NONLINEAR MIXED INTEGER PROGRAMMING MODELS AND ALGORITHMS FOR FAIR AND EFFICIENT LARGE SCALE EVACUATION PLANNING

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We certify that we have read this dissertation and that in our opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

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ABSTRACT

NONLINEAR MIXED INTEGER PROGRAMMING MODELS AND ALGORITHMS FOR FAIR AND EFFICIENT LARGE SCALE EVACUATION PLANNING

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July, 2015

Shelters are safe facilities that protect a population from possible damaging effects of a disaster. Traffic management during an evacuation and the decision of where to locate the shelters are of critical importance to the performance of an evacuation plan. From the evacuation management authority's point of view, the desirable goal is to minimize the total evacuation time by computing a system optimum (SO). However, evacuees may not be willing to take long routes enforced on them by a SO solution; but they may consent to taking routes with lengths not longer than the shortest path to the nearest shelter site by more than a tolerable factor. We develop a model that optimally locates shelters and assigns evacuees to the nearest shelter sites by assigning them to shortest paths, shortest and nearest with a given degree of tolerance, so that the total evacuation time is minimized. As the travel time on a road segment is often modeled as a nonlinear function of the flow on the segment, the resulting model is a nonlinear mixed integer programming model. We develop a solution method that can handle practical size problems using second order cone programming techniques. Using our model, we investigate the trade-off between efficiency and fairness.

Disasters are uncertain events. Related studies and real-life practices show that a significant uncertainty regarding the evacuation demand and the impact of the disaster on the infrastructure exists. The second model we propose is a scenario-based two-stage stochastic evacuation planning model that optimally locates shelter sites and that assigns evacuees to shelters and paths to minimize the expected total evacuation time, under uncertainty. The model considers the uncertainty in the evacuation demand and the disruption in the road network and shelter sites. We present a case study for an impending earthquake in Istanbul, Turkey. We compare the performance of the stochastic programming solutions to solutions based on single scenarios and mean values.

We also propose an exact algorithm based on Benders decomposition to solve the stochastic problem. To the best of our knowledge, ours is the first algorithm that uses duality results for second order cone programming in a Benders decomposition setting. We solve practical size problems with up to 1000 scenarios in moderate CPU times. We investigate methods such as employing a multi-cut strategy, deriving pareto-optimal cuts, using a reduced primal subproblem and preemptive priority multiobjective program to enhance the proposed algorithm. Computational results confirm the efficiency of our algorithm.

This research is supported by TUBITAK, The Scientific and Technological Research Council of Turkey with project number 213M434.

Keywords: Disaster Management, Evacuation Traffic Management, Shelter Location, System Optimal, Constrained System Optimal, User Equilibrium, Nearest Allocation, Two-Stage Stochastic Programming, Second Order Cone Programming, Benders Decomposition, Pareto-optimal Cuts.

ÖZET

ADİL VE ETKİN BÜYÜK ÖLÇEKLİ TAHLİYE PLANLAMASI İÇİN DOĞRUSAL OLMAYAN KARIŞIK TAMSAYILI MODELLER VE ALGORİTMALAR

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Barınaklar bir nüfusu bir felaketin muhtemel yıkıcı etkilerinden koruyan güvenli tesislerdir. Bir tahliye esnasında trafik yönetimi ve barınakların nerelerde açılacağı tahliye planının başarısı açısından önemlidir. Tahliye yönetim birimi açısından arzulanan hedef sistem eniyi yaklaşımını kullanarak toplam tahliye zamanını en azlamaktır. Bununla birlikte, tahliye edilen insanlar, sistem eniyi yaklaşımı tarafından kendilerini uzun bir yola atayan bir çözümü kabul etmekte istekli olmayabilirler; fakat tahliye edilen insanlar, en yakın barınağa giden en kısa yoldan çok fazla uzun olmayan, belli bir tolerans sınırı içinde başka bir rotayı kullanmayı kabul edebilirler. Tahliye edilecek nüfusu, belirli bir tolerans sınırı içerisinde, en yakın barınak noktalarına giden en kısa yolları kullandırarak atayan ve toplam tahliye zamanını en küçükleyecek şekilde barınakları en uygun noktalara yerleştiren bir model geliştiriyoruz. Bir yol kesiti üzerindeki seyahat süresi o yol kesiti üzerindeki trafik yoğunluğunun doğrusal olmayan bir fonksiyonu olarak modellendiğinden, ortaya çıkan model doğrusal olmayan karışık tamsayılı programlama modelidir. Gerçek boyutlu problemleri ikinci seviye konik programlama teknikleri kullanarak çözebilen bir yöntem öneriyoruz. Modelimizi kullanarak etkinlik ve adillik kriterleri arasındaki ödünleşimi araştırıyoruz.

Felaketler belirsizlik içeren olaylardır. Bu konudaki ilgili çalışmalar ve edinilen tecrübeler tahliye talebi ve felaketin yol ağına ve altyapıya etkileri konusunda önemli belirsizliğin olduğunu göstermektedir. Önerdiğimiz ikinci model felaketlerdeki belirsizliği dikkate alarak tahliye edilecek nüfusu, barınak noktalarına ve rotalara atayarak beklenen toplam tahliye zamanını en küçükleyecek şekilde barınakları en uygun noktalara yerleştiren, senaryo tabanlı ve iki aşamalı rassal bir tahliye planlama modelidir. Modelimiz tahliye talebi ile yol ağı ve barınaklardaki bozulma ve yıkım konusundaki belirsizliği dikkate almaktadır.

İstanbul, Türkiye'de beklenen bir depreme yönelik örnek olay incelemesi sunuyoruz. Rassal çözümün sonuçlarını, sadece bir senaryoya dayanan ya da ortalama değerleri dikkate alarak üretilen çözümlerin sonuçları ile karşılaştırıyoruz.

Ayrıca rassal problemi çözmek üzere Benders çözümlemesine dayanan kesin çözümlü bir algoritma sunuyoruz. Algoritmamızın Benders çözümlemesi içinde ikinci seviye konik programlama ikillik sonuçlarını kullanan ilk algoritma olduğunu düşünüyoruz. Gerçek büyüklükteki problemleri 1000 senaryo sayısına kadar kısa süre içerisinde çözebiliyoruz. Önerdiğimiz modelin performansını artırmak için, çoklu kesi stratejisi, pareto-eniyi kesiler elde etme, azaltılmış birincil altproblem kullanma ve öncelikli çok amaçlı programlama gibi yöntemleri araştırıyoruz. Sayısal sonuçlar algoritmamızın etkinliğini kanıtlıyor.

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Chapter 1

Introduction

Natural or man-made disasters such as hurricanes, earthquakes, floods, and terrorist attacks impose a serious risk on the humankind. There has been a significant increase in the number of disasters over the past decades; from fewer than 50 disasters per year reported in 1950 to more than 400 disasters in 2010 [4]. Consequently, the number of people affected and the economic damages caused by disasters increased. The 2004 Indian Ocean tsunami killed more than 225,000 people and dislocated millions of them in countries spread around the Ocean's rim from Kenya to Indonesia. The 2010 Haiti earthquake measured 7.0 magnitude on the Richter scale, caused a massive level of destruction and imposed tremendous operational challenges on the humanitarian agencies and governments. This resulted in a grim situation: three million affected people, 200,000 deaths, and more than one million wounded [1]. The triple disaster that hit the Tohoku region of Japan on 11 March 2011 triggered a massive human displacement: more than 400,000 people evacuated their homes as a gigantic tsunami induced by a magnitude 9.0 earthquake engulfed the coastal areas, and the following nuclear accident in Fukushima released a large amount of radioactive materials into the atmosphere [5]. The significant increase in the number of disasters, and consequently the increase in the number of people affected by them and damages incurred illustrate the importance of an effective disaster management program.

International Federation of Red Cross and Red Crescent Societies (IFRC) [6] defines disasters as "serious disruptions of the functioning of a community through widespread losses that exceed the community's capacity to cope with using its own resources". IFRC classifies disasters as naturally occurring physical phenomena caused either by rapid or slow onset events which can be geophysical, hydrological, climatological, meteorological or biological and as technological or man-made hazards that are caused by humans and occur in or close to human settlements.

Various traffic management problems arise during disasters. Evacuation of the disaster region is the most predominantly used strategy to protect people threatened by a disaster [7]. Traffic management during an evacuation is critical [8] since people's lives are at stake and the unusual surge in traffic demand is generally far beyond the capacity of the road network. U.S. Federal Emergency Management Agency (FEMA) reports that 45 to 75 disasters require an evacuation annually [9]. Whether it is made by the military or civil emergency management authorities, evacuation planning for large scale disasters such as earthquakes, hurricanes, floods, tsunamis or CBRN (Chemical, Biological, Radiological and Nuclear) consequences of conventional or terrorist attacks is of critical importance for disaster management. In 1999 hurricane Floyd [10], and in 2005 hurricanes Katrina and Rita [9] required millions of people to evacuate creating largest traffic jams in the U.S. history. In an interview by CNN [11] after the evacuation for Rita, the top U.S. Democrat on the Senate Homeland Security Committee of the time said "God forbid, there's a terrorist attack of some kind on a major American city that requires evacuation without warning", pointing out requirement for an efficient evacuation plan. If the evacuation is not planned and managed effectively, the surge in evacuation traffic demand can cause congestion and may leave the evacuees in harm's way possibly resulting in further losses. Successful evacuation management not only saves lives but also contributes to the community's regaining functionality in a fast and smooth way [12].

The time to evacuate a disaster region depends on the locations of shelters and on the traffic assignment. Shelters are safe places that protect a population from possible damaging effects of a disaster. They also serve as facilities where evacuees are provided food, medical care and accommodation. FEMA [13], American

Red Cross (ARC) [14], FEMA ([15], [16]) provide the basis for the design and construction of shelters against different types of disasters. Whether it is built as a new shelter or as a retrofit construction, these preparations require time and need to be done before the disaster takes place. For that reason, the decision of which shelters to open are often made before a disaster occurs. Sherali et al. [17] point out in their study that one of the greatest tasks in developing a hurricane evacuation plan is to determine where evacuees should seek shelter in order to retreat from the storm's damaging power. In their study Liu et al. [18] emphasize that improving the local warning system will be effective only if people at risk can be evacuated to safe shelters. And secure shelters are a means to increase evacuation rates [19]. Even though the decision of where to locate the shelters from among potential alternatives is of critical importance to the performance of an evacuation plan (Sherali et al. [17], Kongsomsaksakul et al. [20], Kulshrestha et al. [21]), few evacuation models in the literature decide optimally on the number and location of shelters.

1.1 Aim of the Dissertation

The studies on evacuation planning in the literature generally disregard the decisions of where to locate the safe shelters and they generally assume the shelter locations as given. However, addressing these two problems -locating shelters and planning the evacuation traffic- separately may lead to suboptimal results as the shelter location decisions affect the evacuation traffic management and are critical for an efficient evacuation plan.

The studies that do consider the shelter location decisions simultaneously with evacuation traffic management, suffer from the drawback of not being realistic or implementable. In addition these studies combine the interests of the evacuees and the evacuation management authority only by means of bilevel models which are hard to solve. For that reason the solution methodologies developed for large scale evacuation problems using these models are generally not exact.

Further, the number of studies that take into consideration the uncertainty in evacuation planning is limited. Those studies that take into account uncertainty are not solved with enough number of scenarios for a realistic planning.

The aim of this dissertation is to provide a fair, efficient and a more realistic evacuation planning tool that simultaneously optimizes the shelter locations and the allocations of evacuees to shelters and to routes. The decision of where to locate the shelters is a strategic decision taken before the disaster takes place and we would like to measure the impact of shelter location decisions on routing of evacuees. Our goal is not only to combine shelter location with traffic assignment; we also aim to combine the points of views of the evacuation authority and the evacuees and hence incorporate a fairness consideration into the planning process. The methodology we develop is fair and efficient because we compromise the needs of the evacuees and the evacuation management authority. It is more realistic because, we take into consideration the real conditions in an evacuation during a disaster and the behaviors of the evacuees in such a situation. Further we take into account the uncertainty considering a large number of scenarios and provide exact solution techniques.

1.2 Methodology

In this dissertation we present a novel approach to evacuation planning and propose new models and algorithms as solution methodologies. In an evacuation planning process, from the evacuation management authority's point of view, the desirable goal is to minimize the total evacuation time by computing a system optimum (SO). However, evacuees may not be willing to take long routes enforced on them by a SO solution; but they may consent to taking routes with lengths not longer than the shortest path (shortest geographical distance, shortest free flow travel time or shortest congested time) to the nearest shelter site by more than a tolerable factor. We develop a constrained system optimal (CSO) model that optimally locates shelters and assigns evacuees to the nearest shelter sites by assigning them to shortest paths, shortest and nearest with a given degree of tolerance λ, so that the total evacuation time is minimized.

Even though λ is a parameter that we set a priori, incorporating tolerance into the model is not trivial. We compute an initial set of paths that may be acceptable. But depending on the choice of shelters, some of these paths are in fact not acceptable. We only allow the paths that are not longer than $1+\lambda$ times the length of a shortest path to the closest "open" shelter. Hence the actual set of acceptable paths depends on the locations of open shelters and it is not an input of the model.

We show that our model generalizes p-median facility location problem and SO and nearest allocation (NA) traffic assignment models. As the travel time on a road segment is often modeled as a nonlinear function of the flow on the segment, the resulting model is a nonlinear mixed integer programming model. We develop a solution method that can handle practical size problems using second order cone programming (SOCP) techniques. Clearly, reformulation using SOCP does not change the theoretical complexity of the problem but in practice, it enables us to solve large problems exactly without approximating the nonlinear function with linear functions. Using our model, we investigate the importance of the number and locations of shelter sites and the trade-off between efficiency and fairness. In addition, we present a sensitivity analysis by changing the level of tolerance and the number of shelters to open and make a comparison of the results of SO, user equilibrium (UE), NA and CSO approaches based on system performance and fairness. We measure the efficiency of the system by employing performance measures such as total evacuation time, percentage of evacuees reaching safety up to a specified time T, maximum latency and price of fairness.

Disasters are uncertain events; it is very difficult to anticipate the exact place, time and scale of disasters. Related studies and real-life practices show that a significant uncertainty regarding the evacuation demand and the impact of the disaster on the infrastructure exists. In modeling uncertainty, we opt for the scenario approach to be able to model accurately the impact of the disaster on the population and the infrastructure. If we ignore how vulnerable the road network structure and/or the shelters are, the damage may result in an inefficient evacuation plan possibly resulting in chaos and panic among the evacuees and further losses. We introduce a two-stage stochastic model that decides simultaneously on the locations of shelters and the allocations of evacuees to shelters and routes under uncertainty. In this model we use a stochastic programming approach, and hence instead of planning the evacuation based on a single hazard scenario, we consider a range of hazard scenarios. To the best of our knowledge, this is the first model in the evacuation literature that considers the uncertainty in the evacuation demand, the disruption in the road network, degradation in road capacities and disruption of the shelters simultaneously. We report the results of a case study for an impending earthquake in Istanbul, Turkey and show that the solution of our stochastic model leads to a significant decrease in the total evacuation time compared to the deterministic and mean value solutions. We also analyze the impact of having capacitated shelters on performance measures.

To have a more realistic evacuation planning that considers uncertainty, we need to consider a large number of scenarios. We are not able to solve practical size problems with a large number of scenarios by using an off-the-shelf solver in reasonable CPU times or can not solve the problem at all. To overcome this, we develop exact algorithms based on Benders decomposition. As the second stage of the model is a second order cone programming problem, to the best of our knowledge, ours is the first algorithm that uses duality results for second order cone programming in a Benders decomposition setting. We solve practical size problems with up to 1000 scenarios in moderate CPU times. We investigate methods such as employing a multi-cut strategy, deriving pareto-optimal cuts, using a reduced primal subproblem and preemptive priority multiobjective program to enhance the proposed algorithm. Computational results confirm the efficiency of our algorithm.

Like most evacuation planning models in the literature, and as suggested by [22], our model is not specific to a certain type of disaster. Our model can be used for different types of disasters and the special features are represented in the parameters. One can easily change the risk zone (earthquake affected zone, inundated area in a flood, area predicted to be hit by a hurricane) depending on the type of disaster and potential shelter sites can be modified likewise. Although our model already tries to evacuate everyone to safety as soon as possible, it is easy to assign different risk values to different zones in accordance with when and with how much impact a hurricane is expected to hit, by trying to minimize the total risk evacuees incur as well as the total evacuation time. This can be done by modifying the objective function as in Han et al. [23]. In case of an earthquake, our model can be applied to a post-disaster evacuation or considering the secondary disasters that can emerge after an earthquake it can be used for a pre-disaster evacuation. Hurricanes and floods differ from earthquakes in that they have a warning time to evacuate the population before the disaster hits, with floods having shorter warning times. These are some examples where our model can both be used as a pre/post-disaster evacuation management tool for planning purposes.

