FACILITY LOCATION DECISIONS UNDER VEHICLE ROUTING CONSIDERATIONS

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By
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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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December, 2002

Over the past few decades, the concept of integrated logistics system has emerged as a new management philosophy, which aims to increase distribution efficiency. Such a concept recognizes the interdependence among the location of facilities, the allocation of suppliers and customers to facilities and vehicle route structures around depots. In this study, in order to emphasize the interdependence among these, we build a model for the integration of location and routing decisions. We propose our model on realistic assumptions such as the number of vehicles assigned to each facility is a decision variable and the installing cost of a facility depends on how many vehicles will be assigned to that facility. We also analyze the opportunity cost of ignoring vehicle routes while locating facilities and show the computational performance of integrated solution approach. We propose a greedy type heuristic for the model, which is based on a newly structured savings function.

Key Words: Location – Routing, Integer Programming, Greedy Heuristic, Facility Location, Vehicle Routing.

ÖZET

DAĞITIM ARAÇLARI ROTASI DÜŞÜNÜLEREK TESİS YERLERİ BELİRLENMESİ

Barış Selçuk
Endüstri Mühendisliği Bölümü Yüksek Lisans
Tez Yöneticisi: Doç. Dr. Osman Oğuz
Aralık, 2002

Son yıllarda gelişen ve dağıtım verimliliğini artırmayı amaçlayan bütünleşmiş lojistik sistemleri kavramı yeni bir yönetim felsefesi olarak karşımıza çıkmaktadır. Bu kavram, tesis yerleri, müsterilerin ve tedarikçilerin bu tesislere paylaştırılması ve müsteriler, tedarikçiler ve tesisleri birleştiren araç rotaları arasındaki yakın ilişkiyi ön plana çıkarır. Bu çalışmada araç rotaları ve tesis yerleri kararları arasındaki yakın ilişkinin detaylı analizi yapılmıştır. Araç rotaları ve tesis yerleri kararlarının bütünleşmesine ilişkin bir model kurulmuştur. Üzerinde çalıştığımız problem yapısı gerçekçi tahminlere dayandırılmıştır. Örneğin, herhangi bir tesise bağlı araç sayısı karar değişkeni olarak alınmış ve bir tesisin kurulum maliyeti o tesise verilmiş araç sayısına bağlanmıştır. Araç rotaları göz önüne alınmadan tesis yerleri belirlenmesinin dağıtım sisteminin maliyeti üzerine etkileri araştırılmış ve eş zamanlı çözüm yolunun sayısal performansı gösterilmiştir. Bütün bunlara ek olarak yeni yapılandırılmış bir tasarruf fonksiyonuna dayalı olan bir algoritma sunulmuştur.

Anahtar Sözcükler: Sayısal Proğramlama, Algoritma, Tesis Yeri Problemi, Araç Rotası Problemi.

To my family

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Chapter 1

Introduction

Facility Location Problem is an important research area in industrial engineering and in operations research that encompasses a wide range of problems such as the location of emergency services, location of hazardous materials, location of ATM bank machines, problems in telecommunication networks design, etc. It is a problem that can be encountered in almost all type of industries. Where to locate new facilities is an important strategic issue for decision makers. For instance about $500 billion are spent annually on new facilities in the U.S. This does not include the cost of modification of old facilities. Since the costs incurred to establish new facilities are significantly high, it has become strategically very important for the decision makers to make the location decisions in an optimal way.

Given a set of facility locations and a set of customers who are supposed to be served by one or more of these facilities; the general facility location problem is to determine which facility or facilities should be open and which customers should be served from which facilities so as to minimize the total cost of serving all the customers. The total cost of serving all customers generally formed by two types of costs. The facilities regarded as open are used to serve at least one customer and there is a fixed cost, which is incurred if a facility is open. The distance between a facility and each customer it serves is another term of the cost function. The distance measures can take several forms depending on the structure of the facility location problem. If (x_i, y_i) and (x_j, y_j) are the coordinates of two locations i and j then in general two types of distance measures are most common: Euclidean distance and rectilinear distance.

Euclidean distance is also known as straight – line distance. The Euclidean distance between two points in a two dimensional coordinate system is simply the length of the straight line connecting the points. The Euclidean distance between i and j is:

$$\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

The Euclidean distance measure is used where genuine straight line travel is possible.

The rectilinear distance between i and j is given by the formula

$$\left|x_j - x_i\right| + \left|y_j + y_i\right|$$

The rectilinear distance measure is often used for factories, American cities, etc. which are laid out in the form of a rectangular grid. For this reason it is sometimes called the Manhattan distance measure or metropolitan distance.

Although not as common as Euclidean distance and rectilinear distance measures, there is a third distance measure called the squared Euclidean distance. It can be formulated as:

$$(x_j - x_i)^2 + (y_j - y_i)^2$$

The squared Euclidean distance measure is used where straight line travel is possible but where we wish to discourage excessive distances (squaring a large distance number results in an even larger distance number and it is used in the objective function which we are trying to minimize).

Other factors often encountered in the context of location problems are the demands associated with each customer together with the capacities on the total customer demand that can be served from a facility. With these extensions the problem is called the capacitated facility location problem. In the uncapacitated facility location problem, each facility is assumed to have no limit on its capacity. In this case each customer receives all its demand from exactly one facility. However, in the capacitated facility location problem the customers can be served from more than one facility because of capacity restrictions.

In addition to the facility location problems vehicle routing problems form an important class of combinatorial optimization problems with applications in logistics systems. The well-known vehicle routing problem can be defined as the problem of determining optimal delivery or collection routes from a given depot to a number of geographically dispersed customers. The problem may be subject to some operating restrictions such as fleet size, vehicle capacity, maximum distance traveled and etc. In general, vehicle routing problem is an extension of the famous Traveling Salesman Problem.

In the Traveling Salesman Problem we are given a finite set of vertices V and a cost c_{uv} of travel between each pair u, v $\in$ V. A tour is a circuit that passes exactly once through each vertex in V. The Traveling Salesman Problem is to find a tour of minimal cost. In this context, tours are also called Hamiltonian circuits. Vehicle routing problems can be defined as to find a collection of circuits with minimum cost. Each circuit corresponds to a route for each vehicle starting from a depot and ending at the same depot.

The vehicle routing problem is a well studied combinatorial problem. The problem has attracted a lot of attention in the academic literature for two basic reasons: First; the problem appears in a large number of practical situations and second; the problem is theoretically interesting and not at all easy to solve.

Together with the uncapacitated facility location problems, vehicle routing problems are classified as NP Hard problems in the combinatorial optimization literature. This means, these are among the hardest problems in the context and no one to date has found an efficient (polynomial) algorithm to solve these problems optimally.

Besides their theoretical importance and challenge, both problems arise practically in the area of supply chain management, in logistics network configuration context. Because of the high initial cost of designing and establishing a logistics network, the cost improvements and the efficiency of location and routing decisions plays an important role in the success and failure of a supply chain.

In many logistics environments managers must make 3 basic strategic decisions:

1. Location of factories, warehouses or distribution centers.
2. Allocation of customers to each factory, warehouse or distribution center.
3. Transportation plans or vehicle routes connecting distribution channel members.

These decisions are important in the sense that they greatly affect the level of service for customers and the total system wide cost. For determining the location of newly establishing depots many mathematical models and solution procedures have been developed. However, these models ignore the nature and diversity of transportation types and assume that the transportation costs are determined by the total of distances between each customer and the depot associated with it, which is called moment sum function. This assumption is valid in the case when the customer demand is full truckload (TL). However there occurs a misrepresentation of transportation costs when in fact the customer demand is less than a truckload (LTL). In the TL case each customer can be served by only one vehicle, while in the LTL case, a vehicle stops at more than one customer on its route. This fact introduces the idea that the delivery costs depend on the routing of the delivery vehicles. Using the moment sum function ignores this interdependence between routing and location decisions. For LTL distribution systems, vehicle routing decisions should be incorporated within the location models to represent the whole logistics system more realistically. Otherwise, the problem is usually dealt by first solving a location – allocation problem and given the locations of facilities and allocation of customers from this stage, vehicle routing problem is solved for each facility. This approach produces suboptimal solutions since the moment sum assumption used in the location – allocation phase is not a valid assumption.

In general terms, the combined location and routing model solves the joint problem of determining the optimal number, location and capacity of facilities and optimal allocation of customers to facilities together with the optimal set of vehicle routes departing from facilities.

In this study, our goal is first to show that the integrated model of a location and routing problem produces better results than the sequential approach of first solving a location – allocation problem and then solving a vehicle routing problem for each facility setting the output of the location – allocation problem as fixed. Secondly, we introduce a savings based greedy algorithm to solve a realistic setting of a location – routing problem. We compare the results of our algorithm with the optimal solutions for small sized problems and with the solutions of sequential algorithm of location – allocation and vehicle routing for large sized problems.

We also aim to introduce a distribution model with realistic assumptions. We claim that the initial installation cost of a facility also depends on the number of vehicles departing from that facility. Because, as the number of vehicles assigned to a facility increases, not only the needed storage space increases but also the material handling and work force needs increase. These operational issues should be separately incorporated into the model's objective function. Besides the increase in the storage space, there are also considerations regarding the possible applications of the scale economies for the utilization of vehicles or regarding the huge operating costs resulting from a high demand from a depot due to assigning a large number of vehicles to that depot. We incorporate these considerations into our model and this approach enables us to truly represent the cost figures in designing a distribution network.

The remainder of this thesis can be outlined as follows. In the following chapter, we give a review of the literature on location – routing problems. In Chapter 3, we present the details of our problem definition and give the underlying assumptions and a list of notations we used in our models. We propose integer programming and mixed integer programming formulations for location – routing, location – allocation and vehicle routing problems separately. We used these formulations to make a comparison between the simultaneous solution of location – routing and sequential solution of location – allocation and vehicle routing problems. In Chapter 4, we introduce our heuristic and its performance evaluation results. Finally in Chapter 5, some concluding remarks and suggestions for future research are provided.

Chapter 2

Literature Review

The studies and solution procedures about the integration of facility location and vehicle routing problems are based on the huge literature on various modeling approaches of location – allocation and vehicle routing problems. (instead of the term “integration of facility location and vehicle routing problems”, “location – routing problems” will be used throughout the thesis for the sake of simplicity) However, research in location – routing problems is quite limited compared with the extensive literature on pure location problems and vehicle routing problems and their many variants. Since vehicle routing problems have been recognized as NP Hard problems it was considered impractical to incorporate the vehicle routing decisions into the facility location problems. However, in recent years there is an increasing effort among researchers to analyze location – routing problems and produce efficient solution methods and heuristics.

From a managerial point of view, the location – routing problem is significant because such a problem analysis allows for the distribution system to be considered from a more realistic viewpoint and may enable the firm to achieve higher productivity gains and cost savings.

Conceptually, the idea of incorporating the vehicle routing decisions into location problems dates back to 1960s and was first mentioned by Von Boventer [34], F. E. Maranzana [23], M. H. J. Webb [36], R. M. Lawrence and P. J. Pengilly [22], N. Christofides and S. Eilon [5] and J. C. Higgins [11]. Although, these studies are far from capturing the total complexity and the modeling of location – routing problems, they formed the conceptual foundation of location – routing problems by first pointing the close interdependence between location and transportation decisions. Later, in the early 1970s, L. Cooper [7] generalized the transportation – location problem and aimed to find the optimal locations of supply sources while minimizing the transportation costs from such sources to predetermined destinations. C. S. Tapiero [32] further extended L. Coopers work by incorporating time related considerations.

M. Koksalan, H. Sural and O. Kirca [14], present a location – distribution model where in addition to transportation costs, inventory holding costs are also incorporated in their mixed integer programming model. They consider a multi – stage, multi – period planning environment for determining the location of a single capacitated facility.

Although these studies all mention the impact of transportation costs on location problems, they are not designed to establish vehicle tours on the logistic network. Therefore they might not be considered as the true forms of location – routing problems.

The popularity of location – routing problems has grown in parallel with the development of the integrated logistics concept. The concept of integrated logistics system has emerged as a new management philosophy that aims to increase distribution efficiency. This concept recognizes the location of facilities, the allocation of suppliers and customers to facilities, and vehicle route structure around facilities as the components of a greater system and analyzes the system as a whole by simultaneous approach towards each component.

