NON-COOPERATIVE GAME THEORY UNDER PROSPECT THEORY

A Ph.D. Dissertation

by
KERİM KESKİN

Department of
Economics
İhsan Doğramacı Bilkent University
Ankara
May 2016

To my family

NON-COOPERATIVE GAME THEORY UNDER PROSPECT THEORY

The Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

by

KERİM KESKİN

In Partial Fulfillment of the Requirements for the Degree of
DOCTOR OF PHILOSOPHY IN ECONOMICS

THE DEPARTMENT OF
ECONOMICS
İHSAN DOĞRAMACI BİLKENT UNIVERSITY
ANKARA

May 2016

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

Assist. Prof. Dr. Tarık Kara
Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

Assist. Prof. Dr. Emin Karagözoğlu
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

Assoc. Prof. Dr. Süheyla Özyıldırım
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

Assoc. Prof. Dr. Serkan Küçükşenel
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

Assist. Prof. Dr. Ayça Özdoğan
Examining Committee Member

Approval of the Graduate School of Economics and Social Sciences

---

Prof. Dr. Halime Demirkan
Director

ABSTRACT

NON-COOPERATIVE GAME THEORY UNDER PROSPECT THEORY

KESKİN, Kerim

Ph.D., Department of Economics

Supervisor: Assist. Prof. Dr. Tarık Kara

May 2016

This dissertation consists of three essays in which I study prospect theory preferences in non-cooperative game-theoretic frameworks. In decision-making experiments, it is commonly observed that actual choice behavior might violate the axioms of expected utility theory (EUT). Kahneman and Tversky (1979) argue that such experimental findings invalidate EUT as a descriptive model and propose prospect theory as an alternative representation of preferences. Later, Tversky and Kahneman (1992) propose cumulative prospect theory (CPT). Both of these theories stipulate that individual preferences can be represented by a pair of functions: probability weighting function and value function. These functions capture three key aspects of the theory: subjective probability weighting, reference dependence, and loss aversion. In the first essay of this dissertation, I study mixed strategy equilibrium for finite normal form games in which agents' preferences are represented by the pair of functions suggested in CPT. I introduce the notion of *CPT equilibrium*, prove the existence of CPT equilibrium for finite normal form games, and analyze the set of CPT equilibria for some normal form games. In the second essay, I study correlated equilibrium for finite normal form games in which agents' preferences are represented by the same pair of functions. I relate the notion of *correlated CPT equilibrium* to the notion of CPT equilibrium and investigate the differences between the sets of correlated equilibria under EUT and CPT preferences. Finally, in the third essay, I study a first-price sealed-bid auction. I concentrate on subjective probability weighting and analyze *overbidding* behavior which is commonly observed in first-price auction experiments. I show that inverse S-shaped probability weighting functions cannot completely explain overbidding and that such functions can provide a partial explanation for bidders with high valuations.

Keywords: Correlated Equilibrium, First-price Sealed-bid Auctions, Mixed Strategy Equilibrium, Nash Equilibrium, Prospect Theory.

ÖZET

BEKLENTİ KURAMI VARSAYIMLARI ALTINDA İŞBİRLİKSİZ OYUN KURAMI

KESKİN, Kerim

Doktora, İktisat Bölümü

Tez Yöneticisi: Yard. Doç. Dr. Tarık Kara

Mayıs 2016

Bu çalışma işbirliksiz oyunlarda ve beklenti kuramı varsayımları altında bireylerin nasıl davrandığının incelendiği üç makaleden oluşmaktadır. Karar verme deneylerinde sıklıkla görüldüğü üzere, birey davranışları beklenen fayda kuramının aksiyomları ile çelişebilmektedir. Kahneman ve Tversky (1979) bu tip gözlemlerin beklenen fayda kuramının açıklayıcı bir model olamayacağını işaret ettiğini belirtmiş ve alternatif olarak beklenti kuramını önermiştir. Daha sonra, Tversky ve Kahneman (1992) tarafından kümülatif beklenti kuramı önerilmiştir. Bu kuramlara göre, bireysel tercihler bir çift fonksiyon ile ifade edilebilir: olasılık ayarlama fonksiyonu ve değer fonksiyonu. Bu fonksiyonlar kuramların üç temel ögesi ile ilgilidir: subjektif olasılık ayarlaması, referansa bağlılık, ve kayıptan kaçınma. Bu çalışmanın birinci makalesinde bireylerin kümülatif beklenti kuramı tipi tercihlere sahip oldukları varsayımı altında sonlu normal biçim oyunlarda karma strateji dengesi çalışılmıştır. Çalışmada CPT dengesi adı verilen yeni bir nosyon önerilmiş, tüm sonlu normal biçim oyunlarda dengenin varlığı is-

patlanmış, ve bazı örnek oyunlarda ortaya çıkan denge kümeleri analiz edilmiştir. İkinci makalede bireylerin kümülatif beklenti kuramı tipi tercihlere sahip oldukları varsayımı altında sonlu normal biçim oyunlarda ilişkili denge çalışılmıştır. Burada önerilen ilişkili CPT dengesinin tezin birinci makalesinde önerilmiş olan CPT dengesi ile ilişkisi incelenmiştir. Daha sonra, beklenen fayda kuramı ve kümülatif beklenti kuramı varsayımları altında ortaya çıkan denge kümeleri arasındaki farklar analiz edilmiştir. Üçüncü makalede ise birinci-fiyat kapalı-zarf ihaleleri çalışılmıştır. Sadece subjektif olasılık ayarlamasına odaklanılmış ve deneysel yazında sıklıkla rastlanan aşırı fiyat verme davranışı incelenmiştir. Ters S-şekilli olasılık ayarlama fonksiyonlarının bu davranış şeklini tam olarak açıklayamayacağı, ama satışa sunulan objeye yüksek değer biçen katılımcılar için kısmi bir açıklama getirebileceği gösterilmiştir.

Anahtar Kelimeler: Beklenti Kuramı, Birinci-fiyat Kapalı-zarf İhaleleri, İlişkili Denge, Karma Strateji Dengesi, Nash Dengesi.

ACKNOWLEDGEMENTS

I cannot overstate my gratitude to Tarık Kara for his exceptional supervision and invaluable guidance throughout my graduate studies. He has been a great mentor and for that I will forever be in his debt .

I am also indebted to Emin Karagözoğlu for his insightful comments in almost every stage of this dissertation. I have truly learned a lot from him.

I would like to thank Süheyla Özyıldırım for her helpful comments and suggestions throughout my thesis study. I also thank the other examining committee members, namely Serkan Küçükşenel and Ayça Özdoğan as well as Mürüvvet Büyükboyacı, for their comments and suggestions.

I am truly grateful to Çağrı Sağlam for his continuous support and for keeping me on track when I needed the most.

I thank Zeynep Kantur for being the most patient co-author possible and Tuğba Zeydanlı for making me the most patient co-author I can ever be.

I also wish to thank my other co-authors Takashi Kamihigashi, Elif Özcan-Tok, and Agah Turan for insightful discussions.

I would like to thank Refet Gürkaynak, Selin Sayek Böke, and Erinç Yeldan for their trust and support. I am grateful to all of the professors at the Department of Economics, especially Nuh Aygün Dalkıran, Kevin Hasker, Semih Koray, and Kemal Yıldız, for their guidance throughout my graduate years at the department. I hereby acknowledge the research fund provided by the Department of Economics at Bilkent University for my experimental work.

I would like to thank Mehmet Barlo, Haldun Evrenk, İsa Hafalır, Yusufcan Masatlıoğlu, Peter Wakker, Eyal Winter, and Shmuel Zamir as well as seminar participants at Bilkent University, Ege University, Özyeğin University, TOBB ETÜ, Bosphorus Workshop on Economic Design, Econ Anadolu Conference, and FUR '14 Conference for their comments and suggestions.

Special thanks to my other graduate friends Yıldız Akkaya, Binnur Balkan, Burak Eroğlu, Mehmet Fatih Harmankaya, and Güneş Kolsuz for their generous help and for making my graduate years enjoyable.

I need to mention Nilgün Çorapçıoğlu, Özlem Eraslan, and Neşe Özgür for their help with administrative matters.

Finally, I would like to thank my family for their unconditional love and neverending support.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Expected utility theory (EUT) (von Neumann and Morgenstern, 1944) stipulates that individual preferences can be represented by an expected utility function if it satisfies completeness, transitivity, continuity, and the independence axiom. As EUT is considered to be the standard theory of individual decision making, it is utilized in many subfields of economics, including non-cooperative game theory. As a matter of fact, EUT is quite essential for game theory since it is utilized by well-known solution concepts, such as Nash equilibrium (Nash, 1951) and correlated equilibrium (Aumann, 1974).

Allais (1953) and Ellsberg (1961) are among the first to experimentally analyze certain one-shot choice problems and to observe that EUT has shortcomings in explaining actual choice behavior. Motivated by such experimental findings, Kahneman and Tversky (1979) formulate prospect theory which describes an alternative representation of individual preferences. In the following years, prospect theory is commonly criticized because of its violation of *first order stochastic dominance*. The attempts to overcome this problem lead to the development of cumulative prospect theory (CPT) (see Tversky and Kahneman, 1992).[1]

Cumulative prospect theory stipulates that individual preferences can be represented by a pair of functions rather than a single expected utility function (as in EUT). These functions are called probability weighting function and value function. They capture three key aspects of the theory: (1) Subjective probability weighting: When making decisions, individuals distort probabilities and act as if the probability of an event is higher/lower than the objective probability. (2) Reference dependence: An individual has a *reference point* such that if he/she receives an earning greater than the reference point, then he/she experience this as a gain; but as a loss if the earning is less than the reference point. (3) Loss aversion: A certain amount of loss yields a welfare loss higher than the welfare gain the same amount of gain yields.

As for the experimental analyses in non-cooperative game-theoretic frameworks, Ochs (1995) observes incompatibility between actual choice behavior and Nash equilibrium predictions. Afterwards, Goeree et al. (2003) examine experimental results for a variety of games and show that a structural econometric model with quantal response equilibrium (see McKelvey and Palfrey, 1995) explains the data very well. In a more recent paper, Selten and Chmura (2008) provide an extensive analysis showing for a number of normal form games that there are equilibrium notions which make more accurate predictions than Nash equilibrium does. From another perspective, above-mentioned experimental observations *may* also sup-

[1]The idea to use cumulative probabilities is suggested earlier by Quiggin (1982) for decisions under risk and by Schmeidler (1989) for decisions under uncertainty.

port the idea that agents have non-EUT preferences. More precisely, individual behavior may be consistent with the underlying principle of Nash equilibrium — which is that each agent anticipates the actions of the other agents and responds accordingly— but individuals may not be expected utility maximizers. Stemming from this idea, we aim to understand how agents would behave in non-cooperative game-theoretic frameworks if they have CPT preferences.

The study of prospect-theoretic behavior in non-cooperative game theory attracts less attention than it deserves due to possible complications in the formulation and computational difficulties in the equilibrium analysis.[2] Although there is a number of studies utilizing prospect-theoretic preferences in certain game-theoretic frameworks (see Sonnemans et al., 1998; Shalev, 2002; Lange and Ratan, 2010; Driesen et al., 2012; Rieger, 2014, among others), and there is the notion of loss aversion equilibrium (Shalev, 2000), to the best of our knowledge, there is no *well-established* equilibrium notion that completely incorporates prospect-theoretic behavior.

There is a recent paper studying equilibrium notions under prospect theory preferences (see Metzger and Rieger, 2010). In particular, the authors define mixed strategy equilibria both under prospect theory (for any finite normal form game) and under CPT (only for finite two-person normal form games) preferences. They consider fixed *reference point*s arguing that it is not always clear how the refer-

[2]Unlike prospect theory, non-EUT preferences are widely studied in non-cooperative game theory (see Dekel et al., 1991; Ritzberger, 1996; Chen and Neilson, 1999; Goeree et al., 2002, among others). On the other hand, prospect-theoretic preferences are mostly applied to different frameworks such as those in finance or industrial organization. See Barberis (2013) for a recent review.

ence point should be chosen and that *non-fixed* reference points may lead to the non-existence of equilibrium. In the first two chapters of this dissertation, we define a notion of mixed strategy equilibrium and a notion of correlated equilibrium for agents with CPT preferences. Similar to the notion of Metzger and Rieger (2010), we utilize fixed reference points; but unlike the notion of Metzger and Rieger (2010), the proposed notions are well-defined for any finite normal form game.

In the first chapter, our approach is to convert the strategic framework underlying mixed strategy equilibrium into an *equivalent lottery framework* such that each strategy of an agent induces a lottery (given a mixed strategy profile of the other agents). The agent chooses from the corresponding lottery list, and the strategy which induces the chosen lottery will be described as the agent's best response. In contrast to the standard analysis, we assume that agents exhibit CPT preferences when evaluating these lotteries. We introduce the notion of *CPT* equilibrium, prove the existence of equilibrium for finite normal form games, and analyze the set of CPT equilibria for some normal form games.[3]

In the second chapter, we concentrate on the notion of correlated equilibrium (Aumann, 1974). This notion designates a probability measure, which is commonly referred to as *the correlation device*, on the set of strategy profiles. This pre-defined probability measure induces well-defined lotteries over the set of payoffs. As a result, it is possible to convert the strategic framework underlying correlated equilibrium into another *equivalent lottery framework*. As it turns out,

[3]An earlier version of this chapter was submitted to *Hakan Orbay Research Award*.

this lottery framework is the same as the one we have obtained in the first chapter (for mixed strategy equilibrium). We once again assume that agents exhibit CPT preferences when evaluating these lotteries and define the notion of *correlated CPT equilibrium* for agents with such preferences. We relate this notion to CPT equilibrium we have introduced in the first chapter. Then we investigate the differences between the sets of correlated equilibria under EUT and CPT.

In the third chapter, we turn to Bayesian games. In particular, we study a first-price sealed-bid auction framework. Stemming from the experimental observations that bidders tend to bid higher than the risk neutral Nash equilibrium bid, which is labeled as *overbidding* behavior, we try to understand whether bidders would bid higher if they subjectively weight their winning probabilities using an *inverse S-shaped* probability weighting function. Our results indicate that inverse S-shaped functions cannot completely explain overbidding and that such functions can provide a partial explanation for bidders with high valuations.[4]

Finally, it is worth mentioning that in all three chapters of this dissertation, we mainly concentrate on the subjective probability weighting aspect of CPT. As for reference dependence, we utilize fixed reference points which we normalize to zero. Accordingly, if negative outcomes are possible within the framework that is being considered, this would imply that the loss aversion aspect is also incorporated. On top of that, in the first and the third chapters, we provide detailed arguments on how to introduce non-fixed reference points into the model.

