565	190964,5	256628	0
4	1 1 1 1 0	71,3	459	206924,8	268338	0
4	1 1 1 1 1	71,3	459	206924,8	268338	0
4	2 0 0 0 0	135,7	443	222987,4	279976	2
4	2 0 0 0 1	135,7	443	222987,4	279976	2
4	2 0 0 1 0	130,5	569	237086	279997	1
4	2 0 0 1 1	134,6	569	237086	279997	1
4	2 0 1 0 0	21,7	606	222987,4	279976	0
4	2 0 1 0 1	21,7	606	222987,4	279976	0
4	2 0 1 1 0	70	547	237086	279997	0
4	2 0 1 1 1	70	547	237086	279997	0
4	2 1 0 0 0	205,4	647	223594,2	256528	0
4	2 1 0 0 1	205,4	647	223594,2	256528	0
4	2 1 0 1 0	140,6	569	229151,2	279997	2
4	2 1 0 1 1	138,2	569	229151,2	279997	2
4	2 1 1 0 0	27,7	619	223594,2	256528	0
4	2 1 1 0 1	27,7	619	223594,2	256528	0
4	2 1 1 1 0	70	547	229151,2	279997	0
4	2 1 1 1 1	70	547	229151,2	279997	0
5	0 0 0 0 0	253,5	402	221801	268023	9
5	0 0 0 0 1	253,5	402	221801	268023	9
5	0 0 0 1 0	229	583	250656,6	267357	0
5	0 0 0 1 1	229	583	250656,6	267357	0
5	0 0 1 0 0	141,1	667	221801	268023	0

Seed	(ABCDE)	MIN1	MIN2	MIN3	MIN4	MIN5
5	0 0 1 0 1	141,1	667	221801	268023	0
5	0 0 1 1 0	8,6	688	250656,6	267357	0
5	0 0 1 1 1	8,6	688	250656,6	267357	0
5	0 1 0 0 0	268	422	213992,5	268023	13
5	0 1 0 0 1	268	422	213992,5	268023	13
5	0 1 0 1 0	130,7	584	266576,1	255711	2
5	0 1 0 1 1	130,7	584	266576,1	255711	2
5	0 1 1 0 0	140,1	667	213992,5	268023	0
5	0 1 1 0 1	140,1	667	213992,5	268023	0
5	0 1 1 1 0	4,3	616	266576,1	255711	0
5	0 1 1 1 1	4,3	616	266576,1	255711	0
5	1 0 0 0 0	230	480	184982,9	244824	13
5	1 0 0 0 1	230	480	184982,9	244824	13
5	1 0 0 1 0	55,3	628	244127,9	255711	0
5	1 0 0 1 1	55,3	628	244127,9	255711	0
5	1 0 1 0 0	20,5	751	184982,9	244824	0
5	1 0 1 0 1	20,5	751	184982,9	244824	0
5	1 0 1 1 0	1,5	568	244127,9	255711	0
5	1 0 1 1 1	1,5	568	244127,9	255711	0
5	1 1 0 0 0	130	623	160901,7	221557	0
5	1 1 0 0 1	130	623	160901,7	221557	0
5	1 1 0 1 0	53,1	662	247908,2	255711	0
5	1 1 0 1 1	53,5	636	247908,2	255711	0
5	1 1 1 0 0	26,4	658	160901,7	221557	0
5	1 1 1 0 1	26,4	658	160901,7	221557	0
5	1 1 1 1 0	2,2	568	247908,2	255711	0