Our model is different from classical location models in the sense that it also considers the routing of evacuees on acceptable paths. This is a major difference since the acceptable paths are defined based on the length of a shortest path from a demand node to the closest shelter. As we do not know a priori which shelters will be opened, we do not know the set of acceptable paths. Further, in our model, as opposed to the traffic assignment models in which a set of origin-destination flows, i.e., trip rates are given, we do not have flows with given origins and destinations. The origins are known and destinations are decided optimally.

With our assumptions, possible objectives of our model are to minimize the maximum latency and to minimize the total evacuation time. Minimizing maximum latency, as it considers the performance based on the worst case, is a suitable objective for fairness purposes. In our model, we already incorporate fairness by taking into account the choices of evacuees (in assigning them to closest shelters and shortest paths). For this reason, we minimize the total evacuation time.

Jahn et al. [24] define the normal length of a path as its traversal time in the uncongested network (free flow travel time), its traversal time in UE, its geographic distance, or any other appropriate measure. In our analysis we used the geographical distance as the normal length. We could also use the other measures by changing the parameters accordingly. There are two reasons why we do not use UE travel times as the normal length. First, as the evacuees do not have the opportunity to learn from the past experience which routes minimize their evacuation time, it is unlikely for an equilibrium that distributes demand evenly across the evacuation routes to emerge. Second, since we do not have predefined origin-destination flows and since we also need to decide which shelter sites to open, in an equilibrium there will be UE travel times of the paths that lead only to opened shelter sites in a UE solution. For all the other paths that lead to unopened shelter sites, we will not be able to obtain UE travel times. But clearly, as normal lengths in our models, we can use any estimates of travel times that we believe that people use in making their choices.

1.3 Scope of the Dissertation

In Chapter 2, we present a literature review on disaster management, classical facility location problems, facility location in disaster management, traffic assignment approaches, evacuation planning, travel time modeling and second order cone programming.

In Chapter 3, we show that shelter location and traffic assignment decisions should be considered simultaneously for an efficient evacuation plan. We point out that the location and the number of shelters opened drastically affect the evacuation plan and for a carefully selected number of shelters and tolerance level, an efficient yet fair evacuation plan can be achieved. This part of the dissertation is published in the journal of Transportation Research Part B: Methodological, with reference Bayram et al. [25].

In Chapter 4, we focus on uncertainty and propose a scenario-based two-stage stochastic evacuation planning model that optimally locates shelter sites and that assigns evacuees to the nearest shelters and to shortest paths within a given degree of tolerance to minimize the expected total evacuation time. The model we propose considers the uncertainty in the evacuation demand and the disruption in the road network and shelter sites. We present a case study for an impending earthquake in Istanbul. We compare the performance of the stochastic programming solutions to solutions based on single scenarios and mean values.

In Chapter 5, we consider a large number of scenarios to be able to model a stochastic evacuation problem more realistically. As the number of scenarios grows the extended formulation developed in Chapter 4 may not be solved within reasonable CPU times or can not be solved at all. We propose an exact algorithm based on Benders decomposition.

We conclude and present possible improvement and extension directions in Chapter 6.

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Chapter 2

Literature Review

2.1 Disaster Management Literature

IFRC [6] defines the Disaster Management (DM) as the organization and management of resources and responsibilities for dealing with all humanitarian aspects of emergencies, in particular preparedness, response and recovery in order to lessen the impact of disasters. It includes the strategic, operational and tactical activities that aim at mitigating the possible consequences of a disaster.

"During disasters, due to natural and human-made hazards, the immediate imperative is to save lives, reduce suffering, damage and losses, and to protect, comfort and support affected people. These actions combined with preventive risk reduction, preparedness and resilience building constitute the core components of disaster management" [6].

In order to lessen the possible impacts of a disaster there are measures that are to be taken and tasks to be executed. Tasks prior to a disaster event include predicting and analyzing potential dangers and developing necessary action plans for mitigation. The tasks to be executed after a disaster takes place is about locating, allocating, coordinating and managing available resources [26]. There are four phases of an adequate disaster management program; mitigation, preparedness, response and recovery [27]. Mitigation refers to the activities taken that reduce the long term risk of a disaster to human life and property. Preparedness includes the activities that relate to developing operational capabilities for responding to an emergency. Response phase involves the activities taken immediately before, during and right after the disaster and is related to the deployment of necessary resources and employment of emergency procedures to serve the affected people. Finally, recovery is the short-term activities that restore the vital life support systems to minimum operating standards and long-term activities that stabilize the community. Altay and Green [26] list the typical activities involved in each of these phases.

Caunhye et al. [1] summarize the framework for the main emergency logistics activities and their associated facilities and flows in Figure 2.1.

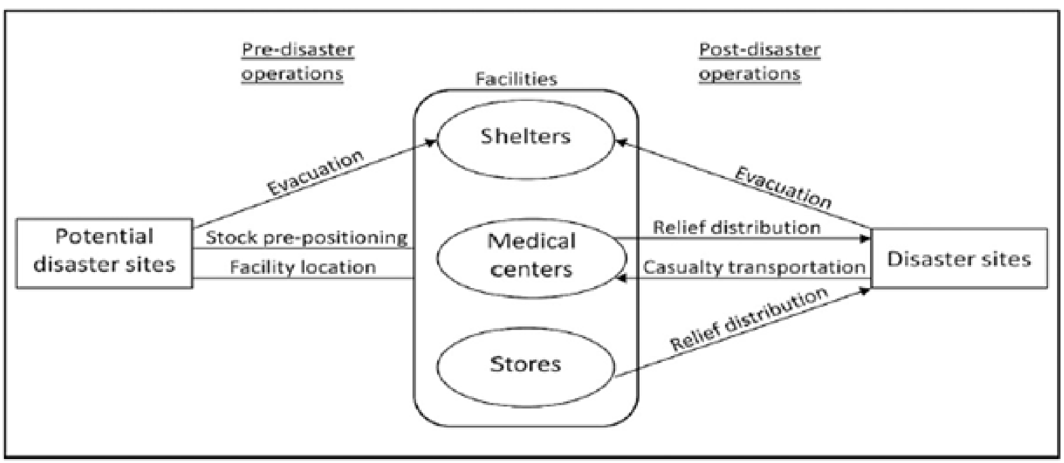


Figure 2.1: Framework for Disaster Operations and Associated Facilities and Flows [1]

Galindo and Batta [28] review recent research in DM as a continuation of a previous review by Altay and Green [26]. In their review they present a list of the most frequent assumptions in the literature and whether these assumptions are realistic or not.

Özdamar et al. [29], Barbarosoğlu and Arda [30], Yi and Özdamar [31], Tomasini and Van Wassenhove [32] and Pedraza Martinez et al. [33] work on problems that aim at developing logistics decision support during disasters.

Özdamar et al. [29], develop a planning model at macro level that is to be integrated into a natural disaster logistics decision support system. The model they propose addresses the dynamic time dependent transportation problem that is solved repetitively at specified time intervals during the ongoing aid delivery. The model incorporates new requests for aid materials, new supplies and transportation means that become available during the current planning time horizon, and regenerates plans accordingly. The plan is a schedule of optimal mixed pick up and delivery times for vehicles within the considered planning time horizon as well as the optimal quantities and types of loads picked up and delivered on these routes. The objective of the model is to minimize the delay in the arrival of commodities at aid centers.

Barbarosoğlu and Arda [30], address in their study the issue of planning the transportation of vital first-aid commodities and emergency personnel to disaster-affected areas by developing a modeling framework to be used in case of earthquakes. They model a resource mobilization system as a probabilistic, multi-commodity, multi-modal network flow problem. The model they develop is a scenario based, two-stage stochastic linear programming model taking into consideration the uncertainty from earthquake magnitude and impact. They validate the model by using the actual data of the August 1999, Marmara earthquake in Turkey.

Yi and Özdamar [31] describe an integrated location-distribution model for coordinating logistics support and evacuation operations in disaster response activities. They propose a multi-commodity network flow model that treats vehicles as integer commodity flows.

Tomasini and Van Wassenhove [32] work on supply chain management problems in disaster relief and the role of new players like the private sector. They compare the differences between commercial and humanitarian supply chains, focusing on preparedness, response and collaboration.

Pedraza Martinez et al. [33] describe the immediate response to the Haiti Earthquake focusing on five clusters: logistics, health, food, shelter and non-food items, and water, sanitation and hygiene. They identify areas where applied OR can have significant impact in supporting humanitarian operations. In terms of the shelter and non-food items they define the main decision problems as; how to maximize demand coverage while minimizing time of response, how to determine the optimal size of the camps, where to locate the camps and how many camps to set up.

Doerner et al. [34] edit a "Special Issue on Optimization in Disaster Relief" with a selection of papers with topics ranging from forecasting of the impact of disasters and the resulting needs for support, to reliability and repair of disaster supply chains, prepositioning of relief items, inventory and other scarce resource allocation and evacuation of victims.

2.2 Facility Location Literature

In strategic planning, facility location has been of critical importance for both public and private firms. The first study of location theory belongs to Weber [35] who considers how to position a single warehouse so as to minimize the total distance between it and several customers [36]. Hakimi [37] seeks to locate switching centers in a communications network and police stations in a highway system. He considers locating one or more facilities on a network so as to minimize the total distance between customers and their closest facility or to minimize the maximum such distance.

Location problems are characterized by four main components; customers, facilities that will be located, a space in which customers and facilities are located, and a metric that indicates distances or times between customers and facilities [38]. In this review we focus on the facility location problems on networks.

2.2.1 The p-Median Problem

The p-median problem is originally designed for and has been extensively used in public facility location, distribution logistics (private facility location), cluster analysis and diversity management. In whichever area it is applied, the p-median location problem basically involves the placement of p facilities on the network in such a way that the total weighted distance of serving all demand is minimized.

Originally defined by Hakimi [39], the p-median location problem finds the optimal location/distribution of switching centers on a communication network. Hakimi assumes that each node represents a point of demand as well as a potential facility site. He allows the facilities to be located anywhere on the network including the interior points of edges. Hakimi proves that there is always a location at a vertex that is optimal to the network 1-median problem. He further proves that there is always a collection of p vertices that minimizes the objective and thus shows that at least one optimal p-median solution to a given problem locates entirely at vertices, although not all optimal solutions to this problem are located at the vertices. The node optimality issue is covered in more detail in the survey by Tansel et al. ([40], [41]).

Kariv and Hakimi [42] show that the problem of finding a p-median is NP-hard even when the network has a simple structure. It is also shown by Kariv and Hakimi [42], Tamir [43] and Benkoczi and Bhattacharya [44] that when the network is a tree, polynomial time algorithms can efficiently solve the problem.

ReVelle and Swain [45] introduce the classical formulation for the p-median problem. Rosing et al. [46] present the only attempt to reformulate the constraints of the original ReVelle-Swain formulation. Church [47] formulates a new model which combines the hybrid structure of Rosing et al. [46] with a variable substitution/elimination concept that is named as COndensed Balinski constraints with the Reduction Assignment variables using equivalent variable substitution (COBRA). Church [48] also proposes a new model formulation for the p-median problem that contains both exact and approximate features, which is called Both

Exact and Approximate Model Representation (BEAMER). There are also formulations based on the cluster analysis, such as by Vinod [49], Rao [50], Mulvey and Crowder [51] and Klastorin [52]. Minoux [53] introduces a set partitioning formulation. Avella and Sassano [54] present a directed graph based formulation. The distance ordered (neighborhood search) formulation of Cornuejols et al. [55] and the improved distance ordered formulation by Elloumi [56] are other formulations of the p-median problem.

The Simple Plant Location Problem (SPLP), also referred to as the Uncapacitated Facility Location Problem (UFL), is similar to the p-median problem, with the number of facilities to be located being endogenous to the problem, which is the only difference. By adding a fixed cost of establishing a facility at a node, p-median problem can easily be transformed into a SPLP. Additionally if the facilities are capacitated, with the addition of the capacity constraints the problem in hand is a Capacitated Plant Location Problem (CPLP).

2.2.2 The p-Center Problem

In p-center problems, coverage of all demands is required, but a given number of facilities are located in such a way that minimizes the coverage distance. Rather than taking an input coverage distance that is used in coverage problems, the minimal coverage distance associated with locating p facilities is determined endogenously. In p-center problems, the maximum distance between any demand and its nearest facility is minimized, for that reason the p-center problem is also known as the minimax problem. If the facility locations are restricted to be at the nodes of the network, then the problem is a vertex center problem. If there is no such restriction, that is, if facilities are allowed to be located anywhere on the network, then this problem is called the absolute center problem.

2.2.3 Covering Problems

In some instances, when decision makers wish to cover customers, the p-median or p-center models may not be satisfactory. A customer or demand node is said to be covered by a facility, if the distance or time between a client and its closest facility is not greater than a prespecified distance or time standard. In the literature on covering problems, coverage is either a requirement in the model, or it is optimized. In Location Set Covering Problem (LSCP) the objective is to minimize the cost of locating facilities in such a way that the required level of coverage is obtained.

Minieka [57] introduces a method for solving the p-center problem by solving a finite series of minimum set covering problems. While successively decreasing distance or time standard, the number of required facilities will remain the same for some iterations and then suddenly increase. At the smallest distance value before the number of facilities increases from p to $p+1$ (or higher), the maximum distance from any demand node to its nearest facility is, by definition, a minimum, that is, the p facilities and their positions at this distance value minimize the maximum distance [38].

Note that, in LSCP formulation each node, irrespective of their demand sizes must be covered within the specified distance, regardless of cost. When the maximal service distance is small, a large number of facilities may have to be located to cover all the demand. Further, the cost/demand ratio of a distant node requiring coverage for a very small demand may be relatively very high, which means basically that a very small demand is covered at a very high cost. A decision maker with a restricted budget, may wish to cover as much demand as possible within the maximal service distance. That is, instead of his main goal of covering all the demand, he may consider leaving as little demand as possible outside the maximal coverage distance. Church and Velle [58] introduce the Maximal Covering Location Problem (MCLP), which seeks the maximum population that can be served within a stated service distance or time given a limited number of facilities. Church and Velle [58] give in their paper an equivalent formulation of the MCLP which seeks to minimize the population left uncovered if p facilities are to be located on a network. Church and Velle [58] further introduce another location problem: Locate p facilities at possible sites on the network to maximize the population that can be covered within a given service distance S while at the same time ensuring that the users at each point of demand will find a facility no more than T distance ($T > S$) away.

2.3 Literature on Facility Location for Disasters

Most emergency logistics models with a facility location aspect combine the process of location with stock pre-positioning, evacuation, or relief distribution [1].

Balçık and Beamon [59] develop a scenario based model for a humanitarian relief chain responding to quick onset disasters. Their model determines the number and locations of distribution centers in a relief network and the amount of relief supplies to be stocked at each distribution center to meet the needs of people affected by disasters. The model they propose is a variant of MCLP. It integrates facility location and inventory decisions for multiple item types along with budgetary and capacity restrictions.

Rawls and Turnquist [60] develop an emergency response planning tool that determines the location and quantities of various types of emergency supplies to be pre-positioned under uncertainty. They present a two-stage stochastic mixed integer program that provides an emergency response pre-positioning strategy for disasters. The model they develop considers uncertainty in demand for the stocked supplies as well as uncertainty regarding transportation network structure. The objective of their model is the minimization of the expected costs over all scenarios resulting from the selection of the pre-positioning location and facility sizes along with the costs from the commodity acquisition and stocking, the shipment of the supplies to demand points, unmet demand penalties and holding costs for unused material.

Doerner et al. [61] present a model they use for multi-criteria decision analysis with respect to the location of public facilities such as schools in coastal areas, by considering the inundation by tsunamis.

Motivated by the observation that large-scale disasters are highly likely to cause the malfunction of service facilities due to destruction, Huang et al. [62] develop a variation of the p-center problem with an additional assumption that the facility at a node fails to respond to demands from that node. Their model basically determines where to locate facilities that minimize the maximum weighted distance to provide service under the condition that the facility located at a given node on the network cannot provide service to the population residing in the same node.

Görmez et al. [63] study the problem of locating disaster response and relief facilities in the city of Istanbul, where a massively destructive earthquake is expected to occur in the near future. They propose a model using a two-stage approach. In the first stage, they solve an integer programming model for each district to locate the temporary facilities in its neighborhoods and they find the number of refugees served from each temporary facility. In the second stage they solve a bi-criteria problem to find the locations of the permanent facilities to serve the demand at the temporary facility locations found in the first stage. They develop mathematical models to decide on the locations of the new facilities with the objectives of minimizing the average-weighted distance between casualty locations and closest facilities, and opening a small number of facilities, subject to distance limits and backup requirements under regional vulnerability considerations. They analyze the trade-offs between these two objectives under various disaster scenarios and investigate the solutions for several modeling extensions.

Duran et al. [64] develop a mixed integer inventory-location model that finds the optimal number and location of pre-positioning warehouses given that demand for relief supplies can be met from both pre-positioned warehouses and suppliers.

Günneç and Salman [65] study a simple vulnerability based stochastic dependency model of link failures in a network prone to disasters. They develop a model that locates facilities on a network whose edges may fail with probability. They consider a link failure dependency which they call as the vulnerability based dependency model. In their model they maximize the expected demand serviced.