[4]This chapter is published in a journal: Inverse S-shaped probability weighting functions in first-price sealed-bid auctions, *Review of Economic Design*: 20(1), 57–67.

CHAPTER 2

A NEW APPROACH TO MIXED STRATEGY EQUILIBRIUM: CPT EQUILIBRIUM

The notion of Nash equilibrium, proposed by Nash (1951), is the most widely used solution concept in non-cooperative game theory. In a normal form game, it is presumed that every agent has beliefs about the actions of the other agents; and given these beliefs, he/she considers every one of his/her strategies to determine a best response. The intersection of these best responses turns out to be a Nash equilibrium of the game.

As mentioned above, in the analysis of Nash equilibrium, it is presumed that every agent takes the actions of the other agents as given and considers each and every one of his/her strategies to determine a best response. Given a mixed strategy profile of the other agents, each strategy of the agent induces a lottery. A strategy and the induced lottery are *equivalent* in the sense that if the agent is given a choice between them, he/she would be indifferent. Utilizing these lotteries, the strategic framework of a normal form game can be converted into an equivalent lottery framework. The idea here is to use the equivalent lottery framework rather than the strategic framework.[1] This idea leaves us with a framework of one-shot choice problems. It is then assumed that agents have CPT preferences over these lotteries rather than EUT preferences. The definition of best response correspondences are accordingly formulated and the notion of *CPT equilibrium* is introduced.

First, we show that a pure strategy Nash equilibrium (under EUT preferences) is also a CPT equilibrium. This implies that such equilibria are *robust* to changes in agents' preferences. Second, we prove that a CPT equilibrium always exists in any normal form game. In this result we refer to a well-known fixed point theorem by Kakutani (1941). Third, in our further analysis, we try to understand the differences between the predictions of CPT equilibrium and mixed strategy Nash equilibrium by analyzing two examples of normal form games. As it turns out, CPT equilibrium makes significantly different predictions which indicates that it might have some explanatory power in experimental analyses.[2]

2.1 The Model

2.1.1 Notation and Definitions

Let $\Gamma = (N, (S_i)_{i\in N}, (h_i)_{i\in N})$ be a finite n-person normal form game where S_i is the finite strategy set of agent $i \in N$ and $h_i : S_1 \times \cdots \times S_n \to \mathbb{R}$ is the payoff

[1] It is worth mentioning that the equilibrium analysis under EUT preferences utilizes this idea implicitly since the expected utility of a strategy is defined to be the expected utility of the lottery which the agent faces when he/she chooses that particular strategy.

[2] Recall that there exist experimental studies which show that the predictions of Nash equilibrium are not sufficiently accurate for some normal form games (see Ochs, 1995; Selten and Chmura, 2008, among others).

function[3] for agent $i \in N$. The set of all strategy profiles is denoted by $S \equiv \bigtimes_{i \in N} S_i$, whereas the set of strategy profiles for agents included in $N \setminus \{i\}$ is denoted by $S_{-i} \equiv \bigtimes_{j \in N \setminus \{i\}} S_j$.

We denote by ΔS_i the set of probability measures on S_i and refer to a member of ΔS_i as a mixed strategy of agent i. In this context, a mixed strategy $\mu_i \in \Delta S_i$ indicates that agent i plays s_i with probability $\mu_i(s_i)$. Additionally, with a slight abuse of notation, the set of all mixed strategy profiles is denoted by $\Delta S \equiv \bigtimes_{i \in N} \Delta S_i$, whereas the set of mixed strategy profiles for agents included in $N \setminus \{i\}$ is denoted by $\Delta S_{-i} \equiv \bigtimes_{j \in N \setminus \{i\}} \Delta S_j$.

When agent i chooses a pure strategy $s_i \in S_i$, as a response to a given mixed strategy profile $\mu_{-i} \in \Delta S_{-i}$ of the remaining agents, he/she faces the lottery[4]

$$L_i(s_i, \mu_{-i}) = \big(\mu_{-i}(s_{-i}^1), h_i(s_i, s_{-i}^1); \cdots; \mu_{-i}(s_{-i}^{|S_{-i}|}), h_i(s_i, s_{-i}^{|S_{-i}|})\big)$$

where $|S_{-i}|$ denotes the cardinality of the set S_{-i}. Accordingly, during the decision process, agent i evaluates all of these lotteries and chooses a strategy that induces *the best* lottery.

Following the assumptions of CPT, an agent's preferences are represented using a pair of functions, *value function* and *probability weighting function*, rather than

[3]Since we study CPT preferences, these are defined to be monetary payoffs. The term 'payoff' is also used to describe *utility* when we refer to EUT preferences or Nash equilibrium under the assumption that $u_i(x) = x$ for every $i \in N$ and every monetary payoff $x \in \mathbb{R}$.

[4]A lottery is defined as a tuple of pairs such that the first component of the pair is the probability of occurrence of an event and the second component is the payoff corresponding to that event.

an expected utility function. For every $i \in N$, we employ an increasing and continuous probability weighting function $w_i : [0,1] \to [0,1]$ satisfying $w_i(0) = 0$ and $w_i(1) = 1$.[5] The value function $v_i : \mathbb{R}^2 \to \mathbb{R}$ is defined for any payoff ($x \in \mathbb{R}$) and any *reference point* ($r \in \mathbb{R}$) as

$$v_i(x,r) = \begin{cases} x - r & , x \geq r \\ \lambda_i(x-r) & , x < r \end{cases} \tag{2.1}$$

where $\lambda_i \geq 1$ is the loss aversion coefficient. This functional form is borrowed from Tversky and Kahneman (1991, 1992).[6]

Following the relevant literature, it is assumed that subjective probability weighting takes place during the evaluation of lotteries. Accordingly, an agent i knows what it means for an event to occur with probability p, but he/she acts as if the probability of this event is $w_i(p)$. To put it differently, subjective probabilities are treated as *decision weights*. As it is explicitly described by Tversky and Kahneman (1992), we assume that a given lottery $L = (p_1, x_1; \ldots; p_k, x_k)$ is first separated into two parts following an increasing order of payoffs. The positive lottery $L^+ = (p_0^+, r_i; p_1^+, x_1^+; \ldots; p_{k_1}^+, x_{k_1}^+)$ is formed with all payoffs of L which are greater than or equal to r_i, and the negative lottery $L^- = (p_1^-, x_1^-; \ldots; p_{k_2}^-, x_{k_2}^-; p_0^-, r_i)$ is

[5] For further reading on subjective probability weighting, see Tversky and Wakker (1995); Wu and Gonzalez (1996); Wakker (2010) among others.

[6] Tversky and Kahneman (1992) propose this function so as to make it consistent with their experimental observations. In fact, their value function is provided in a more general form. For the sake of simplicity, we assume the linear version of the function.

obtained similarly.[7] Also, in order for these to be well-defined lotteries, we set

$$p_0^+ = 1 - \sum_{j=1}^{k_1} p_j^+ \qquad \text{and} \qquad p_0^- = 1 - \sum_{j=1}^{k_2} p_j^- .$$

For every $i \in N$, the expected value of L is defined as

$$EV_i^{\lambda_i, r_i}(L) = EV_i^{\lambda_i, r_i}(L^+) + EV_i^{\lambda_i, r_i}(L^-) \tag{2.2}$$

where

$$EV_i^{\lambda_i, r_i}(L^+) = \sum_{t=1}^{k_1} \left(w_i\Big(\sum_{j=t}^{k_1} p_j^+ \Big) - w_i\Big(\sum_{j=t+1}^{k_1} p_j^+ \Big) \right) v_i(x_t^+, r_i)$$

and

$$EV_i^{\lambda_i, r_i}(L^-) = \sum_{t=1}^{k_2} \left(w_i\Big(\sum_{j=1}^{t} p_j^- \Big) - w_i\Big(\sum_{j=1}^{t-1} p_j^- \Big) \right) v_i(x_t^-, r_i) .$$

Our formulation of mixed strategy equilibrium is consistent with two well-known interpretations of mixed strategies. The first is Harsanyi (1973)'s purification idea stating that every agent is influenced by small perturbations to his/her payoffs which are not observable by the other agents. In every repetition of a particular game, each agent *chooses* a pure strategy depending on the realization of the perturbation. However, considering the long run average of his/her choices, the other agents believe that the agent randomizes between pure strategies. The second interpretation describes agents as a member of a large population of individuals. Accordingly, each agent *chooses* a pure strategy, and a mixed strategy represents

[7] For probabilities and payoffs, the superscript and the subscripts are used only for relabeling. For example, $x_1^+ = x_j$ for some $j \in \{1, \ldots, k\}$ where x_j is the smallest payoff which is greater than or equal to r_i. Note that $p_1^+ = p_j$ in this case.

the distribution of pure strategies chosen by the population (see, e.g., Rosenthal, 1979). Putting these interpretations aside, even when mixed strategies are interpreted as objects of choice, it can be argued that each agent employs a randomization device and *chooses* the pure strategy selected by the device. Relying on the above interpretations, we state that a mixed strategy can be represented by a list of lotteries induced by pure strategies. For a concrete example, consider a normal form game in which agent i has three strategies denoted by s_i^1, s_i^2, and s_i^3. Given $\mu_{-i} \in \Delta S_{-i}$, a mixed strategy $\mu_i = (p, 1-p, 0)$ of agent i does not induce some $L_i(\mu_i, \mu_{-i})$, but rather generates either of the two independent lotteries induced by pure strategies, $L_i(s_i^1, \mu_{-i})$ and $L_i(s_i^2, \mu_{-i})$. Here, the former lottery will be realized with probability p whereas the latter will be realized with probability $1-p$. Consequently, agent i's expected value from choosing μ_i as a response to μ_{-i} is

$$p \cdot EV_i^{\lambda_i, r_i}(L_i(s_i^1, \mu_{-i})) + (1-p) \cdot EV_i^{\lambda_i, r_i}(L_i(s_i^2, \mu_{-i})).$$

These interpretations and the description of mixed strategies that followed yield important insights in the context of CPT: The probabilities of own strategies are not subjectively weighted. This is plausible in the sense that one would expect an agent, who is indifferent between two lotteries induced by two different pure strategies, to be negligent of the result of the randomization between these particular strategies.

As for reference dependence, we simply assume that agents have fixed reference points and normalize each agent i's reference point to $r_i = 0$. In that our notion is similar to the notions proposed by Metzger and Rieger (2010) which utilize fixed reference points. Notice that normalization to 0 does not cause any loss of generality: Given a normal form game $\Gamma = (N, (S_i)_{i\in N}, (h_i)_{i\in N})$ and a reference point profile $r = (r_1, \ldots, r_n)$, we can formulate another normal form game $\Gamma^r = (N, (S_i)_{i\in N}, (h_i^r)_{i\in N})$ such that for every $i \in N$ and every $s \in S$: $h_i^r(s) = h_i(s) - r_i$. Assuming that each agent's reference point is 0 in Γ^r, it is easy to see that the equilibrium predictions will be the same.

To sum up, we can represent an agent's preferences using a single function to be maximized by the agent. For every $i \in N$, every $\mu \in \Delta S$, and every $\mu_i' \in \Delta S_i$, we define the prospect-theoretic utility function $U_i^{\lambda_i,0} : \Delta S \to \mathbb{R}$ as

$$U_i^{\lambda_i,0}(\mu_i', \mu_{-i}) = \sum_{s_i \in S_i} \mu_i'(s_i) EV_i^{\lambda_i,0}\big(L_i(s_i, \mu_{-i})\big). \tag{2.3}$$

Accordingly, the definition of CPT Equilibrium is given below.

2.1.2 Definition of CPT Equilibrium

The best response correspondence for agent $i \in N$, denoted by $\mathcal{BR}_i^{CPT} : \Delta S \to \Delta S_i$, is defined as

$$\mathcal{BR}_i^{CPT}(\mu) = \{\mu_i^* \in \Delta S_i \mid \forall \mu_i' \in \Delta S_i : U_i^{\lambda_i,0}(\mu_i^*, \mu_{-i}) \geq U_i^{\lambda_i,0}(\mu_i', \mu_{-i})\}.$$

The definition of CPT equilibrium is as follows.

Definition 1. A mixed strategy profile μ^* is a cumulative prospect theory equilibrium (CPT equilibrium) if for every $i \in N$:

$$\mu_i^* \in \mathcal{BR}_i^{CPT}(\mu^*).$$

This framework reduces to the expected utility framework when (i) $\lambda_i = 1$ for every $i \in N$; and (ii) $w_i(p) = p$ for every $i \in N$ and every $p \in [0,1]$. As a consequence, CPT equilibrium is a generalization of mixed strategy Nash equilibrium.

Proposition 1. *A pure strategy profile $s \in S$ is a Nash equilibrium if and only if it is a CPT equilibrium.*

Proof. Take any pure strategy profile $s \in S$. Accordingly, for every agent $i \in N$, we have $\mu_{-i}(s_{-i}) = 1$. Therefore, agent i would not deviate from s_i if and only if $h_i(s_i, s_{-i}) \geq h_i(s_i', s_{-i})$ for every $s_i' \in S_i$, i.e., if and only if s is a pure strategy Nash equilibrium. $\square$

This result can be interpreted as the robustness of equilibria. In particular, a pure strategy equilibrium is *robust* in the sense that it is an equilibrium under both types of representation of preferences, EUT and CPT. A non-pure strategy Nash equilibrium, however, is not necessarily an equilibrium under CPT preferences. Related examples are provided below.

2.1.3 Existence of CPT Equilibrium

In this section, we prove the existence of equilibrium which is indubitably the most desirable property of an equilibrium notion. In that regard, we utilize a *fixed point theorem* by Kakutani (1941) which states that a correspondence has a *fixed point*[8] if (*i*) it is nonempty-valued and convex-valued on a non-empty, compact, and convex domain; and (*ii*) it has a closed graph. Noting that a fixed point of the joint best response correspondence $\mathcal{BR}^{CPT} \equiv \mathcal{BR}_1^{CPT} \times \cdots \times \mathcal{BR}_n^{CPT}$ turns out to be an equilibrium, we prove that CPT equilibrium exists for any finite normal form game.