C. Watson – Gandy and P. Dohrn [35] introduced one of the first studies known to consider the multiple – drop property of vehicle routes within a location and transportation network. The examples of true location – routing problems continued to be produced in late 1970s and early 1980s. These studies are I. Or and W. P. Pierskalla [28], S. K. Jacobsen and O. B. G. Madsen [12], H. Harrison [10] and G. Laporte and Y. Nobert [18].

The studies of G. Laporte and Y. Nobert contributed to the literature of location – routing problems in the late 1980s and in the 1990s. In [18], they consider the problem of simultaneously locating one depot among n sites and of establishing m delivery routes from the depot to the remaining n – 1 sites. The problem is formulated as an integer linear program, which is solved by a constraint relaxation method, and integrality is obtained by branch and bound. Later in their study with P. Pelletier [20] a more general location – routing structure is introduced involving simultaneous choice of several depots among n sites and the optimal routing of vehicles through the remaining sites. Different from their previous study integrality is reached through the iterative introduction of Gomory cutting planes. In their study with D. Arpin [19] they further extended the problem by separating the potential depot sites from the customer sites, allowing multiple passages through the same site if this results in a distance savings and assuming the vehicles are capacitated. The formulation of an integer linear program for such a problem involves degree constraints, generalized subtour elimination constraints and chain barring constraints. An exact algorithm using initial relaxation of constraints and branch and cut is employed in their study. The algorithm presented is capable of solving problems with up to 20 sites.

G. Laporte, F Louveaux and H. Mercure [17] first introduce the concept of stochastic supply at collection points into the location – routing problems. Their model assumes that customers have nonnegative indivisible stochastic supplies and all vehicles have the same fixed capacity. An example of this model occurs in the collection of money from bank branches by armoured vehicles where the actual information on supply cannot be known until the time that collection occurs. The model is designed to determine optimal decisions according to expected values of supplies while minimizing some payoff functions generated by the recourse actions taken in case of unexpected failures. An exact solution algorithm is used to solve the problem. The algorithm is an ordinary branch and bound algorithm with depth first approach.

Another example of exact algorithms for location – routing problems is proposed by C. L. Stowers and U. S. Palekar [31]. Their model is a deterministic model with a single uncapacitated facility and single uncapacitated vehicle. Different from the previous studies of G. Laporte they introduce non – linear programming techniques to solve location – routing problems. Their study introduces a model for optimally locating obnoxious facilities such as hazardous waste repositories, dump sites, or chemical incinerators. The model differs from previous models in that it simultaneously addresses the following two issues:

1. The location is not restricted to some known set of potential sites.
2. The risk posed due to the location of the site is considered in addition to the transportation risks.

The former is a rare modeling approach in location – routing literature.

Although exact solution algorithms are helpful in the sense to understand the complexity of the problem, they only can generate efficient results for medium sized problems. Furthermore, when time window and route distance constraints are added, the problems become even harder to solve. For large-scale problems approximate solution algorithms produce close to optimum results in a small amount of time. For this purpose much effort has been spent on heuristics for location – routing problems.

A good example of those studies is the study of R. Srivastava [30], which analyzes the performances of three approximate solution methods with respect to the optimal solution of location – routing problems and the sequential solution of the classical location – allocation and vehicle routing problems. The first heuristic assumes all facilities to be open initially, and uses approximate routing costs for open facilities to determine the facility to be closed. A modified version of the savings algorithm introduced by Clark and Wright [6], for the multiple depot case is used to approximate the routing costs. The algorithm iterates between the routing and facility closing phases until a desired number of facilities remain open. The second heuristic employs the opposite approach, and opens facilities one by one until a stopping criterion is met. The third heuristic is based on a customer clustering technique. It identifies the clusters by generating the minimal spanning tree of customers and then separating it into a desired number of clusters using a density search technique. According to the computational results of the study, all three algorithms perform significantly better than the sequential approach. The sequential approach, which is commonly used in practice, first determines the facility locations using moment sum function, and then solves the multi depot vehicle routing problem applying the modified savings algorithm. A single stage deterministic environment of multiple uncapacitated facilities with single uncapacitated vehicle is analyzed in this study.

A similar savings heuristic is used in P. H. Hansen et al. [9] but in a model structure with multiple vehicles, capacitated facilities and capacitated vehicles different from the model of R. Srivastava [30]. The proposed solution in this study is based on decomposing the problem into three subproblems: The Multi – Depot Vehicle – Dispatch Problem, Warehouse Location – Allocation Problem and The Multi – Depot Routing – Allocation Problem. These subproblems are then solved in a sequential manner while accounting for interdependence between them. The heuristic stops when no further cost improvements are possible.

M. Jamil, R. Batta and M. Malon [13] propose a stochastic repairperson model where the objective is to find the optimum home base for the repairperson that minimizes the average response time to an accepted call. The structure of their model is the same as that of C. L. Stowers and U. S. Palekar [31] with the exception that they consider a stochastic environment. The solution procedure used is a heuristic based on Fibonacci search. Later, I. Averbakh and O. Berman [2] proposed the multiple server case of this model in a deterministic environment. This study differs from the others by its solution technique that it utilizes a dynamic programming algorithm. Further they extended their work and generalized the problem by considering probabilistic and capacitated version of delivery man problem [1]. That is the case where the customer demands are random and the servers have a predefined capacity. Nonlinear programming techniques are used to find an exact solution for this problem.

J. H. Bookbinder and K. E. Reece [3] was the first to consider the distribution of multiple products in a two – stage transportation network. They formulate a capacitated distribution planning model as a non-linear, mixed integer program. Vehicle routes from facilities to customers are established by considering the fleet size mix problem. Its solution yields not only the route for each vehicle but also the capacities of each vehicle and the number of each vehicle type required at the distribution center. The overall algorithm for location – routing problem is based on Bender’s decomposition. Their study employs an iterative algorithm between location and transportation phases. The solution of the location problem is embedded to the routing problem to determine outbound transportation costs.

Unlike the sequential and iterative procedures, the study of G. Nagy and S. Salhi [26] was the first time that a nested heuristic method is applied to location – routing problems. By building on the conceptual knowledge introduced in the previous work of S. Salhi and K. Rand [29], they introduced a new solution procedure to a location – routing problem with single – stage uncapacitated facilities and multiple vehicles with capacities in a deterministic environment. Their approach is different from the previous approaches that they treat the routing element as a sub-problem within the larger problem of location. They observe that a location – routing problem is essentially a location problem, with the vehicle routing element taken into consideration. Instead of treating the two sub-problems as if they were on the same footing, which is applied in iterative approaches, they propose a hierarchical structure, with location as the main problem and routing as a subordinate one. A neighborhood structure is defined by three moves; add, drop and shift for location of the facilities. Each time a change occurs in the location of facilities a multi-depot vehicle routing heuristic is applied to find the appropriate vehicle routing structure. The neighborhood search algorithm is combined with a variant of tabu search incorporated into the model. The heuristic is capable to solve problems consisting 400 customers.

The study of D. Tuzun and L. I. Burke [33] is also based on a tabu search approach but they present an iterative algorithm. Their study is the first that applies a two – phase tabu search architecture for the solution of location – routing problems. This two – phase approach coordinates two tabu search mechanisms; one seeks for a good facility location configuration and the other finds a good vehicle routing structure that corresponds to this configuration. A comparison of this new two – phase tabu search approach with the heuristic proposed in R. Srivastava [30] is presented in this study. The solution quality of tabu search algorithm is better than that of Srivastava's algorithm however; tabu search algorithm requires more cpu time than Srivastava's algorithm. The comparison of these two algorithms initiates a basis for evaluating the performance of location – routing heuristics, which is lacking in the location – routing literature. The problem in this study is modeled as a multiple, uncapacitated facility with multiple, capacitated vehicles in a single stage deterministic environment.

In a very recent study by T. H. Wu, C. Low, and J. W. Bai [38] the location routing problem is divided into two sub – problems; the location – allocation problem and the general vehicle routing problem, respectively. Each sub – problem is then solved in a sequential and iterative manner by the simulated annealing algorithm. In the first iteration of the algorithm the solution of the location – allocation problem is some selected depots and a plan for allocating customers to each chosen depot. These solutions are then used as input to the vehicle routing problem to generate a starting feasible set of routes. Starting from the second iteration each current route consisting of several customers is viewed as a single node with demand represented by the sum of demands of all customers in that route. These aggregated nodes are then consolidated for reducing the number of depots established and, thus, the total cost. A new savings matrix for improving the location – allocation solutions starting from the second iteration is defined. The algorithm performs good results for problems of sizes 50, 75, 100 and 150 nodes.

Although the literature on location – routing problems is quite limited, a classification of studies can be done based on some characteristics of the problem structures and the solution procedures presented in the papers.

In general, the location – routing problems presented thus far have two distinct structures; single – stage and two – stage. In the single – stage problems the primary concern is on the location of facilities and the outbound delivery routes to the customers around these facilities. A simple example is pictured in Figure 2.1. In two – stage location – routing problems the structure is expanded to consider two layers of production – distribution network where both outbound and inbound distribution processes are involved. A simple example of this type can be seen in Figure 2.2.

The classification can be further developed to consider deterministic and stochastic environments. In deterministic models the nature of location and routing parameters such as demand and supply values are known with certainty while in stochastic models these values are represented by random variables. In addition, we can further express the differences and closeness between different models by considering the number and capacity of facilities and number and capacity of vehicles.

Exact algorithms and heuristics are the two distinct types of solution methods in location – routing literature. In analogy with the vehicle routing classification scheme suggested by G. Laporte [21] we can further classify the exact algorithms into three groups:

1.Direct tree search / branch and bound.
2.Dynamic Programming.
3.Non – linear Programming.

The heuristic algorithms can be categorized into three groups based on the structure of the algorithm. These are:

1.Iterative algorithms.
   a. Location – allocation first, route second.
   b. Route first, location – allocation second.
2.Sequential algorithms.
   a. Savings / insertion.

3.Nested heuristics.

In Table 2.1 a classification of the previous studies is presented based on the set of criteria listed above.

A survey of location – routing problems is studied in H. Min, V. Jayaraman and R. Srivastava [24]. A frequency listing of location – routing problems by publication outlets is presented in this study, which we figured in Table 2.2 by the addition of some recent studies and some conceptual studies, which are not considered in this review.

In our model, we aim to include more realistic assumptions than the studies presented thus far. We consider a single stage, deterministic structure consisting of multiple uncapacitated depots serving to a number of geographically dispersed customers. The number of vehicles used to serve those customers is considered as a decision variable in our model because we believe that if the decision makers are to decide on the location of facilities then they should also be able to decide on the number of vehicles assigned to each open facility. If it is profitable to close an open depot by assigning more vehicles to a nearby depot than this decision should be taken in advance of the construction of the distribution network. Because, as well as the location of open depots, number of vehicles can be considered as a strategic decision and needs long term planning. The decision for the number of vehicles assigned to each open depot is as important as the decision for which depot to open and these two strategic decisions should be considered simultaneously in designing a distribution network. The vehicles in our model are assumed to have a fixed capacity. With these characteristics the model structure that we present is similar to the structures of Jacobsen and Madsen[12], and Nagy and Salhi[26]. However, the number of vehicles utilized for each open depot is not a decision variable in these models and it is determined a priori. Depending on the characteristics of the cost function our model can account for single vehicle case as well as multiple vehicle cases.

Another interesting part of our study is that we propose a different cost structure to the location – routing problem. None of the studies presented thus far has considered such a structure. We believe that the initial cost of opening a facility is dependent on the number of vehicles that are designed to depart from that facility and serve its customers. In the light of this approach we develop a new model for the problem.