2.4 Traffic Assignment Models

The existing models used for assigning evacuees to routes are mostly based on three traffic assignment models, namely, the User Equilibrium (UE, also known as User Optimal or Nash Equilibrium), the System Optimal (SO) and the Nearest Allocation (NA) models. These models differ in assumed driver behaviors.

UE is known as the Wardrop's [66] first principle which states "the journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route". In accordance with that principle, travellers' aim is to minimize their individual travel times. Under the identical traffic conditions every vehicle in the traffic will experience the same travel times. It is assumed that, all travelers have perfect information on all possible routes and traffic conditions in the network. In UE models each individual acts only in his interest, not necessarily in the interest of the whole system. A stable condition is reached when no traveler can improve his travel time by unilaterally changing routes [67].

UE traffic assignment can be stated equivalently as the following complementary inequality for the route flow:

$$\begin{aligned} f_k^{rs} &> 0 \text{ if } C_k^{rs} = C^{rs*}, && \forall k \in P_{rs}, \forall r, s \\ f_k^{rs} &= 0 \text{ if } C_k^{rs} \geq C^{rs*}, && \forall k \in P_{rs}, \forall r, s. \end{aligned}$$

This can equivalently be stated as:

$$\begin{aligned} f_k^{rs}(C_k^{rs} - C^{rs*}) &= 0, && \forall k \in P_{rs}, \forall r, s \\ (C_k^{rs} - C^{rs*}) &\geq 0, && \forall k \in P_{rs}, \forall r, s \end{aligned}$$

where f_k^{rs} is the flow assigned to route k, P_{rs} is the set of all routes from origin r to destination s, C_k^{rs} is the travel time along route k, i.e. $C_k^{rs} = \sum_{a \in k} t_a \ \forall k \in P_{rs}$ and C^{rs^*} is the minimum travel time from origin r to destination s [68].

In an equilibrium, either a path carries flow in which case the travel time on this path equals the minimum origin-destination (O-D) travel time, or there is no flow on it, in which case the travel time on this path is greater than or equal to the minimum travel time in the same O-D pair. Hence, no traveler can be better off by unilaterally changing routes [67].

Below is the UE formulation , known as the Beckmann et al.'s [69] transformation Sheffi [67]:

$$\min z(x) = \sum_a \int_0^{x_a} t_a(w)dw \tag{2.1}$$

$s.t.$

$$\sum_k f_k^{rs} = q_{rs}, \qquad \forall r, s \tag{2.2}$$

$$f_k^{rs} \geq 0, \qquad \forall k, r, s \tag{2.3}$$

$$x_a = \sum_{r,s,k} f_k^{rs} \delta_{a,k}^{r,s}, \qquad \forall a \tag{2.4}$$

The notation x_a represents the traffic flow on link a, $t_a(x_a)$ is the travel time along link a when the traffic flow is x_a, q_{rs} represents the traffic demand (trip rate) from origin r to destination s, in other words the origin-destination matrix is denoted by q with entries q_{rs} and $\delta_{a,k}^{r,s} = 1$ if link a is a part of path k connecting O-D pair $r - s$ and $\delta_{a,k}^{r,s} = 0$ otherwise.

The objective function is the sum of the integrals of the link performance functions. Constraint (2.2) is the flow conservation which ensures that the flow on all paths connecting each O-D pair is equal to the O-D trip rate. This implies that all O-D trip rates have to be assigned to the network. Constraint (2.3) are the nonnegativity constraints.

While a UE satisfies the travelers, it does not necessarily minimize the total evacuation time in the system. From the evacuation traffic management authority's point of view, the desirable goal is to minimize the total evacuation time by computing a SO. Wardrop's second principle which is also known as SO principle states that "the average journey time is minimum." In a SO approach, some travelers may be assigned to routes that have lengths or travel time higher than the minimal that they would choose to travel along. Although these travelers will incur additional travel time or length, overall there will be greater savings in the system that will outweigh these costs, that is under SO conditions some travelers may travel longer than they would to the benefit of the overall system. In a SO assignment, the model is like a central system manager distributing the traffic over the road network so that the total, rather than individual, benefit of all travelers in the system is maximized. That way the traffic network will be able to carry more flow then it would compared to the UE solution.

The system optimal assignment can be formulated as a problem that minimizes the total travel time spent in the network. Below is the SO formulation [67]:

$$\min z(x) = \sum_a x_a t_a(x_a) \tag{2.5}$$

$s.t.$

$$\sum_k f_k^{rs} = q_{rs}, \qquad \forall r, s \tag{2.6}$$

$$f_k^{rs} \geq 0, \qquad \forall k, r, s \tag{2.7}$$

$$x_a = \sum_{r,s,k} f_k^{rs} \delta_{a,k}^{r,s}, \qquad \forall a \tag{2.8}$$

The optimality conditions of the system optimal assignment are given in Sheffi [67] and Chow [68] as:

$$f_k^{rs} > 0 \quad \text{if} \qquad C_k^{rs} + \sum_a \delta_{a,k} x_a \frac{\partial t_a(x_a)}{\partial x_a} = C^{rs*}, \qquad \forall k \in P_{rs}, \forall r, s$$

$$f_k^{rs} = 0 \quad \text{if} \qquad C_k^{rs} + \sum_a \delta_{a,k} x_a \frac{\partial t_a(x_a)}{\partial x_a} \geq C^{rs*}, \qquad \forall k \in P_{rs}, \forall r, s.$$

The quantity, $C_k^{rs} + \sum_a \delta_{a,k} x_a \frac{\partial t_a(x_a)}{\partial x_a}$ represents the marginal contribution of an additional traveler on route k from origin node r to destination node s, to the total travel time on that route k, $t_a(x_a)$ is the travel time experienced by that additional traveler when the total link flow is x_a. If we take the derivative of link travel time with respect to the link flow, i.e., $\frac{\partial t_a(x_a)}{\partial x_a}$, we find the additional travel time induced by an additional traveler to each of the existing travelers already on that link. When the network is at SO, the marginal travel time on all used routes connecting each origin-destination pair in the network is equal [67]. On the other hand, if the marginal total travel time on a route is greater than or equal to the marginal total travel times of the used routes, then the flow on this route is zero.

These formulations assume that the travel time on a given road segment is a function of the flow on that road segment only. In addition the travel time functions are assumed to be positive and increasing [67].

Barrett et al. [70] classifies destination choices of evacuees as nearest safe destination, soonest safe destination and easiest safe destination. A similar classification is made by Southworth [71]. In a disaster situation, where there is limited information on the road network and congestion levels, evacuees show selfish behavior, as people do even under normal daily traffic conditions (Roughgarden [72], Jahn et al. [73], Schulz and Moses [74], Correa et al. [75], Schulz and Stier-Moses [76], Correa et al. [77], Olsthoorn [78]) and they tend to select routes that take them to the nearest shelter site, as proposed and implemented by Yamada [79], Cova and Johnson [80], Alçada-Almeida et al. [81], Coutinho-Rodrigues et al. [82] and Sheu and Pan [83]. In the NA model, each evacuee uses a shortest path based on geographical distance or free flow travel time to reach the nearest shelter.

2.5 Literature on Evacuation Planning

Evacuation models are categorized mainly into two groups; microscopic models and macroscopic ones. If an evacuation model represents traffic as flows, then it is a macro-level model. These models generally attempt to answer how long it takes to evacuate an area (network clearance time or total evacuation time) and they are generally used for large-scale evacuations. Transportation engineering approaches on the other hand consider traffic at more detailed levels and focus on individual entities (vehicles). These kind of models are classified as micro and meso models.

Depending on the type and peculiarity of the disaster, and in accordance with the aim of the evacuation planning authority, different objectives can be employed [23] for an evacuation. Most frequently used objectives in the literature are minimizing the clearance time, minimizing the maximum latency, minimizing the total evacuation time, minimizing average evacuation time, maximizing the number of people reaching safety up to a specified critical time T, which are directly related to the quickest flow problem [84], minimization of the weighted sum problem ([85], [86]) and the earliest arrival problem ([87], [88]) respectively within the context of dynamic network flows [89].

Clearance time is the time that the last vehicle in the network leaves the danger zone and reaches safety. Latency is defined as the total time it takes a vehicle to complete its trip on a given route, and maximum latency is the maximum of the total journey times incurred by the vehicles. Since we assume in our model that all the evacuees (demand) enter the network at the same time, minimizing the network clearance time and minimizing the maximum latency are equivalent objectives for our case.

Since the average evacuation time is equal to the total evacuation time divided by the total number of evacuees, minimizing average evacuation time is equivalent to minimizing total evacuation time.

Evacuation models are generally based on traffic assignment models, static models mostly originating from the formulation introduced by Beckmann et al. [69]. The dynamic evacuation models are mostly modified versions of the model proposed by Merchant and Nemhauser [90] and the Cell Transmission Model (CTM) based Dynamic Traffic Assignment (DTA) model which is a discretization of the differential equations of hydrodynamic model of Lighthill and Whitham [91] and Richards [92] used to model traffic evolution as introduced by Daganzo ([93], [94]) and developed into an LP by Ziliaskopoulos [95]. Also flow based evacuation models in time-space networks are used as shown in Hamacher and Tjandra [85] and Bretschneider [89].

Few of the evacuation planning models we have encountered in the literature optimally decide on the number and location of shelters to minimize the total system cost or to maximize the benefit.

Yamada [79] uses the shortest path (nearest allocation) and minimum cost flow approaches to assign pedestrian evacuees to shelters and to routes. These approaches minimize the total distance traveled and disregard the evacuation traffic congestion. Cova and Johnson [80] propose to use lane-based routing to reduce the delays at the intersections. They present a network flow model to minimize the total distance traveled. Yazıcı and Özbay [96] and Chiu et al. [97] use a CTM based SO DTA approach. Ng et al. [98] present a bi-level model that assigns evacuees to shelters in a SO manner in the upper level, and in the lower level evacuees reach their assigned shelters in a UE manner. Hu et al. [99] propose a mixed-integer linear programming model that considers a multi-step evacuation and temporary resettlement. The model minimizes panic-induced psychological penalty cost, psychological intervention cost, transportation cost and cost of building shelters. These studies do not consider optimal selection of shelter sites among potential ones. Yazıcı and Özbay [96] perform sensitivity analysis to find out the appropriate locations of shelter sites and Chiu et al. [97] consider all the nodes at the boundary between the danger zone and the safe zone but inside the safe zone as potential shelter sites and suggest that a shelter is opened at a node if there is a flow into it at the optimal solution.

The location-allocation models that consider the optimal selection of shelter sites are either single level models with a SO approach or bi-level models that specify the locations of shelter sites in a SO manner at the upper level, while assigning evacuees to shelters and routes in a UE manner at the lower level. Sherali et al. [17] develop a location-allocation model in which the selection of shelter sites and the assignment of the evacuees to the routes are specified in a SO manner. Kongsomsaksakul et al. [20] study the impact of the shelter locations on evacuation traffic flow management. At the upper level their model determines the number and locations of the shelter sites with the objective of minimizing the total network evacuation time. The lower level is a static UE formulation and given the number and location of the shelter sites, the evacuees choose their routes and the shelter sites that they travel to. Kulshrestha et al. [21] develop a robust bi-level model that considers demand uncertainty and minimizes the total cost to establish and operate shelters at the upper level while assigning evacuees to shelters and routes in a UE manner at the lower level. Shen et al. [100] develop scenario based, stochastic, bi-level models that minimize the maximum UE travel time among all node shelter pairs by locating shelters at the upper level and assigning evacuees to shelters and routes in a UE manner at the lower level. Li et al. [101] propose a scenario based location model for identifying a set of shelter locations that are robust for a range of hurricane events. Their model is a DTA-based stochastic bi-level programming model in which at the upper level the central authority selects the shelter sites for a particular scenario. The objective of the upper-level problem is to minimize the weighted sum of the expected unmet shelter demand and the expected total network travel time. In the lower level, evacuees choose their routes in a dynamic UE manner. Sheu and Pan [83] propose a method for designing an integrated emergency supply network that utilizes a three-stage multi-objective programming problem. The first stage of their method designs the shelter network for evacuation with a nearest allocation approach as one of the objectives.

Alçada-Almeida et al. [81] and Coutinho-Rodrigues et al. [82] introduce a multi-objective approach to identify the number and location of rescue facilities (shelters) and primary and secondary evacuation routes. Their models can be regarded as a multi-objective extension of the p-median model. No congestion effect is included in these models, instead average travel velocity is used.

The location allocation models proposed by Kongsomsaksakul et al. [20], Shen et al. [100], Ng et al. [98], and Li et al. [101] are solved using heuristic algorithms and the ones developed by Alçada-Almeida et al. [81], Coutinho-Rodrigues et al.

[82] and Sheu and Pan [83] are solved to optimality by exact solution methodologies. Kulshrestha et al. [21] employ an approximation based cutting plane algorithm. Hamacher et al. [86] introduce a model and heuristic algorithms using a time expanded network for their problem. Sherali et al. [17] develop both a heuristic and an exact algorithm to solve their model.

Jahn et al. [24] propose a SO traffic assignment model that includes user constraints to be fair to drivers. They define unfairness as a measure of the detriment for users as the ratio of the traversal time of the recommended path to that of the shortest possible path the user could have taken. The normal length of a path, is defined as either its free flow travel time, its traversal time in UE, its geographic distance, or any other measure that does not depend on the actual flow on the path. They look for a constrained system optimum in which no path carrying positive flow between a certain origin-destination pair is allowed to exceed the normal length of a shortest path between the same origin-destination pair by more than a tolerable factor. They use a variant of the convex combination algorithm of Frank and Wolfe [102] combined with column generation method to solve their problem. Jahn et al. ([103], [73]), Schulz and Stier-Moses [76], Li and Zhao [104], Zhou and Li [105] develop models and algorithms that consider user needs while trying to achieve the system optimal to find solutions that are fair and efficient at the same time. These models are developed for traffic management in every day normal traffic conditions and do not consider the location of facilities and allocation of drivers.

A related notion is that of satisficing, suggested by Simon [106], as a model of bounded rational decision making that seeks an acceptable solution rather than a necessarily optimal one, where acceptability is generally defined in relation to some threshold or aspiration level [107], [108]. Following the notion explained by Mahmassani and Chang [107] and Chen et al. [109], Lou et al. [110] define travelers with bounded rationality as those who always choose routes with no cycle and do not necessarily switch to the shortest routes when the difference between the travel times on their current routes and the shortest one is no larger than a threshold value. Szeto and Lo [111] call this tolerance based dynamic user optimal principle. To find the bounded rational user equilibrium they formulate and solve mathematical programs with complementarity constraints and propose heuristic algorithms.

There are also microscopic and macroscopic simulation based evacuation models such as NETVAC [112], MASSVAC [113], [114], REMS [115], OREMS [116], DYNASMART [117] which can be used to decide on the locations of shelter sites. Although these models are informative, they require relatively much more time, extensive data and effort to set up and computer resources to run properly. Further, the heuristic approaches used in them always brings the possibility of converging at a suboptimal solution. These models are generally better fit for real time evacuation planning purposes.

Due to its simplicity and relatively good results in most cases, static traffic assignment has been used by traffic planners to estimate current and future use of transportation networks [118]. The use and advantage of dynamic models in a time-varying environment for real time evacuation management is unquestionable. But as for the evacuation planning, the trade-off between getting more realistic solutions to a given evacuation problem on a road network of a certain size by solving a dynamic model and the advantage of being able to solve evacuation problems on larger size road networks can be discussed. Although dynamic models represent the traffic flow over time more realistically, they suffer from the drawback of solving large instances exactly and lose tractability as the evacuation road network size grows. Liu et al. [119], Chiu et al. [97], Ng et al. [98], Yazıcı and Özbay [120] and Bish et al. [121] propose dynamic evacuation models in their studies and solve exactly evacuation problems of size (10, 24), (8, 14), (8, 14), (26, 30), (87, 757) respectively, where (., .) represents the number of nodes and arcs in the evacuation network. Mostly heuristic methods are employed to solve larger instances of size (728, 2127), (99, 200), and (165, 263) by Tüydeş and Ziliaskopoulos [122], Xie et al. [123], and Kimms and Maassen [124] respectively. Static models give very good estimations for planning purposes and instances with very large evacuation networks can be solved to optimality using exact solution methodology in a short time. Further, the static evacuation problems are useful for planning purposes in that they can give important and sometimes counter-intuitive insights taking into consideration different performance measures.

2.6 Travel Time Modeling

We define the evacuation traffic flow as an *uninterrupted-flow*, i.e., the traffic flow with no causes of delay or interruption external to the traffic stream [125]. The traffic stream on uninterrupted-flow facilities such as freeways, multi-lane highways and two-lane highways is the result of individual vehicles interacting with each other and the facility's geometric characteristics [125].