5	1 1 1 1 1	2,2	568	247908,2	255711	0
5	2 0 0 0 0	199,1	526	217759,7	268023	6
5	2 0 0 0 1	199,1	526	217759,7	268023	6
5	2 0 0 1 0	66,6	637	231310,1	267357	0
5	2 0 0 1 1	66,6	637	231310,1	267357	0
5	2 0 1 0 0	20,1	748	217759,7	268023	0
5	2 0 1 0 1	20,1	748	217759,7	268023	0
5	2 0 1 1 0	5,5	595	231310,1	267357	0

Seed	(ABCDE)	MIN1	MIN2	MIN3	MIN4	MIN5
5	2 0 1 1 1	5,5	595	231310,1	267357	0
5	2 1 0 0 0	181,1	526	204932,9	268023	7
5	2 1 0 0 1	181,1	526	204932,9	268023	7
5	2 1 0 1 0	116,9	561	226519,2	267357	7
5	2 1 0 1 1	116,9	561	226519,2	267357	7
5	2 1 1 0 0	20,9	748	204932,9	268023	1
5	2 1 1 0 1	19,4	748	204932,9	268023	1
5	2 1 1 1 0	8,2	635	226519,2	267357	0
5	2 1 1 1 1	7,9	635	226519,2	267357	0
6	0 0 0 0 0	182,2	473	219614,1	267610	8
6	0 0 0 0 1	182,2	473	219614,1	267610	8
6	0 0 0 1 0	414,6	503	223758,3	279793	5
6	0 0 0 1 1	414,6	503	223758,3	279793	5
6	0 0 1 0 0	62,4	570	219614,1	267610	0
6	0 0 1 0 1	62,4	570	219614,1	267610	0
6	0 0 1 1 0	127,7	480	223758,3	279793	0
6	0 0 1 1 1	127,7	480	223758,3	279793	0
6	0 1 0 0 0	230,4	485	214866,3	279231	15
6	0 1 0 0 1	230,4	485	214866,3	279231	15
6	0 1 0 1 0	418,5	503	220279	291480	11
6	0 1 0 1 1	418,5	503	220279	291480	11
6	0 1 1 0 0	46,5	634	214866,3	279231	0
6	0 1 1 0 1	46,5	634	214866,3	279231	0
6	0 1 1 1 0	126,6	480	220279	291480	0
6	0 1 1 1 1	127,6	480	220279	291480	0
6	1 0 0 0 0	112	617	218264,4	290837	0
6	1 0 0 0 1	112	617	218264,4	290837	0
6	1 0 0 1 0	151,9	421	210119	279793	2
6	1 0 0 1 1	163,7	421	210119	279793	2
6	1 0 1 0 0	63,4	555	218264,4	290837	0
6	1 0 1 0 1	63,4	555	218264,4	290837	0
6	1 0 1 1 0	22,3	457	210119	279793	0
6	1 0 1 1 1	22,3	457	210119	279793	0
6	1 1 0 0 0	92,9	617	216777,2	255875	0

Seed	(ABCDE)	MIN1	MIN2	MIN3	MIN4	MIN5
6	1 1 0 0 1	92,9	617	216777,2	255875	0
6	1 1 0 1 0	166,2	421	205047,1	279793	2
6	1 1 0 1 1	166,2	421	205047,1	279793	2
6	1 1 1 0 0	49,6	565	216777,2	255875	0
6	1 1 1 0 1	49,6	565	216777,2	255875	0