Proposition 2. *CPT equilibrium exists for any finite normal form game.*

Proof. This proof relies on Kakutani (1941) fixed point theorem. It is clear that each strategy set ΔS_i is a non-empty, compact, and convex subset of a Euclidean space. We then check for the conditions on the joint best response correspondence, $\mathcal{BR}^{CPT}$.

Note that probability weighting function, value function, and reference point function are continuous by definition; that the expected value function $EV_i^{\lambda_i, r_i}$ is obtained from these functions by using addition, multiplication, and composition; and that continuity is preserved under these operators. Accordingly, $EV_i^{\lambda_i, 0}$ is continuous for every $i \in N$. Following a similar reasoning, also the prospect-theoretic utility function $U_i^{\lambda_i, 0}$ turns out to be continuous for every $i \in N$. As a consequence, the individual best response set $\mathcal{BR}_i^{CPT}(\mu)$ is non-empty for every $i \in N$ and every $\mu \in \Delta S$. Therefore, the joint best response correspondence $\mathcal{BR}^{CPT}$ is nonempty-valued.

[8] A point $x \in X$ is a fixed point of a correspondence $F : X \to X$ if and only if $x \in F(x)$.

We now show that each individual best response correspondence is convex-valued: Take any $i \in N$ and any $\mu \in \Delta S$. The result trivially follows if $\mathcal{BR}_i^{CPT}(\mu)$ is a singleton. Otherwise, take two different strategies $\mu_i', \mu_i'' \in \mathcal{BR}_i^{CPT}(\mu)$. Then any convex combination $\bar{\mu}_i = \alpha\mu_i' + (1-\alpha)\mu_i''$ of these strategies is a mixed strategy for agent i and satisfies

$$\begin{aligned}
U_i^{\lambda_i,0}(\bar{\mu}_i, \mu_{-i}) &= \sum_{s_i \in S_i} \bar{\mu}_i(s_i) EV_i^{\lambda_i,0}\big(L_i(s_i, \mu_{-i})\big) \\
&= \alpha \sum_{s_i \in S_i} \mu_i'(s_i) EV_i^{\lambda_i,0}\big(L_i(s_i, \mu_{-i})\big) \\
&\qquad\qquad + (1-\alpha) \sum_{s_i \in S_i} \mu_i''(s_i) EV_i^{\lambda_i,0}\big(L_i(s_i, \mu_{-i})\big) \\
&= \alpha\, U_i^{\lambda_i,0}(\mu_i', \mu_{-i}) + (1-\alpha)\, U_i^{\lambda_i,0}(\mu_i'', \mu_{-i}) \\
&= U_i^{\lambda_i,0}(\mu_i', \mu_{-i}) = U_i^{\lambda_i,0}(\mu_i'', \mu_{-i}).
\end{aligned}$$

Therefore, $\bar{\mu}_i$ is a best response to μ as well, implying that $\mathcal{BR}_i^{CPT}$ is convex-valued. Hence $\mathcal{BR}^{CPT}$ is convex-valued.

Finally, we need to show that the graph of $\mathcal{BR}^{CPT}$ is closed: Take any convergent sequence $(\mu^t, \nu^t) \to (\mu, \nu)$ from the graph of the joint best response correspondence. This convergent sequence can be written as two convergent sequences $(\mu^t) \to \mu$ and $(\nu^t) \to \nu$ from the set ΔS satisfying $\nu^k \in \mathcal{BR}^{CPT}(\mu^k)$ for every $k \in \mathbb{N}$. It then follows for every $i \in N$, every $\xi_i \in \Delta S_i$, and every $k \in \mathbb{N}$ that

$$U_i^{\lambda_i,0}(\nu_i^k, \mu_{-i}^k) \geq U_i^{\lambda_i,0}(\xi_i, \mu_{-i}^k).$$

Since $U_i^{\lambda_i,0}$ is continuous at any element of ΔS, we conclude that

$$U_i^{\lambda_i,0}(\nu_i, \mu_{-i}) \geq U_i^{\lambda_i,0}(\xi_i, \mu_{-i})$$

for every $i \in N$ and every $\xi_i \in \Delta S_i$. Hence $\nu_i \in \mathcal{BR}_i^{CPT}(\mu)$ for every $i \in N$. Then, it eventually follows that $\mathcal{BR}^{CPT}$ has a closed graph.

Since all of the conditions are satisfied, Kakutani (1941) fixed point theorem applies: There exists a fixed point of $\mathcal{BR}^{CPT}$ which turns out to be a CPT equilibrium. □

2.1.4 Further Analysis

Introducing Non-fixed Reference Points: The major difference between CPT equilibrium and the notion proposed by Metzger and Rieger (2010) is that the notion of CPT equilibrium is well-defined for n-player games whereas the latter is well-defined only for two-player games. This is due to the fact that Metzger and Rieger (2010) cannot find a method to obtain cumulative probabilities (which are subjectively weighted) if there are more than two players. In this chapter we fill this gap by introducing the equivalent lottery framework and by extendig the analysis to n-player games. Now, we further argue that it is possible to introduce non-fixed reference points into the definition of CPT equilibrium. This is another problem highlighted by Metzger and Rieger (2010) as these authors claim that non-fixed reference points may lead to the non-existence of equilibrium.

We refer to a well-known interpretation of mixed strategy equilibrium. Mixed strategy profiles can be interpreted as beliefs: A mixed strategy profile $\mu \in \Delta S$ is realized as an equilibrium if, under the case in which all agents believe that the outcome of the game will be μ, every agent i prefers not to deviate from μ_i. Accordingly, we can assume that agents evaluate payoffs with respect to their reference points which are influenced by these beliefs (or expectations). For every $i \in N$, we can define the reference point $r_i : \Delta S \to \mathbb{R}$ as a continuous function of mixed strategy profiles. A natural example is

$$r_i^*(\mu) = \sum_{s \in S} \mu(s) h_i(s) \tag{2.4}$$

for every $\mu \in \Delta S$. This definition can be motivated by past experiences: Agents play the game sufficiently many times, their expectations are accordingly formed, and their average earnings represent their reference points. The best response correspondence for agent $i \in N$, denoted by $\mathcal{BR}_i^{CPT^*} : \Delta S \to \Delta S_i$, is then defined as

$$\mathcal{BR}_i^{CPT^*}(\mu) = \{\mu_i^* \in \Delta S_i \mid \forall \mu_i' \in \Delta S_i : U_i^{\lambda_i, r_i^*(\mu)}(\mu_i^*, \mu_{-i}) \geq U_i^{\lambda_i, r_i^*(\mu)}(\mu_i', \mu_{-i})\}.$$

where r_i^* is defined as in (2.4). The definition of CPT* equilibrium follows similarly.

Definition 2. A mixed strategy profile μ^* is a CPT* equilibrium if for every $i \in N$:

$$\mu_i^* \in \mathcal{BR}_i^{CPT^*}(\mu^*).$$

Below we analyze equilibrium predictions of both CPT equilibrium and CPT* equilibrium for two examples of normal form games.

Equilibrium Predictions: There are several experimental studies arguing that Nash equilibrium predictions are not sufficiently accurate for some normal form games. We choose our examples from these experimental studies. For instance, Game 1 is one of the normal form games analyzed in two of these studies (see Ochs, 1995; McKelvey et al., 2000). As another example, we have Game 2 which is analyzed by Selten and Chmura (2008).

Game 1

	L	R
U	0 , 4	1 , 0
D	1 , 0	0 , 1

Game 2

	L	R
U	10 , 8	0 , 18
D	9 , 9	10 , 8

Figure 2.1: Two Examples

Tables 1 and 2 include the predictions of CPT equilibrium and CPT* equilibrium for Game 1 and Game 2, respectively (see pg. 20). At this stage, we utilize a probability weighting function proposed by Prelec (1998):

$$w(p) = \exp\{-(-\ln p)^{\alpha}\}. \tag{2.5}$$

The parameter α is chosen to be 0.65, 0.85, and 1. The first value is the estimate found by Prelec (1998) and the third yields the case of Nash equilibrium. The second value is utilized to capture the behavioral change in between. As for loss aversion, notice that it would be ineffective since both games always yield non-negative payoffs to agents.[9] Moreover, for CPT* equilibrium, we choose the loss aversion coefficient λ as 1.25 and 2.25. The latter is in line with Tversky and Kahneman (1992)'s suggestion and the former is used for technical purposes.

In this exercise, our aim is to understand how the unique equilibrium[10] changes depending on the equilibrium notion and the relevant parameters. We do not aim to compare the accuracies of these equilibrium notions. As a matter of fact, such a comparison would necessitate a more thorough analysis in which the parameters are chosen to fit the data.[11]

It is also worth mentioning here that our notion of mixed strategy equilibrium is not a substitute for *other* types of equilibrium notions under EUT preferences; including those analyzed by Selten and Chmura (2008), such as *action-sampling equilibrium* and *payoff-sampling equilibrium*. In fact, our approach to incorporate CPT preferences can rather be seen as a complement to those notions. For example, it is possible to utilize the underlying principle of action-sampling equilibrium —which is that each agent takes a sample of seven observations of the strategies played on the other side and optimizes against this sample—

[9] In games with non-negative payoffs, CPT equilibrium turns out to be similar to the notion of Ritzberger (1996) which is defined under *rank-dependent expected utility theory* (see Quiggin, 1982). Our difference lies within the formulation of probability weighting. Accordingly, Ritzberger (1996)'s existence result additionally necessitates the concavity of the probability weighting function, hence does not cover *inverse S-shaped functions* which are consistently suggested in many empirical studies (see Camerer and Ho, 1994; Wu and Gonzalez, 1996; Abdellaoui, 2000, among others). On the other hand, CPT equilibrium exists also for inverse S-shaped functions.

[10] Note that equilibrium is unique for both games and for any given values of parameters.

[11] For such a comparison, it may be misleading to use the observations from the aforementioned experimental studies. The reason is that the experimental procedure becomes crucial under CPT preferences. For example, in his experiments, Ochs (1995) converts *payoff*s into tickets for a lottery through which the actual earnings of the participants are determined. Under EUT preferences, this is a distinction without a difference; but under CPT preferences, the outcome *may* dramatically change.

Table 2.1: Predictions for Game 1

Equilibrium Notion	α	λ	p_U	p_L
CPT Equilibrium	0.65	1	0.1167	0.5000
	0.85	1	0.1700	0.5000
	1*	1	0.2000	0.5000
CPT* Equilibrium	0.65	2.25	0.2199	0.5000
	0.85	2.25	0.2712	0.5000
	1	2.25	0.2962	0.5000
	0.65	1.25	0.1386	0.5000
	0.85	1.25	0.1934	0.5000
	1	1.25	0.2231	0.5000

p_U: The probability assigned to U
p_L: The probability assigned to L
* : The case of Nash equilibrium

Table 2.2: Predictions for Game 2

Equilibrium Notion	α	λ	p_U	p_L
CPT Equilibrium	0.65	1	0.0221	0.9738
	0.85	1	0.0609	0.9390
	1*	1	0.0909	0.9091
CPT* Equilibrium	0.65	2.25	0.0777	0.9955
	0.85	2.25	0.1277	0.9729
	1	2.25	0.1590	0.9490
	0.65	1.25	0.0329	0.9857
	0.85	1.25	0.0761	0.9516
	1	1.25	0.1071	0.9230

p_U: The probability assigned to U
p_L: The probability assigned to L
* : The case of Nash equilibrium

and to assume that agents have CPT preferences when they consider their options. Stemming from this availability, we argue that if one aims to compare an equilibrium notion under CPT preferences with an equilibrium notion under EUT preferences, it would be more informative to compare the notions with the same underlying principle.

Finally, for the introduction of CPT preferences, the equivalent lottery framework is of utmost importance. We remind the reader that these lotteries can only be obtained within the equilibrium analysis, that is, only when the actions of the other agents are taken as given. At this point, one can simply argue the possibility of agents being unable to perceive the lottery framework they are supposed to perceive. Such an argument, however, would be a question of bounded rationality; hence it is not in the scope of our analysis. Though, it may pose an interesting question for experimental economists to test whether individual behavior differs in a strategic framework and in its equivalent lottery framework. In fact, we are aware of two experimental studies by Bohnet and Zeckhauser (2004) and by Ivanov (2011) in which normal form games are analyzed together with their equivalent lottery frameworks. Although such an interpretation is not provided by the authors, the findings indicate that subjects do not necessarily perceive the lottery framework they are supposed to perceive.

2.2 Conclusion

In an extensive literature review, Starmer (2000) writes

> [I]t is precisely the simplicity and economy of EUT that has made

> it such a powerful and tractable modeling tool. My concern, however, is with the descriptive merits of the theory and, from this point of view, a crucial question is whether EUT provides a sufficiently accurate representation of actual choice behavior.

Having the same concern, non-EUT preferences are widely studied in non-cooperative game-theoretic environments (see Dekel et al., 1991; Ritzberger, 1996; Chen and Neilson, 1999; Goeree et al., 2002, among others); and as a well-known non-EUT, prospect-theoretic preferences are specifically utilized in some of these studies (see Sonnemans et al., 1998; Shalev, 2002; Lange and Ratan, 2010; Driesen et al., 2012; Rieger, 2014, among others).

In this chapter, we study normal form games in which agents' preferences are represented by the pair of functions suggested in CPT. A new definition of mixed strategy equilibrium is introduced: CPT equilibrium. As it turns out, CPT equilibrium is a generalization of Nash equilibrium. We show that an equilibrium exists for any finite normal form game and additionally prove that a pure strategy equilibrium is robust in the sense that it is an equilibrium under both types of representation of preferences, EUT and CPT.

The introduction of CPT preferences may give rise to many interesting questions for both experimental and theoretical literature. If one believes that the assumption of CPT preferences within a non-cooperative game-theoretic framework is realistic, one can analyze the predictions of above-described equilibrium notions for well-known normal form games. As a matter of fact, these predictions can further be tested experimentally. Finally, such experimental observations can be utilized to estimate the function parameters and to analyze whether these estimates are different than those found in the prospect theory literature.