Volume and *flow rate* terms are defined as measures that quantify the number of vehicles that pass a point on a road segment during a specific time interval. Volume is the number of vehicles observed or predicted to pass a point during a time interval. Flow rate represents the number of vehicles passing a point during a time interval less than 1 hour, but expressed as an equivalent hourly rate, that is Flow Rate = Volume / Time Interval [125].

Free-flow (base) speed is the average speed of vehicles on a given road segment, measured under low-volume conditions (when the density and flow rate are both zero), i.e., when drivers are free to drive at their desired speed and are not constrained by the presence of other vehicles or downstream traffic control devices such as traffic signals, roundabouts and stop signs.

Density is defined as the number of vehicles present on a road segment at a particular instant. In Highway Capacity Manual (HCM) density is expressed as: Density (veh/mi) = Flow Rate (veh/h) / Average Travel Speed (mi/h). Density is a critical parameter for uninterrupted-flow facilities because it characterizes the quality of traffic operations. The density at which all movement stops is called the *jam density* [125]. When density increases to the level of jam density, the speed and the flow rate are zero because there is no movement and vehicles can not pass a point on the roadway [125].

HCM [125] defines *capacity* as the maximum sustainable hourly flow rate at which vehicles are expected to traverse a point or a uniform section of a roadway during a given time period under prevailing roadway, environmental, traffic and control conditions. Capacity is reached when the product of density and speed results in maximum flow rate. The density at capacity is referred to as *critical density*.

Generally, travel times are considered to be positive and monotonically increasing functions of traffic flow, since an increase in a road segment's traffic volume will normally decrease the travel speed due to congestion and hence increase the travel time along the road segment. Therefore, rather than an average travel time measure, a travel time function should be associated with each of the road segments representing the evacuation network. Link travel time functions are also referred to as link performance functions, link capacity functions, volume-delay curves, link impedance functions and latency functions. In his study, Branston [126] investigates the link capacity functions in the literature reviewing more than 20 of them. A typical link performance function is shown in Figure 2.2.

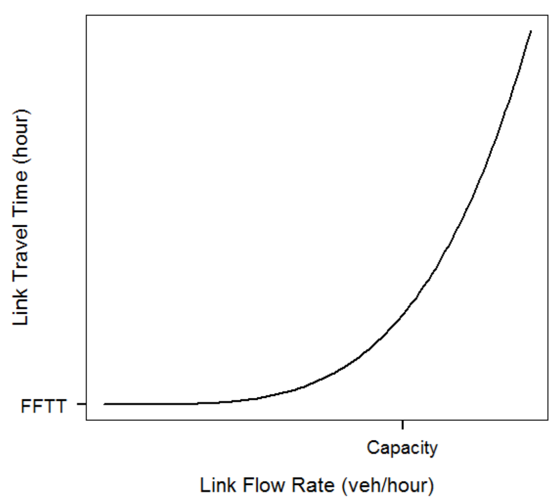


Figure 2.2: A Typical Link Performance Function

U.S. Department of Commerce Bureau of Public Roads expresses the relationship between travel time (or speed) and the volume of traffic on a link by the following function (refered to as the BPR function):

$$t(x) = t^0 \left(1 + \alpha \left(\frac{x}{c}\right)^{\beta}\right)$$

where $t(x)$ is the travel time at which assigned volume x can travel on the link, c is the practical capacity (maximum flow rate) and t^0 is the base travel time or free flow travel time at zero volume. The parameters $\alpha \geq 0$ and $\beta \geq 0$ are the tuning parameters defined in accordance with the road characteristics and they are taken as 0.15 and 4 by the U.S. Department of Commerce Bureau of Public Roads, respectively [127].

In the BPR function, α is a parameter that basically specifies the ratio of free-flow travel time to the travel time at capacity. Parameter β determines the steepness of the function, i.e., it specifies how rapidly travel time increases from the free-flow travel time.

Speed-flow relationships have been studied to a large extent in the literature. Singh [128], Kurth et al. [129], Dowling et al. [130], Skabardonis and Dowling [131], Singh and Dowling [132], Hansen et al. [133], Dowling and Skabardonis [134], Kalaee [118], Systematics [135] and Huntsinger and Rouphail [136] investigate and evaluate empirically and theoretically the accuracy of different speed-flow models used in planning for traffic assignment applications including the BPR function. In these studies the BPR function is fitted against the real field data by searching the best values of α and β. It is shown by these studies that different freeway lanes and segments may have substantially different traffic characteristics and parameters such as free-flow speed, speed at capacity, and capacity, and α and β values specifically for the BPR function. To represent the speed-flow relationships correctly with the the best fit parameters, one needs to collect real field data for every lane of every road segment of a road network. Since disasters are rare events and evacuations are different from normal daily traffic conditions, it is not possible to collect field data that will serve this purpose. Among the evacuation planning literature, the studies that employ BPR function to represent the speed-flow relationship in a given road segment (Sherali et al. [17], Kongsomsaksakul et al. [20], Ng and Waller [137], Li et al. [101]) all use the same α and β parameters, i.e., $\alpha = 0.15$, $\beta = 4$. For the reasons discussed above we also have employed the same values for these parameters.

2.7 Second Order Cone Programming

Let $\mathbf{K}$ be a cone in $\mathbf{R}^m$ (convex, pointed, closed and with a nonempty interior). Given an objective $c \in \mathbf{R}^n$, an $m \times n$ constraint matrix A, and a right hand side $b \in \mathbf{R}^m$, the optimization problem

$\min_x \{c^T x : Ax - b \geq_{\mathbf{K}} 0\}$

is a conic problem associated with the cone $\mathbf{K}$ [138].

When $\mathbf{K}$ is the nonnegative orthant, i.e.,

$\mathbf{K} = \mathbf{R}^m_+ = \left\{x = (x_1, ..., x_m)^T \in \mathbf{R}^m : x_i \geq 0, i = 1, ..., m\right\}$,

this is called an LP problem, when $\mathbf{K}$ is the Lorentz (or the second order or the ice cream) cone, i.e.,

$\mathbf{K} = \mathbf{L}^m = \left\{x = (x_1, ..., x_{m-1}, x_m)^T \in \mathbf{R}^m : x_m \geq \sqrt{\sum_{i=1}^{m-1} x_i^2}\right\}$,

this is called convex quadratic or second order cone programming problem, and when $\mathbf{K}$ is the positive semidefinite cone, i.e.,

$\mathbf{K} = \mathbf{S}^m_+ = \left\{x = (x_1, ..., x_m)^T \in \mathbf{R}^m : x^T A x \geq 0, A = A^T\right\}$,

this is called semidefinite programming problem.

A conic quadratic (second order cone) problem is a conic problem $\min_x \{c^T x : Ax - b \geq_{\mathbf{K}} 0\}$ for which the cone $\mathbf{K}$ is a direct product of several ice cream cones [138]. In other words, a conic quadractic problem is an optimization problem with linear objective and finitely many, say k, ice cream constraints [138].

If we partition the data matrix $[A_i; b_i] = \begin{bmatrix} D_i & d_i \\ p_i^T & q_i \end{bmatrix}$ with submatrix $[D_i; d_i]$ consisting of the first $m_i - 1$ rows and $[p_i^T; q_i]$ being the last row, the conic quadratic programming problem can alternatively be stated as:

$\min_x \{c^T x : [\![D_i x - d_i]\!]_2 \leq p_i^T x - q_i, i = 1, ..., k\}$.

When restricted to nice generic convex programs, like LP, CQP, and SDP, convex analysis becomes an algorithmic calculus-as algorithmic as in the LP case [138].

Fruitful research achievements on SOCP have been gained in the past decade both in theoretical findings and in industrial applications [139], [140]. Lobo et al. [141], Nemirovski and Tal [138], Alizadeh and Goldfarb [139] give a variety of conic quadratic representable sets and functions.

With the theoretical findings in the last two decades and the applicability of efficient interior point algorithms, SOCP has become a state-of-the-art technique in mathematical programming. Most of the interior-point methods that have been developed for linear (or semidefinite) programming can be generalized (or specialized) to handle SOCPs as well [141]. The book by Nesterov et al. [142] is the main reference on interior point methods on SOCP problems. The efficient interior point algorithms can usually solve SOCP models to optimality in polinomial time [139], which stimulates the appearance of dozens of SOCP solvers like CPLEX and MOSEK [143]. Lobo et al. [141], Nemirovski and Tal [138], and Alizadeh and Goldfarb [139] show that various kinds of nonlinear objectives and constraints can be reformulated as SOCP constraints and that way finding the global optima at an acceptable computational cost becomes possible.

We say a convex set $\mathbf{C} \subseteq \mathbf{R}^n$ is *second-order cone representable* if it can be represented by a number of second order cone constraints, possibly after introducing auxiliary variables, and we say a function $\mathbf{f}$ is second-order cone representable if its epigraph $\{(x,t) \mid f(x) \leq t\}$ has a second-order cone representation and if $\mathbf{f}$ and $\mathbf{C}$ are SOC-representable, then the convex optimization problem can be cast as an SOCP and efficiently solved via interior-point methods [141].

Consider an inequality of the form,

$$r^{2^l} \leq s_1 s_2 ... s_{2^l} \; for \; r, \; s_1, \; s_2, \; ..., \; s_{2^l} \geq 0 \tag{2.9}$$

An equivalent representation of the inequality (2.9) can be achieved by using $O(2^l)$ variables and $O(2^l)$ hyperbolic inequalities of the form

$$u^2 \leq v_1 v_2, \; u, \; v_1, \; v_2 \geq 0. \tag{2.10}$$

as shown by Lobo et al. [141], Nemirovski and Tal [138], Alizadeh and Goldfarb [139] and Günlük and Linderoth [144]. Then each hyperbolic inequality can easily be transformed into a second-order cone inequality

$$[\![2u, v_1 - v_2]\!] \leq v_1 + v_2. \tag{2.11}$$

Chapter 3

Compromising System and User Interests in Shelter Location and Evacuation Planning

The material in this chapter was published as a part of the paper by Bayram et al. [25] in the journal of Transportation Research Part B: Methodological.

The evacuation planning and management models are basically based on traffic assignment models. However, the existing traffic assignment approaches may not directly be used for evacuation planning purposes, either because they are hard to implement in practice or they are not realistic.

The UE approach is not realistic to plan an evacuation during a disaster for the following reasons. In the UE model, it is assumed that the evacuees have full information on travel times on every possible route and they are able to find the optimal routes. This assumption may not be valid in the turmoil of an evacuation during a disaster, as observed by other researchers. Disasters and evacuations are rare events with unusual traffic demand resulting in different from normal traffic conditions. As a result, evacuees do not have the opportunity to learn from the past experience which routes minimize their evacuation time [145]. It is unlikely for an equilibrium that distributes demand evenly across the evacuation routes to emerge [146]. Galindo and Batta [28] and Faturechi and Miller-Hooks [147] also state that the assumption that evacuees have perfect information about the road network and the traffic conditions is unrealistic, since it takes a while to state the traffic conditions. Such knowledge hardly exists for emergency evacuation for which the evacuees will have very limited if any prior experience regarding the travel patterns [148].

On the other hand, the SO model, in which a central authority assigns evacuees to routes to minimize the total evacuation time, may route some evacuees on paths much longer than the ones they could take if they had a choice. In a disaster, where the aim of an evacuee is to leave the endangered zone as soon as possible and to reach safety at a shelter point, people may not show conscientious behavior to accept routes that are much longer than the shortest ones they would take. It is likely that some may not abide by the evacuation rules imposed on them; instead they may choose routes to reach the closest shelter site as quickly as possible without considering the adverse affects of their choice on others.

The NA may be a reasonable approach since the information on path lengths or free flow travel times is more accessible to evacuees compared to actual travel times during an evacuation. However, such a traffic assignment may lead to poor system efficiency.

To develop a route guidance system, Jahn et al. [24] propose to honor the individual needs by imposing additional constraints to ensure that drivers are assigned to "acceptable" paths only. Such a traffic assignment model is referred to as Constrained System Optimal (CSO).

As we pointed out in Chapter 2, most of the studies on evacuation planning in the literature generally disregard the shelter location decisions, which may lead to suboptimal results as the shelter location decisions affect the evacuation traffic management and are critical for an efficient evacuation plan.

Our aim is to propose a CSO model that optimally locates shelter sites and that assigns evacuees to the nearest shelter sites and to shortest paths to those shelter sites, shortest and nearest within a given degree of tolerance. As our model already considers fairness among evacuees by assigning them to close shelter sites, we use the overall system performance in our objective and minimize the total evacuation time. We note here that our model generalizes both SO and NA traffic assignment models, as these correspond to the cases of infinite and zero tolerance levels, respectively. The solution of the model evacuates the disaster region as quickly as possible, with a "fair" assignment of evacuees to shelters and to routes. To this end, we propose a nonlinear mixed integer programming model and solve practical size problems in reasonable times by representing the nonlinear objective function with second order cone programming. In addition, we present a sensitivity analysis by changing the level of tolerance and the number of shelters to open and make a comparison of the results of SO, UE, NA and CSO approaches based on system performance and fairness. We measure the efficiency of the system by employing performance measures such as total evacuation time, percentage of evacuees reaching safety up to a specified time T, maximum latency and price of fairness. As most evacuation planning models in the literature, and as suggested by FEMA [22], our model is not specific to a certain type of disaster. The specifics of a disaster are represented in the parameters. Consequently, the model can be used both for pre and post disaster management.

In our experiments, we observe that the SO solution may have unacceptable levels of unfairness whereas the solution in which the evacuees travel to the nearest shelter using a shortest path may result in substantial deterioration in system performance. Even small levels of tolerance on the side of the evacuees improves both the system performance and the unfairness measures significantly. Overall, we can conclude that the location and the number of shelters opened drastically affect the evacuation plan and for a carefully selected number of shelters and tolerance level, an efficient yet fair evacuation plan can be achieved.

The rest of the chapter is organized as follows. In Section 3.1, we state our contributions. In Section 3.2, we define our problem formally, show that it is NP-Hard and give a second order conic mixed integer programming formulation. We compare the results with different traffic assignment models in Section 3.3

and conclude in Section 3.4.

3.1 Our Contribution

In this study, we propose a CSO model that locates the shelter sites in a SO manner and that assigns evacuees to the nearest shelter sites by assigning them to the shortest paths to their nearest shelter sites, shortest and nearest with a given degree of tolerance, so that the total evacuation time is minimized. Our aim is to meet both the system needs and the user interests in an evacuation by finding an efficient solution that evacuates the disaster region as quickly as possible and that is fair to the evacuees. Our contributions are: 1. We introduce a novel model that combines the decision making of location of shelters and allocation of evacuees to shelters and routes. In that sense our model is a location-allocation model and in contrast to most of the location-allocation models in the literature that take into account congestion, ours is a single level model that compromises system and user needs. 2. We reformulate our problem using second order conic constraints and solve real size problems exactly using a commercial solver. 3. We present a sensitivity analysis by changing the level of tolerance and the number of shelters to open and make a comparison of the results of SO, NA and CSO approaches based on system performance and fairness. 4. We analyze the impact of having capacitated shelters on performance measures.

3.2 Models

3.2.1 Problem Formulation under CSO Traffic Assignment Model

We define our problem on a directed network $G = (N, A)$, where N is the set of nodes and A is the set of arcs (links) in the network. Each arc a is associated with a convex travel time function (BPR function) t_a. We define O as the set of origin (demand) nodes from where the evacuees at risk are to be evacuated and F as the set of destination nodes (potential shelter sites) where evacuees reach to safety. Without loss of generality, we assume that O and F are disjoint subsets of N. The amount of demand at origin $r \in O$, w_r, is the number of passenger vehicles that will be evacuated.

We denote the set of alternative paths between origin-destination pair $r-s$ by P_{rs}. The values d^*_{rs} is the shortest path distance from demand node r to shelter site s. The number p is a predetermined parameter that restricts the number of shelter sites that can be opened due to budgetary and/or management issues.

We denote a driver's level of tolerance (or indifference) by λ. In other words, λ is the level that a driver perceives two paths as equal to each other. We define the set of shortest paths from origin r to destination s of tolerance degree λ as: $P^{\lambda}_{rs} = \{\pi \in P_{rs} : d^{\pi} \leq (1+\lambda)d^*_{rs}\}$, where d^{π} is the length of path π (one can also define this set based on free flow travel times if these reflect better the drivers' behavior). We compute this set using an algorithm developed by Byers and Waterman [149].

The aim of our evacuation planning problem is to decide on the locations of p shelters and the assignment of evacuees to shelters and routes so that the region is evacuated as quickly as possible.

We define the following variables to formulate the problem with CSO traffic assignment: v_{π} is the fraction of demand that uses path $\pi \in P^{\lambda}_{rs}$ from origin $r \in O$ to destination $s \in F$; x_a is the number of vehicles on arc $a \in A$; the binary variable y_s is 1 if a shelter site is opened at node $s \in F$, 0 otherwise. Using these variables, we formulate the evacuation planning problem as follows.