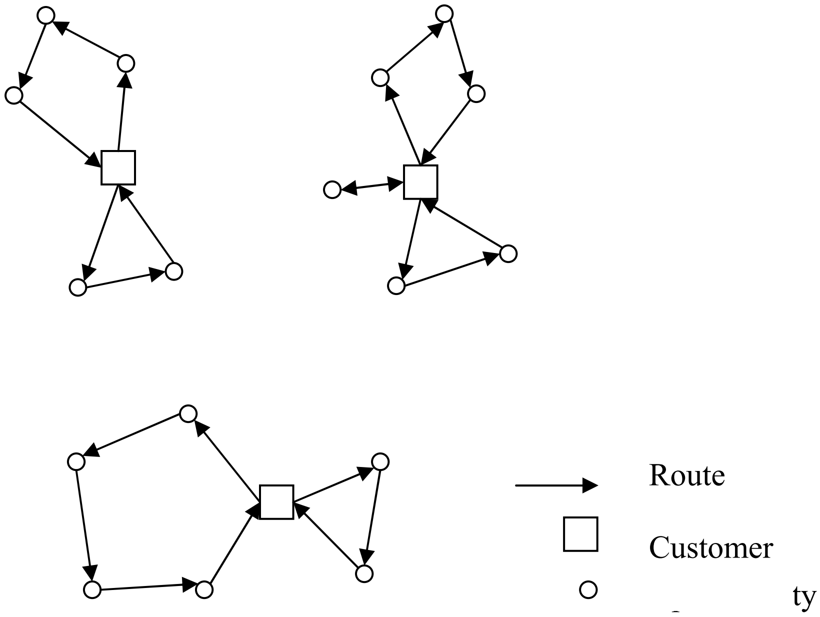
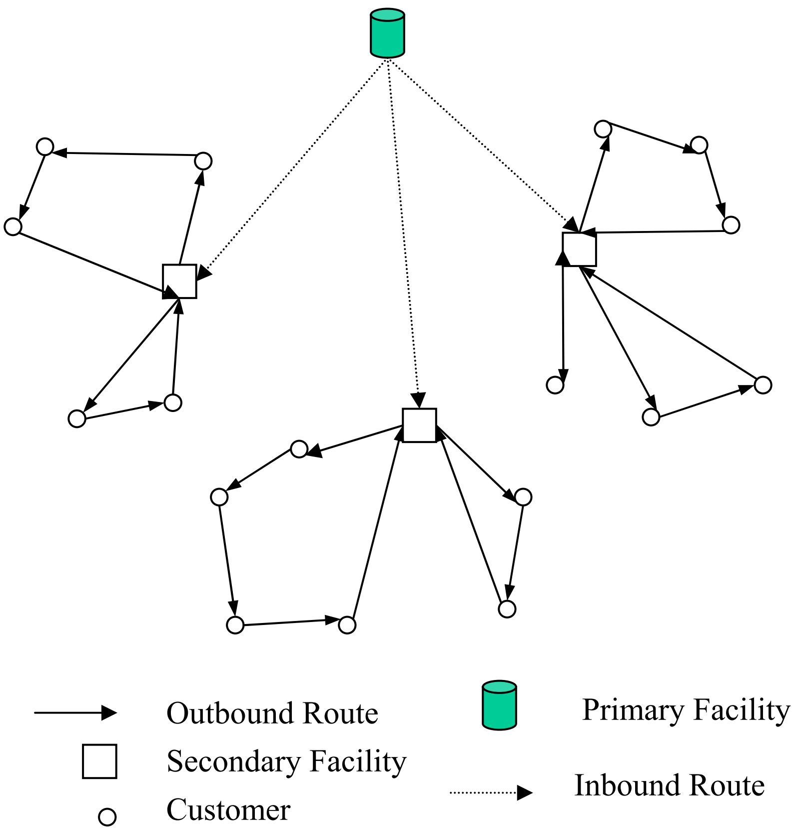


Figure 2.1: Single – stage distribution network.



Two – stage distribution network.

	Stages		Parameters		Facilities				Vehicles				Heuristic			Exact		
	Single	Two	Stoc	Deter	Multi	Single	Cap	Uncap	Multi	Single	Cap	Uncap	Iter	Seq	Nested	B&B	DP	NLP
Averbakh [2]	✔			✔		✔		✔	✔			✔					✔	
Averbakh [1]	✔		✔			✔		✔	✔			✔						✔
Bookbinder [3]		✔		✔	✔		✔		✔		✔		LA - VRP					
Hansen [9]	✔			✔	✔		✔		✔		✔			✔				
Harrison [10]	✔		✔		✔			✔	✔			✔	LA - VRP					
Jacobsen [12]	✔			✔	✔			✔	✔		✔		LA - VRP					
Jamil [13]	✔		✔			✔		✔		✔		✔		✔				
Laporte [17]	✔		✔		✔			✔	✔		✔					✔		
Laporte [19]	✔			✔	✔		✔		✔		✔					✔		
Nagy [26]	✔			✔	✔			✔	✔		✔				✔			
Or [28]		✔		✔	✔			✔	✔		✔		LA - VRP					
Salhi [29]	✔			✔	✔	✔		✔	✔			✔		✔				
Srivastava [30]	✔			✔	✔			✔		✔		✔		✔				
Stowers [31]	✔			✔		✔		✔		✔		✔						✔
Tuzun [33]	✔			✔	✔			✔	✔		✔		VRP - LA					
Wu [38]	✔			✔	✔		✔		✔		✔		LA - VRP					

Deter: Deterministic
Stoc: Stochastic
Cap: Capacitated
Uncap: Uncapacitated
Iter: Iterative
Seq: Sequential
B&B: Branch and Bound
DP: Dynamic Programming
NLP: Non - linear Programming

Table 2.1: Classification of studies mentioned in the literature review.

Journal	Total number of LRP articles published
European Journal of Operational Research	14
Transportation Science	9
Omega	3
Journal of Business Logistics	2
Computers and Operations Research	2
Journal of Operational Research Society	2
Operations Research	2
Journal of Regional Science	2
AIIE Transactions	1
Annals of Operations Research	1
Decision Sciences	1
Interfaces	1
Transportation Research	1
Transportation Research Board	1

Table 2.2: Frequency listing of location – routing articles by publication outlets.

Chapter 3

Opportunity Cost of Ignoring Vehicle Routes While Locating Facilities

The location – routing problem (LRP) is defined to find the optimal number and locations of the distribution centers (depots) simultaneously with the allocation of customers to these depots and vehicle schedules and distribution or collection routes from the depots to the customers so as to minimize the total system costs. With this definition LRP is considered as a combination of two well – known problems; location – allocation problem and vehicle routing problem.

The location – allocation decisions are strategic decisions and needs long term planning while the vehicle routing decisions are operational decisions. It is a common approach both in industry and in the operations research literature that these problems are considered as separate and independent from each other. A common solution procedure for such a distribution system design problem is to solve a location – allocation problem first and then solving a vehicle routing problem given the location of open depots and customers and the allocation of customers to these depots.

In this chapter, we will analyze the effect of incorporating routing decisions into the location problems. We show that by solving location – allocation and vehicle routing problems simultaneously we can get better solutions in comparison to the sequential approach commonly used. We show that the solution of the sequential approach is sub-optimal because of the misrepresentation of transportation costs.

This chapter is organized as follows: In Section 3.1 we give the problem definition and the location – routing problem formulation. The integer and mixed integer programming formulations for the associated location – allocation and vehicle routing problems are also presented in this section. In Section 3.2 we emphasize the assumption that we made on the relation between the number of vehicles assigned to a depot and its initial installation cost. We introduce three types of cost structure and then depending on these cost structures we propose different models for the problem. In Section 3.3 we compare the solutions of the location – routing models with the solutions of the sequential approach.

3.1 Problem Definition and Formulation

In our model we consider a single stage distribution environment where there is an outbound transportation between the depots and the customers served by these depots. Each customer will be assigned to a depot and served by a single vehicle departing from that depot. The vehicles have predetermined capacities and the total demand of customers served by a vehicle can not exceed vehicle capacity. The locations of the customers and the locations of the potential depot sites are known a priori. The locations are expressed by their coordinates in a two dimensional coordinate system. The number of vehicles utilized in a depot is a decision variable that determines the total storage space, material handling and labor force needs in that depot and the total demand associated with that depot. In our model the number of vehicles departing from a depot is a significant term in the cost function. We also set the depots uncapacitated. In other words, a huge number of vehicles can be assigned to a given depot but there will be restrictions about this issue since we introduce a new cost term depending on the number of vehicles departing from a depot.

Our objective is to find the number and locations of open depots, allocation of customers to these depots with the number of vehicles departing from each depot and their distribution routes to the associated customers that yields minimum systemwide costs.

To sum up, our model have a structure of a single stage distribution environment with multiple uncapacitated facilities and multiple capacitated vehicles.

Below, we present a mixed integer programming formulation of the location – routing problem described above. We assume that the cost function that relates the number of vehicles with the initial installation cost of a facility is a linear function. For more complex cost structures we leave the formulations to the following section.

The initial installation cost of a facility is a long term cost while the other cost figures like operating cost of a vehicle or travelling costs are considered to be operational and short term costs. Therefore, we need to adjust the cost figures in the objective function to remove this inappropriateness. We assume that the cost parameters we use in the models are generated to represent their annual equivalent.

Sets:

I = Set of all potential depot sites.

J = Set of all customers.

K = Set of all vehicles that can be utilized. $|K| \leq |J|$

Parameters:

C_{ij} = Annual travelling cost between locations i and j (based on the Euclidean distance between locations i and j); i, j $\in$ I $\cup$ J.

F_i = Annual equivalent cost of opening a depot at location i; i $\in$ I.

D_j = Demand of customer j; j $\in$ J.

V = Capacity of each vehicle.

G = Annual cost of utilizing a vehicle.

N = Number of customers.

Decision Variables:

$x_{ijk} = \begin{cases} 1, & \textit{if i precedes j on route k} \\ 0, & \textit{otherwise} \end{cases}$ $\quad$ i, j ∈ I ∪ J, k ∈ K

$y_i = \begin{cases} 1, & \textit{if a depot is opened at location i} \\ 0, & \textit{otherwise} \end{cases}$ $\quad$ i ∈ I

z_i = Number of vehicles assigned to depot i. $\quad$ i ∈ I

U_{lk} = Auxiliary variable for subtour elimination constraint on route k. l ∈ J, k ∈ K

(LRP)

Minimize $\sum_{i \in I} F_i y_i + \sum_{i \in I} G z_i + \sum_{i \in I \cup J} \sum_{j \in I \cup J} \sum_{k \in K} C_{ij} x_{ijk}$

Subject to:

$$\sum_{k \in K} \sum_{i \in I \cup J} x_{ijk} = 1, \ j \in J \qquad (1)$$

$$\sum_{j \in J} D_j \sum_{i \in I \cup J} x_{ijk} \le V, \ k \in K \qquad (2)$$

$$\sum_{j \in I \cup J} x_{ijk} - \sum_{j \in I \cup J} x_{jik} = 0, \ i \in I \cup J, k \in K \qquad (3)$$

$$\sum_{i \in I} \sum_{j \in I \cup J} x_{ijk} \le 1, \ k \in K \qquad (4)$$

$$U_{lk} - U_{jk} + N x_{ljk} \le N - 1, \ l, j \in J, k \in K \qquad (5)$$

$$\sum_{j \in J} \sum_{k \in K} x_{ijk} - z_i = 0, \ i \in I \qquad (6)$$

$$z_i \le N y_i, \ i \in I \qquad (7)$$

$$x_{ijk} = \{0,1\}, \ i, j \in I \cup J, k \in K \qquad (8)$$

$$y_i = \{0, 1\}, \ i \in I \qquad (9)$$

$$z_i \ge 0, \ i \in I \qquad (10)$$

$$U_{lk} \ge 0, \ l \in J, k \in K \qquad (11)$$

The first term in the objective function indicates the total cost of open depots. It is a linear function of the number of open depots. The second term of the objective function is the total dispatching cost of vehicles used in the distribution system. Note here that, as the number of vehicles assigned to a depot increases the throughput in that depot also increases which will yield an increase in the storage space of that depot. Besides storage cost, the number of vehicles also affects the labor force and material handling equipment needs in that depot and hence can yield additional increases in the cost of installing a depot. This increase in the installation cost based on the number of vehicles is incorporated within the vehicle dispatching cost "G" because we assume a linear cost function here. The third term of the objective function represents the total transportation cost given that the transportation is done by vehicle routes.

The constraint set (1) assures that each customer can be served by only one vehicle. Besides, it also indicates that no customer can have more than one precedecesors on a given route or we can more clearly state that vehicles can not visit a customer more than once on its route. The satisfaction of customer demands and the vehicle capacity restrictions are modeled in the constraint set (2). That is; the total demand of customers served by a vehicle is smaller than or equal to the vehicles capacity. Together with the constraint set (1), constraint set (3) assures that each vehicle route is a closed loop that starts and ends at the same location. Since a vehicle can not be at more than one location at the same time, constraint set (4) is there to indicate that a vehicle can not be used to travel more than one route. Constraint set (5) represents the subtour elimination constraints for each route. The description of these subtour elimination constraints is as follows:

Consider the route structure with a subtour pictured in Figure 3.1.