CHAPTER 3

A NEW APPROACH TO CORRELATED EQUILIBRIUM: CORRELATED CPT EQUILIBRIUM

In the early 1950s, Nash (1951) introduces the notion of Nash equilibrium which is the most prominent solution concept in non-cooperative game theory. For every normal form game, Nash equilibrium conveniently provides insights into the outcome of the game. Despite its convenience, however, there may be cases in which these insights do not contribute much to our understanding. When there are multiple equilibria, for instance, it is not always clear which Nash equilibrium will be realized. More importantly, unless every agent perfectly anticipates the actions of the other agents, the realized outcome of the game may be different than the predictions of Nash equilibrium. With these in mind, Aumann (1974) introduces the notion of correlated equilibrium after arguing for the necessity of a *correlation device* that facilitates coordination between agents. The correlation device, by recommending that agents play certain strategies under certain events, may even lead agents to more desirable outcomes. Moreover, correlated equilibria are computationally simpler objects than Nash equilibria as they need not be fixed points of a correspondence but solutions of a system of linear inequalities. As a result of these nice properties, the notion of correlated equilibrium has many areas of use and applications both within and outside the scope of economics.[1]

Now that we have studied mixed strategy equilibrium for agents with CPT preferences in the first chapter, it is natural to utilize the same approach for the other well-known equilibrium notions, such as the notion of correlated equilibrium. As a matter of fact, one may argue that the notion of correlated equilibrium is more convenient for this task: In the framework of correlated equilibrium, there exists a correlation device defined as a probability measure on the set of strategy profiles. This pre-defined device induces well-defined lotteries over the set of payoffs and can be employed as an anchor that influences agents' reference points.[2]

Our approach is very similar to the one we have utilized in the previous chapter. In the analysis of correlated equilibrium, an agent considers each and every one of his/her strategies to decide on his/her action, given that the other agents act in line with a certain probability measure on their set of strategy profiles. Given this probability measure, each strategy of the agent induces a lottery. A strategy and the induced lottery are *equivalent* in the sense that if the agent is given a choice between them, he/she would be indifferent. The idea here is to use the

[1] See Myerson (1986); Maskin and Tirole (1987); Dhillon and Mertens (1996); Liu (1996); Cavaliere (2001); Solan and Vieille (2002); Teague (2004); Altman et al. (2006); Cason and Sharma (2007); Ramsey and Szajowski (2008); Lin and Van der Schaar (2009), among others.

[2] For simplicity, we normalize reference points to 0 in this chapter. However, as it is the case in the first chapter, our results can be generalized to any fixed reference point. If done so, the correlation device would become a *natural* status-quo for the agents.

equivalent lottery framework rather than the strategic framework.[3] This idea leads us to a framework of one-shot choice problems. It is then assumed that agents have CPT preferences over these lotteries rather than EUT preferences.

After introducing correlated CPT equilibrium, we analyze the differences between the sets of correlated CPT equilibria and correlated equilibria. First, we show that if a correlation device is induced by a pure strategy Nash equilibrium, then it is a correlated CPT equilibrium. This result is in fact the counterpart of the *robustness* result we have shown in the first chapter. Second, we provide an example in which such a robustness result fails to hold for mixed strategy Nash equilibrium. Third, we prove that if a correlation device is induced by a CPT equilibrium, then it is a correlated CPT equilibrium. With this result, we highlight a very strong relation between the notions of mixed strategy equilibrium and correlated equilibrium under CPT preferences. On top of that, this relation implies the existence of a correlated CPT equilibrium for every normal form game. Fourth, after noting that the set of correlated equilibria is always convex, we show that the set of correlated CPT equilibria is not necessarily convex. Fifth, we prove a weaker result that the set of correlated CPT equilibria always includes the set of probability measures induced by the convex hull of pure strategy Nash equilibria.

[3] It is worth mentioning that the equilibrium analysis under EUT preferences utilizes this idea implicitly since the expected utility of a strategy is defined to be the expected utility of the lottery which is faced by the agent when he/she chooses that particular strategy.

3.1 The Model

3.1.1 Notation and Definitions

Let $\Gamma = (N, (S_i)_{i\in N}, (h_i)_{i\in N})$ be a finite n-person normal form game where S_i is the finite strategy set of agent $i \in N$ and $h_i : S_1 \times \cdots \times S_n \to \mathbb{R}$ is the payoff function[4] for agent $i \in N$. We define a correlation device as a probability measure π on the set of strategy profiles $S \equiv S_1 \times \cdots \times S_n$. Accordingly, the nature chooses $s \in S$ with probability $\pi(s)$ and suggests agent i play the strategy $s_i \in S_i$. Each agent i observes the suggestion privately and updates his/her probabilistic beliefs via Bayes' rule. Given that the other agents play the strategies signaled to them, each strategy of agent i induces a lottery consisting of these conditional probabilities. For instance, if agent i observes s_i, then each strategy $s'_i \in S_i$ induces the following lottery:[5]

$$L_i^\pi(s'_i|s_i) = \left(\pi(s^1_{-i}|s_i), h_i(s'_i, s^1_{-i}); \ldots; \pi(s^{|S_{-i}|}_{-i}|s_i), h_i(s'_i, s^{|S_{-i}|}_{-i})\right).$$

During the decision process, agent i evaluates these lotteries and chooses a strategy that induces *the best* lottery. A correlated equilibrium is achieved if, for every probable suggestion, nobody can gain by deviating from the suggested strategy given that the others follow their suggested strategies. The following is the formal definition of correlated equilibrium where $E(\cdot)$ denotes the expected value as in the expected utility framework.

[4]Since we study CPT preferences, it is more convenient to define these as monetary payoffs. The term 'payoff' is also used to describe *utility* when we refer to EUT preferences or correlated equilibrium based on the assumption that a monetary payoff of $x \in \mathbb{R}$ yields a utility equal to x.

[5]A lottery is defined as a tuple of pairs such that the first component of the pair is the probability of occurrence of an event and the second component is the corresponding payoff.

Definition 3. A correlated equilibrium of Γ is a probability measure π^* on S such that for every $i \in N$, every $s_i^* \in S_i$ with $\sum_{s_{-i} \in S_{-i}} \pi^*(s_i^*, s_{-i}) > 0$, and every $s_i \in S_i \setminus \{s_i^*\}$

$$E(L_i^{\pi^*}(s_i^*|s_i^*)) \geq E(L_i^{\pi^*}(s_i|s_i^*)).$$

As in the first chapter, we assume that agents' preferences are represented by the pair of functions suggested in CPT. Below we explain these assumptions and how they are incorporated to our framework of correlated equilibrium.

First, for every $i \in N$, we employ an increasing and continuous probability weighting function $w_i : [0,1] \to [0,1]$ satisfying $w_i(0) = 0$ and $w_i(1) = 1$. As for the timing of subjective probability weighting, we assume that agents first update their beliefs via Bayes' rule and then subjectively weight these conditional probabilities. According to this assumption, an agent i knows what it means for an event to occur with probability p, but he/she acts as if the probability of this event is $w_i(p)$. To put it differently, subjective probability weighting takes place when decisions are being made. It is really worth noting that this is a standard assumption due to Kahneman and Tversky (1979), as they refer to transformed probabilities as *decision weights* rather than beliefs.[6]

Second, we recall the value function given in the first chapter (see Eqn 2.1). In

[6]See Wakker (2010) for further discussion on the mechanics of probability weighting.

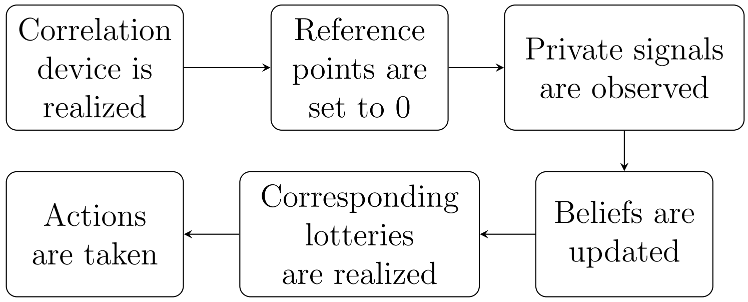

Figure 3.1: The Order of Events

particular, for every $i \in N$, the value function $v_i : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$v_i(x, r) = \begin{cases} x - r & , \quad x \geq r \\ \lambda_i(x - r) & , \quad x < r \end{cases} \tag{3.1}$$

where $x \in \mathbb{R}$ denotes the payoff, $r \in \mathbb{R}$ denotes the *reference point*, and $\lambda_i \geq 1$ is the loss aversion coefficient.

Third, as it can be seen from the value function, payoffs are evaluated with respect to a reference point. Here we simply assume that each agent's reference point is fixed and normalized to 0. This also helps us to relate our notion of correlated CPT equilibrium to the notion of CPT equilibrium we have introduced in the first chapter.

In this setting, the order of events is summarized in Figure 3.1. Our assumptions regarding the timing of the determination of the reference point and the timing of subjective probability weighting rely on the idea that prospect-theoretic behavior takes place during the decision process, i.e., between the last two nodes of the figure. This idea is supported by our assumption that agents are able to convert the strategic framework into the equivalent lottery framework and that they employ CPT preferences only when evaluating the lotteries.

To sum up, our definition of expected value function is similar to Eqn 2.2. Here we restate this definition for a given lottery $L = (p_1, x_1; \ldots; p_k, x_k)$ which is separated into positive and negative prospects following an increasing order of payoffs. The positive lottery $L^+ = (p_0^+, r_i; p_1^+, x_1^+; \ldots; p_{k_1}^+, x_{k_1}^+)$ is formed with all payoffs of L which are greater than or equal to r_i, and the negative lottery $L^- = (p_1^-, x_1^-; \ldots; p_{k_2}^-, x_{k_2}^-; p_0^-, r_i)$ is obtained similarly. In order for these to be well-defined lotteries, we set

$$p_0^+ = 1 - \sum_{j=1}^{k_1} p_j^+ \qquad \text{and} \qquad p_0^- = 1 - \sum_{j=1}^{k_2} p_j^- .$$

For every $i \in N$, the expected value of L is defined as

$$EV_i^{\lambda_i, r_i}(L) = EV_i^{\lambda_i, r_i}(L^+) + EV_i^{\lambda_i, r_i}(L^-) \tag{3.2}$$

where

$$EV_i^{\lambda_i, r_i}(L^+) = \sum_{t=1}^{k_1} \left(w_i\Big(\sum_{j=t}^{k_1} p_j^+ \Big) - w_i\Big(\sum_{j=t+1}^{k_1} p_j^+ \Big) \right) v_i(x_t^+, r_i)$$

and

$$EV_i^{\lambda_i, r_i}(L^-) = \sum_{t=1}^{k_2} \left(w_i\Big(\sum_{j=1}^{t} p_j^- \Big) - w_i\Big(\sum_{j=1}^{t-1} p_j^- \Big) \right) v_i(x_t^-, r_i) .$$

We note once again that the definition above relies on the probability weighting procedure suggested by Tversky and Kahneman (1992).

3.1.2 Definition of Correlated CPT Equilibrium

For any finite n-person normal form game $\Gamma = (N, (S_i)_{i\in N}, (h_i)_{i\in N})$, the notion of correlated CPT equilibrium is defined as follows.

Definition 4. A correlated CPT equilibrium of Γ is a probability measure π^* on S such that for every $i \in N$, every $s_i^* \in S_i$ with $\sum_{s_{-i}\in S_{-i}} \pi^*(s_i^*, s_{-i}) > 0$, and every $s_i \in S_i \setminus \{s_i^*\}$:

$$EV_i^{\lambda_i,0}(L_i^{\pi^*}(s_i^*|s_i^*)) \geq EV_i^{\lambda_i,0}(L_i^{\pi^*}(s_i|s_i^*)).$$

This framework reduces to the expected utility framework when (i) $\lambda_i = 1$ for every $i \in N$; and (ii) $w_i(p) = p$ for every $i \in N$ and every $p \in [0, 1]$. Consequently, correlated CPT equilibrium is a generalization of correlated equilibrium.

3.2 The Results

Correlated equilibrium has a number of desirable properties. First, an equilibrium exists for any finite normal form game. Second, every mixed strategy Nash equilibrium induces a correlated equilibrium. As a matter of fact, any convex combination of mixed strategy Nash equilibria induces a correlated equilibrium. Third, outside the set of probability measures induced by the convex hull of the set of mixed strategy Nash equilibria, there may exist correlated equilibrium as well. Nevertheless, these additional equilibria are such that the convexity of the equilibrium set is not violated. In this part, we analyze the set of correlated CPT equilibria to investigate (i) its differences from the set of correlated equilibria; and (ii) whether the above-mentioned properties are preserved under CPT preferences.

The first proposition focuses on pure strategy Nash equilibrium. In what follows, π^s denotes the degenerate probability measure induced by $s \in S$; i.e., $\pi^s(s) = 1$ and for every $s' \in S \setminus \{s\}$: $\pi^s(s') = 0$.

Proposition 3. *For any finite normal form game, s is a pure strategy Nash equilibrium if and only if π^s is a correlated CPT equilibrium.*

Proof. Take any pure strategy profile $s \in S$ and let the corresponding probability measure π^s be the correlation device. For every agent $i \in N$, we have $\pi^s(s_{-i}|s_i) = 1$. Therefore, agent i would obey the signal if and only if $h_i(s) \geq h_i(s'_i, s_{-i})$ for every $s'_i \in S_i$, i.e., if and only if s is a pure strategy Nash equilibrium. $\square$

The above result implies that the sets of correlated equilibria and correlated CPT equilibria have a non-empty intersection if the normal form game possesses a pure strategy Nash equilibrium. The next step is to examine whether the same relationship exists when the game has no pure strategy Nash equilibrium.

In order to have more concrete ideas about the equilibrium set, we concentrate on a particular special case of correlated CPT equilibrium. More precisely, we only consider subjective probability weighting by assuming that there is no loss aversion. In particular, we assume that $\lambda_i = 1$ for every $i \in N$.