6	1 1 1 1 0	22,8	493	205047,1	279793	0
6	1 1 1 1 1	22,8	493	205047,1	279793	0
6	2 0 0 0 0	204,1	514	228665,5	267610	6
6	2 0 0 0 1	204,1	514	228665,5	267610	6
6	2 0 0 1 0	125,9	586	222672,9	279793	3
6	2 0 0 1 1	124	586	222672,9	279793	3
6	2 0 1 0 0	65	608	228665,5	267610	0
6	2 0 1 0 1	65	608	228665,5	267610	0
6	2 0 1 1 0	97,7	383	222672,9	279793	1
6	2 0 1 1 1	97,7	383	222672,9	279793	1
6	2 1 0 0 0	202,9	506	232462,5	279237	7
6	2 1 0 0 1	202,9	506	232462,5	279237	7
6	2 1 0 1 0	104,4	586	219296,1	279793	3
6	2 1 0 1 1	104,4	586	219296,1	279793	3
6	2 1 1 0 0	76,1	546	232462,5	279237	0
6	2 1 1 0 1	76,1	546	232462,5	279237	0
6	2 1 1 1 0	98,1	385	219296,1	279793	1
6	2 1 1 1 1	98,1	385	219296,1	279793	1

Appendix I

CPU Times

		Hybrid Algorithm			Dedicated Algorithm		
Seed	(ABCDE)	Initial	b=3	b=6	Initial	b=3	b=6
1	0 0 0 0 0	6,8	325,5	619,5	6,4	114,5	230,7
1	0 0 0 0 1	6,8	322,5	615,3	6,3	114,7	228,5
1	0 0 0 1 0	6,6	218,8	424,8	6,3	111,3	218,6
1	0 0 0 1 1	6,7	218,6	423,0	6,2	110,6	219,6
1	0 0 1 0 0	6,1	84,9	156,7	6,3	113,9	228,2
1	0 0 1 0 1	6,0	84,4	183,3	6,3	113,7	227,7
1	0 0 1 1 0	6,2	41,2	73,2	6,3	110,9	219,3
1	0 0 1 1 1	6,1	41,0	73,1	6,1	110,8	219,6
1	0 1 0 0 0	7,1	250,9	524,4	6,7	61,1	118,3
1	0 1 0 0 1	7,0	249,7	528,8	6,6	60,9	118,5
1	0 1 0 1 0	6,7	199,5	417,3	6,4	93,3	158,2
1	0 1 0 1 1	6,8	199,4	419,1	6,4	92,8	159,1
1	0 1 1 0 0	6,0	52,2	101,3	6,7	61,0	118,8
1	0 1 1 0 1	6,1	52,3	101,1	6,8	61,0	118,3
1	0 1 1 1 0	6,1	47,1	88,0	6,3	92,7	156,8
1	0 1 1 1 1	6,1	47,0	87,4	6,3	93,2	158,2
1	1 0 0 0 0	6,4	58,3	114,0	6,4	105,6	205,3

		Hybrid Algorithm			Dedicated Algorithm		
Seed	(ABCDE)	Initial	b=3	b=6	Initial	b=3	b=6
1	1 0 0 0 1	6,5	58,2	114,2	6,4	105,5	206,0
1	1 0 0 1 0	6,3	189,4	355,7	6,3	101,9	189,7
1	1 0 0 1 1	6,3	190,7	358,3	6,3	101,4	189,8
1	1 0 1 0 0	6,0	47,9	93,5	6,5	104,9	205,1
1	1 0 1 0 1	6,1	48,1	94,0	6,5	104,6	206,7
1	1 0 1 1 0	6,0	82,9	161,0	6,4	102,0	189,4
1	1 0 1 1 1	5,9	83,4	160,9	6,4	102,1	189,6
1	1 1 0 0 0	6,1	108,2	218,3	6,4	80,7	153,0