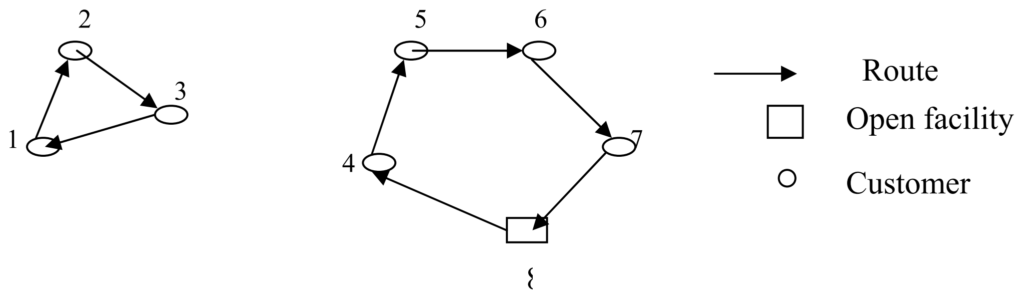


Figure 3.1: A vehicle route structure with subtour.

Let's call this route k. The auxiliary variables considering this route are; U_{1k}, U_{2k}, U_{3k}, U_{4k}, U_{5k}, U_{6k} and U_{7k}. If we write the subtour elimination constraints for route k:

$$U_{1k} - U_{2k} + N \leq N - 1 \qquad (5.1)$$
$$U_{2k} - U_{3k} + N \leq N - 1 \qquad (5.2)$$
$$U_{3k} - U_{1k} + N \leq N - 1 \qquad (5.3)$$
$$U_{4k} - U_{5k} + N \leq N - 1 \qquad (5.4)$$
$$U_{5k} - U_{6k} + N \leq N - 1 \qquad (5.5)$$
$$U_{6k} - U_{7k} + N \leq N - 1 \qquad (5.6)$$

If we sum up (5.1), (5.2) and (5.3) we end up with:

$$0 \leq -3$$

which makes the construction of a subtour infeasible. When we turn our attention to (5.4), (5.6) and (5.7) we see that the subtour elimination constraints do not affect the construction of a valid route. We can conclude that the new subtour elimination constraints make the routes with subtours infeasible while keeping the others feasible.

Note that introducing a new set of subtour elimination constraints does result in a smaller number of constraints in the model. However, with the addition of auxiliary variables, the number of variables in the model increases.

Constraint set (6) is to express the definition of the number of vehicles assigned to each depot and in order to avoid assigning vehicles to closed depots and to set an upper limit on the number of vehicles there included constraint set (7). (8), (9), (10) and (11) are sign restrictions for the decision variables.

The location – allocation model for this problem is a general model for allocating customers to uncapacitated depots. Besides the costs of open depots, the travel cost is also an important cost figure for this formulation. However, the travel cost is assumed to be represented by moment sum function, which is the case when a vehicle departing from the depot visits only one customer and then returns back. The location – allocation problem (LAP) based on this assumption is presented below:

Sets:

I = Set of potential depot sites.

J = Set of customers.

Parameters:

C_{ij} = Annual travelling cost between locations i and j (based on the Euclidean distance between locations i and j); i, j $\in$ I $\cup$ J.

F_i = Annual equivalent cost of opening a depot at location i; i $\in$ I.

Decision variables:

$y_i = \begin{cases} 1, \mathit{if\ a\ depot\ is\ opened\ at\ location\ i} \\ 0, \mathit{otherwise} \end{cases}$ $\quad$ $i \in I$

$z_{ij} = \begin{cases} 1, \mathit{if\ customer\ j\ is\ assigned\ to\ depot\ i} \\ 0, \mathit{otherwise} \end{cases}$ $\quad$ $i \in I, j \in J$

(LAP)

Minimize $\sum_{i\in I}\sum_{j\in J} C_{ij} z_{ij} + \sum_{i\in I} F_i y_i$

Subject to:

$\sum_{i\in I} z_{ij} = 1, j \in J$ (12)

$z_{ij} \le y_i, i \in I, j \in J$ (13)

$z_{ij} = \{0, 1\}, i \in I, j \in J$ (14)

$y_i = \{0, 1\}, i \in I$ (15)

This is a typical formulation of an uncapacitated location – allocation problem where (12) and (13) assures allocation of each customer to one of the open depots. As it is understood from the sign restrictions (14) and (15) LAP is a binary integer programming problem.

If we want to solve the location – routing problem by applying the sequantial approach then, we should first generate optimal customer assignments and location of open depots from LAP, and then formulate and solve a vehicle routing problem (VRP) for each open facility.

The VRP we need for this stage is presented below:

Sets:

J = Set of customers.

K = Set of vehicles that can be used. $|K| \leq |J|$

Parameters:

n = Depot in use.

V = Vehicle capacity.

G = Annual cost of utilizing a vehicle.

C_{ij} = Annual travelling cost between locations i and j (based on the Euclidean distance between locations i and j); i, j ∈ {n} ∪ J.

D_j = Demand of customer j. j ∈ J.

N = Number of customers.

Decision variables:

$x_{ijk} = \begin{cases} 1, \textit{if i precedes j on route k} \\ 0, \textit{otherwise} \end{cases}$ i, j ∈ {n} ⋃ J, k ∈ K

z = Number of vehicles assigned to the depot in use.

U_{lk} = Auxiliary variable for subtour elimination constraint on route k. l ∈ J, k ∈ K

(VRP)

Minimize $\sum_{i \in \{n\} \cup J} \sum_{j \in \{n\} \cup J} \sum_{k \in K} C_{ij} x_{ijk} + G.z$

Subject to:

$$\sum_{k \in K} \sum_{i \in \{n\} \cup J} x_{ijk} = 1, \ j \in J \tag{16}$$

$$\sum_{j \in J} D_j \sum_{i \in \{n\} \cup J} x_{ijk} \leq V, \ k \in K \tag{17}$$

$$\sum_{j \in \{n\} \cup J} x_{ijk} - \sum_{j \in \{n\} \cup J} x_{jik} = 0, \ i \in \{n\} \cup J, k \in K \tag{18}$$

$$\sum_{j \in J} x_{njk} \leq 1, \ k \in K \tag{19}$$

$U_{lk} - U_{jk} + N x_{ljk} \leq N - 1,\ l, j \in J,\ k \in K$ (20)

$\sum_{j \in J} \sum_{k \in K} x_{njk} - z = 0$ (21)

$x_{ijk} = \{0, 1\},\ i, j \in \{n\} \cup J,\ k \in K$ (22)

$z \geq 0$ (23)

$U_{lk} \geq 0,\ l \in J,\ k \in K$ (24)

3.2. A Realistic Structure for the Facility Opening Cost

As we mentioned before, in a realistic point of view the initial facility installing cost should be dependent on the number of vehicles departing from that facility. Because, the number of vehicles departing from a facility determines the storage space, material handling structure in that facility together with the total demand from that facility.

Considering these interdependences between the facility installing cost and the number of vehicles assigned to that facility, we introduce an additional cost function to the objective function of the LRP.

We believe that the relation between the number of vehicles and the facility cost may yield different cost functions depending on the structure of the distribution environment. We consider three types of cost structures:

1. *Linear cost function:* The above formulation LRP represents a linear relationship between the facility cost and the number of vehicles. The additional cost of assigning a vehicle to a facility is incorporated to the vehicle dispatching cost in that formulation.
2. *Convex cost function:* As the number of vehicles assigned to a facility increases the storage space and the space for needed material handling equipments and the needed labor force also increases. This can result in an excess inventory kept at the facility due to the high demand from that

facility or may result in large operating cost due to the large labor force needed. Therefore, assigning a huge number of vehicles to a single facility can cause congestion and problems. To represent such a structure we propose a convex function to determine the additional cost incurred with the additional assignment of a vehicle to a facility. Let's say that the number of vehicles departing from a specific facility is s. Then the associated cost function is represented as: $A(s) = 20(s–1)^2$.

3. *Concave cost function:* Economies of scale can be applied in the utilization of resources of the facility as the number of vehicles departing from that facility increases. This can cause cost savings and result in an effective utilization of resources like storage space, material handling, labor force etc. To represent such an environment we propose a concave cost function: $A(s) = 50(s – 1)^{1/2}$.

We proposed LRP model for linear cost function between the number of vehicles and additional installation cost of a facility in the previous section. In case of convex and concave cost functions the model needs additional variables, constraints and objective function terms. Below we present the location – routing model and vehicle routing model modified for the convex and concave installation cost functions.

Mixed integer programming formulation for the LRP with modified cost function:

Sets:

I = Set of all potential depot sites.

J = Set of all customers.

K = Set of all vehicles that can be utilized. $|K| \le |J|$

Parameters:

C_{ij} = Annual travelling cost between locations i and j (based on the Euclidean distance between locations i and j); $i, j \in I \cup J$.

F_i = Annual equivalent cost of opening a depot at location i; $i \in I$.

D_j = Demand of customer j; $j \in J$.

A(s) = Additional cost due to assigning s vehicles to a depot.

V = Capacity of each vehicle.

G = Annual cost of utilizing a vehicle.

N = Number of customers.

Decision Variables:

$x_{ijk} = \begin{cases} 1, & \textit{if i precedes j on route k} \\ 0, & \textit{otherwise} \end{cases}$ $\quad i, j \in I \cup J, k \in K$

$m_{ki} = \begin{cases} 1, & \textit{if k vehicles are assigned to location i} \\ 0, & \textit{otherwise} \end{cases}$ $\quad i \in I, k \in K$

U_{lk} = Auxiliary variable for subtour elimination constraint on route k. $l \in J, k \in K$

(LRPm)

Minimize $\sum_{i\in I}\sum_{k\in K}(F_i + A(k))m_{ki} + \sum_{i\in I} G \sum_{k\in K} k m_{ki} + \sum_{i\in I\cup J}\sum_{j\in I\cup J}\sum_{k\in K} C_{ij}x_{ijk}$

Subject to:

$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1, \; j \in J \tag{25}$$

$$\sum_{j\in J} D_j \sum_{i\in I\cup J} x_{ijk} \le V, \; k \in K \tag{26}$$

$$\sum_{j\in I\cup J} x_{ijk} - \sum_{j\in I\cup J} x_{jik} = 0, \; i \in I \cup J, k \in K \tag{27}$$

$$\sum_{i\in I}\sum_{j\in I\cup J} x_{ijk} \le 1, \; k \in K \tag{28}$$

$$U_{lk} - U_{jk} + N x_{ljk} \le N - 1, \; l, j \in J, k \in K \tag{29}$$

$$\sum_{k\in K} m_{ki} \le 1, \; i \in I \tag{30}$$

$$\sum_{k\in K} k m_{ki} \ge \sum_{j\in J}\sum_{k\in K} x_{ijk}, \; i \in I \tag{31}$$

$$x_{ijk} = \{0,1\}, \; i, j \in I \cup J, k \in K \tag{32}$$

$$m_{ki} = \{0, 1\}, \; i \in I, k \in K \tag{33}$$

$$U_{lk} \ge 0, \; l \in J, k \in K \tag{34}$$

Different from LRP, here we introduce new binary variables to represent the number of vehicles assigned to a depot. With the addition of constraints sets (30) and (31) our model is ready to represent the new cost structures we mentioned above.

Constraint set (30) claims that only one of the m_{ki} variables can take the value 1 for each i. Constraints set (31) sets the value of m_{ki} equal to 1 and all other m_{ni}, $n \neq k$ equal to 0 if the number of vehicles departing from depot i is k.

We set A(s) values in advance according to the functions defined above.

LAP formulation is not affected by this new cost structure and remains the same as in the previous section. Because in LAP, we assume moment sum function to represent the transportation costs and hence vehicle route structures are not incorporated into this model. Although the number of customers assigned to a facility can be considered to cause additional installation costs, we choose to build this cost on the number of vehicles and leave it to the VRP module. Therefore, VRP module should be modified to VRPm as presented below:

Sets:

J = Set of customers.

K = Set of vehicles that can be used. $|K| \leq |J|$

Parameters:

n = Depot in use.

V = Vehicle capacity.