Definition 5. A cumulatively weighted correlated equilibrium (CWCE) of Γ is a probability measure π^* on S such that for every $i \in N$, every $s_i^* \in S_i$ with

$\sum_{s_{-i} \in S_{-i}} \pi^*(s_i^*, s_{-i}) > 0$, and every $s_i \in S_i \setminus \{s_i^*\}$:

$$EV_i^{1,0}(L_i^{\pi^*}(s_i^*|s_i^*)) \geq EV_i^{1,0}(L_i^{\pi^*}(s_i|s_i^*)).$$

In the following example, we also employ a particular probability weighting function which is originally suggested by Prelec (1998). For every $i \in N$:

$$w_i(p) = \exp\{-(-\ln p)^{\alpha_i}\} \quad (3.3)$$

for some $\alpha_i \in (0, 1)$. We show that a mixed strategy Nash equilibrium does not necessarily induce a CWCE. Since the following game possesses a unique equilibrium, we obtain an even stronger result:[7]

Example 1. For the game Γ represented by the following game matrix

1, 0	0, 1
0, 2	1, 0

the sets CE(Γ) and CWCE(Γ) are disjoint. For instance, consider the probability measure

a	b
c	d

which constitutes the unique correlated equilibrium when $a = 1/3$, $b = 1/3$, $c = 1/6$, and $d = 1/6$. In fact, this is the probability measure induced by the unique mixed strategy Nash equilibrium $((2/3, 1/3), (1/2, 1/2))$. However, as it

[7]In the other extreme, there exist games in which both sets of equilibria coincide. For example, in a standard 2×2 matching pennies game, the probability measure that assigns $1/4$ to each strategy profile is the unique correlated equilibrium and the unique CWCE.

is discussed below, it is not a CWCE:

For this probability measure to be a CWCE, in addition to $a+b+c+d=1$, the following conditions should be satisfied:

$$\begin{array}{rcl} w_1\left(\frac{a}{a+b}\right) \geq w_1\left(\frac{b}{a+b}\right) & \text{and} & w_1\left(\frac{d}{c+d}\right) \geq w_1\left(\frac{c}{c+d}\right) \\ 2w_2\left(\frac{c}{a+c}\right) \geq w_2\left(\frac{a}{a+c}\right) & \text{and} & w_2\left(\frac{b}{b+d}\right) \geq 2w_2\left(\frac{d}{b+d}\right) \end{array}$$

Considering the probability weighting function given by Eqn (3.3), this problem has a unique solution for which $a=b$, $c=d$, and $ax=(1-x)c$ where x satisfies $2w_2(x)=w_2(1-x)$. $\Diamond$

This is not an unexpected result. The notion of mixed strategy Nash equilibrium is compatible with EUT; hence, a probability measure induced by some mixed strategy Nash equilibrium might not constitute an equilibrium under non-EUT preferences. However, we can expect to find a relation between correlated CPT equilibrium and the notion of CPT equilibrium introduced in the first chapter.

In the following proposition, we let π^μ denote the probability measure induced by the mixed strategy profile $\mu \in \Delta S$, i.e., $\pi^\mu(s) = \mu(s) \equiv \prod_{i \in N} \mu_i(s_i)$ for every pure strategy profile $s \in S$.

Proposition 4. *A mixed strategy profile $\mu \in \Delta S$ is a CPT equilibrium if and only if π^μ is a correlated CPT equilibrium.*

Proof. Take any $\mu \in \Delta S$. Assume that π^μ is a correlated CPT equilibrium. We want to show that μ is a CPT equilibrium. Take any agent $i \in N$. Consider the case in which s_i is signaled to agent i which is possible only if $\mu_i(s_i) > 0$. For this case, agent i faces the lottery $L_i^{\pi^\mu}(s_i|s_i)$ under obedience and faces the lottery $L_i^{\pi^\mu}(s_i'|s_i)$ if he/she disobeys by deviating to some $s_i' \in S_i$. Since π^μ is a correlated CPT equilibrium, we have

$$EV_i^{\lambda_i,0}\big(L_i^{\pi^\mu}(s_i|s_i)\big) \geq EV_i^{\lambda_i,0}\big(L_i^{\pi^\mu}(s_i'|s_i)\big)$$

for every $s_i' \in S_i$. And since $\pi^\mu(s_{-i}'|s_i) = \mu_{-i}(s_{-i}')$ for every $s_{-i}' \in S_{-i}$, we have

$$EV_i^{\lambda_i,0}\big(L_i(s_i, \mu_{-i})\big) \geq EV_i^{\lambda_i,0}\big(L_i(s_i', \mu_{-i})\big)$$

for every $s_i' \in S_i$. Since s_i was arbitrary, the above inequality holds for every $s_i \in S_i$ satisfying $\mu_i(s_i) > 0$. This implies that

$$U_i^{\lambda_i,0}(\mu_i, \mu_{-i}) \geq U_i^{\lambda_i,0}(\mu_i', \mu_{-i})$$

for every $\mu_i' \in \Delta S_i$. Hence μ is a CPT equilibrium.

Conversely, assume that μ is a CPT equilibrium. Take any agent $i \in N$ and assume that $\mu_i(s_i^*) = 1$ for some $s_i^* \in S_i$. This means

$$U_i^{\lambda_i,0}(\mu_i, \mu_{-i}) = EV_i^{\lambda_i,0}\big(L_i(s_i^*, \mu_{-i})\big).$$

Now, only s_i^* can be signaled to agent i by the correlation device π^μ, and in that case

$$EV_i^{\lambda_i,0}\big(L_i^{\pi^\mu}(s_i^*|s_i^*)\big) \geq EV_i^{\lambda_i,0}\big(L_i^{\pi^\mu}(s_i'|s_i^*)\big)$$

for every $s_i' \in S_i$ since μ is a CPT equilibrium. If μ_i is not as such, then define $\sigma_i(\mu_i)$ as the set of all pure strategies of agent i satisfying $\mu_i(\cdot) > 0$. Noting that

$$EV_i^{\lambda_i,0}\big(L_i(s_i,\mu_{-i})\big) = EV_i^{\lambda_i,0}\big(L_i(\tilde{s}_i,\mu_{-i})\big) \geq EV_i^{\lambda_i,0}\big(L_i(s_i',\mu_{-i})\big)$$

for every $s_i, \tilde{s}_i \in \sigma_i(\mu_i)$ and every $s_i' \in S_i$, we have

$$U_i^{\lambda_i,0}(\mu_i,\mu_{-i}) = EV_i^{\lambda_i,0}\big(L_i(s_i,\mu_{-i})\big) = U_i^{\lambda_i,0}(s_i,\mu_{-i})$$

for every $s_i \in \sigma_i(\mu_i)$. This implies that $s_i \in \mathcal{BR}_i^{CPT}(\mu)$ and that

$$EV_i^{\lambda_i,0}\big(L_i^{\pi^\mu}(s_i|s_i)\big) \geq EV_i^{\lambda_i,0}\big(L_i^{\pi^\mu}(s_i'|s_i)\big)$$

for every $s_i \in \sigma_i(\mu_i)$ and every $s_i' \in S_i$. Since $s_i'' \notin \sigma_i(\mu_i)$ cannot be signaled to agent i by the correlation device π^μ, we conclude that π^μ is a correlated CPT equilibrium. $\square$

The following existence result follows as a corollary.

Proposition 5. *Correlated CPT equilibrium exists for any finite normal form game.*

Proof. In the first chapter, we have shown that a CPT equilibrium exists for any finite normal form game (see Proposition 2). Above we further show that a probability measure induced by any CPT equilibrium turns out to be a correlated CPT equilibrium. It directly follows that correlated CPT equilibrium exists for any finite normal form game. □

Finally, we focus on the convexity of the equilibrium set. We start by presenting an example indicating that the set of correlated CPT equilibria is not necessarily convex. In this example, we once again refer to the notion of CWCE and to Prelec (1998)'s probability weighting function.

Example 2. Consider the game represented by the following matrix

1, $2a$	0, 2
1, 0	0, 2
1, $a+1$	0, 2
1, 1	0, 2

for which the following correlation devices are CWCE given that $a = 1/w_2(.5)$:

$1/2$	0
$1/2$	0
0	0
0	0

0	0
0	0
$1/2$	0
$1/2$	0

However, a convex combination (e.g., with coefficient 1/2) of these devices does not constitute a CWCE if $\alpha_2 = 0.67$. ◊

Now, we verify a weaker property for correlated CPT equilibrium.

Proposition 6. *For any finite normal form game, the set of correlated CPT equilibria includes the set of probability measures induced by the convex hull of the set of pure strategy Nash equilibria.*

Proof. Take any collection of pure strategy Nash equilibria $s^1, \ldots, s^k$. Consider the induced probability measures which are respectively denoted by $\pi^1, \ldots, \pi^k$. Take any $\varphi \in [0,1)^k$ satisfying $\sum_1^k \varphi^j = 1$. Set a correlation device as $\pi^* = \sum_1^k \varphi^j \pi^j$. Consider any $i \in N$. Without loss of generality, assume that s_i^1 is signaled to i. Let M be a subset of $\{s^1, \ldots, s^k\}$ such that each $s^j \in M$ satisfies $s_i^j = s_i^1$. Assume that $M = \{s^1\}$. Then the conditional probability $\pi^*(s_{-i}^1 | s_i^1) = 1$. Since s^1 is a Nash equilibrium, agent i obeys the signal. As for the complementary case, assume that $M \setminus \{s^1\}$ is non-empty and let $\sum_{j \in M} \varphi^j = m$. Then if agent i obeys, he/she faces the lottery

$$\left(p_1, h_i(s_i^1, s_{-i}^1); p_2, h_i(s_i^1, s_{-i}^2); \ldots; p_k, h_i(s_i^1, s_{-i}^k)\right),$$

where $p_j = \varphi^j / m$ if $s^j \in M$ and $p_j = 0$ if otherwise. When agent i disobeys the signal s_i^1 by deviating to some strategy $s_i' \in S_i$, then he/she faces the lottery

$$\left(p_1, h_i(s_i', s_{-i}^1); p_2, h_i(s_i', s_{-i}^2); \ldots; p_k, h_i(s_i', s_{-i}^k)\right).$$

Note that $p_j = 0$ for every j with $s^j \notin M$. Moreover, $(s_i^1, s_{-i}^j) = s^j$ for every $s^j \in M$ which implies that the corresponding payoffs under obedience are the payoffs from Nash equilibria. These jointly imply that the latter lottery is first order stochastically dominated by the former lottery. Hence, agent i prefers to obey the signal.[8] Therefore, any probability measure induced by an element of the convex hull of the set of pure strategy Nash equilibria is a correlated CPT

[8] CPT preferences satisfy the axiom of first order stochastic dominance. This means that if a lottery L first order stochastically dominates another lottery L', then an agent with CPT preferences would choose L over L'.

equilibrium. □

The set of probability measures induced by the convex hull of the set of pure strategy Nash equilibria is included in the set of correlated equilibria as well. Therefore, every such equilibrium is *robust* in the sense that it is an equilibrium under both types of representation of preferences, EUT and CPT.

3.3 Conclusion

This chapter studies correlated equilibrium for normal form games in which agents' preferences are represented by the pair of functions suggested in CPT. The motivation is based on the studies in the prospect theory literature, and the findings and arguments therein. If one finds it realistic that agents' preferences are represented by CPT rather than EUT, then it will be better if the analysis of correlated equilibrium is performed under CPT preferences. Our results indicate that the equilibrium set significantly depends on this assumption regarding agents' preferences.

Regarding our results, the differences between the sets of equilibria may prove to be promising in experimental analysis. In particular, it sets the stage to analyze whether the observed individual behavior are in line with the predictions of correlated equilibrium or with the predictions of a specific correlated CPT equilibrium. Additionally, in the application areas of correlated equilibrium, the notion(s) under CPT preferences can be studied to understand the effect of such preferences on the existing results.

CHAPTER 4

INVERSE S-SHAPED PROBABILITY WEIGHTING FUNCTIONS IN FIRST-PRICE SEALED-BID AUCTIONS

This chapter is already published in a journal as an original research article: Inverse S-shaped probability weighting functions in first-price sealed-bid auctions, *Review of Economic Design*: 20(1), 57–67. It is reprinted in this thesis after the permission of the copyright holder.

It is often observed in first-price sealed-bid auction experiments that subjects tend to bid above the *risk neutral Nash equilibrium* (RNNE) predictions (see Cox et al., 1988; Kagel, 1995, among others). This *overbidding* phenomenon has often been explained using models with risk averse bidders. However, for such an explanation to be valid, bidders should be excessively risk averse. Accordingly, it is argued that risk aversion cannot be the only factor and may well not be the most important factor behind overbidding (see Kagel and Roth, 1992). Along this line, several alternative explanations have been provided: ambiguity aversion (Salo and Weber, 1995), regret theory (Filiz-Ozbay and Ozbay, 2007), level-k thinking (Crawford and Iriberri, 2007), and loss aversion (Lange and Ratan, 2010).[1]

In addition to the above studies, a number of papers suggest subjective probability weighting as an alternative explanation for overbidding. To the best of our knowledge, Cox et al. (1985) are the first to present the idea of using subjective probability weighting in first price auctions. They propose that a power probability weighting function (PWF) is observationally equivalent to a model with risk aversion. Afterwards, Goeree et al. (2002) employ this idea utilizing a functional form which is originally suggested by Prelec (1998). They estimate that the PWF should be essentially convex over the whole range if it were to explain their experimental observations. Finally, Armantier and Treich (2009b) experimentally show that bidders tend to overbid as they underestimate their winning probabilities, whereas Armantier and Treich (2009a) analytically show that a star-shaped PWF[2] can explain overbidding in first-price auctions.

The above-mentioned PWFs imply the underweighting of all probabilities. Hence they are not in accordance with the PWFs commonly used in the literature (i.e., inverse S-shaped functions) (see Tversky and Kahneman, 1992; Camerer and Ho, 1994; Wu and Gonzalez, 1996; Prelec, 1998, among others).[3] In this

[1]Salo and Weber (1995) show that greater aversion for ambiguity leads to higher bid amounts. Filiz-Ozbay and Ozbay (2007) introduce the concepts of winner and loser regret, and they explain overbidding by claiming that loser regret is more dominant. Crawford and Iriberri (2007) propose level-k thinking as a cause of overbidding. Finally, Lange and Ratan (2010) analyze overbidding in auctions using a multi-dimensional reference-dependent model.

[2]A function $F : [0,1] \to [0,1]$ with $F(0) = 0$ and $F(1) = 1$ is star-shaped if $F(x)/x$ is increasing in x.

[3]As a matter of fact, subjective probability weighting is suggested earlier by Karmarkar (1978) and by Kahneman and Tversky (1979). It is worth noting here that the PWF described by Kahneman and Tversky (1979) is essentially similar to an inverse S-shaped function; a

chapter we introduce inverse S-shaped PWFs into first-price sealed-bid auctions and investigate the extent to which such weighting functions explain overbidding.