1	1 1 0 0 1	6,2	112,9	201,5	6,4	80,4	152,6
1	1 1 0 1 0	6,5	188,0	366,5	6,3	97,1	170,7
1	1 1 0 1 1	6,5	187,5	366,9	6,3	96,5	170,5
1	1 1 1 0 0	6,0	47,1	86,8	6,3	80,1	152,2
1	1 1 1 0 1	6,1	47,2	87,5	6,4	80,4	152,4
1	1 1 1 1 0	6,1	147,9	306,9	6,3	97,1	170,4
1	1 1 1 1 1	6,2	148,6	308,2	6,3	97,1	171,4
1	2 0 0 0 0	6,4	202,6	378,2	6,3	116,1	217,9
1	2 0 0 0 1	6,5	202,6	374,5	6,3	116,3	217,7
1	2 0 0 1 0	6,4	199,4	360,5	6,1	151,9	295,6
1	2 0 0 1 1	6,3	198,6	359,0	6,1	151,6	294,5
1	2 0 1 0 0	6,1	59,7	105,3	6,2	116,5	217,6
1	2 0 1 0 1	5,9	59,5	105,9	6,3	116,0	217,3
1	2 0 1 1 0	6,0	213,7	155,8	6,1	151,7	294,7
1	2 0 1 1 1	5,9	215,5	155,8	6,1	152,0	296,2
1	2 1 0 0 0	6,5	203,5	369,2	6,3	101,2	278,5
1	2 1 0 0 1	6,4	204,0	369,2	6,3	100,9	279,5
1	2 1 0 1 0	6,3	195,9	376,5	6,1	131,1	260,0
1	2 1 0 1 1	6,4	196,2	376,5	6,1	130,9	258,0
1	2 1 1 0 0	6,1	37,1	156,8	6,3	101,1	279,1
1	2 1 1 0 1	6,1	37,2	157,6	6,3	100,7	279,4
1	2 1 1 1 0	5,9	77,1	151,7	6,1	130,3	259,4
1	2 1 1 1 1	5,9	76,7	151,7	6,0	131,0	259,4

		Hybrid Algorithm			Dedicated Algorithm		
Seed	(ABCDE)	Initial	b=3	b=6	Initial	b=3	b=6
3	0 0 0 0 0	6,4	63,8	99,6	6,2	123,3	221,9
3	0 0 0 0 1	6,4	63,6	100,0	6,3	123,1	221,5
3	0 0 0 1 0	6,6	156,2	309,8	6,3	88,4	169,6
3	0 0 0 1 1	6,5	156,9	309,7	6,3	88,4	169,8
3	0 0 1 0 0	5,9	67,9	127,4	6,2	123,9	221,5
3	0 0 1 0 1	6,0	69,5	126,7	6,1	123,2	221,2
3	0 0 1 1 0	6,2	57,4	113,3	6,3	88,9	170,0
3	0 0 1 1 1	6,2	57,3	113,1	6,3	88,8	169,9
3	0 1 0 0 0	6,4	71,3	141,2	6,3	110,1	218,4
3	0 1 0 0 1	6,3	71,5	141,2	6,2	109,6	218,3
3	0 1 0 1 0	7,2	142,2	275,5	6,5	68,0	125,3
3	0 1 0 1 1	7,1	142,3	275,6	6,5	67,7	125,2
3	0 1 1 0 0	6,0	81,9	242,2	6,4	110,2	219,9
3	0 1 1 0 1	6,0	80,7	241,8	6,3	110,7	219,4
3	0 1 1 1 0	7,2	29,2	46,4	6,5	68,2	125,3
3	0 1 1 1 1	7,1	29,2	46,6	6,4	67,9	125,6
3	1 0 0 0 0	6,1	58,8	123,6	6,3	96,8	187,8
3	1 0 0 0 1	5,9	57,3	122,7	6,4	96,8	187,9
3	1 0 0 1 0	7,0	87,8	160,2	6,5	85,2	162,2
3	1 0 0 1 1	7,0	88,2	160,2	6,4	85,5	162,0