G = Annual cost of utilizing a vehicle.

C_{ij} = Annual travelling cost between locations i and j (based on the Euclidean distance between locations i and j); $i, j \in I \cup J$.

D_j = Demand of customer j. $j \in J$.

A(s) = Additional cost due to assigning s vehicles to the open depot.

N = Number of customers.

Decision variables:

$x_{ijk} = \begin{cases} 1, \textit{if i precedes j on route k} \\ 0, \textit{otherwise} \end{cases}$ $\quad i, j \in \{n\} \bigcup J, k \in K$

$m_k = \begin{cases} 1, \quad \textit{if k vehicles are assigned to the depot} \\ 0, \quad \textit{otherwise} \end{cases}$ $\quad k \in K$

U_{lk} = Auxiliary variable for subtour elimination constraint on route k. $l \in J, k \in K$

(VRPm)

Minimize $\sum_{i\in\{n\}\bigcup J} \sum_{j\in\{n\}\bigcup J} \sum_{k\in K} C_{ij} x_{ijk} + \sum_{k\in K} A(k) m_k + G \sum_{k\in K} k m_k$

Subject to:

$$\sum_{k\in K} \sum_{i\in\{n\}\bigcup J} x_{ijk} = 1, \; j \in J \tag{35}$$

$$\sum_{j\in J} D_j \sum_{i\in\{n\}\bigcup J} x_{ijk} \leq V, \; k \in K \tag{36}$$

$$\sum_{j\in\{n\}\bigcup J} x_{ijk} - \sum_{j\in\{n\}\bigcup J} x_{jik} = 0, \; i \in \{n\} \bigcup J, k \in K \tag{37}$$

$$\sum_{j\in J} x_{njk} \leq 1, \; k \in K \tag{38}$$

$$U_{lk} - U_{jk} + N x_{ljk} \leq N - 1, \; l, j \in J, k \in K \tag{39}$$

$$\sum_{k\in K} m_k = 1 \tag{40}$$

$$\sum_{k\in K} k m_k = \sum_{j\in J} \sum_{k\in K} x_{njk} \tag{41}$$

$$x_{ijk} = \{0, 1\}, \; i, j \in \{n\} \bigcup J, k \in K \tag{42}$$

$$m_k = \{0, 1\}, \; k \in K \tag{43}$$

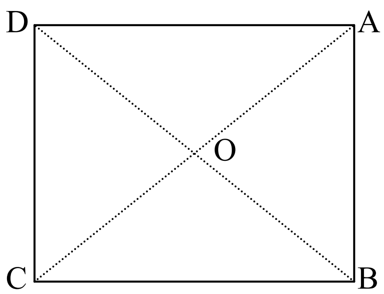
$$U_{lk} \geq 0, \; l \in J, k \in K \tag{44}$$

3.3 Comparison of Simultaneous and Sequential Solution Approaches

We claim that ignoring the multiple drop properties of a vehicle in designing a distribution network results in improper representation of the problem and thus yields suboptimal solutions. In order to simply justify our thesis let's consider a very simple example.

3.3.1 An example on the opportunity cost of ignoring vehicle routes while locating facilities.

Let's have four customers A, B, C and D located at the corners of a square. All four customers can be served by one vehicle. There are two potential depot locations: one is at the center of the square and the other is at the same location as customer A. The depot installing costs are the same for both locations. Therefore, only the transportation costs differ between each alternative. A simple picture of this distribution network is depicted in Figure 3.2.



$|AB| = |BC| = |CD| = |DA|$

$|OD| = |OA| = |OB| = |OC|$

Figure 3.2: A distribution structure with customers A, B, C, D and potential depot locations A and O.

Let's say $|AB| = |BC| = |CD| = |AD| = 2x$. Then, by using the theorems of geometry we can state that $|OA| = |OB| = |OC| = |OD| = x\sqrt{2}$.

Let's solve the problem by the sequential approach first. The depot will be opened at the location that gives the smallest total moment sum distances between that location and the customers. The total moment sum distances for location O is:

$$|OA| + |OC| + |OB| + |OD| = 4x\sqrt{2} = 5.64x.$$

The total moment sum distances for location A is:

$$|AB| + |AC| + |AD| = 4x + 2\,x\sqrt{2} = 6.82x.$$

It is clear that 5.64x < 6.82x and the optimal location of the depot given by the LAP is O. When we solve VRP for depot O and its associated customers A, B, C and D then the transportation cost is:

$$\begin{aligned} T_{seq} = \min \{ & |OA| + |AB| + |BC| + |CD| + |OD|, \\ & |OB| + |BC| + |CD| + |DA| + |OA|, \\ & |OC| + |CD| + |AD| + |AB| + |OB|, \\ & |OD| + |AD| + |AB| + |BC| + |OC| \} = 8.82x \end{aligned}$$

However, when depot is opened at location A instead of location O, then the transportation cost will be:

$$T_{sim} = |AB| + |BC| + |CD| + |AD| = 8x$$

It is clear that $T_{sim} < T_{seq}$. Also the triangular inequality leads to $T_{sim} < T_{seq}$. This means that the simultaneous approach for this problem will produce less cost solution than the sequential approach.

A similar example is also mentioned in Salhi and Rand [29].

3.3.2 Computational Results

After introducing the effect of ignoring vehicle routes in distribution system design in a simple example, we now present a number of computational results that further prove our claim in this thesis.

We solve LRP and LRPm and then compare their optimum solutions with the solutions of the sequential approach. We use CPLEX to solve LRP, LAP, VRP, LRPm and VRPm. Since CPLEX do not allow us to solve problems of size greater than 10, we restrict the simulation environment to a network structure with 3 potential depot sites and 7 customers. All the parameters needed to solve the problems are randomly generated. The distance values are assumed to be euclidean distances in a two dimensional geographic structure. The locations are represented by x and y coordinates whose values are choosen at random from (0, 150) range. Vehicle capacity is fixed to 200 and the demand of each customer is choosen at random from (0, 200). Therefore, we assure that none of the customer demands can ever exceed vehicle capacity and this avoids a customer to be served by more than one vehicle.

We present 100 simulation runs in five different experimental environments depending on the characteristics of the cost structure. First we develop a simulation environment based on the facility costs (F_i) where the facility installing costs have linear structure. Later we consider the cases when it have convex and concave structures.

We divide the facility costs into three cathegories:

Low facility cost: Choosen at random from [25, 200].
Medium facility cost: Choosen at random from [50, 400].
High facility cost: Choosen at random from [200, 600].

It is intuitive from the mixed integer and integer programming models presented thus far that sequential approach always produces optimal solutions if customer – depot assignment is the same in the solutions of both LRP and LAP. Therefore the differences between the solutions of LRP and the sequential approach will be based on the wrong assignment of customers to depots, which results from the moment sum assumption of transportation costs in the sequential approach.

The computational results are presented in Tables 3.1 – 3.5.

We can see from Table 3.1 to Table 3.3 that as the cost of opening a facility decreases the gap between the optimum solution of LRP and the solution of sequential approach increases. This results from the fact that as the cost of opening a facility decreases the models are eager to open more depots to decrease system costs. Because, the fixed cost of a facility is small when compared to the transportation cost in the LAP module. According to the structure of the LAP module it can be profitable to open more depots to save the transportation cost. However, in LRP module vehicle dispatching costs will restrict to open more depots since it means to operate more vehicles in the system. This difference between the philosophy of LRP and that of LAP – VRP shows its strength mostly when facility cost is low.

On the other hand, it is seen that the average gap in Table 3.3 is greater than the average gap in Table 3.2, although the facility cost in the data set of Table 3.3 is greater than that of the data set of Table 3.2. This is due to the large differences between facility costs and it still supports our claim when we look at the number of LAP – VRP runs that is not optimal in both tables.

Our argument about the opportunity cost of ignoring vehicle routes is strengthened by the computational results in Table 3.4 and Table 3.5. We apply simultaneous and sequential solution approaches to a problem setting that has a convex relationship between the opening cost of a facility and the number of vehicles assigned to it in Table 3.4. In Table 3.5, there is a concave relationship between these two terms. In both of the problem settings the fixed cost of opening a facility and vehicle dispatching cost are of medium size.

From the computational results it is verified that the sequential approach performs worse when there is a convex cost function of initial establishing cost of a facility. We see gaps of greater than 10% in such a case. We can state that the contribution of simultaneous approach is more significant when facility opening cost has a convex structure.

When the facility opening cost is concave, the gap is smaller than the gap when it is convex. However, it is still greater than the case where the facility opening cost has a linear structure.

Data Set	LRP	LAP - VRP	Gap %
1	1589	1589	0.0
2	1384	1384	0.0
3	1132	1132	0.0
4	1369	1369	0.0
5	1377	1377	0.0
6	1774	1774	0.0
7	1475	1521	3.1
8	1811	1811	0.0
9	890	890	0.0
10	1814	1814	0.0
11	918	918	0.0
12	1228	1228	0.0
13	1005	1008	0.3
14	1680	1686	0.4
15	793	793	0.0
16	1160	1160	0.0
17	1218	1218	0.0
18	1494	1494	0.0
19	1122	1122	0.0
20	1204	1204	0.0
	Average Gap % =		0.2

Table 3.1: Problem size 10; Medium facility cost, medium vehicle cost.

Data Set	LRP	LAP - VRP	Gap %
1	916	916	0.0
2	1465	1465	0.0
3	1483	1509	1.8
4	1000	1000	0.0
5	1179	1179	0.0
6	1216	1216	0.0
7	1001	1037	3.6
8	1401	1455	3.9
9	1178	1178	0.0
10	937	937	0.0
11	1096	1096	0.0
12	940	940	0.0
13	1407	1581	12.4
14	1247	1247	0.4
15	1458	1628	11.7
16	950	1016	6.9
17	1253	1369	9.3
18	1147	1147	0.0
19	1512	1526	0.9
20	1128	1128	0.0
	Average Gap % =		2.5

Table 3.2: Problem size 10; Low facility cost; medium vehicle cost.

Data Set	LRP	LAP - VRP	Gap %
1	1511	1511	0.0
2	1846	1846	0.0
3	1342	1342	0.0
4	1149	1149	0.0
5	1439	1439	0.0
6	1353	1353	0.0
7	1445	1445	0.0
8	1489	1489	0.0
9	1446	1446	0.0
10	1231	1306	6.1
11	1636	1636	0.0
12	1113	1113	0.0
13	1351	1351	0.0
14	1241	1241	0.0
15	1181	1181	0.0
16	1319	1319	0.0
17	1256	1256	0.0
18	1303	1370	5.1
19	2077	2077	0.0
20	1261	1261	0.0
	Average Gap % =		0.6

Table 3.3: Problem size 10; High facility cost, medium vehicle cost.

Data Set	LRPm	LAP - VRPm	Gap %
1	1810	2089	15.4
2	1620	1704	5.2
3	1312	1312	0.0
4	1522	1688	10.9
5	1979	2273	14.9
6	1477	1477	0.0
7	1655	1701	2.8
8	2072	2131	2.8
9	970	970	0.0
10	2176	2533	16.4
11	998	998	0.0
12	1408	1408	0.0
13	1025	1088	6.1
14	1979	2007	1.4
15	852	873	2.5
16	1240	1371	10.6
17	1298	1298	0.0
18	1618	1813	12.1
19	1264	1302	3.0
20	1384	1384	0.0
	Average Gap % =		5.2

Table 3.4: Problem size 10; Convex installation function.

Data Set	LRPm	LAP - VRPm	Gap %
1	1593	1593	0.0
2	1388	1388	0.0
3	1136	1136	0.0
4	1373	1373	0.0
5	1778	1778	0.0
6	1385	1435	3.6
7	1479	1525	3.1
8	1815	1815	0.0
9	894	894	0.0
10	1818	1818	0.0
11	922	922	0.0
12	1232	1232	0.0
13	1012	1012	0.0
14	1684	1691	0.4
15	797	797	0.0
16	1164	1164	0.0
17	1222	1222	0.0
18	1498	1498	0.0
19	1126	1126	0.0
20	1208	1208	0.0
	Average Gap % =		0.4

Table 3. 5: Problem size 10; Concave installation cost.