Model CSO

$$\min \sum_{a \in A} t_a^0 \left(1 + \alpha \left(\frac{x_a}{c_a}\right)^{\beta}\right) x_a \tag{3.1}$$

$$\text{s.t.} \sum_{s \in F} \sum_{\pi \in P_{rs}^{\lambda}} v_\pi = 1 \qquad \forall r \in O, \tag{3.2}$$

$$\sum_{\pi \in P_{rs}^{\lambda}} v_\pi \leq y_s \qquad \forall r \in O, s \in F, \tag{3.3}$$

$$\sum_{s \in F} y_s = p, \tag{3.4}$$

$$\sum_{s \in F} \sum_{\pi \in P_{rs}^{\lambda} : d^{\pi} > (1+\lambda) d_{ri}^*} v_\pi + y_i \leq 1 \qquad \forall r \in O, i \in F, \tag{3.5}$$

$$x_a = \sum_{r \in O} \sum_{s \in F} \sum_{\pi \in P_{rs}^{\lambda} : a \in \pi} w_r v_\pi \qquad \forall a \in A, \tag{3.6}$$

$$v_\pi \geq 0 \qquad \forall \pi \in \cup_{r \in O, s \in F} P_{rs}^{\lambda}, \tag{3.7}$$

$$y_s \in \{0, 1\} \qquad \forall s \in F. \tag{3.8}$$

Objective function (3.1) minimizes the total travel time that evacuees spend in the network. Constraints (3.2) ensure that all demand is evacuated. Constraints (3.3) forbid assigning demand to non-open shelter sites. Constraint (3.4) limits the number of shelter sites open to a pre-specified number p. Constraints (3.5) ensure that if the shelter at site i is open, then the demand at origin node r cannot be routed on any path whose length is longer than $(1 + \lambda)d_{ri}^*$. The set of constraints (3.6) computes the traffic flow on every arc and constraints (3.7) and (3.8) are variable restrictions.

If the central authority planning the evacuation requires the entire demand at a given origin to be routed on the same path to the same shelter, then one can define the variables v_π as binary variables.

Evacuation management authority may also require the evacuees of the same origin to be allocated to the same shelter while allowing the traffic from an origin to a shelter site to be distributed between alternative routes. To enable having separate control levels over the assignment of demand to shelters and to alternative paths we define an additional variable z_{rs}, which takes value one if origin r is assigned to shelter s and zero otherwise. We add the constraints $\sum_{\pi \in P_{rs}^{\lambda}} v_\pi = z_{rs}$ for all $r \in O$ and $s \in F$.

Note that if $\alpha = 0$, G is a complete bipartite graph where $N = O \cup F$ and arcs go from nodes in O to nodes in F, then our problem reduces to the p-median problem, which is NP-hard [42]. Hence, we can conclude that the evacuation planning problem under CSO traffic assignment model is NP-hard even when $\alpha = 0$ and G is bipartite.

The CSO approach generalizes both the SO and the NA traffic assignment approaches. When $\lambda = 0$, the above formulation models the evacuation planning problem under the nearest allocation traffic assignment model. When $\lambda = \infty$, we obtain a model for the SO traffic assignment.

Finally, note that our model is different from a traffic assignment model for a given set of origin-destination flows since in our case, the origins are known and the destinations are decided optimally.

3.2.2 Formulation for the SO Traffic Assignment Model

To compare the results of the CSO model, we also solve the same evacuation planning problems with SO traffic assignment model. The SO model decides on the locations of shelter sites and assigns the evacuees to shelters and routes so that the total evacuation time is minimized.

One can formulate the SO evacuation planning problem as done in Sherali et al. [17]. We use $\delta^-(i)$ and $\delta^+(i)$ to denote the sets of incoming and outgoing arcs of node $i \in N$, respectively. In addition to the variables defined above, we define f_s to be the number of evacuees who arrive in shelter $s \in F$.

Model SO

$$\min \sum_{a \in A} t_a^0 \left(1 + \alpha \left(\frac{x_a}{c_a}\right)^{\beta}\right) x_a \tag{3.9}$$

$$\text{s.t. (3.4) and (3.8),} \tag{3.10}$$

$$\sum_{a \in \delta^-(i)} x_a - \sum_{a \in \delta^+(i)} x_a = \begin{cases} -w_i & \forall i \in O \\ 0 & \forall i \in N \setminus (O \cup F) \\ f_i, & \forall i \in F \end{cases} \tag{3.11}$$

$$0 \leq f_s \leq \sum_{r \in O} w_r y_s \qquad \forall s \in F \tag{3.12}$$

$$x_a \geq 0 \qquad \forall a \in A. \tag{3.13}$$

Objective function (3.9) minimizes the total evacuation time. Constraints (3.11) are flow balance constraints. Finally, constraints (3.12) ensure that if a shelter site is not open, then no evacuee can be assigned to it.

We also use a multi-commodity version of this model to have the routes of evacuees in an optimal solution.

3.2.3 Second Order Cone Programming Approach

Most of the approaches for evacuation planning problems with a congestion related nonlinear objective are heuristics. Alternatively the nonlinear objective function may be approximated with a piecewise linear function. Here we avoid these two approaches. Motivated by the advances in second order cone programming (see, e.g., Nemirovski and Tal [138] and Alizadeh and Goldfarb [139]), we reformulate the nonlinear mixed integer programming models presented in the previous sections as second order conic mixed integer programs.

Gürel [150] shows that a multi-commodity network flow problem with congestion and capacity expansion can be efficiently formulated by using second order conic programming. He states that his approach can be adopted to problems with the BPR function.

We first define auxiliary variables μ_a for each $a \in A$ and move the nonlinearity from the objective function to the constraints, i.e., the objective function of the CSO model becomes $\sum_{a\in A}\left(t_a^0 x_a + \frac{t_a^0\alpha}{c_a^\beta}\mu_a\right)$ and we add the constraints $x_a^{\beta+1} \le \mu_a$ for all $a \in A$.

We take $\beta = 4$ and represent $x_a^5 \le \mu_a$ with

$$x_a^2 \le \theta_a h_a, \tag{3.14}$$

$$\theta_a^2 \le u_a x_a, \tag{3.15}$$

$$u_a^2 \le \mu_a x_a, \tag{3.16}$$

$$h_a = 1, \theta_a, u_a, x_a, \mu_a \ge 0. \tag{3.17}$$

And these hyperbolic inequalities are represented by their respective quadratic cone constraints:

$$\|2x_a, \theta_a - h_a\| \le \theta_a + h_a, \tag{3.18}$$

$$\|2\theta_a, u_a - x_a\| \le u_a + x_a, \tag{3.19}$$

$$\|2u_a, \mu_a - x_a\| \le \mu_a + x_a, \tag{3.20}$$

$$h_a = 1, \theta_a, u_a, x_a, \mu_a \ge 0. \tag{3.21}$$

3.3 Computational Study

3.3.1 Instances

We solved the models CSO and SO with different test networks. The test problems we used are from three sources. The first source is an online library called *Transportation Network Test Problems* [151] and the second is the *OR-Library* [152], originally described in Beasley [153]. We got the data for Istanbul road network from the masters thesis of Kırıkçı [154] who worked in collaboration with the Disaster Coordination Center of Istanbul Metropolitan Municipality. Computational tests were performed on a laptop with a 2.4 GHz. Intel i7-3630QM CPU and 16 GB of RAM using Java ILOG CPLEX version 12.5.1.

As geographical distances and free flow travel times are highly correlated [24], we used the geographical distances as arc lengths in our analysis. We employed the standard BPR function and assumed that the parameters of the function are the same for all arcs (road segments), i.e., $\alpha = 0.15$ and $\beta = 4$ for all $a \in A$.

Sioux Falls and Anaheim networks were downloaded from TNTP [151]. In these instances, the demands are for origin-destination pairs. We take the demand at node r as the sum of the demands whose origin is node r. For Sioux Falls we also performed runs with demand at each origin $r \in O$ as one tenth of the original demand. The original and modified instances are refered to as Sioux Falls 1 and 2, respectively.

We downloaded the P-median1 and P-median6 instances from the OR-Library [152] and used their network structure. We created the demand for each origin node randomly between 1000-2000. We also generated potential shelters sites randomly on the network for these instances. We assumed all arcs (road segments) have two lanes and three lanes for P-median1 and P-median6 instances, respectively, with a maximum traffic flow rate (capacity) of 2000 vehicles per hour per lane and with a free flow speed of 80 kms/h in an uninterrupted traffic flow.

Istanbul houses a population of more than 14 million people, which is above one-sixth of the total population of Turkey [155]. In addition to the high earthquake hazard of the city, the risk for earthquake has increased due to the improper land-use planning, construction, overpopulation and other reasons [156]. The efforts for earthquake preparedness and response planning for an impending major earthquake in Istanbul were motivated by the massively destructive 1999 Marmara (Turkey) earthquake, followed by a disaster prevention and mitigation study conducted by the Istanbul Metropolitan Municipality (IMM) in collaboration with the Japan International Cooperation Agency (JICA) [2]. The report by IMM and JICA points out that it is imperative that a community evacuation system be established. For the Istanbul network, we assumed that each road segment has three lanes with a maximum flow rate of 2000 vehicles per hour per lane and with a free flow speed of 90 km/h. Each vehicle was assumed to carry three passengers on the average. In the report by the IMM and JICA [2], it is stated that all residents in heavily damaged buildings, half of the residents in moderately damaged buildings and 10 % of the residents in partially damaged buildings adds up to 1.3 million citizens who require shelters in accordance with the most probable scenario. A similar number is given for each district of Istanbul by Kırıkçı [154]. We assumed that only those people in need of a shelter will be evacuated. The potential shelter sites are determined by interviews with the IMM as stated by Kırıkçı [154]. As there are two bridges that connect the European and Anatolian sides of Istanbul City, we assumed that the population living on each side of the Bosphorus will be evacuated to shelters on their own side.

The specifics of the instances used in the computational study are shown in Table 3.1. Here $|O - F|$ is the number of origin destination pairs that are connected with a directed path.

Table 3.1: Specifics of the Instances Used in the Computational Study

Instance	|N|	|A|	|O|	|F|	Total Demand	|O-F|
Sioux Falls 1	24	76	15	9	234,600	135
Sioux Falls 2	24	76	15	9	23,460	135
P-median1	100	396	90	10	132,212	900
P-median6	200	1572	176	24	260,520	4,224
Anaheim	416	914	37	17	104,698	408
Istanbul Anatolian	124	362	13	17	110,843	221
Istanbul European	158	448	25	32	363,865	800

3.3.2 Computational Performance

In Tables 3.2 and 3.3, we report the results of the CSO model. For each instance, we report the number of paths in the network, the number of paths with positive flow in the optimal solution, the gap between the optimal value and the continuous relaxation at the root node, the number of nodes enumerated and the solution time in seconds. All instances are solved to optimality and the longest computation time is slightly more than half an hour. We observe that, in general, the gap, the number of nodes enumerated and the solution time decrease with increasing p. We also observe that even though the gaps tend to decrease as λ increases, the solution times increase as the number of paths increase. If p is not very small or very large and if $\lambda > 0$, then the number of paths with positive flow decreases as p increases and the gap and the solution times also decrease. For Sioux Falls 1 with $\lambda = 0.2$, the computation time increases when we increase p from four to five. The same happens for Istanbul Anatolian network with $\lambda = 0.3$ when p is increased from 10 to 13 and in both cases the number of paths with positive flow also increases. We also experimented on the version of the formulation when constraint (3.3) is expressed as an inequality, i.e., open less than or equal to p shelters. In that case, generally solution times gets worse compared to the equality version as we increase p.

3.3.3 The Impact of the Number and Locations of Shelters on the Total Evacuation Time

In Tables 3.2 and 3.3, we also report the total evacuation time for all instances. It is interesting to see that increasing the number of shelters improves the system performance up to a point. For example, for Sioux Falls 1 instance, when p is four and the tolerance level is 0.2, the total evacuation time is about two million hours and when we increase p to seven and nine, the total evacuation time increases to more than four million and 74 million hours, respectively. Figure 3.1 illustrates how this happens. The potential shelter sites are at nodes 2, 6-8, 16-20. When p is four, the demands at nodes 9, 10 and 11 are assigned to two different shelter sites (8 and 19) through eight different routes. But when p is nine, the new shelter site 16 is much closer to those demand nodes than others, so nodes 9, 10, 11 all get assigned to shelter site 16 and the evacuees at each of these nodes use a single path to reach shelter 16. The total demand at these three nodes constitutes approximately % 35.7 of the total evacuation demand and the three

Table 3.2: Computational Performance I

p	λ	# of paths	# of paths with positive flow	gap	nodes	total evacuation time	solution time
Sioux Falls 1							
2	0	138	15	66.74	32	18,050,148	1.23
3	0	138	17	49.04	46	9,363,128	1.46
4	0	138	16	50.96	23	9,497,033	1.82
5	0	138	15	34.69	8	7,556,851	1.56
7	0	138	16	0.00	0	8,122,617	0.14
9	0	138	16	0.00	0	76,375,938	0.07
2	0.1	214	20	80.52	43	15,040,993	1.81
3	0.1	214	20	60.47	30	8,550,802	2.05
4	0.1	214	16	52.24	37	9,497,033	1.73
5	0.1	214	15	37.22	20	7,556,851	1.54
7	0.1	214	16	22.59	3	8,122,617	1.36
9	0.1	214	16	0.00	0	76,375,938	0.09
2	0.2	389	33	81.37	44	4,852,731	2.47
3	0.2	389	23	71.71	29	3,242,163	1.65
4	0.2	389	25	52.89	27	2,109,087	1.47
5	0.2	389	26	45.93	28	1,998,505	2.09
7	0.2	389	22	4.38	2	4,081,764	1.52
9	0.2	389	21	0.00	0	74,137,933	0.07
P-median1							
2	0	906	90	97.52	38	1,276,280	12.21
5	0	906	90	91.75	59	306,667	7.87
8	0	906	90	82.11	15	255,896	5.23
10	0	906	90	0.00	0	258,510	0.16
2	0.1	1,450	94	96.54	45	692,957	14.11
5	0.1	1,450	102	67.80	56	57,023	8.83
8	0.1	1,450	105	37.99	10	35,674	4.82
10	0.1	1,450	97	0.00	0	39,708	0.18
2	0.2	2,450	92	91.63	49	269,051	17.05
5	0.2	2,450	136	48.30	116	34,165	10.90
8	0.2	2,450	137	20.50	31	23,371	7.08
10	0.2	2,450	125	0.00	0	24,881	0.27
2	0.5	13,006	118	77.12	83	90,884	38.56
5	0.5	13,006	235	21.19	120	21,793	23.05
8	0.5	13,006	220	5.50	32	16,842	8.96
10	0.5	13,006	193	0.00	0	15,864	0.46
P-median6							
5	0	4,335	176	71.55	893	54,386	325.92
10	0	4,335	178	27.09	1,016	18,002	111.47
15	0	4,335	176	17.11	448	14,753	35.18
20	0	4,335	177	16.09	107	14,288	18.07
5	0.1	8,160	194	45.33	1,563	28,225	445.71
10	0.1	8,160	194	13.75	844	15,122	97.43
15	0.1	8,160	191	8.30	156	13,120	22.50
20	0.1	8,160	197	8.19	54	12,660	11.70
5	0.2	15,896	221	29.62	1,499	21,827	607.14
10	0.2	15,896	228	9.60	1,351	14,484	154.00
15	0.2	15,896	221	5.69	728	12,728	41.82
20	0.2	15,896	223	3.88	18	11,976	13.18
5	0.4	63,471	264	18.61	2,052	18,934	1,066.59
10	0.4	63,471	278	8.14	1,937	14,203	412.40
15	0.4	63,471	260	3.07	654	12,293	85.96
20	0.4	63,471	219	0.86	27	11,465	17.00

Table 3.3: Computational Performance II

p	λ	# of paths	# of paths with positive flow	gap	nodes	total evacuation time	solution time
Anaheim							
5	0	602	26	56.92	802	29,140	8.58
8	0	602	26	54.26	373	28,334	22.62
10	0	602	26	51.20	333	28,334	14.53
12	0	602	26	44.44	98	28,335	7.11
15	0	602	26	34.00	14	29,852	3.38
5	0.1	44,789	44	37.51	347	15,511	49.05
8	0.1	44,789	46	35.08	238	14,895	22.86
10	0.1	44,789	44	31.84	179	14,815	28.09
12	0.1	44,789	33	23.37	44	15,239	16.22
15	0.1	44,789	32	48.02	16	24,738	10.74
5	0.2	787,198	78	24.45	276	11,892	1,907.59
8	0.2	787,198	71	22.14	272	11,615	1,618.45
10	0.2	787,198	79	20.42	179	11,594	457.89
12	0.2	787,198	73	20.50	138	11,630	171.78
15	0.2	787,198	40	5.76	13	12,324	66.24
Istanbul European							
12	0	815	25	88.09	1,300	1,080,069	13.68
17	0	815	25	87.86	1,064	1,063,162	10.79
22	0	815	25	87.99	1,290	1,062,640	25.21
27	0	815	25	86.77	358	1,062,560	8.91
32	0	815	25	0.01	0	1,070,171	0.16
12	0.1	33,723	36	91.05	3,487	1,065,615	125.12
17	0.1	33,723	30	90.99	2,026	1,055,381	63.81
22	0.1	33,723	29	90.99	1,763	1,054,913	55.07
27	0.1	33,723	29	89.54	590	1,054,888	18.39
32	0.1	33,723	26	0.00	0	1,070,163	0.56
12	0.15	225,449	52	78.62	1,705	445,586	392.41
17	0.15	225,449	41	78.19	728	433,460	157.14
22	0.15	225,449	38	78.08	627	432,234	137.79
27	0.15	225,449	38	76.73	381	432,239	129.89
32	0.15	225,449	29	0.01	0	462,698	3.29
Istanbul Anatolian							
7	0	221	13	67.44	109	58,731	5.00
10	0	221	13	68.20	60	57,446	3.24
13	0	221	13	64.78	51	57,445	3.52
15	0	221	13	44.77	18	57,445	2.88
7	0.1	7,151	20	75.10	124	55,404	7.29
10	0.1	7,151	20	74.08	62	54,254	5.66
13	0.1	7,151	18	67.91	39	54,173	4.48
15	0.1	7,151	18	42.80	13	54,173	4.55
7	0.2	133,183	29	52.51	62	26,460	18.03
10	0.2	133,183	26	54.50	41	25,992	13.61
13	0.2	133,183	28	49.89	21	25,992	12.70
15	0.2	133,183	21	32.60	11	51,054	12.17
7	0.3	1,123,027	39	48.36	138	23,423	207.10
10	0.3	1,123,027	39	46.54	78	22,956	724.21
13	0.3	1,123,027	41	46.06	52	22,957	1,427.39
15	0.3	1,123,027	24	56.62	13	49,475	263.41

paths used to reach the shelter 16 share a common arc, (10,16), which causes a bottleneck due to over congestion, thus increasing the total evacuation time. This example shows that the choice of potential shelter locations and the number of shelters to open is critical for the efficiency of an evacuation plan. Clearly, using distances instead of real travel times in modeling evacuees' choices may result in a congested system as seen in the above example. However, as pointed out in the Introduction, it is not reasonable to assume that the evacuees have full information on travel times on every possible route. Instead, one may try to estimate the congested travel times and use them as normal lengths.