3	1 0 1 0 0	6,0	74,6	152,5	6,2	97,0	188,1
3	1 0 1 0 1	6,1	74,7	274,7	6,3	96,8	188,0
3	1 0 1 1 0	6,1	65,8	208,1	6,4	85,5	162,4
3	1 0 1 1 1	6,1	79,8	297,5	6,4	85,4	162,6
3	1 1 0 0 0	6,0	67,3	123,8	6,4	95,8	185,2
3	1 1 0 0 1	6,0	67,1	124,0	6,4	95,5	184,8
3	1 1 0 1 0	7,5	51,7	240,3	6,6	116,5	95,3
3	1 1 0 1 1	7,3	51,5	241,7	6,6	116,9	95,0
3	1 1 1 0 0	6,2	83,7	146,2	6,5	95,9	185,5
3	1 1 1 0 1	6,0	81,8	142,1	6,4	95,7	185,0
3	1 1 1 1 0	6,5	72,5	162,3	6,6	117,1	95,1
3	1 1 1 1 1	6,4	72,9	164,6	6,7	117,3	95,1

		Hybrid Algorithm			Dedicated Algorithm		
Seed	(ABCDE)	Initial	b=3	b=6	Initial	b=3	b=6
3	2 0 0 0 0	6,2	15,4	25,2	6,1	144,1	283,7
3	2 0 0 0 1	6,2	15,3	25,4	6,2	143,1	283,2
3	2 0 0 1 0	6,2	81,9	152,7	6,2	99,2	195,9
3	2 0 0 1 1	6,2	81,8	166,5	6,2	99,0	195,9
3	2 0 1 0 0	6,0	57,6	105,5	6,1	142,4	283,7
3	2 0 1 0 1	5,9	57,6	106,4	6,1	141,5	283,5
3	2 0 1 1 0	6,1	58,7	110,2	6,3	98,9	196,7
3	2 0 1 1 1	6,0	58,3	110,0	6,2	99,3	197,0
3	2 1 0 0 0	6,3	9,2	14,9	6,2	123,5	213,0
3	2 1 0 0 1	6,2	9,0	15,1	6,2	123,5	212,6
3	2 1 0 1 0	6,9	132,4	256,6	6,3	76,5	151,7
3	2 1 0 1 1	7,0	131,5	255,8	6,3	76,4	151,7
3	2 1 1 0 0	6,0	59,2	118,5	6,1	123,2	212,3
3	2 1 1 0 1	5,9	59,4	115,9	6,2	122,5	213,2
3	2 1 1 1 0	6,5	101,4	201,3	6,2	75,9	151,4
3	2 1 1 1 1	6,4	101,5	201,7	6,4	76,5	151,2
4	0 0 0 0 0	6,0	32,0	59,7	6,2	132,5	295,7
4	0 0 0 0 1	6,1	32,1	59,8	6,2	132,4	295,9
4	0 0 0 1 0	6,2	49,2	297,9	6,1	130,9	243,6
4	0 0 0 1 1	6,2	49,0	296,5	6,1	130,5	244,1
4	0 0 1 0 0	6,0	95,2	216,4	6,1	131,4	296,2
4	0 0 1 0 1	5,9	171,9	221,8	6,1	131,3	296,1
4	0 0 1 1 0	6,2	59,3	134,6	6,1	130,1	243,0
4	0 0 1 1 1	6,2	59,5	135,3	6,2	130,6	244,0
4	0 1 0 0 0	6,3	31,9	160,9	6,4	81,7	159,5
4	0 1 0 0 1	6,4	32,0	158,5	6,5	81,6	159,5
4	0 1 0 1 0	7,1	137,1	284,1	6,3	83,4	169,7
4	0 1 0 1 1	7,2	137,6	284,6	6,3	82,9	169,0
4	0 1 1 0 0	6,0	102,3	231,8	6,5	81,6	159,0
4	0 1 1 0 1	5,9	98,5	230,6	6,5	81,5	159,6
4	0 1 1 1 0	6,1	180,8	212,7	6,3	83,0	170,0