Our results indicate that bidders with low valuations underbid if all bidders use the same inverse S-shaped PWF. We also show that (*i*) there exist cases under which all bidders always underbid and (*ii*) if the number of participants is sufficiently low, there exists a threshold valuation such that any bidder with a valuation higher than this threshold will overbid.[4] Therefore, we conclude that inverse S-shaped PWFs provide a *partial* explanation for overbidding. It is worth noting that these findings are somewhat consistent with the aforementioned experimental studies since overbidding is mostly observed for bidders with high valuations, whereas the submitted bids of subjects with low valuations are close to the RNNE predictions (see Filiz-Ozbay and Ozbay, 2007; Armantier and Treich, 2009b, among others).

4.1 The Model

4.1.1 On Subjective Probability Weighting

Subjective probability weighting is supported by numerous individual decision-making experiments (see Camerer, 1995, for a detailed review). It constitutes one of the key aspects of prospect theory (Kahneman and Tversky, 1979) and cumulative prospect theory (Tversky and Kahneman, 1992). Moreover, it is the

function that overweights low probabilities and underweights moderate to high probabilities. Later, hints about inverse S-shaped PWFs are also given by Quiggin (1982).

[4]To obtain these overbidding results, we assume that the valuations are distributed according to the uniform distribution and we utilize a specific PWF originally suggested by Prelec (1998).

main aspect of rank-dependent expected utility theory (Quiggin, 1982) and dual theory (Yaari, 1987). The bulk of relevant literature argues that a PWF appears to be inverse S-shaped (see Tversky and Kahneman, 1992; Camerer and Ho, 1994; Wu and Gonzalez, 1996; Prelec, 1998, among others). An increasing function $w : [0,1] \to [0,1]$ is inverse S-shaped if

(i) $w(0) = 0$ and $w(1) = 1$; and

(ii) there exists a unique $\bar{p} \in (0,1)$ for which

- $w(\bar{p}) = \bar{p}$;
- $w(p) > p$ for every $p \in (0, \bar{p})$; and
- $w(p) < p$ for every $p \in (\bar{p}, 1)$.

Following this line of research, we study inverse S-shaped PWFs in this chapter. In particular, we assume that all bidders employ an increasing, differentiable, and inverse S-shaped PWF when making their bidding decisions.

4.1.2 The Auction Framework

There is a single object to be sold, and there are n bidders in the player set N. Each bidder $i \in N$ assigns a monetary value to the auctioned object. The valuation v_i represents the maximum amount bidder i is willing to pay for the object and is only known to bidder i. In addition, each bidder knows that the valuations of other bidders are identically and independently distributed according to a cumulative distribution function F over $[0,1]$. In this first-price sealed-bid auction framework, bidders simultaneously submit their bids. The bidder with the highest bid wins the auction and gets the object. For the case in which there are multiple bidders with the highest bid, the winner is determined randomly and with equal probabilities. The winner pays an amount equal to his/her bid, whereas the remaining bidders do not make any payment.

Consider a bidder $i \in N$ with valuation v_i and assume that each bidder $j \in N \backslash \{i\}$ follows[5] some increasing bid function $\beta_j : [0,1] \to [0,\infty)$. Then if bidder i bids some $b \in [0,\infty)$, he/she wins the auction with probability $\prod_{j\neq i} F(\beta_j^{-1}(b))$, because b turns out to be greater than the bid of some $j \in N \backslash \{i\}$ with probability $F(\beta_j^{-1}(b))$. Consequently, the bidder faces the following lottery

$$L^{v_i}(b, (\beta_j)_{j\in N\backslash\{i\}}) = \left(\prod\nolimits_{j\neq i} F(\beta_j^{-1}(b)) \; , \; v_i - b \; ; \; 1 - \prod\nolimits_{j\neq i} F(\beta_j^{-1}(b)) \; , \; 0\right)$$

which describes a situation in which the bidder either wins the auction and receives a payoff of $v_i - b$ or does not win the auction and receives a payoff of zero. In this context, given the bid functions of other bidders, a best response of bidder i is the bid amount b^* that induces the lottery with the highest expected utility.

In this chapter, we assume that all bidders subjectively weight probabilities with an inverse S-shaped PWF when evaluating these lotteries.[6] To fully concentrate

[5] A bidder is said to *follow* β if he/she bids $\beta(v)$ when his/her valuation is v.

[6] A natural question that arises is whether first-degree stochastic dominance relationships are preserved when we apply subjective probability weighting directly to the winning probabilities. The answer is provided by Goeree et al. (2002). Noting that the preferred solution would be to apply the weights to the cumulative distribution function, it is emphasized that the lotteries have only two outcomes in a first-price auction. Then, since the weighted probability of losing will be multiplied by 0 (which is the earning from losing), it is argued that applying the weights to the cumulative distribution function is equivalent to directly weighting the winning probabilities.

on the effect of such weighting functions on bid amounts, we employ a standard linear utility function. Hence our model is in line with Yaari (1987)'s dual theory. Furthermore, it is assumed to be common knowledge that bidders have the same utility function and the same PWF.

4.1.3 The Equilibrium Analysis

We analyze symmetric equilibrium throughout the chapter. The probability of winning the auction can then be represented by a function $G \equiv F^{n-1}$. Accordingly, when subjective probability weighting steps in, bidders have $w(G(\cdot))$ as their weighted probability of winning.

For the equilibrium analysis, take any bidder $i \in N$ with valuation v_i. His/Her expected utility from bidding $b \in [0, v_i]$ while all other bidders $j \in N \backslash \{i\}$ follow the same increasing, differentiable bid function $\beta : [0,1] \to [0,\infty]$ is[7]

$$w\left(G(\beta^{-1}(b))\right)(v_i - b) \ .$$

The analysis yields the following equilibrium bid function.

Proposition 7. *In a first-price sealed-bid auction with subjective probability weighting, the unique symmetric equilibrium is given by*

$$\beta^*(v_i) = v_i - \frac{\displaystyle\int_0^{v_i} w(G(y))dy}{w(G(v_i))} \tag{4.1}$$

[7]It is straightforward that bidding any amount higher than own valuation is dominated by bidding 0. Accordingly, we do not consider those values of b in our analysis although they are in the bidder's strategy set.

if all bidders weight probabilities with the same inverse S-shaped PWF, w.

Proof. The first order condition with respect to b is

$$\frac{\partial w(G(\beta^{-1}(b)))}{\partial \beta^{-1}(b)} \frac{\partial \beta^{-1}(b)}{\partial b}(v_i - b) - w(G(\beta^{-1}(b))) = 0 \ .$$

As we search for symmetric equilibrium, $b = \beta(v_i)$ should be the maximizer of the objective function; that is, $b = \beta(v_i)$ should solve the equation above. Thus,

$$\frac{\partial w(G(v_i))}{\partial v_i} \frac{1}{\beta'(v_i)}(v_i - \beta(v_i)) = w(G(v_i)) \ .$$

After arranging terms, we obtain

$$\frac{\partial}{\partial v_i}(w(G(v_i))\beta(v_i)) = v_i \frac{\partial w(G(v_i))}{\partial v_i} \ ,$$

which implies

$$\beta^*(v_i) = \frac{1}{w(G(v_i))} \int_0^{v_i} y \frac{\partial w(G(y))}{\partial y} dy = v_i - \frac{\int_0^{v_i} w(G(y))dy}{w(G(v_i))} \ .$$

By differentiating, we see that β^* is increasing in v_i. Moreover, it is straightforward that $\beta^*(v_i)$ is not greater than v_i for any $v_i \in [0, 1]$. Thus β^* is the only candidate for a symmetric equilibrium.

To verify that β^* is an equilibrium, we first assume that every $j \in N \setminus \{i\}$ follows β^*. Note that bidding above $\beta^*(1)$ is dominated for bidder i. Suppose that bidder i acts as if his/her valuation is $z \in [0, 1]$ rather than v_i. Then it turns out that $z = v_i$ is a best response. Consequently, β^* is the unique symmetric equilibrium. $\square$

It is worth noting that the above equilibrium bid function reduces to the *risk neutral Nash equilibrium* (RNNE) if the PWF is the identity function. Also note that the fraction in Eqn (4.1) is bidder i's net earning when he/she wins the auction; and it will be the only relevant part of the bid function when comparing β^* with the RNNE (denoted by β^*_{RN}).

4.1.4 The Results on Overbidding

Considering the results of the earlier studies on subjective probability weighting in first-price auctions, one can conjecture that inverse S-shaped functions cannot *completely* explain overbidding. That said, our first objective is to check whether there exist valuations for which bidders underbid.

Proposition 8. *Consider a first-price sealed-bid auction with subjective probability weighting in which all bidders use the same inverse S-shaped PWF. Then there exists a valuation $\hat{v}$ such that $\beta^*(\hat{v}) < \beta^*_{RN}(\hat{v})$.*

Proof. Take any $n \in \mathbb{N}$. Since w is inverse S-shaped, there exists a unique $\hat{v} \in (0,1)$ such that $w(F(\hat{v})^{n-1}) = F(\hat{v})^{n-1}$. Consider a bidder with valuation $\hat{v}$. First note that his/her weighted probability of winning equals to his/her winning probability. Also, since every probability lower than $F(\hat{v})^{n-1}$ is being overweighted,

$$\int_0^{\hat{v}} w(F(y)^{n-1})dy > \int_0^{\hat{v}} F(y)^{n-1}dy \ .$$

It then follows that $\beta^*(\hat{v}) < \beta_{RN}(\hat{v})$ for every $n \in \mathbb{N}$. □

The above proposition indicates that the above-mentioned conjecture is true. However, to what extent inverse S-shaped PWFs explain overbidding remains unanswered. We answer this question by checking the existence of valuations for which bidders overbid. At this point, we make two additional assumptions. First, we employ a functional form which is originally suggested by Prelec (1998) and is defined as

$$w(p) = \exp\{-(-\ln p)^{\alpha}\} \tag{4.2}$$

where $\alpha \in (0, 1)$. Second, we assume that F is the uniform distribution.[8]

Letting β^U and β^U_{RN} denote the corresponding unique symmetric equilibria under the uniform distribution, we first show that underbidding is possible for all values of valuations.

Example 3. Consider a first-price sealed-bid auction with subjective probability weighting in which valuations are distributed according to the uniform distribution. Assume that all bidders weight probabilities with the inverse S-shaped PWF given by Eqn (4.2). Further assume that $n = 10$ and $\alpha = 0.67$. Then for every $v \in (0, 1]$,

$$\beta^U(v) < \frac{9v}{10} = \beta^U_{RN}(v).$$

Therefore, there may be cases under which all bidders always underbid. ◊

The next proposition states that if a certain regularity condition is satisfied,

[8]The results of the following analysis depend on the distribution. In what follows, we prefer to adopt the uniform distribution, because overbidding is observed under the uniform distribution in the aforementioned experimental studies.

overbidding is *partially* explained under WAC.

Proposition 9. *Consider a first-price sealed-bid auction with subjective probability weighting in which valuations are distributed according to the uniform distribution. Assume that all bidders weight probabilities with the inverse S-shaped PWF given by Eqn* (4.2)*. If*

$$\int_0^1 \exp\left\{-(n-1)^\alpha \left(-\ln y\right)^\alpha\right\} dy \le \frac{1}{n} \ ,$$

then there exists a unique critical valuation $v^* \in (0,1]$ *such that* $\beta^U(v^*) = \beta^U_{RN}(v^*)$ *and any bidder with valuation* v *underbids if* $v < v^*$ *whereas he/she overbids if* $v > v^*$.

Proof. Given the regularity condition, a bidder with valution 1 overbids. Recall that a bidder with valuation $\hat{v}$ underbids; i.e., $\beta^U(\hat{v}) < \beta^U_{RN}(\hat{v})$. Since β^U and β^U_{RN} are both continuous, there exists v^* such that $\beta^U(v^*) = \beta^U_{RN}(v^*)$. The proof of uniqueness follows as in the proof of Proposition 12. □

In addition, we have the following equation that uniquely characterizes the critical valuation v^*:

$$\frac{\displaystyle\int_0^{v^*} \exp\left\{-(n-1)^\alpha(-\ln y)^\alpha\right\} dy}{\exp\left\{-(n-1)^\alpha(-\ln v^*)^\alpha\right\}} = \frac{v^*}{n} \ .$$

We know by Propositions 8 and 9 that all overweighters underbid if all bidders weight probabilities with the inverse S-shaped PWF given by Eqn (4.2). In other words, subjective probability weighting causes bidders with low valuations to overestimate their chances of winning the auction, to which they respond by lowering their bids. Thus we relate our underbidding results with the overweighting interval of the PWF, which is given by $(0, \bar{p})$. On the other hand, the underweighting interval of the function gives bidders an incentive to increase their bid amounts. However, since the bids of overweighters are already below the RNNE predictions, there are some underbidding underweighters.[9] The effect of the underweighting interval becomes dominant for bidders with sufficiently high valuations, and there occurs overbidding.

4.2 Utilizing Another Weighting Method

In first-price sealed-bid auctions, a participant wins the auction if every other bidder submits a bid less than that of the participant. Hence his/her winning probability is calculated by compounding the probabilities of other bidders' submitting such bids. Naturally, the timing of subjective probability weighting may lead to different theoretical predictions. In this chapter, we assume that bidders directly weight their wining probabilities. This is in line with the standard method employed in earlier studies on subjective probability weighting in first-price auctions. In this Appendix A, we propose an alternative method: *weighting before compounding* (WBC). The following example demonstrates the difference between the standard method and WBC.

Example 4. Consider a lottery in which a fair coin is flipped twice. The lottery yields \$1 if both outcomes are heads and yields nothing otherwise. Obviously, the probability of winning the lottery is 1/4. For this lottery, two possible weight-

[9] This follows by the continuity of equilibrium bid functions.

ing methods are as follows: (*i*) weighting the winning probability, which yields $w(1/4)$; or (*ii*) weighting the probabilities of each independent event separately and then compounding the weighted probabilities, which yields $w(1/2)^2$. $\Diamond$

Clearly, (*i*) corresponds to the standard method. It stipulates that the probabilities of independent events are first compounded, and then this compounded probability will be distorted. On the other hand, (*ii*) corresponds to WBC. The idea behind this method resembles the one behind *prospective reference theory* introduced by Viscusi (1989).[10] According to this theory, a compound lottery is treated differently than the corresponding reduced lottery.[11] Under this method, the probability of each independent event is first distorted, and then these weighted probabilities will be compounded.