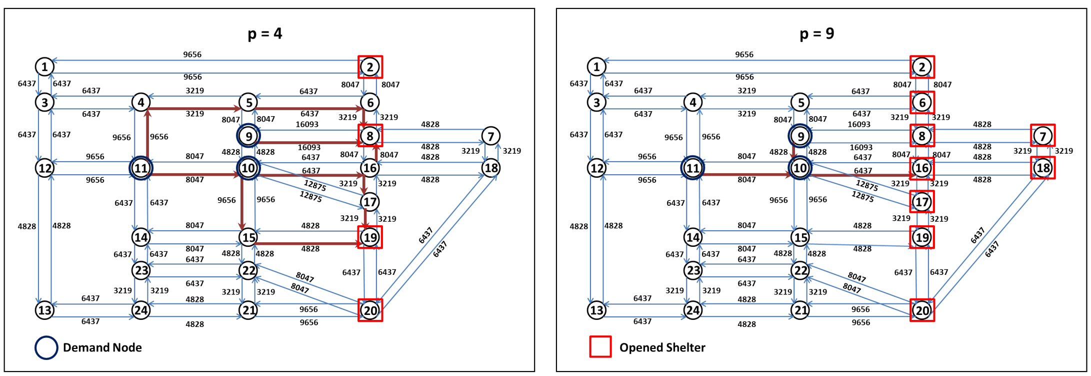


Figure 3.1: Sioux Falls 1: Allocation of Demand Nodes when $p = 4$ and $p = 9$

Figure 3.2 depicts the effect of the number of shelters and the level of tolerance on the total evacuation time for Sioux Falls 1 and 2. We observe that when the network is overloaded, which is the case of Sioux Falls 1, increasing the number of shelters to nine has an adverse effect for all tolerance levels. We also observe a similar behavior for Sioux Falls 2 when λ is small. For this instance, when $\lambda \geq 0.3$, we do not observe an adverse effect when the number of shelters is increased to nine, however there is no improvement. We also observe that when p is two, the change in the level of tolerance has little or no effect on the total evacuation time for Sioux Falls 2 whereas an opposite result is observed for Sioux Falls 1.

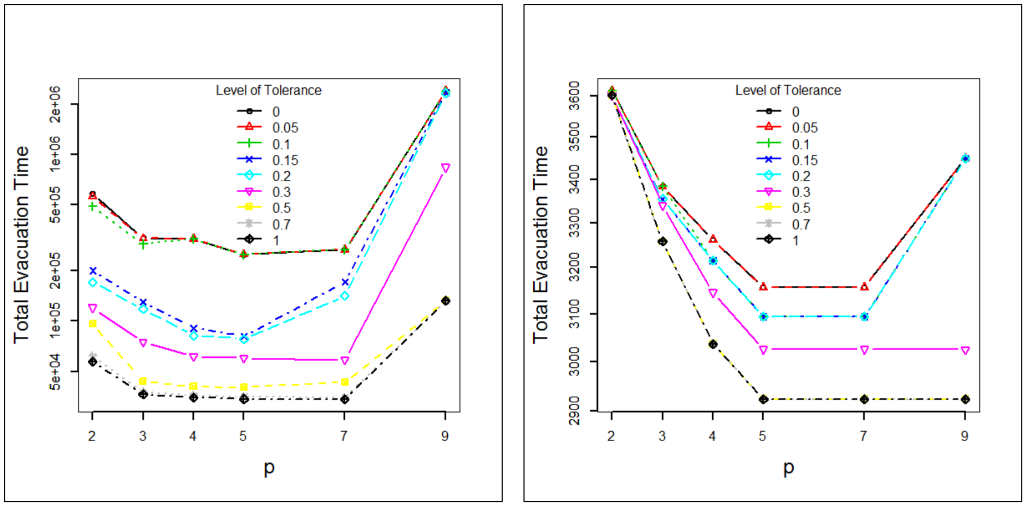


Figure 3.2: The Effect of p and the Level of Tolerance on the Total Evacuation Time, Sioux Falls 1 and 2

3.3.4 Efficiency and Fairness

While deciding on the number and location of shelters and assigning evacuees to shelters and to routes, our goal is to establish an efficient evacuation plan without losing fairness among evacuees. We measure the efficiency of an evacuation plan with regard to performance criteria such as the total evacuation time and the maximum latency.

The *price of anarchy* measures the impact of selfishness. In the literature [157, 72, 73, 74, 75, 76, 77, 78], it is defined as the worst possible proportion between the social utility from a User Equilibrium and the System Optimal. In our setting, we do not have evacuees acting on their own, but the system compromises efficiency for fairness. Hence we define *price of fairness* to measure the difference between the total evacuation times of our CSO solutions and the SO solution. Let $\tau_{CSO}(\lambda)$ and τ_{SO} be the optimal total evacuation times for the CSO model with λ level of tolerance and the SO model, respectively. The price of fairness for tolerance level λ is

$$\rho(\lambda) = \frac{\tau_{CSO}(\lambda)}{\tau_{SO}}.$$

We need to be fair to evacuees in two ways; first with respect to the travel times to their shelter sites and second with respect to the lengths of the routes they take. We employ two *unfairness* notions defined by Jahn et al. [24], namely, normal unfairness and loaded unfairness. Let F^* be the set of open shelter sites and v^* be the routing in an optimal solution.

Normal unfairness with respect to routes: Ratio of the length of an evacuee's route to the length of the shortest route for the same origin-destination pair, both measured with respect to normal arc lengths:

$$NUR = \max_{r \in O, s \in F^*} \max_{\pi \in P^{\lambda}_{rs} : v^*_{\pi} > 0} \frac{d^{\pi}}{d^*_{rs}}.$$

Normal unfairness with respect to shelters: Ratio of the length of an evacuee's route to a shelter site, to the length of the shortest route to the nearest shelter site for the same origin, both measured with respect to normal arc lengths:

$$NUS = \max_{r \in O} \max_{\pi \in \cup_{s \in F^*} P^{\lambda}_{rs} : v^*_{\pi} > 0} \frac{d^{\pi}}{d^*_{r}},$$

where $d^*_r = \min_{s \in F^*} d^*_{rs}$.

Loaded unfairness with respect to routes: Ratio of the experienced travel time of an evacuee to the experienced travel time of the fastest evacuee for the same origin-destination pair, where experienced travel time is the travel time measured in terms of the current congestion level:

$$LUR = \max_{r \in O, s \in F^*} \max_{\pi \in P^{\lambda}_{rs} : v^*_{\pi} > 0} \frac{t^{\pi}}{t^*_{rs}},$$

where t^{π} is the congested travel time on route π and t^*_{rs} is the shortest congested travel time from origin r to destination s.

Loaded unfairness with respect to shelters: Ratio of the experienced travel time of an evacuee to a shelter site, to the experienced travel time of an evacuee that is assigned to the most quickly reached shelter site for the same origin:

$$LUS = \max_{r \in O} \max_{\pi \in \cup_{s \in F^*} P^{\lambda}_{rs} : v^*_{\pi} > 0} \frac{t^{\pi}}{t^*_r},$$

where $t^*_r = \min_{s \in F^*} t^*_{rs}$.

Another measure we use is the maximum latency. The latency of a path π is the experienced travel time on path π and *maximum latency* is

$$ML = \max_{r \in O, s \in F^*} \max_{\pi \in P^{\lambda}_{rs} : v^*_{\pi} > 0} t^{\pi}.$$

Note that since we assume that all the evacuees enter the network at the same time, the network clearance time is equal to the maximum latency.

In Tables 3.4 and 3.5, for each instance, we report the total evacuation time, price of fairness, maximum latency, normal and loaded unfairness with respect to routes and shelters. We observe that when the demand is high compared to the capacity of the network, as is the case for Sioux Falls 1 and Istanbul European, the price of fairness is very high. Even though this improves with increasing levels of tolerance, the difference between the performances of SO and CSO solutions is still significant. On the other hand, for Istanbul European with $p = 17$, the normal unfairness of the SO solution with respect to routes and shelters is 18 and 63, respectively. In other words, there exist evacuees who are assigned to routes that are 63 times longer than the shortest route to the closest shelter. Hence the SO solution, even though very efficient compared to CSO solutions, is difficult to execute in practice. For the other networks, when $\lambda = 0.2$, we obtain solutions that are almost as efficient as the SO solution and that are fair to evacuees. For instance, for the P-median1 network with $p = 5$, the NUR and NUS for SO are 5.5 and 11.125, respectively, whereas they are 1.184 and 1.2 for the CSO solution with $\lambda = 0.2$. The price of fairness for this tolerance level is 1.641. The last evacuee to reach safety takes 0.487 hours, which is 0.417 hours in the SO solution. For P-median6, the performance of SO and CSO solutions are very close in terms of total evacuation time and maximum latency, whereas the SO solutions are unfair, assigning some evacuees to shelters that are four times more distant compared to the closest ones. We also observe that the NA traffic assignment ($\lambda = 0$) is fair but may result in significant increase in the total evacuation time and the maximum latency. For P-median1, the price of fairness for these solutions are about 15 and the maximum latency may be as high as 20 to 30 times the one of the SO solution.

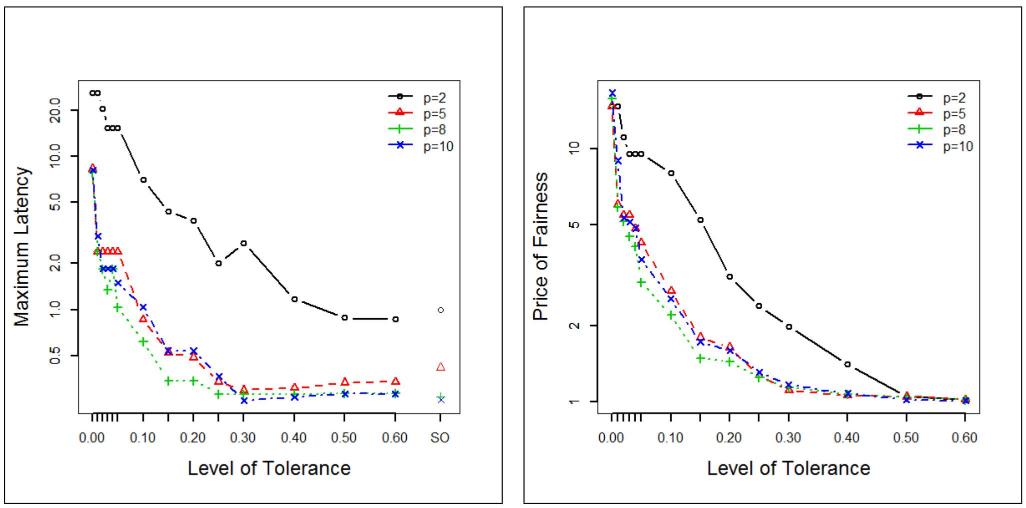


Figure 3.3: The Effect of Level of Tolerance on Maximum Latency and the Trade-off between Level of Tolerance and Price of Fairness, P-median1

Figure 3.3 illustrates the effect of level of tolerance on the maximum latency and the tradeoff between price of fairness and level of tolerance for the P-median1 network. As we increase the level of tolerance, the maximum latency tends to decrease for every p value. When p is two and the level of tolerance is zero, the maximum latency a vehicle incurs is more than 25 hours. When we open eight shelter sites and convince evacuees for a level of tolerance of 0.15, maximum latency drops to 20 minutes. Since the traffic is distributed more evenly across the network as we increase the level of tolerance, the price of fairness tends to decrease for every value of p. Overall, we observe that having even some small tolerance has a positive impact on performance measures when p is not very small. Figure 3.4 depicts the same results for Istanbul Anatolian. In this case, small tolerance does not improve the performance measures. However, as λ increases to 0.2, the efficiency of the solution improves significantly.

Table 3.4: Efficiency and Fairness I

p	λ	total evacuation time	$\rho(\lambda)$	ML	NUR	LUR	NUS	LUS
Sioux Falls 1								
3	SO	484,808	-	2.457	3.000	1.092	4.125	1.161
	0	9,363,128	19.313	78.764	1.000	1.000	1.000	1.003
	0.05	9,363,063	19.313	78.764	1.000	1.000	1.000	1.003
	0.1	8,550,802	17.638	78.376	1.083	1.003	1.083	1.013
	0.15	3,634,100	7.496	20.673	1.125	1.001	1.125	1.001
	0.2	3,242,163	6.688	17.776	1.182	1.002	1.200	1.002
5	SO	472,219	-	2.423	5.250	1.116	6.000	1.163
	0	7,556,851	16.003	75.106	1.000	1.000	1.000	1.000
	0.05	7,556,851	16.003	75.106	1.000	1.000	1.000	1.000
	0.1	7,556,851	16.003	75.106	1.000	1.000	1.000	1.000
	0.15	2,107,745	4.463	18.447	1.125	1.001	1.125	1.002
	0.2	1,998,505	4.232	12.049	1.167	1.002	1.200	1.003
Sioux Falls 2								
3	SO	3,258	-	0.282	1.250	1.190	1.333	1.242
	0	3,383	1.038	0.243	1.000	1.000	1.000	1.000
	0.05	3,383	1.038	0.243	1.000	1.000	1.000	1.000
	0.1	3,383	1.038	0.243	1.000	1.000	1.000	1.000
	0.15	3,354	1.030	0.269	1.125	1.097	1.143	1.110
	0.2	3,354	1.030	0.269	1.125	1.097	1.200	1.153
5	SO	2,923	-	0.232	1.000	1.000	1.330	1.247
	0	3,157	1.080	0.232	1.000	1.000	1.000	1.000
	0.05	3,157	1.080	0.232	1.000	1.000	1.000	1.000
	0.1	3,094	1.058	0.232	1.000	1.000	1.091	1.000
	0.15	3,094	1.058	0.232	1.000	1.000	1.111	1.087
	0.2	3,094	1.058	0.232	1.182	1.000	1.200	1.154
P-median1								
5	SO	20,821	-	0.417	5.500	2.503	11.125	2.816
	0	306,667	14.729	8.271	1.000	1.000	1.000	1.000
	0.05	88,666	4.259	2.387	1.046	1.000	1.046	1.044
	0.1	57,023	2.739	0.864	1.085	1.016	1.099	1.021
	0.15	37,472	1.800	0.525	1.135	1.039	1.148	1.046
	0.2	34,165	1.641	0.487	1.184	1.055	1.200	1.085
8	SO	16,202	-	0.266	1.611	1.399	11.125	3.377
	0	255,896	15.794	7.822	1.000	1.000	1.000	1.000
	0.05	48,139	2.971	1.037	1.000	1.000	1.050	1.002
	0.1	35,674	2.202	0.617	1.085	1.007	1.099	1.027
	0.15	24,081	1.486	0.344	1.135	1.071	1.148	1.079
	0.2	23,371	1.442	0.344	1.151	1.072	1.200	1.096
P-median6								
10	SO	14,121	-	0.132	1.514	1.359	4.000	2.221
	0	18,002	1.275	0.173	1.000	1.000	1.000	1.000
	0.05	16,078	1.139	0.134	1.033	1.025	1.043	1.029
	0.1	15,122	1.071	0.140	1.091	1.057	1.100	1.057
	0.15	14,684	1.040	0.127	1.139	1.097	1.146	1.111
	0.2	14,484	1.026	0.129	1.161	1.124	1.200	1.147
15	SO	12,239	-	0.127	1.282	1.000	2.600	1.964
	0	14,753	1.205	0.154	1.000	1.000	1.000	1.000
	0.05	13,524	1.105	0.131	1.015	1.011	1.048	1.037
	0.1	13,120	1.072	0.126	1.041	1.011	1.100	1.060
	0.15	12,928	1.056	0.127	1.139	1.101	1.145	1.109
	0.2	12,728	1.040	0.130	1.161	1.126	1.200	1.145