4	0 1 1 1 1	6,1	180,7	273,1	6,3	83,1	169,9

		Hybrid Algorithm			Dedicated Algorithm		
Seed	(ABCDE)	Initial	b=3	b=6	Initial	b=3	b=6
4	1 0 0 0 0	6,2	77,0	162,8	6,6	62,6	108,1
4	1 0 0 0 1	6,3	77,1	157,3	6,6	62,4	107,9
4	1 0 0 1 0	6,2	50,0	250,3	6,4	94,7	185,1
4	1 0 0 1 1	6,1	49,4	98,9	6,3	94,5	185,1
4	1 0 1 0 0	5,9	91,8	202,4	6,7	63,0	108,0
4	1 0 1 0 1	5,8	95,8	202,7	6,6	62,5	107,6
4	1 0 1 1 0	5,9	58,2	113,2	6,4	94,8	185,2
4	1 0 1 1 1	5,8	57,9	113,0	6,4	94,8	185,9
4	1 1 0 0 0	6,1	117,2	250,5	6,8	45,6	72,6
4	1 1 0 0 1	6,3	117,4	171,5	6,8	45,7	72,5
4	1 1 0 1 0	6,1	57,6	111,4	6,4	74,2	143,2
4	1 1 0 1 1	6,0	57,8	111,2	6,5	74,5	144,0
4	1 1 1 0 0	5,8	99,5	203,8	6,8	45,9	72,8
4	1 1 1 0 1	5,8	99,7	279,2	6,9	46,0	72,6
4	1 1 1 1 0	5,9	62,1	122,3	6,4	74,8	144,1
4	1 1 1 1 1	5,8	61,6	122,2	6,4	74,3	144,4
4	2 0 0 0 0	6,2	125,1	338,3	6,0	161,8	284,2
4	2 0 0 0 1	6,1	127,5	339,0	5,8	162,6	284,3
4	2 0 0 1 0	6,2	56,9	116,8	5,9	168,7	329,4
4	2 0 0 1 1	6,1	56,3	106,8	5,9	168,6	329,4
4	2 0 1 0 0	5,9	45,5	80,6	5,9	162,3	282,5
4	2 0 1 0 1	6,0	45,6	80,7	6,0	162,2	283,0
4	2 0 1 1 0	5,7	49,3	97,7	6,0	169,1	330,2
4	2 0 1 1 1	5,8	49,3	98,0	5,9	168,7	329,0
4	2 1 0 0 0	6,1	15,0	27,0	6,0	159,6	282,0
4	2 1 0 0 1	6,1	14,9	27,0	6,0	159,7	283,2
4	2 1 0 1 0	6,2	61,3	118,3	5,9	162,6	314,3
4	2 1 0 1 1	6,2	61,9	118,3	5,9	162,2	313,3
4	2 1 1 0 0	6,1	79,4	166,1	6,0	159,7	282,2
4	2 1 1 0 1	6,1	79,3	166,0	6,0	160,1	283,3
4	2 1 1 1 0	5,8	48,1	96,3	5,8	162,5	315,3
4	2 1 1 1 1	5,8	48,6	95,3	5,9	162,3	314,5

		Hybrid Algorithm			Dedicated Algorithm		
Seed	(ABCDE)	Initial	b=3	b=6	Initial	b=3	b=6
5	0 0 0 0 0	6,8	179,7	345,5	6,2	103,6	245,3
5	0 0 0 0 1	6,7	179,0	349,6	6,1	103,5	245,1
5	0 0 0 1 0	6,1	14,0	23,4	6,2	105,2	201,3
5	0 0 0 1 1	6,1	14,0	23,2	6,2	105,0	201,4
5	0 0 1 0 0	6,1	68,2	131,5	6,2	103,2	245,4
5	0 0 1 0 1	6,2	68,3	131,5	6,1	103,0	244,9
5	0 0 1 1 0	5,8	47,7	88,6	6,0	104,7	201,1
5	0 0 1 1 1	5,9	47,8	88,6	6,1	104,9	200,9
5	0 1 0 0 0	7,2	101,1	251,4	6,3	107,9	204,6