4.2.1 The Equilibrium Analysis

First, at a symmetric equilibrium, the weighted probability of winning is $w(F(\cdot))^{n-1}$ rather than $w(G(\cdot)) = w(F(\cdot)^{n-1})$. The equilibrium analysis follows similarly: Take an arbitrary bidder $i \in N$ with valuation v_i. His/Her expected utility from bidding $b \in [0, v_i]$ while all other bidders $j \in N \backslash \{i\}$ follow the same increasing, differentiable bid function $\beta : [0, 1] \to [0, \infty]$ is

$$w\left(F(\beta^{-1}(b)\right)^{n-1}(v_i - b) \ .$$

[10]Prospective reference theory is able to predict several phenomena such as premiums for certain eliminations of a risk, the representativeness heuristic, the isolation effect, and the Allais paradox and related violations of the substitution axiom.

[11]Note that the lottery in Example 1 can also be described as a compound lottery.

The analysis yields the following equilibrium bid function.

Proposition 10. *In a first-price sealed-bid auction with subjective probability weighting, the unique symmetric equilibrium is given by*

$$\beta_B^*(v_i) = v_i - \frac{\displaystyle\int_0^{v_i} w(F(y))^{n-1}dy}{w(F(v_i))^{n-1}} \tag{4.3}$$

if all bidders weight probabilities before compounding with the same inverse S-shaped PWF, w.

Proof. The first order condition with respect to b is

$$\frac{\partial w(F(\beta^{-1}(b)))^{n-1}}{\partial \beta^{-1}(b)} \frac{\partial \beta^{-1}(b)}{\partial b}(v_i - b) - w(F(\beta^{-1}(b)))^{n-1} = 0 \ .$$

It then follows that

$$\beta_B^*(v_i) = v_i - \frac{\displaystyle\int_0^{v_i} w(F(y))^{n-1}dy}{w(F(v_i))^{n-1}} \ .$$

This bidding function is increasing in v_i, and $\beta_B^*(v_i)$ is not greater than v_i for any $v_i \in [0, 1]$. Hence β_B^* is the only candidate for a symmetric equilibrium. For verification, one can show that a bidder with valuation v_i bids $\beta_B^*(v_i)$ given that other bidders follow β_B^*. Thus β_B^* is the unique symmetric equilibrium. □

4.2.2 The Results on Overbidding

First, we prove the existence of an underbidder.

Proposition 11. *Consider a first-price sealed-bid auction with subjective probability weighting in which all bidders use the same inverse S-shaped PWF. Then there exist a valuation $\hat{v}_B$ such that $\beta^*_B(\hat{v}_B) < \beta^*_{RN}(\hat{v}_B)$.*

Proof. Assume that bidders weight probabilities before compounding. Since w is inverse S-shaped, there exists a unique $\hat{v}_B \in (0,1)$ such that $w(F(\hat{v}_B)) = F(\hat{v}_B)$. Consider a bidder with valuation $\hat{v}_B$. His/Her weighted probability of winning equals to his/her winning probability. Also, since every probability lower than $F(\hat{v}_B)$ is being overweighted,

$$\int_0^{\hat{v}_B} w(F(y))^{n-1} dy > \int_0^{\hat{v}_B} F(y)^{n-1} dy$$

for every $n \in \mathbb{N}$. It then follows that $\beta^*_B(\hat{v}_B) < \beta^*_{RN}(\hat{v}_B)$. □

For the following proposition, let β^U_B denote the equilibrium bid function under the assumption that F is the uniform distribution.

Proposition 12. *Consider a first-price sealed-bid auction with subjective probability weighting in which valuations are distributed according to the uniform distribution. Assume that all bidders weight probabilities before compounding with the inverse S-shaped PWF given by Eqn* (4.2)*. Then there exists a unique critical valuation $v^*_B \in (0,1]$ such that $\beta^U_B(v^*_B) = \beta^U_{RN}(v^*_B)$ and any bidder with valuation v underbids if $v < v^*_B$ whereas he/she overbids if $v > v^*_B$.*

Proof. To show the existence of v^*_B, we first prove that a bidder with valuation 1 overbids. To do this, we first take the derivative of $\beta_B^U(1)$ with respect to α:

$$(1-n)\int_0^1 \exp\{-(n-2)(-\ln y)^\alpha\}\frac{\partial \exp\{-(-\ln y)^\alpha\}}{\partial\alpha}dy \ .$$

This expression turns out to be negative which implies that $\beta_B^U(1)$ is decreasing in α. Noting that $\beta_B^U(1) = \beta_{RN}^U(1)$ when $\alpha = 1$, we have $\beta_B^U(1) > \beta_{RN}^U(1)$ when $\alpha \in (0,1)$. Recall that a bidder with valuation $\hat{v}_B$ underbids; i.e., $\beta_B^U(\hat{v}_B) < \beta_{RN}^U(\hat{v}_B)$. Since β_B^U and β_{RN}^U are both continuous, there exists v_B^* such that $\beta_B^U(v_B^*) = \beta_{RN}^U(v_B^*)$.

As for uniqueness, one needs to show that there exists a unique $v \in (0,1]$ satisfying $\beta_B^U(v) - \beta_{RN}^U(v) = 0$. This expression is zero when $v = 0$, negative when $v = 1/e$, and positive when $v = 1$. Furthermore, it has a single extremum at some point in the interval $(1/e, 1]$. These jointly imply our claim that the critical valuation v_B^* is unique. In addition to this, any bidder with valuation v underbids if $v < v_B^*$ whereas he/she overbids if $v > v_B^*$. $\square$

In addition, we have the following equation that uniquely characterizes the critical valuation v_B^*:

$$\frac{\int_0^{v_B^*} \exp\left\{-(n-1)(-\ln y)^\alpha\right\} dy}{\exp\left\{-(n-1)(-\ln v_B^*)^\alpha\right\}} = \frac{v_B^*}{n} \ .$$

Given the uniform distribution, notice that a regularity condition is no longer necessary in order for inverse S-shaped PWFs to *partially* explain overbidding. This is because a bidder with valuation 1 always overbids under WBC. Accordingly, we can conclude that overbidding is explained for a wider range of valuations under WBC in comparison to the standard method.

4.3 Further Remarks

Prospect Theory Preferences: Throughout this chapter, we have only considered subjective probability weighting.[12] Now, we discuss the differences that would arise if one additionally considers reference dependence and loss aversion. In this part, we use the value function (see Eqns 2.1 or 3.1) instead of the standard linear utility function used in the previous section.

In our framework, the reference point of every bidder $i \in N$ can be defined as his/her expected earning which is directly affected by the valuation v_i and the number of bidders n. Thus it is natural to define the reference point as a function of these variables: $R(n, v_i)$. Then the reference point is (i) well-defined since every bidder i knows v_i and n; (ii) positive-valued since otherwise the bidder would not participate the auction; (iii) non-decreasing in v_i; (iv) non-increasing in n; and (v) less than v_i for every $v_i \in (0, 1]$.

Under such preferences, the analysis of symmetric equilibrium yields

$$\beta_A^R(v_i) = \beta_A(v_i) + (\lambda - 1) \int_0^{v_i} R(n, y) \frac{\partial w(G(y))}{\partial y} dy$$

if all bidders weight probabilities after compounding with the same inverse S-

[12] One may argue that we have also considered reference dependence when the reference point is fixed and normalized to 0 for every bidder $i \in N$. This would not make a difference in our analysis since a negative payoff is not possible for the bids we have considered.

shaped PWF and

$$\beta_B^R(v_i) = \beta_B(v_i) - \int_0^{v_i} R(n,y)\frac{\partial w(F(y))^{n-1}}{\partial y}dy - \lambda \int_0^{v_i} R(n,y)\frac{\partial H(y)}{\partial y}dy$$

if all bidders weight probabilities before compounding with the same inverse S-shaped PWF, where $H(\cdot)$ represents the weighted probability of losing. Since $\lambda > 1$ and $-\partial H(y)/\partial y > \partial w(F(y))^{n-1}/\partial y$, we have $\beta_A^R(v_i) > \beta_A(v_i)$ and $\beta_B^R(v_i) > \beta_B(v_i)$ for every $v_i \in (0,1]$. Therefore, inverse S-shaped PWFs seem to be more successful in explaining overbidding when used together with loss aversion and positive reference points.

Experimental Analysis: In this chapter, we have shown that inverse S-shaped functions cannot completely explain overbidding. This is in line with the earlier findings that a probability weighting function should be underweighting all probabilities if it were to explain overbidding behavior. However, a considerable number of experimental studies on one-shot choice problems indicate that individuals employ inverse S-shaped functions. This calls for a new experimental analysis to unravel the reasons behind the discordance between the PWFs suggested to explain overbidding and inverse S-shaped PWFs commonly used in the literature.

A possible experimental design for such an analysis proceeds as follows: Following the design by Armantier and Treich (2009b), subjects are divided into groups of two. They participate in a two-player first-price sealed-bid auction. They are asked to submit a bid and to guess their winning probabilities. This will reveal their perception of the framework they are in. Then a lottery framework is formulated which is equivalent to the strategic framework of first-price auctions.[13] Subjects are asked to choose *the best* lottery from this lottery framework. This part of the experiment is borrowed from Dorsey and Razzolini (2003).

Now, if subjects' choices in the auction framework and the lottery framework are consistent with each other, then we would say that subjects *correctly* perceive the situation and we would expect them to *correctly* guess their winning probabilities. Accordingly, we would conclude that subjective probability weighting does not have a major influence on subjects' bidding behavior. If otherwise; that is, if we observe situations where subjects prefer a strategy to another strategy but prefer the lottery induced by the latter strategy to the one induced by the former strategy; then we would say that subjects misperceive the auction framework. In such a case, we would expect them to make inaccurate predictions. Accordingly, we would conclude that subjective probability weighting may be the reason behind the observed overbidding behavior. Surely, a further empirical analyses of actions in the auction framework, actions in the lottery framework, and the predictions of winning probabilities would be necessary. A possible hypothesis is that subjects act in line with the two-stage model proposed by Fox and Tversky (1995, 1998).[14]

[13] The lottery framework is obtained in a similar manner to those in the first two chapters of this dissertation. Given the bid function of the other player, each strategy of an agent induces an equivalent lottery.

[14] As a matter of fact, such an experimental project is already in progress. The author thanks to the Department of Economics at Bilkent University for providing a financial support for that project.

4.4 Conclusion

In this chapter, we have introduced inverse S-shaped PWFs into a first-price sealed-bid auction framework. We have shown that inverse S-shaped PWFs cannot *completely* explain overbidding as bidders with low valuations underbid and that such weighting functions can *partially* explain overbidding as bidders with sufficiently high valuations overbid. It appears that the reason behind underbidding is the overweighting interval of inverse S-shaped PWFs.

This study is the first to use inverse S-shaped PWFs in first-price auctions. It can be considered a first step towards analyzing the reason(s) behind the discordance between the PWFs suggested to explain overbidding and inverse S-shaped PWFs commonly used in the literature. Our findings indicate that the level of discordance is greater for bidders with low valuations and if the number of bidders is high. Hence these issues may require special emphasis if one aims to unravel the reason(s) behind this discordance.

BIBLIOGRAPHY

Abdellaoui, M. (2000). Parameter-free Elicitation of Utility and Probability Weighting Functions. *Management Science*, 46(11):1497–1512.

Allais, M. (1953). Le Comportement de l'Homme Rationnel devant le Risque, Critique des Postulats et Axiomes de l'Ecole Americane. *Econometrica*, 21:503–546.

Altman, E., Bonneau, N., and Debbah, M. (2006). Correlated Equilibrium in Access Control for Wireless Communications. In *Proceedings of the 5th International IFIP-TC6 Networking Conference (NETWORKING)*, pages 173–183.

Armantier, O. and Treich, N. (2009a). Star-shaped Probability Weighting Functions an Overbidding in First-price Auctions. *Economics Letters*, 104:83–85.

Armantier, O. and Treich, N. (2009b). Subjective Probabilities in Games: An Application to the Overbidding Puzzle. *International Economic Review*, 50(4):1079–1102.

Aumann, R. J. (1974). Subjectivity and Correlation in Randomized Strategies. *Journal of Mathematical Economics*, 1:67–96.

Barberis, N. C. (2013). Thirty Years of Prospect Theory in Economics: A Review and Assessment. *Journal of Economic Perspectives*, 27:173–196.

Bohnet, I. and Zeckhauser, R. (2004). Trust, Risk and Betrayal. *Journal of Economic Behavior and Organization*, 55(4):467–484.

Camerer, C. F. (1995). Individual Decision Making. In Kagel, J. H. and Roth, A. E., editors, *The Handbook of Experimental Economics. Ch. 7*, pages 587–703.

Camerer, C. F. and Ho, T.-H. (1994). Violations of the Betweenness Axiom and Nonlinearity in Probability. *Journal of Risk and Uncertainty*, 8(2):167–196.

Cason, T. and Sharma, T. (2007). Recommended Play and Correlated Equilibria: An Experimental Study. *Economic Theory*, 33:11–27.

Cavaliere, A. (2001). Coordination and the Provision of Discrete Public Goods by Correlated Equilibria. *Journal of Public Economic Theory*, 3(3):235–255.

Chen, H.-C. and Neilson, W. S. (1999). Pure-strategy Equilibria with Non-expected Utility Players. *Theory and Decision*, 46:199–209.

Cox, J. C., Smith, V. L., and Walker, J. L. (1988). Theory and Individual Behavior of First-Price Auctions. *Journal of Risk and Uncertainty*, 1:61–99.

Cox, J. C., Smith, V. L., and Walker, J. M. (1985). Experimental Development of Sealed-Bid Auction Theory; Calibrating Controls for Risk Aversions. *The American Economic Review*, 75(2):160–165.

Crawford, V. P. and Iriberri, N. (2007). Level-k Auctions: Can a Nonequilibrium Model of Strategic Thinking Explain the Winner's Curse and Overbidding in Private Value Auctions? *Econometrica*, 75(6):1721–1770.

Dekel, E., Safra, Z., and Segal, U. (1991). Existence and Dynamic Consistency of Nash Equilibrium with Non-expected Utility Preferences. *Journal of Economic Theory*, 55:423–437.

Dhillon, A. and Mertens, J. F. (1996). Perfect Correlated Equilibria. *Journal of Economic Theory*, 68:279–302.