Chapter 4

A Heuristic for Location – Routing Problem

The solution procedures for location – routing problems are quite limited when compared to the ones found in facility location literature and vehicle routing literature. Most of the successful location – routing heuristics are iterative algorithms that are based on decomposing the problem into location – allocation and vehicle routing phases or their variants. Therefore, we can state that the development of the efficient algorithms for location – routing problems is based on the research on the existing facility location and vehicle routing heuristics.

The study of G. Clarke and J. Wright [6] introduced a savings concept to the single depot vehicle routing problems and produced a greedy type heuristic to find a vehicle routing structure that is close to the optimum structure. An equally valid greedy approach for the uncapacitated facility location problem is to start with all facilities open and then, one – by – one, close a facility whose closing leads to the greatest increase in profit as stated in A. A. Kuehn and M. J. Hamburger [15].

In our study, inspiring from the savings algorithm of Clarke and Wright [6] and the study of Kuehn and Hamburger [15], we propose a greedy type heuristic algorithm that will approximately solve the location – routing problems presented in Chapter 3.

This chapter can be outlined as follows: In Section 4.1 we describe the heuristic algorithm for the location – routing problem that have a single stage, multiple uncapacitated facility and multiple capacitated vehicle structure with deterministic supplies and demands. In Section 4.2 we present the computational results based on a number of experimental and hypothetical data sets. We compare the results of our heuristic with the optimum solutions of LRP and LRPm for small sized problems. We also compare the solutions of our heuristic with the solutions of the sequential approach commonly used, that is the approach of first solving a location – allocation and then a vehicle routing problem.

4.1 A Greedy Heuristic for LRP

In the previous chapters we suggest that the number of vehicles used is a decision variable and this makes a facility to be closed if its customers can be served by the vehicles of a nearby facility and if this leads to a decrease in system – wide costs. Based on this assumption our heuristic is developed on a savings concept. We introduced savings functions for both the construction of vehicle routes and facility closing phase. We claim that using a cleverly established savings function can result in close to optimum solutions or at least better solutions than the solutions of the sequential approach.

The heuristic we propose here starts with an initial feasible solution where a depot is opened at all potential depot sites. Here, each customer is assigned to the depot, which is closest to it in terms of the Euclidean distance. After getting the initial solution, our algorithm applies two main subalgorithms. These are: Combining vehicle routes and closing open depots until no more cost improvement can be possible. The procedures to combine vehicle routes and close open depots are mentioned in the following sections.

4.1.1 Cost Savings Realized From Combining Vehicle Routes

The way we compute cost savings realized from combining two vehicle routes is similar to the savings function proposed by G. Clarke and J. Wright [6]. Different from that, our savings function incorporates vehicle capacities and cost savings realized from utilizing one less vehicle in the distribution system.

Consider routes k and l. The cost saving when k combines with l, which means l serves for the customers initially at k, is depicted in Figure 4.1.

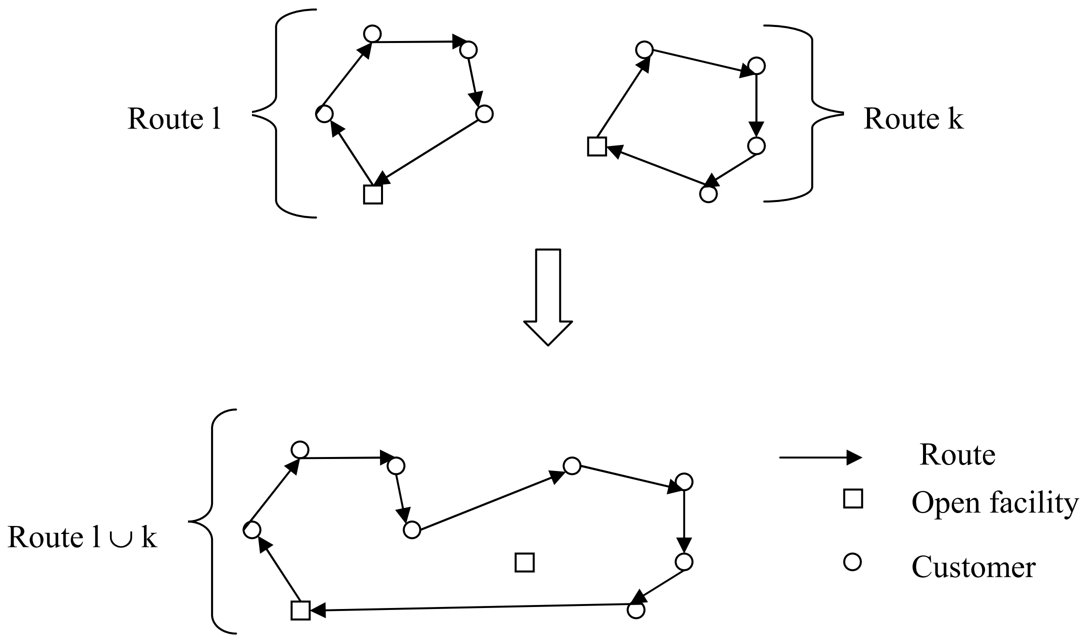


Figure 4.1: Procedure to combine two routes subject to capacity restrictions.

In order to define the cost savings realized from combining two vehicle routes we need to define some of the important parameters, which can be seen in Figure 4.1 above.

Definitions:

S_{lk}: Cost savings when route k is combined with route l.

End[k]: The last customer that vehicle k visits on its route.

Start[k]: The first customer that vehicle k visits on its route.
Assign[k]: Depot that vehicle k departs from.
VC: Cost of operating one more vehicle.

We can now define our savings function as:

$$S_{lk} = C_{End[l]Assign[l]} + C_{End[k]Assign[k]} + C_{Assign[k]Start[k]} + VC - C_{End[l]Start[k]} - C_{End[k]Assign[l]}.$$

The cost figures that we present in the savings function can be easily seen in Figure 4.1. By combining route l and route k we delete the paths from End[l] to Assign[l], from End[k] to Assign[k] and from Assign[k] to Start[k]. On the other hand, we add the paths from End[l] to Start[k] and from End[k] to Assign[l]. We also add the saving in vehicle dispatching cost resulting from using one less vehicle. This term in the savings function can be easily modified for representing the convex and concave cost functions proposed in Chapter 3.

Note here that the savings function presented above is also valid for combining vehicle routes that are assigned to the same depot. In such a case:

$$C_{End[k]Assign[k]} = C_{End[k]Assign[l]}$$

And

$$S_{lk} = C_{End[l]Assign[l]} + C_{Assign[k]Start[k]} + VC - C_{End[l]Start[k]}.$$

which is the savings function when both vehicle k and vehicle l are assigned to the same depot.

4.1.2 Cost Savings Realized From Closing an Open Depot

In order to close a depot we have to reassign its customers to the other depots. The reassigning of a depot's customers can be done by combining the routes of its vehicles with the routes of other depots' vehicles and/or shifting its vehicles to another depot.

How to combine two different routes is described in the previous section. The shifting of a vehicle from its depot to another depot can be defined in a similar way but the structure of the route changes depending on the location of the candidate depot. We assume that the closest customers to a depot will be the starting and ending customers on a vehicle's route departing from that depot. We take into account this assumption while shifting vehicles. In order to define it more clearly, a picture of a vehicle shift can be seen in Figure 4.2.

The definitions below are used to define the savings concept when shifting a vehicle route from one depot to the other. These definitions are understood more clearly when analyzed together with the Figure 4.2.

Definitions:
Shift[k][i] = Cost savings when vehicle k is shifted to depot i.
Min[k][i] = The customer on route k that is closest to depot i.
Follower[k][i] = The customer that follows Min[k][i] on route k.
End[k] = The last customer that vehicle k visits on its route.
Start[k] = The first customer that vehicle k visits on its route.
Assign[k] = The depot that vehicle k departs from.

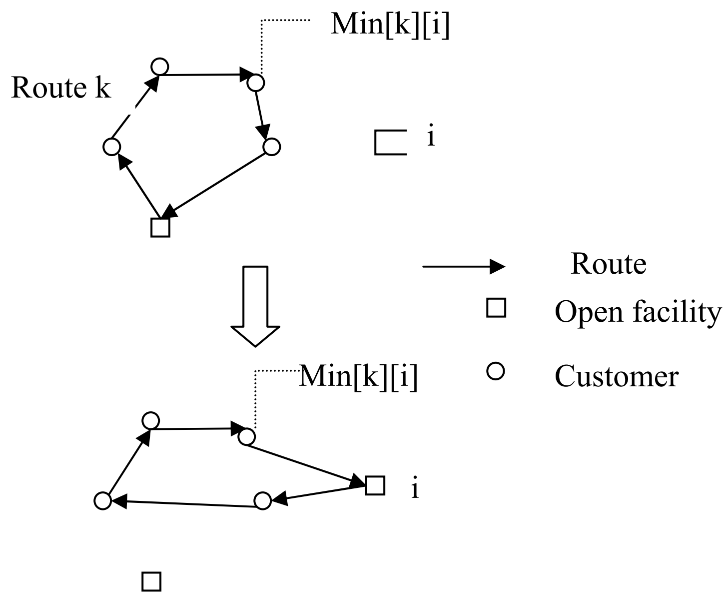


Figure 4.2: Vehicle shift between depots.

The cost savings obtained by shifting vehicle k to depot i is dependent on the location of the customer, which we call Min[k][i], and can be expressed as follows under two possible scenarios:

If Min[k][i] is not End[k]:

$$Shift[k][i] = C_{Assign[k]Start[k]} + C_{End[k]Assign[k]} + C_{Min[k][i]Follower[k][i]} - C_{Min[k][i]i} - C_{iFollower[k][i]} - C_{End[k]Start[k]}$$

If Min[k][i] is End[k]:

$$Shift[k][i] = C_{End[k]Assign[k]} + C_{Assign[k]Start[k]} - C_{End[k]i} - C_{iStart[k]}$$

Cost savings realized from closing an open depot result from the total savings from the combination of that depot's vehicle routes to other depots' routes and from the vehicle shifts from that depot to the other depots. Besides, the cost of opening a depot at the corresponding location is incorporated into the savings realized from closing that depot as follows: In order to obtain the cost saving realized from closing depot i we first need to find the maximum savings for each vehicle of depot i when we shift that vehicle or combine its route to another depot's vehicle route. When depot i has more than one vehicle, we first find the vehicle and its required action (combining or shifting) that yields the greatest saving. After applying this change into the distribution structure, we recompute the savings and apply the same procedure for each of the vehicle routes. After all, we add the fixed cost of opening a depot at location i to the total of the maximum savings realized from shifting or combining depot i's vehicles to the other depots or vehicle routes.

When we consider the convex and concave functions representing the effect of the number of vehicles assigned to a depot on the initial installing cost of that depot, we should consider the cost savings resulting from the changes in the number of vehicles assigned to each depot. When we shift a vehicle from depot i to depot j, the number of vehicles departing from depot i will decrease by 1 and the number of vehicles departing from depot j will increase by 1. Similarly, when we combine two vehicle routes assigned to different depots the number of vehicles departing from one of the depots will decrease by 1 and for the other depot it will remain the same. If the vehicles are assigned to the same depot then the number of vehicles departing from that depot will decrease by 1 when we combine two of its vehicle routes.

4.1.3 Algorithm

In the light of the savings functions defined above our heuristic algorithm for solving the location – routing problem can be stated as follows:

Step 1: Obtain an initial feasible solution with customers assigned to the closest depots around them.

Step 2: Assign one vehicle to each customer and construct routes between the customers and the associated depots.

Step 3: Compute S_{lk} for each vehicle route pairs. Find the vehicle route pair that have the largest cost saving. If the maximum saving is greater than zero and if it is feasible in terms of vehicle capacity then combine route k to route l and return to Step 3. If it is not feasible then set maximum saving equal to zero and return to

Step 3. If maximum saving is smaller than or equal to zero then continue with Step 4.

Step 4: Compute S_{lk}'s where vehicle l and vehicle k are assigned to different depots. Compute Shift[k][i]'s where vehicle k is not assigned to depot i and a depot is opened at location i. For each open depot compute the cost saving realized from closing that depot. If maximum saving is greater than zero then close the depot that yields the maximum savings and return to Step 4. Else continue with Step 5.

Step 5: Compute S_{lk}'s for each vehicle route pairs. Compute Shift[k][i]'s for all k and i where vehicle k is not assigned to depot i and a depot is opened at location i. Find the maximum of S_{lk}'s and Shift[k][i]'s. If maximum is greater than zero then make the appropriate change in the feasible solution and continue with Step 5. Else stop.