5	0 1 0 0 1	7,3	101,2	251,3	6,4	108,2	204,3
5	0 1 0 1 0	6,5	57,0	112,6	6,4	64,1	123,3
5	0 1 0 1 1	6,5	56,8	113,1	6,4	64,2	123,7
5	0 1 1 0 0	6,3	89,5	163,5	6,4	108,0	204,1
5	0 1 1 0 1	6,3	88,4	163,5	6,3	107,9	204,3
5	0 1 1 1 0	5,9	135,7	214,2	6,4	64,1	123,6
5	0 1 1 1 1	6,0	136,1	213,5	6,4	64,0	123,6
5	1 0 0 0 0	6,9	126,0	366,5	6,7	64,8	125,4
5	1 0 0 0 1	7,0	126,0	366,0	6,8	64,7	125,6
5	1 0 0 1 0	6,1	153,4	97,4	6,1	126,9	220,2
5	1 0 0 1 1	6,0	153,1	97,7	6,1	127,2	221,1
5	1 0 1 0 0	5,8	106,4	226,3	6,7	64,9	125,9
5	1 0 1 0 1	5,9	108,7	226,2	6,7	65,2	126,1
5	1 0 1 1 0	6,1	97,9	189,9	6,1	127,4	221,8
5	1 0 1 1 1	6,0	98,3	251,5	6,0	127,0	221,4
5	1 1 0 0 0	7,0	68,0	127,0	6,8	72,8	138,4
5	1 1 0 0 1	6,9	66,8	126,9	6,7	72,7	138,3
5	1 1 0 1 0	6,1	48,3	266,4	6,0	131,1	238,1
5	1 1 0 1 1	6,0	48,2	104,2	6,0	131,4	237,5
5	1 1 1 0 0	6,5	96,5	311,9	6,8	73,1	138,5
5	1 1 1 0 1	6,5	95,8	311,8	6,9	72,9	137,9
5	1 1 1 1 0	6,2	103,6	207,0	6,1	130,5	237,7
5	1 1 1 1 1	6,2	103,3	207,4	6,1	130,6	237,9

		Hybrid Algorithm			Dedicated Algorithm		
Seed	(ABCDE)	Initial	b=3	b=6	Initial	b=3	b=6
5	2 0 0 0 0	6,7	100,8	202,6	6,1	116,0	257,7
5	2 0 0 0 1	6,8	100,8	202,3	6,3	115,6	256,5
5	2 0 0 1 0	6,1	57,7	112,3	6,0	133,4	272,6
5	2 0 0 1 1	6,0	58,1	112,2	5,9	133,2	272,9
5	2 0 1 0 0	5,8	98,4	249,7	6,2	115,8	256,6
5	2 0 1 0 1	5,9	98,2	191,6	6,2	115,3	256,5
5	2 0 1 1 0	5,9	95,7	250,4	6,1	133,4	273,6
5	2 0 1 1 1	6,1	96,1	254,4	6,0	133,4	273,6
5	2 1 0 0 0	6,7	83,1	170,0	6,3	132,1	236,6
5	2 1 0 0 1	6,7	83,7	167,3	6,3	132,9	236,8
5	2 1 0 1 0	6,5	142,2	275,7	6,0	130,1	224,3
5	2 1 0 1 1	6,4	142,5	275,2	6,1	130,4	224,6
5	2 1 1 0 0	6,1	208,3	300,7	6,3	132,4	236,9
5	2 1 1 0 1	6,2	95,0	295,7	6,2	131,9	236,4
5	2 1 1 1 0	5,9	80,9	159,0	6,0	130,0	225,0
5	2 1 1 1 1	6,0	81,2	147,9	6,0	129,6	224,3
6	0 0 0 0 0	6,4	193,2	386,8	6,3	107,3	196,4
6	0 0 0 0 1	6,4	192,0	387,9	6,1	107,0	196,5
6	0 0 0 1 0	6,5	185,0	359,3	6,0	119,9	226,6
6	0 0 0 1 1	6,6	185,1	359,5	6,1	120,5	226,9