Dorsey, R. and Razzolini, L. (2003). Explaining Overbidding in First Price Auctions Using Controlled Lotteries. *Experimental Economics*, 6:123–140.

Driesen, B., Perea, A., and Peters, H. (2012). Alternating Offers Bargaining with Loss Aversion. *Mathematical Social Sciences*, 64(2):103–118.

Ellsberg, D. (1961). Risk, Ambiguity, and the Savage Axioms. *The Quarterly Journal of Economics*, 75(4):643–669.

Filiz-Ozbay, E. and Ozbay, E. Y. (2007). Auctions with Anticipated Regret: Theory and Experiment. *The American Economic Review*, 97:1407–1418.

Fox, C. R. and Tversky, A. (1995). Ambiguity Aversion and Comparative Ignorance. *The Quarterly Journal of Economics*, 110(3):585–603.

Fox, C. R. and Tversky, A. (1998). A Belief-based Account of Decision Under Uncertainty. *Management Science*, 44(7):879–895.

Goeree, J. K., Holt, C. A., and Palfrey, T. R. (2002). Quantal Response Equilibrium and Overbidding in Private Value Auctions. *Journal of Economic Theory*, 104:247–272.

Goeree, J. K., Holt, C. A., and Palfrey, T. R. (2003). Risk Averse Behavior in Generalized Matching Pennies Games. *Games and Economic Behavior*, 45(1):97–113.

Harsanyi, J. C. (1973). Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-strategy Equilibrium Points. *International Journal of Game Theory*, 2:1–23.

Ivanov, A. (2011). Attitudes to Ambiguity in One-shot Normal-form Games: An Experimental Study. *Games and Economic Behavior*, 71(2):366–394.

Kagel, J. H. (1995). Auctions: A Survey of Experimental Research. In Kagel, J. H. and Roth, A. E., editors, *The Handbook of Experimental Economics. Ch. 7*, pages 501–586.

Kagel, J. H. and Roth, A. E. (1992). Theory and Misbehavior of First-Price Auctions: Comment. *The American Economic Review*, 82:1379–1391.

Kahneman, D. and Tversky, A. (1979). Prospect Theory: An Analysis of Decision Under Risk. *Econometrica*, 47(2):263–291.

Kakutani, S. (1941). A Generalization of Brouwer's Fixed Point Theorem. *Duke Mathematical Journal*, 8(3):457–459.

Karmarkar, U. S. (1978). Subjectively Weighted Utility: A Descriptive Extension of the Expected Utility Model. *Organizational Behavior and Human Performance*, 21(1):61–72.

Lange, A. and Ratan, A. (2010). Multi-dimensional Reference-dependent Preferences in Sealed-bid Auctions – How (Most) Laboratory Experiments Differ from the Field. *Games and Economic Behavior*, 68(2):634–645.

Lin, Z. and Van der Schaar, M. (2009). On the Correlated Equilibrium Selection for Two-User Channel Access Games. *Signal Processing Letters, IEEE*, 16(3):156–159.

Liu, L. (1996). Correlated Equilibrium of Cournot Oligopoly Competition. *Journal of Economic Theory*, 68(2):544–548.

Maskin, E. and Tirole, J. (1987). Correlated Equilibria and Sunspots. *Journal of Economic Theory*, 43(2):364–373.

McKelvey, R. and Palfrey, T. R. (1995). Risk Averse Behavior in Generalized Matching Pennies Games. *Games and Economic Behavior*, 10(1):6–38.

McKelvey, R. D., Palfrey, T. R., and Weber, R. A. (2000). The Effects of Payoff Magnitude and Heterogeneity on Behavior in 2×2 Games with Unique Mixed Strategy Equilibria. *Journal of Economic Behavior and Organization*, 42:523–548.

Metzger, L. P. and Rieger, M. O. (2010). Equilibria in Games with Prospect Theory Preferences. Working Paper.

Myerson, R. (1986). Acceptable and Predominant Correlated Equilibria. *International Journal of Game Theory*, 15(3):133–154.

Nash, J. F. (1951). Non-cooperative Games. *Annals of Mathematics*, 54:286–295.

Ochs, J. (1995). Games with Unique, Mixed Strategy Equilibria: An Experimental Study. *Games and Economic Behavior*, 10(1):202–217.

Prelec, D. (1998). The Probability Weighting Function. *Econometrica*, 66(3):497–527.

Quiggin, J. (1982). A Theory of Anticipated Utility. *Journal of Economic Behavior and Organization*, 3:323–343.

Ramsey, D. M. and Szajowski, K. (2008). Selection of a Correlated Equilibrium in Markov Stopping Games. *European Journal of Operational Research*, 184(1):185–206.

Rieger, M. O. (2014). Evolutionary Stability of Prospect Theory Preferences. *Journal of Mathematical Economics*, 50:1–11.

Ritzberger, K. (1996). On Games Under Expected Utility with Rank Dependent Probabilities. *Theory and Decision*, 40:1–27.

Rosenthal, R. W. (1979). Sequences of Games with Varying Opponents. *Econometrica*, 47:1353–1366.

Salo, A. A. and Weber, M. (1995). Ambiguity Aversion in First-price Sealed-bid Auctions. *Journal of Risk and Uncertainty*, 11:123–137.

Schmeidler, D. (1989). Subjective Probability and Expected Utility without Additivity. *Econometrica*, 57:571–587.

Selten, R. and Chmura, T. (2008). Stationary Concepts for Experimental 2×2 Games. *The American Economic Review*, 98(3):938–966.

Shalev, J. (2000). Loss Aversion Equilibrium. *International Journal of Game Theory*, 29:269–287.

Shalev, J. (2002). Loss Aversion and Bargaining. *Theory and Decision*, 52(3):201–232.

Solan, E. and Vieille, N. (2002). Correlated Equilibrium in Stochastic Games. *Games and Economic Behavior*, 38(2):362 – 399.

Sonnemans, J., Schram, A., and Offerman, T. (1998). Public Good Provision and Public Bad Prevention: The Effect of Framing. *Journal of Economic Behavior and Organization*, 34:143–161.

Starmer, C. (2000). Developments in Non-expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk. *Journal of Economic Literature*, 38(2):332–382.

Teague, V. (2004). Selecting Correlated Random Actions. In *Financial Cryptography*, pages 181–195.

Tversky, A. and Kahneman, D. (1991). Loss Aversion in Riskless Choice: A Reference-dependent Model. *The Quarterly Journal of Economics*, 106(4):1039–1061.

Tversky, A. and Kahneman, D. (1992). Advances in Prospect Theory: A Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty*, 5:297–323.

Tversky, A. and Wakker, P. P. (1995). Risk Attitudes and Decision Weights. *Econometrica*, 63:1255–1280.

Viscusi, W. K. (1989). Prospective Reference Theory: Toward an Explanation of the Paradoxes. *Journal of Risk and Uncertainty*, 2:235–264.

von Neumann, J. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior*. Princeton University Press.

Wakker, P. P. (2010). *Prospect Theory: For Risk and Ambiguity*. Cambridge University Press.

Wu, G. and Gonzalez, R. (1996). Curvature of the Probability Weighting Function. *Management Science*, 42:1676–1690.

Yaari, M. E. (1987). The Dual Theory of Choice Under Risk. *Econometrica*, 55(1):95–115.

APPENDIX

The following pages include the agreement between the author and Springer about the permission for reprinting the article "Inverse S-shaped probability weighting functions in first-price sealed-bid auctions" in this thesis (see Chapter 4).

SPRINGER LICENSE TERMS AND CONDITIONS

May 26, 2016

This Agreement between Kerim Keskin ("You") and Springer ("Springer") consists of your license details and the terms and conditions provided by Springer and Copyright Clearance Center.

License Number	3871820930518
License date	May 18, 2016
Licensed Content Publisher	Springer
Licensed Content Publication	Review of Economic Design
Licensed Content Title	Inverse S-shaped probability weighting functions in first-price sealed-bid auctions
Licensed Content Author	Kerim Keskin
Licensed Content Date	Jan 1, 2015
Licensed Content Volume Number	20
Licensed Content Issue Number	1
Type of Use	Thesis/Dissertation
Portion	Full text
Number of copies	1
Author of this Springer article	Yes and you are the sole author of the new work
Order reference number	None
Title of your thesis / dissertation	Non-cooperative Game Theory under Prospect Theory
Expected completion date	May 2016
Estimated size(pages)	76
Requestor Location	Kerim Keskin Bilkent University None None Ankara, Turkey 06800 Attn: Kerim Keskin
Billing Type	Invoice
Billing Address	Kerim Keskin Bilkent University None None Ankara, Turkey 06800 Attn: Kerim Keskin
Total	**0.00 USD**
Terms and Conditions	

Introduction
The publisher for this copyrighted material is Springer. By clicking "accept" in connection with completing this licensing transaction, you agree that the following terms and conditions apply to this transaction (along with the Billing and Payment terms and conditions established by Copyright Clearance Center, Inc. ("CCC"), at the time that you opened your Rightslink account and that are available at any time at http://myaccount.copyright.com).
Limited License
With reference to your request to reuse material on which Springer controls the copyright, permission is granted for the use indicated in your enquiry under the following conditions:

- Licenses are for one-time use only with a maximum distribution equal to the number stated in your request.
- Springer material represents original material which does not carry references to other sources. If the material in question appears with a credit to another source, this permission is not valid and authorization has to be obtained from the original copyright holder.
- This permission
• is non-exclusive
• is only valid if no personal rights, trademarks, or competitive products are infringed.
• explicitly excludes the right for derivatives.
- Springer does not supply original artwork or content.
- According to the format which you have selected, the following conditions apply accordingly:
• **Print and Electronic:** This License include use in electronic form provided it is password protected, on intranet, or CD-Rom/DVD or E-book/E-journal. It may not be republished in electronic open access.
• **Print:** This License excludes use in electronic form.
• **Electronic:** This License only pertains to use in electronic form provided it is password protected, on intranet, or CD-Rom/DVD or E-book/E-journal. It may not be republished in electronic open access.
For any electronic use not mentioned, please contact Springer at permissions.springer@spi-global.com.
- Although Springer controls the copyright to the material and is entitled to negotiate on rights, this license is only valid subject to courtesy information to the author (address is given in the article/chapter).
- If you are an STM Signatory or your work will be published by an STM Signatory and you are requesting to reuse figures/tables/illustrations or single text extracts, permission is granted according to STM Permissions Guidelines: http://www.stm-assoc.org/permissions-guidelines/
For any electronic use not mentioned in the Guidelines, please contact Springer at permissions.springer@spi-global.com. If you request to reuse more content than stipulated in the STM Permissions Guidelines, you will be charged a permission fee for the excess content.
Permission is valid upon payment of the fee as indicated in the licensing process. If permission is granted free of charge on this occasion, that does not prejudice any rights we might have to charge for reproduction of our copyrighted material in the future.
-If your request is for reuse in a Thesis, permission is granted free of charge under the following conditions:
This license is valid for one-time use only for the purpose of defending your thesis and with a maximum of 100 extra copies in paper. If the thesis is going to be published, permission needs to be reobtained.
- includes use in an electronic form, provided it is an author-created version of the thesis on his/her own website and his/her university's repository, including UMI (according to the definition on the Sherpa website: http://www.sherpa.ac.uk/romeo/);
- is subject to courtesy information to the co-author or corresponding author.
Geographic Rights: Scope
Licenses may be exercised anywhere in the world.
Altering/Modifying Material: Not Permitted
Figures, tables, and illustrations may be altered minimally to serve your work. You may not alter or modify text in any manner. Abbreviations, additions, deletions and/or any other alterations shall be made only with prior written authorization of the author(s).
Reservation of Rights
Springer reserves all rights not specifically granted in the combination of (i) the license details provided by you and accepted in the course of this licensing transaction and (ii) these terms and conditions and (iii) CCC's Billing and Payment terms and conditions.
License Contingent on Payment
While you may exercise the rights licensed immediately upon issuance of the license at the end of the licensing process for the transaction, provided that you have disclosed complete and accurate details of your proposed use, no license is finally effective unless and until full payment is received from you (either by Springer or by CCC) as provided in CCC's Billing and Payment terms and conditions. If full payment is not received by the date due, then any license preliminarily granted shall be deemed automatically revoked and shall be void as if never granted. Further, in the event that you breach any of these terms and conditions or any of CCC's Billing and Payment terms and conditions, the license is automatically revoked and shall be void as if never granted. Use of materials as described in a revoked license, as well as any use of the materials beyond the scope of an unrevoked license, may constitute copyright infringement and Springer reserves the right to take any and all action to protect its copyright in the materials.
Copyright Notice: Disclaimer
You must include the following copyright and permission notice in connection with any reproduction of the licensed material: "Springer book/journal title, chapter/article title, volume, year of publication, page, name(s) of author(s), (original copyright notice as given in the publication in which the material was originally published) "With permission of Springer"
In case of use of a graph or illustration, the caption of the graph or illustration must be included, as it is indicated in the original publication.
Warranties: None
Springer makes no representations or warranties with respect to the licensed material and adopts on its own behalf the limitations and disclaimers established by CCC on its behalf in its Billing and Payment terms and conditions for this licensing transaction.

Indemnity
You hereby indemnify and agree to hold harmless Springer and CCC, and their respective officers, directors, employees and agents, from and against any and all claims arising out of your use of the licensed material other than as specifically authorized pursuant to this license.
No Transfer of License
This license is personal to you and may not be sublicensed, assigned, or transferred by you without Springer's written permission.
No Amendment Except in Writing
This license may not be amended except in a writing signed by both parties (or, in the case of Springer, by CCC on Springer's behalf).
Objection to Contrary Terms
Springer hereby objects to any terms contained in any purchase order, acknowledgment, check endorsement or other writing prepared by you, which terms are inconsistent with these terms and conditions or CCC's Billing and Payment terms and conditions. These terms and conditions, together with CCC's Billing and Payment terms and conditions (which are incorporated herein), comprise the entire agreement between you and Springer (and CCC) concerning this licensing transaction. In the event of any conflict between your obligations established by these terms and conditions and those established by CCC's Billing and Payment terms and conditions, these terms and conditions shall control.
Jurisdiction
All disputes that may arise in connection with this present License, or the breach thereof, shall be settled exclusively by arbitration, to be held in the Federal Republic of Germany, in accordance with German law.

Other conditions:

V 12AUG2015

Questions? customercare@copyright.com or +1-855-239-3415 (toll free in the US) or +1-978-646-2777.