Table 3.5: Efficiency and Fairness II

p	λ	total evacuation time	$\rho(\lambda)$	ML	NUR	LUR	NUS	LUS
Anaheim								
10	SO	8,490	-	0.180	2.286	1.794	2.633	1.813
	0	28,334	3.337	1.203	1.000	1.000	1.000	1.103
	0.05	20,104	2.368	0.556	1.000	1.000	1.047	1.198
	0.1	14,815	1.745	0.415	1.096	1.146	1.099	1.149
	0.15	12,105	1.426	0.239	1.147	1.323	1.148	1.323
	0.2	11,594	1.366	0.236	1.196	1.290	1.200	1.380
12	SO	8,480	-	0.180	2.286	1.794	2.633	1.813
	0	28,335	3.341	1.203	1.000	1.000	1.000	1.103
	0.05	25,270	2.980	1.203	1.049	1.068	1.049	1.110
	0.1	15,239	1.797	0.450	1.089	1.018	1.099	1.131
	0.15	13,025	1.536	0.373	1.147	1.114	1.148	1.199
	0.2	11,630	1.371	0.236	1.197	1.290	1.197	1.380
Istanbul European								
17	SO	94,529	-	0.940	18.000	1.717	63.000	2.516
	0	1,063,162	11.247	14.546	1.000	1.000	1.000	1.000
	0.05	1,055,691	11.168	14.546	1.000	1.000	1.039	1.097
	0.1	1,055,381	11.165	14.546	1.056	1.229	1.075	1.229
	0.15	433,460	4.585	7.382	1.116	1.229	1.145	1.229
22	SO	91,516	-	0.918	18.000	1.655	18.000	2.764
	0	1,062,640	11.612	14.546	1.000	1.000	1.000	1.000
	0.05	1,055,124	11.529	14.546	1.000	1.000	1.039	1.095
	0.1	1,054,913	11.527	14.546	1.000	1.000	1.075	1.046
	0.15	432,234	4.723	7.382	1.116	1.057	1.145	1.062
Istanbul Anatolian								
7	SO	13,805	-	0.491	3.531	2.242	4.000	2.242
	0	58,731	4.254	1.314	1.000	1.000	1.000	1.000
	0.05	58,295	4.223	1.314	1.041	1.039	1.041	1.039
	0.1	55,404	4.013	1.314	1.041	1.039	1.061	1.062
	0.15	47,506	3.441	0.704	1.147	1.085	1.147	1.085
	0.2	26,460	1.917	0.401	1.191	1.146	1.194	1.146
10	SO	11,788	-	0.355	4.778	2.104	4.778	2.104
	0	57,446	4.873	1.314	1.000	1.000	1.000	1.000
	0.05	57,206	4.853	1.314	1.041	1.022	1.041	1.022
	0.1	54,254	4.602	1.314	1.053	1.053	1.061	1.053
	0.15	47,295	4.012	0.704	1.147	1.106	1.147	1.106
	0.2	25,992	2.205	0.401	1.191	1.098	1.194	1.098

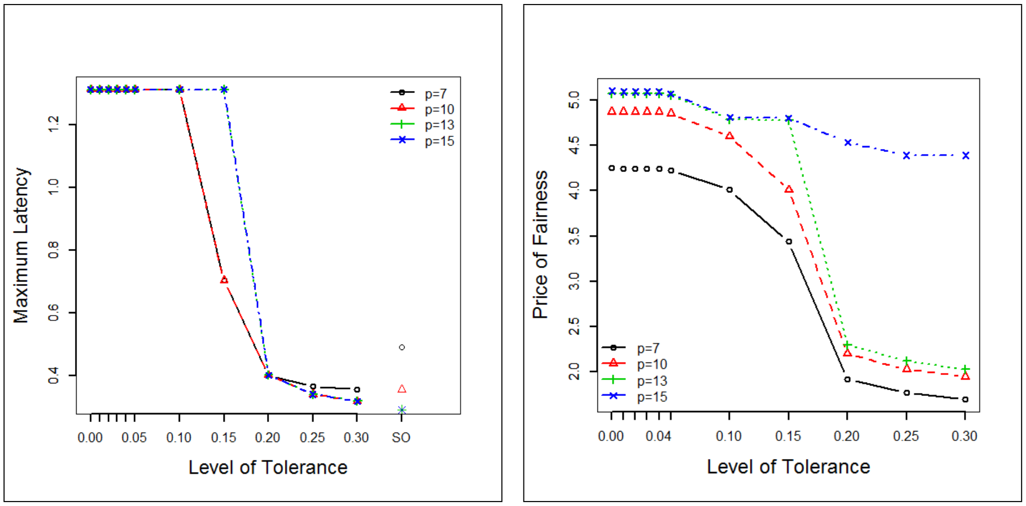


Figure 3.4: The Effect of Level of Tolerance on Maximum Latency and the Trade-off between Level of Tolerance and Price of Fairness, Istanbul Anatolian

Figure 3.5 illustrates the effect of tolerance level on the percentage of demand that can be evacuated up to a specific time T. With the level of tolerance $\lambda = 0$ and $p = 17$ for the Istanbul European instance, it takes almost 8 hours to evacuate % 88 of the people in danger. When we increase λ to only 0.15 the percentage of people evacuated in 2 hours is % 90. The case with Istanbul Anatolian instance is similarly striking. When $\lambda = 0$ and $p = 7$, it takes 1 hour and 18 minutes to clear as much as % 83 percent of people from the danger zone. If we can convince the evacuees for a level of tolerance of 0.2, every evacuee reaches safety in 30 minutes.

When we closely examine the Istanbul European instance we see that the demand at two origin nodes are very close to two shelter sites each and with a level of tolerance of 0.1 there are no other shelter sites at their proximity nor alternative paths within that limit. So each of these two nodes is assigned to a single shelter through a single path and since the amount of demand at these origin nodes is relatively large (% 21 of total evacuation demand), this assignment causes a congestion on these paths causing a latency of more than 7 hours and 14 hours, respectively. With an increase of level of tolerance to 0.15, the demand at one of these two origins, specifically the one causing a latency of 14 hours, is

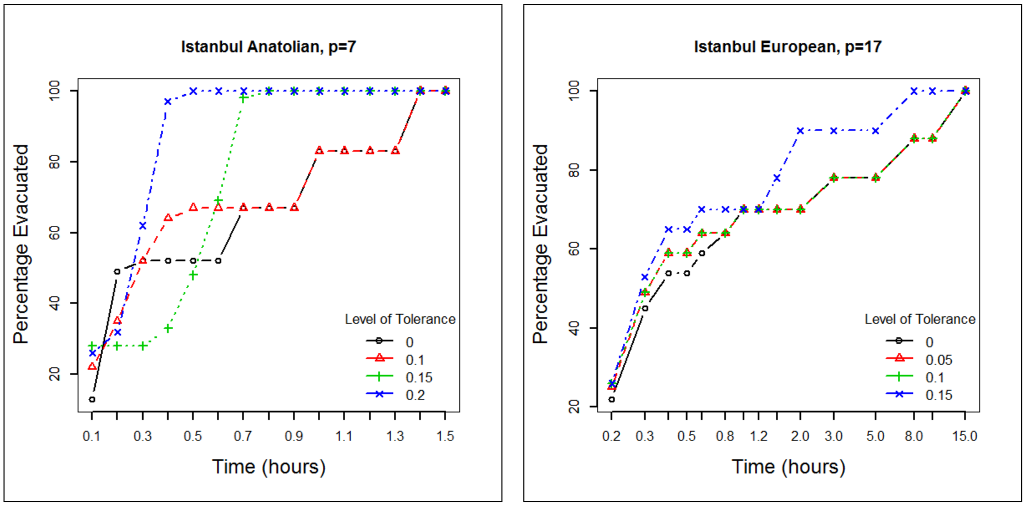


Figure 3.5: The Effect of the Level of Tolerance on Percentage Evacuated by a Given Time, Istanbul

distributed almost evenly to two shelter sites decreasing the congestion to a level that causes a latency of only an hour and a half, which in turn contributes by % 12 to the % 20 increase in the amount of people evacuated within 2 hours.

Finally, Figure 3.6 illustrates the normal unfairness distributions with respect to paths and shelters for Istanbul Anatolian instance. Here, we depict the ratio of the population whose unfairness is at most a certain level. For instance, % 89 of all evacuees for Istanbul Anatolian instance incur a normal unfairness of 1.16 with respect to paths when we employ a tolerance level of 0.2 with $p = 7$. To rephrase, the route length of just % 11 of evacuees is at least % 16 more than that of the shortest path for their demand node - shelter site pairs. One can observe that there is substantial improvement in the unfairness distribution compared to the SO approach when $\lambda = 0.3$. The improvement gets larger as λ gets smaller.

To conclude, we compare our results with the case where evacuees would have perfect information and make their choices optimally in a UE setting. We use the Istanbul Anatolian instance for this purpose. We obtain results that are comparable with those of UE when $\lambda = 0.3$. The results are reported in Table 3.6.

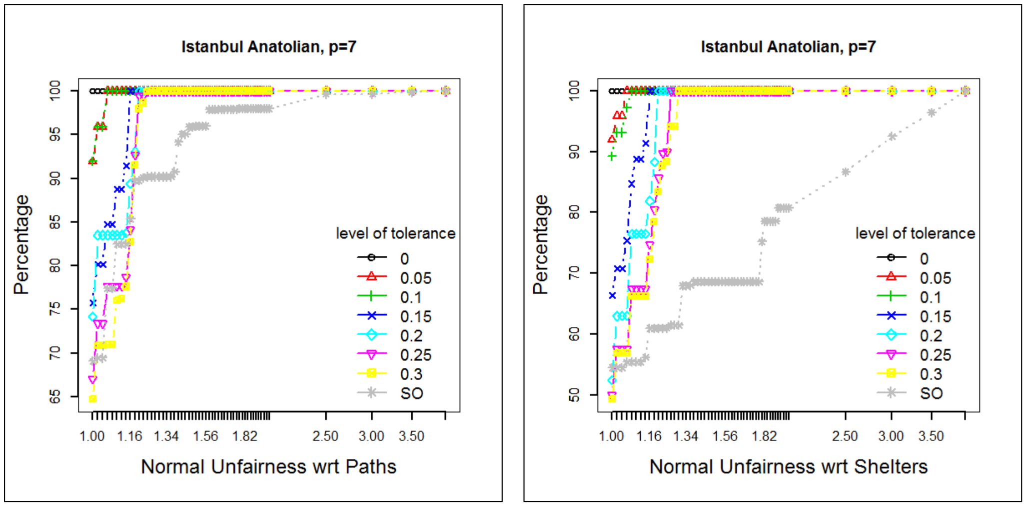


Figure 3.6: Normal Unfairness Distributions with respect to Paths and Shelters for Various Tolerance Levels, Istanbul Anatolian

Table 3.6: Comparison of SO, CSO and UE Solutions for Istanbul Anatolian Network

p	model	total evacuation time	ML	NUR	LUR	NUS	LUS
7	SO	13,805	0.491	3.531	2.242	4.000	2.242
	CSO $\lambda = 0.3$	23,423	0.355	1.259	1.116	1.3	1.141
	UE	16,749	0.355	1.471	1.001	3.403	1.001
10	SO	11,788	0.355	4.778	2.104	4.778	2.104
	CSO $\lambda = 0.3$	22,956	0.317	1.259	1.116	1.3	1.141
	UE	14,356	0.355	1.381	1.000	3.403	1.001

When $\lambda = 0.3$ and $p = 7$ the performance of CSO is 69.7 percent worse than that of SO. The performance of UE is only 21.3 percent worse than that of SO for the same p. For the same instance, the time when the last evacuee leaves the network in CSO is the same as the one for UE but CSO performs better when $p = 10$. The normal unfairness ratio with respect to shelters for UE when $p = 7$ is more than 3.4 which is very close to the value of 4 for SO, i.e., there are evacuees who are assigned to routes that are 3.4 times longer than the shortest route to the closest shelter, which may be an unacceptable result for the evacuees without any knowledge of traffic conditions on the evacuation road network. The value for this ratio for CSO is 1.3. As expected the UE solution is very good for the loaded unfairness ratios since an equilibrium state prevails. But the values for these ratios for CSO are close to those of UE and are much better than those of the SO. We also compared the percentage of demand that can be evacuated up to a specific time T for various levels of tolerance for CSO model and for UE model when $p = 7$. UE evacuates % 99.6 of the demand in 18 minutes whereas CSO evacuates % 87 of the demand within the same time. Both models evacuate everyone to safety within 24 minutes.

3.3.5 Capacitated Shelters

In our CSO model, we assume that the shelters have unlimited capacity and that their capacity can be planned according to the allocated demands. In this section, we analyze how fixed capacities affect the evacuation times and other performance measures.

To add capacity constraints to the CSO model, we omit the constraint $\sum_{s \in F} y_s = p$ and add the capacity constraints $\sum_{r \in O} \sum_{\pi \in P_{rs}^{\lambda}} w_r v_\pi \leq K_s y_s, \forall s \in F$ where K_s is the capacity of shelter s. We refer to the resulting model as "constrained system optimal model with capacitated shelters" and abbreviate it with CSO-CS. Likewise, we modified the SO model by omitting the constraint $\sum_{s \in F} y_s = p$ and adding the capacity constraints $0 \leq f_s \leq K_s y_s \; \forall s \in F$. The resulting model is called SO-CS.

We use the Istanbul European and Istanbul Anatolian networks in this experiment. We take the shelter capacities from Kırıkçı [154]. With the original capacities, there is no feasible solution for the Istanbul European network for all λ values up to 0.2. For that reason, we use two times the original capacities for that network. With these updated shelter capacities we find optimal solutions when $\lambda = 0.15$ and $\lambda = 0.2$. For the Istanbul Anatolian network, the problem is feasible for $\lambda \geq 0.05$.

In Table 3.7, we report the results for Istanbul European and Istanbul Anatolian networks for various levels of tolerance. For each instance, we report the number of shelters opened ($\#shelters$), total evacuation time, price of fairness, maximum latency, normal and loaded unfairness with respect to routes and shelters. Clearly, the effect of tolerance on the total evacuation time is much higher when the shelters are capacitated. Having no tolerance results in infeasibility for both networks. For the Anatolian network, increasing the tolerance level from 0.05 to 0.1 decreases the total evacuation time to its quarter and decreases the maximum latency to its half. The maximum latency values for SO-CS model are similar compared to the uncapacitated case for both networks, but they tend to increase for CSO-CS model. The normal unfairness ratio with respect to shelters for SO-CS model for Istanbul European network increases drastically. For the uncapacitated case the worst value over any p for normal unfairness ratio with respect to shelters is 18 and for the capacitated case this number is 30.5. With the SO model, the loaded unfairness ratio with respect to shelters for the Istanbul Anatolian network shows a large increase compared to the uncapacitated case, assigning evacuees to shelters more than 8 times longer in travel time compared to the shelter with the shortest travel time. For the CSO model there is a significant increase in the loaded unfairness ratio with respect to shelters for large values of λ, but the model still performs much better than SO model in this respect.

Table 3.7: Efficiency and Fairness with Capacitated Shelters

λ	# shelters	tot. evac. time	$\rho(\lambda)$	ML	NUR	LUR	NUS	LUS
Ist. Eur.								
SO	28	90,426	-	0.943	18	1.658	30.5	2.883
0.15	23	473,085	5.232	7.382	1.116	1.057	1.145	5.329
0.2	22	463,249	5.123	7.382	1.116	1.057	1.156	5.329
Ist. Ana.								
SO	14	11,755	-	0.338	4.778	2.601	15.444	8.225
0.05	8	207,162	17.624	2.835	1.049	1.004	1.050	1.032
0.1	11	54,795	4.662	1.314	1.008	1.006	1.061	1.265
0.15	11	54,795	4.662	1.314	1.058	1.058	1.146	1.265
0.2	10	39,855	3.391	0.935	1.191	1.044	1.194	4.244
0.25	10	38,333	3.261	0.939	1.191	1.067	1.247	4.261
0.3	10	38,320	3.260	0.939	1.259	1.080	1.288	4.258

Figure 3.7 illustrates the percentage of demand that can be evacuated up to a specific time T for various levels of tolerance for CSO model with capacitated shelters for the Istanbul Anatolian network. In the uncapacitated case, when $\lambda = 0.2$ everyone reaches safety within 30 minutes, for the capacitated case the time to evacuate everyone is almost one hour.