6	0 0 1 0 0	6,3	40,5	76,2	6,3	107,6	197,0
6	0 0 1 0 1	6,2	40,5	76,4	6,3	107,6	196,8
6	0 0 1 1 0	5,9	42,3	77,2	6,1	120,3	225,6
6	0 0 1 1 1	5,9	42,2	77,0	6,0	120,5	225,6
6	0 1 0 0 0	6,9	107,8	226,9	6,6	51,0	93,2
6	0 1 0 0 1	6,9	107,8	227,8	6,5	50,8	93,6
6	0 1 0 1 0	6,8	120,8	236,1	6,3	103,8	202,2
6	0 1 0 1 1	6,8	120,7	235,7	6,3	103,6	202,1
6	0 1 1 0 0	6,4	44,5	87,4	6,6	50,7	93,7
6	0 1 1 0 1	6,6	44,9	87,6	6,5	50,3	93,6
6	0 1 1 1 0	6,0	52,2	100,1	6,4	104,1	201,6
6	0 1 1 1 1	6,0	49,6	94,2	6,3	104,3	201,3
6	1 0 0 0 0	6,3	67,4	122,3	6,4	68,4	132,8

		Hybrid Algorithm			Dedicated Algorithm		
Seed	(ABCDE)	Initial	b=3	b=6	Initial	b=3	b=6
6	1 0 0 0 1	6,3	67,1	311,6	6,4	68,4	132,8
6	1 0 0 1 0	6,2	70,3	134,5	6,4	67,0	124,7
6	1 0 0 1 1	6,3	74,2	253,1	6,5	67,1	124,9
6	1 0 1 0 0	6,3	37,1	69,7	6,3	68,5	133,2
6	1 0 1 0 1	6,4	37,3	69,8	6,5	68,4	132,9
6	1 0 1 1 0	6,5	44,1	98,0	6,5	67,0	125,0
6	1 0 1 1 1	6,4	44,2	98,0	6,5	66,8	124,8
6	1 1 0 0 0	6,5	68,8	130,4	6,5	47,8	95,8
6	1 1 0 0 1	6,5	64,3	130,1	6,5	47,7	95,9
6	1 1 0 1 0	6,4	71,6	135,7	6,6	61,3	112,5
6	1 1 0 1 1	6,3	71,9	139,8	6,5	61,4	112,5
6	1 1 1 0 0	6,3	42,4	83,7	6,6	47,9	95,8
6	1 1 1 0 1	6,3	42,4	83,8	6,5	48,0	96,1
6	1 1 1 1 0	6,5	56,8	114,7	6,4	61,4	112,0
6	1 1 1 1 1	6,5	56,8	114,3	6,6	61,4	112,4
6	2 0 0 0 0	6,4	159,9	286,0	6,1	133,1	247,6
6	2 0 0 0 1	6,4	159,8	285,6	6,1	133,1	249,5
6	2 0 0 1 0	6,1	106,0	137,6	6,1	134,7	253,9
6	2 0 0 1 1	6,2	70,2	134,9	6,1	134,9	253,6
6	2 0 1 0 0	6,2	35,3	154,3	6,1	133,0	247,7
6	2 0 1 0 1	6,2	35,3	154,0	6,1	132,4	247,8
6	2 0 1 1 0	6,1	126,8	178,9	6,0	133,7	253,2
6	2 0 1 1 1	6,0	128,3	181,4	6,0	134,6	253,5
6	2 1 0 0 0	6,5	167,0	326,7	6,2	86,2	226,7
6	2 1 0 0 1	6,4	167,6	325,6	6,2	86,4	226,4
6	2 1 0 1 0	6,2	67,9	136,4	6,0	133,4	256,7
6	2 1 0 1 1	6,2	68,4	127,8	6,1	133,1	257,0
6	2 1 1 0 0	6,2	121,7	71,1	6,1	85,9	226,8
6	2 1 1 0 1	6,1	121,7	70,8	6,2	86,1	226,6
6	2 1 1 1 0	6,1	60,4	103,9	6,0	132,3	256,6
6	2 1 1 1 1	5,9	60,2	104,5	6,0	132,8	256,2