Step 1 and 2 construct an initial feasible solution for our algorithm. After executing these steps we have each customer to be assigned to the closest depot and each vehicle is assigned to one customer. Step 3 applies a procedure that is similar to the savings algorithm presented in G. Clarke and J. Wright [6]. After Step 3 we are sure that no more cost savings can be possible by combining two vehicle routes. However, further improvements in the cost function are possible by considering to close some of the open depots and this is done in Step 4. Finally, Step 5 concludes the algorithm. The changes in the allocation of vehicle routes and customers in the previous sections can make further cost improvements possible by the combination or shifting of the existing vehicle routes.

4.2 Computational Results

We propose a variety of experimental environments to analyze the strength of our greedy heuristic. We randomly generate data for problem sizes 10, 50 100. These problem sizes refer to the total number of locations in the experimental model. In all of the problem settings the location of potential depot sites are kept separate from the customer locations.

In a problem setting of size 10, 3 of the locations are potential depot locations while the others are customer locations. In a problem setting of size 50 we have 5 potential depot locations and in a problem setting of size 100 there are 10 potential locations for depot sites while all the others are customer locations.

We present the comparison of the results of our heuristic with optimal solutions of LRP and LRPm for problems of size 10 since the state of the art software CPLEX can allow us to solve location – routing problems of size up to 10. We also present comparisons of our heuristic and the sequential approach for problems of sizes 50 and 100. In building the sequential algorithm, we first solve a LAP formulation and then apply the savings algorithm of Clarke and Wright [6] for each open facility and its customers.

We introduce experimental settings depending on the importance of the fixed cost of opening a facility and of the vehicle dispatching cost. As mentioned in Chapter 3 we divide the facility costs into three categories:

Low facility cost: Choosen at random from [25, 200].
Medium facility cost: Choosen at random from [50, 400].
High facility cost: Choosen at random from [200, 600].

In a similar manner we categorize the vehicle dispatching cost into three groups:

Low vehicle cost: 50.
Medium vehicle cost: 100.
High vehicle cost: 200.

The computational results are presented through the Tables 4.1 – 4.15. The most interesting issue that results from the computational experiments is that as the problem size increases our heuristic outperforms the sequential approach. The improvement of the heuristic over the sequential approach increases as the problem size increases. This can be due to the fact that the moment sum assumption of the sequential approach causes improper assignment of customers to depots and the degree it biases from the optimal solution increases as the number of potential depot sites and the number of customers increase. On the other hand our heuristic aims to solve the location and routing problems in an integrated manner.

The computational results based on the fixed costs of opening facilities are presented from Table 4.6 to Table 4.8. As it is seen, the heuristic we have proposed performs better than sequential approach in all three of the experimental environments. Especially, the heuristic is superior when the facility cost is low. As the facility cost decreases the average improvement of our heuristic over the sequential approach also increases.

A similar result can be obtained when Tables 4.1 – 4.3 are analyzed. In these tables the comparison of the solution of our heuristic and the optimal solution of LRP is done for three different experimental environments based on the facility cost. These results agree with our observation stated above. As the costs of opening facilities decreases our heuristic gets closer to optimum solutions. In Table 4.2 our heuristic finds 8 optimal solutions in 20 runs where the facility costs are taken as low facility costs for these problem parameters. This performance is the best in all three simulation sets.

The performance of the heuristic in relation to the vehicle dispatching cost is also analyzed. The computational results based on three different experimentation environments are presented in Tables 4.6, 4.9 and 4.10. It is seen that the heuristic outperforms the sequential approach especially when the vehicle dispatching cost is high.

Our heuristic performs best when the function that relates facility opening cost to the number of vehicles is convex. The associated computational results are presented in Tables 4.4, 4.5, 4.11, 4.12, 4.14 and 4.15. It even outperforms the sequential method for problem size of 10 as seen in Table 4.4. The improvement that the solution of our greedy heuristic implies is larger when there is a convex relationship between the initial facility establishing cost and the number of vehicles assigned to that facility.

When we analyze the performance of the heuristic according to the structure of the facility opening cost, we see that the improvement is smaller in a problem setting of concave cost function. Then it gets larger as we applied linear and convex cost functions to the model.

Data Set	LRP	Heuristic	Gap %
1	1589	1589	0.0
2	1384	1494	7.9
3	1132	1132	0.0
4	1369	1415	3.4
5	1377	1439	4.5
6	1774	1774	0.0
7	1475	1511	2.4
8	1811	1834	1.3
9	890	903	1.5
10	1814	1814	0.0
11	918	961	4.7
12	1228	1240	1.0
13	1005	1027	2.2
14	1680	1687	0.4
15	793	808	1.9
16	1160	1188	2.4
17	1218	1260	3.4
18	1494	1510	1.0
19	1122	1177	4.9
20	1204	1246	3.5
	Average Gap % =		2.3

Table 4.1: Problem size 10; Medium facility cost, medium vehicle cost.

Data Set	LRP	Heuristic	Gap %
1	916	916	0.0
2	1465	1482	1.1
3	1483	1495	0.8
4	1000	1000	0.0
5	1179	1179	0.0
6	1216	1239	1.9
7	1001	1037	3.6
8	1401	1401	0.0
9	1178	1224	3.9
10	937	948	1.2
11	1096	1137	3.7
12	940	1004	6.8
13	1407	1407	0.0
14	1247	1247	0.0
15	1458	1546	6.0
16	950	1016	6.9
17	1253	1253	0.0
18	1147	1147	0.0
19	1512	1512	0.1
20	1128	1144	1.4
	Average Gap % =		1.9

Table 4.2: Problem size 10; Low facility cost, medium vehicle cost.

Data Set	LRP	Heuristic	Gap %
1	1511	1540	1.9
2	1846	1909	3.4
3	1342	1455	8.4
4	1149	1149	0.0
5	1439	1495	3.9
6	1353	1355	0.1
7	1445	1486	2.8
8	1489	1489	0.0
9	1446	1477	2.1
10	1231	1252	1.7
11	1636	1636	0.0
12	1113	1131	1.6
13	1351	1559	15.4
14	1241	1271	2.4
15	1181	1189	0.6
16	1319	1353	4.9
17	1256	1325	5.5
18	1303	1368	5.0
19	2077	2077	0.0
20	1261	1304	3.4
	Average Gap % =		3.2

Table 4.3: Problem size 10; High facility cost, medium vehicle cost.

Data Set	LRPm	Heuristic	Gap %
1	1810	1810	0.0
2	1620	1808	11.6
3	1312	1312	0.0
4	1522	1583	4.0
5	1979	1979	0.0
6	1477	1601	8.4
7	1655	1717	3.7
8	2072	2227	7.5
9	970	989	2.0
10	2176	2268	4.2
11	998	1052	5.4
12	1408	1420	0.9
13	1025	1047	2.1
14	1979	2007	1.4
15	852	852	0.0
16	1240	1268	2.3
17	1298	1340	3.2
18	1618	1618	0.0
19	1264	1312	3.8
20	1384	1426	3.0
	Average Gap % =		3.2

Table 4.4: Problem size 10; Convex cost function.

Data Set	LRPm	Heuristic	Gap %
1	1593	1636	2.7
2	1388	1447	4.3
3	1136	1201	5.7
4	1373	1373	0.0
5	1778	1817	2.2
6	1385	1449	4.6
7	1479	1521	2.8
8	1815	1874	3.3
9	894	908	1.6
10	1818	1818	0.0
11	922	922	0.0
12	1232	1287	4.5
13	1012	1028	1.6
14	1684	1695	0.7
15	797	830	4.1
16	1164	1269	9.0
17	1222	1309	7.1
18	1498	1579	5.4
19	1126	1205	7.0
20	1208	1325	9.7
	Average Gap % =		3.8

Table 4.5: Problem size 10; Concave cost function.

Data Set	Sequential	Heuristic	Improvement %
1	5831	5652	-3.1
2	6016	5802	-3.6
3	5680	5602	-1.4
4	6534	6547	0.2
5	5036	4886	-3.0
6	6957	6880	-1.1
7	6319	6303	-0.3
8	6657	6647	-0.2
9	5638	5642	0.0
10	5974	5828	-2.4
11	5010	4877	-2.7
12	5870	5782	-1.5
13	6833	6734	-1.4
14	6446	6329	-1.8
15	5364	5332	-0.6
16	5536	5460	-1.4
17	5387	5303	-1.6
18	6238	6238	0.0
19	6656	6641	-0.2
20	6499	6338	-2.5
	Average Improvement =		-1.4

Table 4.6: Problem size 50; Medium facility cost, medium vehicle cost.

Data Set	Sequential	Heuristic	Improvement %
1	5615	5543	-1.3
2	4964	4779	-3.7
3	5157	5101	-1.1
4	5378	5273	-2.0
5	5506	5368	-2.5
6	6842	6715	-1.9
7	6108	5948	-2.6
8	5424	5413	-0.2
9	6062	5986	-1.3
10	5475	5328	-2.7
11	7198	7205	0.1
12	5913	5767	-2.5
13	4580	4546	-0.7
14	5308	5256	-1.0
15	6624	6481	-2.2
16	5266	5145	-2.3
17	6826	6758	-1.0
18	5265	5156	-2.0
19	5432	5335	-1.8
20	5050	5005	-0.9
	Average Improvement =		-1.7

Table 4.7: Problem size 50; Low facility cost, medium vehicle cost.

Data Set	Sequential	Heuristic	Improvement %
1	6542	6404	-2.1
2	6306	6302	-0.1
3	7141	7100	-0.6
4	7172	7160	-0.2
5	5449	5409	-0.7
6	6772	6634	-2.0
7	6813	6912	1.4
8	5843	5792	-0.9
9	6995	6947	-0.7
10	5431	5396	-0.6
11	5636	5588	-0.8
12	5068	5047	-0.4
13	5444	5496	0.9
14	6391	6378	-0.2
15	5740	5642	-1.7
16	6524	6560	0.5
17	6209	6213	0.0
18	6759	6566	-2.9
19	5492	5426	-1.2
20	6343	6242	-1.6
	Average Improvement =		-0.7

Table 4.8: Problem size 50; High facility cost, medium vehicle cost.

Data Set	Sequential	Heuristic	Improvement %
1	4481	4352	-2.9
2	4566	4442	-2.7
3	4330	4252	-1.8
4	4884	4897	0.2
5	3986	3886	-2.5
6	5507	5480	-0.5
7	4819	4853	0.7
8	5307	5297	-0.2
9	4288	4292	-0.1
10	4574	4478	-2.1
11	3660	3577	-2.3
12	4520	4482	-0.8
13	5283	5234	-0.9
14	4996	4879	-2.3
15	4064	4032	-0.8
16	4286	4260	-0.6
17	4087	4033	-1.3
18	4688	4688	0.0
19	5106	5091	-0.3
20	4899	4788	-2.2
	Average Improvement =		-1.2

Table 4.9: Problem size 50; Medium facility cost, low vehicle cost.

Data Set	Sequential	Heuristic	Improvement %
1	8531	8252	-3.3
2	8916	8502	-4.6
3	8380	8302	-0.9
4	9834	9847	0.1
5	7136	6886	-3.5
6	9857	9680	-1.8
7	9319	9203	-1.2
8	9357	9347	-0.1
9	8338	8342	0.0
10	8774	8505	-3.1
11	7710	7477	-3.0
12	8570	8382	-2.2
13	9933	9734	-2.0
14	9346	9168	-1.9
15	7964	7897	-0.8
16	8036	7860	-2.2
17	7987	7840	-1.8
18	9338	9338	0.0
19	9756	9741	0.1
20	9699	9438	-2.7
	Average Improvement =		-1.7

Table 4.10: Problem size 50; Medium facility cost, high vehicle cost.

Data Set	Sequential	Heuristic	Improvement %
1	6315	6348	0.5
2	5880	5826	-0.9
3	6427	6273	-2.5
4	6055	5904	-2.6
5	6286	6286	0.0
6	6502	6453	-0.8
7	5326	5239	-1.7
8	7345	7264	-1.1
9	6666	6500	-2.6
10	6848	6610	-3.6
11	6009	5840	-2.9
12	6431	6298	-2.1
13	6091	6138	0.8
14	5927	5811	-2.0
15	6497	6344	-2.4
16	7039	6902	-2.0
17	7374	7287	-1.2
18	5894	5894	0.0
19	5458	5376	-1.5
20	5917	5894	-0.4
	Average Improvement =		-1.4

Table 4.11: Problem size 50; Concave cost function.

Data Set	Sequential	Heuristic	Improvement %
1	7056	6709	-5.2
2	6681	6744	0.9
3	7125	6834	-4.3
4	5755	5679	-1.3
5	6464	6386	-1.2
6	8086	7749	-4.3
7	6887	6560	-5.0
8	8028	7492	-7.2
9	5634	5796	2.8
10	7299	7015	-4.0
11	7467	7258	-2.9
12	8079	7731	-4.5
13	6019	5938	-1.4
14	7099	6852	-3.6
15	6922	6812	-1.6
16	5960	5927	-0.6
17	8318	8136	-2.2
18	6553	6408	-2.3
19	6770	6591	-2.7
20	7535	7369	-2.3
	Average Improvement =		-2.6

Table 4.12: Problem size 50; Convex cost function.

Data Set	Sequential	Heuristic	Improvement %
1	10015	9845	-1.7
2	9421	9311	-1.2
3	9714	9612	-1.1
4	9801	9609	-2.0
5	8220	7901	-4.0
6	10159	9844	-3.2
7	10205	10013	-1.9
8	9833	9598	-2.4
9	9440	9239	-2.2
10	9584	9392	-2.0
11	9763	9561	-2.1
12	11094	10888	-1.9
13	9817	9628	-2.0
14	11012	10847	-1.5
15	10653	10482	-1.6
16	10269	10103	-1.6
17	11530	11381	-1.3
18	10694	10481	-2.0
19	10936	10792	-1.3
20	10210	9979	-2.3
	Average Improvement =		-2.0

Table 4.13: Problem size 100; Medium facility cost, medium vehicle cost.

Data Set	Sequential	Heuristic	Improvement %
1	11284	10835	-4.1
2	11179	10818	-3.3
3	10746	10539	-2.0
4	10544	10362	-1.8
5	10531	10206	-3.2
6	11002	10494	-4.8
7	10800	10320	-4.7
8	9861	9731	-1.3
9	11034	10865	-1.6
10	12355	12090	-2.2
11	11665	11174	-4.4
12	12985	12677	-2.4
13	10810	10616	-1.8
14	10746	10619	-1.2
15	11991	11669	-2.8
16	10435	10244	-1.9
17	9995	9805	-1.9
18	11392	11038	-3.2
19	12447	11455	-8.7
20	11464	11313	-1.3
	Average Improvement =		-2.9

Table 4.14: Problem size 100: Convex cost function.

Data Set	Sequential	Heuristic	Improvement %
1	10751	10558	-1.8
2	10002	9983	-0.2
3	10858	10613	-2.3
4	11632	11558	-0.6
5	11606	11443	-1.4
6	9951	9892	-0.6
7	11177	11052	-1.1
8	11134	10889	-2.2
9	11157	11037	-1.1
10	10696	10319	-3.7
11	10938	10760	-1.7
12	10982	10601	-3.6
13	10945	10660	-2.7
14	11645	11389	-2.2
15	10654	10359	-2.8
16	11870	11756	-1.0
17	9929	9638	-3.0
18	11411	11251	-1.4
19	10915	10858	-0.5
20	10379	10101	-2.8
	Average Improvement =		-1.8

Table 4.15: Problem size 100; Concave cost function.

Chapter 5

Conclusion

This chapter provides a brief summary of the contributions of this thesis and addresses a wide range of directions for future research. In this thesis we have considered the analysis of the integration of vehicle routing decisions with the facility location decisions. The assumptions that we made throughout this study were:

- The distribution system is a single stage distribution system. The transportation costs are restricted to outbound transportation only.
- Locations of potential depot sites are separate from the locations of customers.
- The number of facilities to open and the number of vehicles to operate are not fixed to some value and are decision variables.
- Facilities are uncapacitated.
- Vehicles have a predetermined capacity.
- Each customer can be served by only one vehicle.
- All the parameters are deterministic.
- The initial installing cost of a facility is dependent on the number of vehicles departing from that facility.

In the following section, we make a short summary of the contributions of this thesis to the location – routing literature.

5.1 Contributions

We analyzed the opportunity cost of ignoring vehicle routes while locating depots both in a conceptual way and in a computational way. We claimed that in designing a distribution system the whole system should be represented by a single model and it should be seen as an integration of interdependent components. As a result of our study we computationally showed that the solution derived from applying the sequential approach is a sub – optimal solution and the optimal solution can be obtained by formulating the routing decisions and location decisions under a single model and then solving this model. We showed that simultaneous solution approach has more realistic assumptions then the moment sum assumption and produces better results.

We introduced a realistic cost structure to the problem that the initial establishing cost of opening a new depot is dependent on the number of vehicles that will depart from that depot and serve its customers. We analyzed the effect of this new cost structure under different scenarios and presented its effects on the solution of the problem. The location – routing model for this modified cost structure was also presented. Besides, taking the number vehicles that the system will operate as a decision variable is a rare modeling approach in the location – routing literature. We believe that this decision is as important as the location decisions in a distribution environment.

The heuristic that we applied is an interesting study that it is helpful to analyze possible greedy solution methods for location – routing problems. We introduced a new savings structure and built our greedy algorithm on this structure. We showed that our heuristic produces close to optimum results and performs better than the sequential approach, especially for large sized problems. It can serve as a reliable benchmark for other algorithms in the context of this problem.

5.2 Future Research Directions

At the end, there are several future research directions emanating from this research study as such:

- The number, demand and location of customers as well as travel times of vehicles may not be known a priori and consequently can be treated as random variables. In such cases the complexity of the problem will increase.
- Improved transportation performance in terms of greater speed and reliability has decreased the significance of spatial parameters such as distance in logistics model design. Some customers may impose service deadlines and desirable service time restrictions. The model can be extended to further consider time windows.
- The parameters may have changing natures. For example, readjusting of vehicle routes may be needed periodically due to changing demand patterns or depot location cost may vary over time with the fluctuations of employee wages and interest rates. Hence, the incorporation of the dynamic nature into the location – routing problems may improve the realism associated with the distribution system design.
- The location – routing model may be further extended to examine the interactions among location, routing and inventory control decisions. Because, the level of inventory has a significant effect on the capacity and number of depots as well as the choice and route of transportation modes.

Bibliography

[1] Averbakh, I., and Berman, O., “Probabilistic sales – delivery man and sales – delivery facility location problems on a tree”, Transportation Science 29 (1995) 184 – 197.

[2] Averbakh, I., and Berman, O., “Routing and Location – Routing p – delivery men problems on a path”, Transportation Science 28 (1994) 162 – 167.

[3] Bookbinder, J. H., and Reece K. E., “Vehicle routing considerations in distribution system design”, European Journal of Operational Research 37 (1988) 204 – 213.

[4] Bruno, G., Ghiani, G., and Improta G., “A multi – modal approach to the location of a rapid transit line”, European Journal of Operational Research 104 (1998) 321 – 332.

[5] Christofides N. and Eilon, S., “An algorithm for the vehicle dispatching problem”, Operational Research Quarterly 20 (1969) 309 – 318.

[6] Clarke, G., and Wright, J., “Scheduling of vehicles from a central depot to a number of delivery points”, Operations Research 12 (1964) 568 – 581.

[7] Cooper, L., “The transportation – location problem”, Operations Research 20 (1972) 309 – 315.

[8] Ghiani, G., and Improta, G., “An efficient transformation of the generalized vehicle routing problem”, European Journal of Operational Research 122 (2000) 11 – 17.

[9] Hansen, P. H., Hegedahl, B., Hjortkjaer S., and Obel B., “A heuristic solution to the warehouse location – routing problem”, European Journal of Operational Research 76 (1994) 111 – 127.

[10] Harrison, H., “A planning system for facilities and resources in distribution networks”, Interfaces 9 (1979) 6 – 22.

[11] Higgins, J. C., “On the merits of simple models in distribution planning”, International Journal of Physical Distribution 2 (1972) 144 – 148.

[12] Jacobsen, S. K., and Madsen, O. B. G., “A comparative study of heuristics for a two level routing – location problem”, European Journal of Operational Research 5 (1980) 378 – 387.

[13] Jamil, M., Batta, R., and Malon, D. M., “The traveling repairperson home base location problem”, Transportation Science 28 (1994) 150 – 161.

[14] Koksalan, M., Sural H., and Kirca, O., “A location – distribution application for a beer company”, European Journal of Operational Research 80 (1995) 16 – 24.

[15] Kuehn, A. A., Hamburger, M. J., “A heuristic program for locating warehouses”, Management Science 9 (1963) 643 – 666.

[16] Laporte, G., Gendreau, M., Potvin, J., and Semet, F., “Classical and modern heuristics for the vehicle routing problem”, International Transactions in Operational Research 7 (2000) 285 – 300.

[17] Laporte, G., Louveaux, F., and Mercure, H., “Models and exact solutions for a class of stochastic location – routing problems”, European Journal of Operational Research 39 (1989) 71 – 78.

[18] Laporte, G., and Nobert, Y., “An exact algorithm for minimizing routing and operating costs in depot location”, European Journal of Operational Research 6 (1981) 224 – 236.

[19] Laporte, G., Nobert, Y., and Arpin, D., “An exact algorithm for solving capacitated location – routing problem”, Annals of Operations Research 6 (1986) 293 – 310.

[20] Laporte, G., Nobert, Y., and Pelletier, P., “Hamiltonian location problems”, European Journal of Operational Research 12 (1983) 82 – 96.

[21] Laporte, G., “The Vehicle Routing Problem: An overview of exact and approximate algorithms”, European Journal of Operational Research, 59 (1992) 345 – 358.

[22] Lawrence, R. M., and Pengilly P. J., “The number and location of depots required for handling products for distribution to retail stores in South – East England”, Operational Research Quarterly 20 (1969) 23 – 32.

[23] Maranzana, F. E., “On the location of supply points to minimize transport costs”, Operational Research Quarterly 15 (1965) 261 – 270.

[24] Min, H., Jayaraman, V., and Srivastava, R., “Combined location – routing problems: A synthesis and future research directions”, European Journal of Operational Research, 108 (1998) 1 – 15.

[25] Minoux, M., “Mathematical Programming: Theory and Algorithms”, John – Wiley & Sons (1986).

[26] Nagy, G., and Salhi, S., “Nested heuristic methods for the location – routing problem”, Journal of the Operational Research Society 47 (1996) 1166 – 1174.

[27] Ogryczak, W., Studzinski K., and Zorychta K., “A solver for the multi objective transshipment problem with facility location”, European Journal of Operational Research 43 (1989) 53 – 64.

[28] Or, I., and Pierskalla, W. P., “A transportation location – allocation model for regional blood banking”, AIIE Transactions 11 (1979) 86 – 95.

[29] Salhi, S., and Rand G. K., “The effect of ignoring routes when locating depots”, European Journal of Operational Research 39 (1989) 150 – 156.

[30] Srivastava, R., “Alternate solution procedures for the location – routing problem”, OMEGA International Journal of Management Science 21 (1993) 497 – 506.

[31] Stowers, C. L., and Palekar U. S., “Location models with routing considerations for a single obnoxious facility”, Transportation Science 27 (1993) 350 – 362.

[32] Tapiero, C. S., “Transportation – location – allocation problems over time”, Journal of Regional Science 11 (1971) 377 – 384.

[33] Tuzun, D., and Burke, L. I., “A two – phase tabu search approach to the location routing problem”, European Journal of Operational Research 116 (1999) 87 – 99.

[34] Von Boventer, “The relationship between transportation costs and location rent in transportation problems”, Journal of Regional Science 3 (1961) 27 – 40.

[35] Watson – Gandy, C., and Dohrn, P., “Depot location with one salesman – a practical approach”, OMEGA International Journal of Management Science 1 (1973) 321 – 329.

[36] Webb, M. H. J., “Cost functions in the location of depots for multiple delivery journeys”, Operational Research Quarterly, 19 (1968) 311 – 320.

[37] Wolsey. L. A., “Integer Programming”, John Wiley & Sons Toronto (1998).

[38] Wu, T., Low, C., and Bai, J., "Heuristic solutions to multi – depot location – routing problems", Computers and Operations Research 29 (2002) 1393 – 1415.

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