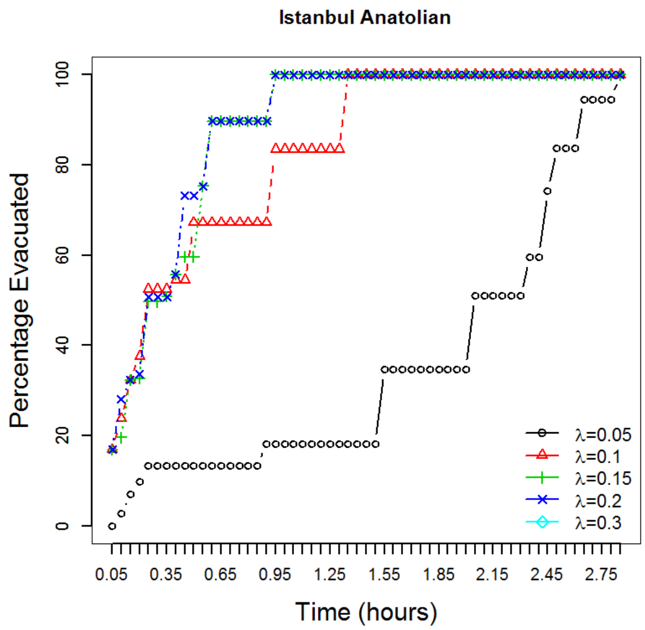


Figure 3.7: The Effect of the Level of Tolerance on Percentage Evacuated by a given time with Capacitated Shelters, Istanbul Anatolian

Overall, for our networks, we can conclude that in the presence of capacity restrictions, it is crucial to have some tolerance for a better system performance.

3.4 Conclusions

When planning for an evacuation the desirable goal is to minimize the total evacuation time by considering a SO approach. Since this approach may guide evacuees to paths that are longer than the ones they would take, its solution may be inapplicable. On the other hand, since disasters are rare events and it is not possible for evacuees to know the traffic conditions on the road network, they will rather tend to reach the nearest shelter by taking a shortest route (or shortest free flow time route). However, if evacuees know that they are being treated fairly among others and also that their relatively small sacrifice by taking a route within a tolerance level instead of the shortest route will contribute to them and to the overall benefit of the evacuation process in a great deal, they can consent to the CSO solution.

We propose a novel model that captures this human behavior by also taking into account the impact of location of shelter sites. It turns out that the decision of how many shelter sites to open and where to locate them is critical to the evacuation planners. For our instances, we observed that as we open more shelter sites and convince the evacuees for a higher level of tolerance, the total evacuation time and the maximum latency decrease and the percentage of people evacuated up to a specified time increases. This guarantees having a more efficient evacuation plan, compared to the case where evacuees act selfishly and take the shortest route to the nearest shelter site. However, one needs to be careful about opening more shelter sites, i.e., if the potential shelter sites are not chosen properly it may not be advantageous to open more shelters.

As the level of tolerance increases, so does unfairness, both in terms of paths and shelters. By considering the trade off between unfairness and price of fairness, a carefully chosen level of tolerance can be a balance between these two conflicting objectives.

Chapter 4

A Stochastic Programming Approach for Shelter Location and Evacuation Planning

When preparing for a disaster, it is very difficult to predict the evacuation demand accurately. Although the number of evacuees during Hurricane Rita from the Galvestan and Harris Counties was predicted to be less than 700,000, the actual number turned out to be 1.8 million people [146]. It is highly probable that the traffic network will be affected and lose capacity in case of a hurricane because of flood. Road segments in the traffic network can be blocked by debris caused by collapsed buildings or landslides, or damaged completely or partially in an earthquake, viaducts and bridges may collapse or can be unusable. The 1985 Mexico City earthquake wiped out approximately 70 percent of the city's central transportation network [158]. Likewise, the predetermined shelter sites can be affected.

In this chapter, our aim is to propose a scenario-based evacuation planning model that optimally locates shelter sites and assigns evacuees to the nearest shelter sites and to shortest paths (shortest geographical distance, shortest free flow travel time or shortest congested time) to those shelter sites within a given degree of tolerance to minimize the expected total evacuation time. Our model considers the uncertainty about future realizations of the evacuation demand, the disruption in the road network and degraded road capacities and disruption of the shelter sites. It is a two-stage stochastic nonlinear mixed integer programming model. We solve practical size problems in reasonable times by representing the nonlinear objective function with second order cone programming. We present a case study for evacuation planning of a potential major earthquake in Istanbul, Turkey. In our case study the population at risk is evacuated against the possible impact of aftershocks and the secondary disasters following the major earthquake. We compare the results of the stochastic programming solutions with those of wait and see solutions and mean value solutions in terms of robustness and efficiency, using criteria such as the expected total evacuation time, maximum regret, expected value of perfect information, value of stochastic solution, and performance measures such as the total evacuation time, maximum latency and percentage of evacuees reaching safety up to a specified time T. We observe that stochastic programming provides robust and effective solutions compared to models based on a single scenario.

The rest of the chapter is organized as follows. In Section 4.1 and 4.2 we present a literature review on uncertainty in facility location and evacuation planning respectively. In Section 4.3 we state our contributions. In Section 4.4 we describe the problem and give a two-stage stochastic nonlinear mixed integer programming formulation. We present the results of our case study in Section 4.5 and conclude in Section 4.6.

4.1 Uncertainty in Facility Location

The stochasticity regarding the demand, road capacities, facilities and other factors have been widely studied in classical facility location literature and the location literature for disaster management. Frank [159], Cooper [160], Louveaux [161], Laporte et al. [162], Chan et al. [163], Berman and Krass [164], Daskin et al. [165], Berman and Wang [166], Chang et al. [167] study the uncertainty in demand in classical facility location problems and Belardo et al. [168], Psaraftis et al. [169], Wilhelm and Srinivasa [170], Barbarosoğlu and Arda [30], Chang et al. [167], Balçık and Beamon [59], Rawls and Turnquist [60], Mete and Zabinsky [171], Duran et al. [64] and Noyan [172] in disaster management.

Nel and Colbourn [173], Eiselt et al. [174] take into consideration the uncertainty regarding the disruption and/or degradation of edges. Barbarosoğlu and Arda [30], Rawls and Turnquist [60], Günneç and Salman [65] and Noyan [172] propose stochastic models employing similar uncertainty issues in humanitarian relief and disaster management problems.

Mirchandani and Odoni [175], Mirchandani and Oudjit [176], Berman and Odoni [177], Ingolfsson et al. [178] work on facility location in networks using stochastic edge lengths or travel times. Mirchandani and Odoni [175] employ a nonlinear utility function for the random edge length. Mete and Zabinsky [171] take into account this same notion in their medical supply location and distribution model they develop for disaster management.

The effects of facility disruption has been studied to a large extent in the classical facility location literature (Bundschuh et al. [179], Snyder and Daskin [180], Snyder et al. [181], Berman et al. [182], Snyder and Daskin [183], Cui et al. [184], Li and Ouyang [185], and Lim et al. [186], Huang et al. [62], Peng et al. [187]) and in disaster management (Rawls and Turnquist [60], Noyan [172]).

To the best of our knowledge, the only study in location literature that takes into consideration the congestion on the road segments is by Chang and Wang [188] for site selection of solid waste facilities.

4.2 Uncertainty in Evacuation Planning

As for the studies taking into account uncertainty in the evacuation literature, most focus on demand (Rui et al. [189], Yao et al. [8], Huibregtse et al. [190], Ng and Waller [137], Yazıcı and Özbay [120], Kulshrestha et al. [21]) and/or capacity (Shen et al. [100], Ng and Waller [137], Yazıcı and Özbay [120]) uncertainty. Shen et al. [100] develop scenario-based, stochastic, bilevel models that minimize the maximum UE travel time among all node shelter pairs by locating shelters at the upper level and assigning evacuees to shelters and routes in a UE manner at the lower level. Their model is an α-reliable mean-excess regret model in the context of the p-median problem [191] in which the distance between the demand nodes and the shelter sites as well as the demand are taken as uncertain parameters. Yao et al. [8] consider the evacuation on a network under demand uncertainty. They model a deterministic LP model based on a modified CTM in which they introduce a measure called, coefficient of threat level, which is an estimate of susceptibility of an area to disaster at a particular time and by means of which they can capture the spatio-temporal priorities during evacuation. By focusing on this coefficient of threat level, they develop a robust optimization model. Huibregtse et al. [190] optimize evacuation measures to increase the efficiency of an evacuation plan by considering uncertainty in demand, the behavior of people and the hazard (location, time, and intensity). Ng and Waller [137] present an evacuation planning model that considers demand and capacity uncertainty. They provide a framework that determines the amount of demand inflation and supply deflation necessary to ensure a user-specified reliability level. Yazıcı and Özbay [120] propose a SO DTA formulation with probabilistic constraints that takes into account uncertainties in demand and roadway capacities. The model they propose is a CTM-based SO DTA formulation that uses chance constrained programming. They assume that demand and capacity distributions follow discrete probability distributions. They analyze the effects of probabilistic roadway capacities on clearance and average travel times, and the spatial shelter utilization is discussed in terms of shelter management. Their model does not endogenously select the number and location of shelter sites, rather they do sensitivity analysis. Kulshrestha et al. [21] develop a robust bi-level model that considers demand uncertainty and minimizes the total cost to establish and operate shelters at the upper level while assigning evacuees to shelters and routes in a UE manner at the lower level. They confine the uncertain demand to an uncertainty set and determine the optimal locations and capacities of shelters from the worst case scenario that can be realized from this set.

To the best of our knowledge, there is only a single study that considers the disruption in shelters in the evacuation literature. Li et al. [101] propose a scenario based location model for identifying a set of shelter locations that are robust for a range of hurricane events, by considering disruption in shelter sites. Their model is a DTA-based stochastic bi-level programming model in which at the two-stage stochastic upper level, the central authority selects the shelter sites for a particular scenario. The objective of the upper-level problem is to minimize the weighted sum of the expected unmet shelter demand and the expected total network travel time. In the lower level, evacuees choose their routes in a dynamic UE manner.

4.3 Our Contribution

In this study, we propose a two-stage stochastic scenario-based evacuation planning model. Considering the literature on evacuation planning, the contributions of our study are the following: We introduce a novel model that decides simultaneously on the locations of shelters and the allocations of evacuees to shelters and routes under uncertainty. The model incorporates evacuees' preferences and fairness considerations by routing the evacuees on paths that are not much longer than the shortest paths to the nearest shelters. We use a stochastic programming approach, and hence instead of planning the evacuation based on a single hazard scenario, our model considers a range of hazard scenarios. To the best of our knowledge, our model is the first model in the evacuation literature that considers the uncertainty in the evacuation demand, the disruption in the road network, degradation in road capacities and disruption of the shelters simultaneously. We report the results of a case study and we show that the solution of our stochastic model leads to a significant decrease in the total evacuation time compared to the deterministic and mean value solutions. We also analyze the impact of having capacitated shelters on performance measures.

4.4 Problem Description and Model Formulation

We introduce a two-stage stochastic CSO model, that we refer to as SCSO. We develop our model as an "expected value model", which is the predominant approach in the stochastic programming literature [192].

Our model decides on the number and locations of p shelters and the assignment of evacuees to shelters and routes so that the region is evacuated as quickly as possible. The problem is defined on a directed network $G = (N, A)$, where N is the set of nodes and A is the set of arcs (road segments) in the network.

We define the travel time spent on a given road segment as a positive and monotonically increasing function of traffic flow, since an increase in traffic volume will normally decrease the travel speed due to congestion and hence increase the travel time along the road segment.

Each arc a is associated with a convex travel time function (BPR function) t_a. O is defined as the set of origin (demand) nodes from where the evacuees at risk are to be evacuated and F as the set of destination nodes (potential shelter sites) where evacuees reach to safety, O and F being disjoint subsets of N. The number p is a predetermined parameter that restricts the number of shelter sites that can be opened due to budgetary and/or management issues.

When dealing with uncertainty it is important to consider what happens before (first-stage) and after (second-stage) the uncertainty is revealed [193]. Considering the evacuation problem in the framework of two-stage stochastic programming, the first stage is about where to locate the shelters before a disaster takes place, and given the location of shelters and realization of the evacuation demand and the disruption in the infrastructure, the second stage assigns evacuees to shelters and to routes.

We define Ω as the set of disaster scenarios and denote with $p(\omega)$ the probability that disaster scenario $\omega \in \Omega$ occurs. We define $F(\omega) \subseteq F$ as the set of potential shelter sites that are not disrupted in scenario $\omega \in \Omega$. Likewise, $A(\omega) \subseteq A$ is the set of arcs (road segments) that are not disrupted in scenario $\omega \in \Omega$. The binary variable y_s is 1 if a shelter site is opened at node $s \in F$, 0 otherwise. Let $P_{rs}(\omega)$ be the set of alternative paths for the pair (r, s), in scenario $\omega \in \Omega$. The amount of demand at origin $r \in O$, $w_r(\omega)$, is the number of passenger vehicles that will be evacuated in scenario $\omega \in \Omega$.

We model the tolerance of evacuees using parameter λ. An evacuee can be persuaded to take a path whose length is at most $(1 + \lambda)$ times the length of a shortest path to the closest shelter in a given scenario. The set of acceptable paths from origin r to destination s of tolerance level λ in a given scenario $\omega \in \Omega$ is defined as: $P^{\lambda}_{rs}(\omega) = \{\pi \in P_{rs}(\omega) : d^{\pi} \leq (1 + \lambda)d^{*}_{rs}(\omega)\}$, where d^{π} is the length of path π and $d^{*}_{rs}(\omega)$ is the length of a shortest path from r to s in scenario $\omega \in \Omega$. This set is computed using an algorithm developed by Byers and Waterman [149].

In our model, we also use the following decision variables: $v_{\pi}(\omega)$ is the fraction of origin r's demand that uses path $\pi \in P^{\lambda}_{rs}(\omega)$ from origin $r \in O$ to destination $s \in F(\omega)$ in scenario $\omega \in \Omega$ and $x_a(\omega)$ is the amount of traffic on arc $a \in A(\omega)$ in scenario $\omega \in \Omega$.

SCSO is formulated as follows:

$$\min \sum_{\omega \in \Omega} p(\omega) \sum_{a \in A(\omega)} t_a^0 \left(1 + \alpha \left(\frac{x_a(\omega)}{c_a(\omega)}\right)^{\beta}\right) x_a(\omega) \tag{4.1}$$

$$\text{s.t.} \sum_{s \in F} y_s \le p, \tag{4.2}$$

$$\sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^{\lambda}(\omega)} v_\pi(\omega) = 1 \qquad \forall r \in O, \omega \in \Omega, \tag{4.3}$$

$$\sum_{\pi \in P_{rs}^{\lambda}(\omega)} v_\pi(\omega) \le y_s \qquad \forall r \in O, \omega \in \Omega, s \in F(\omega), \tag{4.4}$$

$$\sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^{\lambda}(\omega): d^\pi > (1+\lambda) d_{ri}^*(\omega)} v_\pi(\omega) + y_i \le 1 \qquad \forall r \in O, \omega \in \Omega, i \in F(\omega), \tag{4.5}$$

$$x_a(\omega) = \sum_{r \in O} \sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^{\lambda}(\omega): a \in \pi} w_r(\omega) v_\pi(\omega) \qquad \forall \omega \in \Omega, a \in A(\omega), \tag{4.6}$$

$$v_\pi(\omega) \ge 0 \qquad \forall \omega \in \Omega, \pi \in \bigcup_{r \in O, s \in F(\omega)} P_{rs}^{\lambda}(\omega), \tag{4.7}$$

$$y_s \in \{0, 1\} \qquad \forall s \in F. \tag{4.8}$$

Objective function (4.1) is equal to the expected total evacuation time. Constraint (4.2) limits the number of shelter sites open to a pre-specified number p. Constraints (4.3) ensure that all demand is evacuated in each scenario. Constraints (4.4) forbid assigning demand to non-open shelter sites for each scenario. Constraints (4.5) ensure that if the shelter at site i is open and functioning in scenario ω, then the demand at origin node r cannot be routed on any path whose length is longer than $(1 + \lambda)d_{ri}^*(\omega)$. Let i_r be the shelter that is open and that is closest to origin r. Due to constraints (4.5), the demand at node r cannot be routed on a path whose length is more than $(1 + \lambda)d_{ri_r}^*(\omega)$. As i_r is the closest shelter to r and $d_{ri_r}^*(\omega)$ is the length of a shortest path from r to i_r in scenario ω, these constraints forbid the use of paths whose lengths are longer than $(1+\lambda)$ times the length of the shortest path. Constraints (4.5) for open shelters i other than i_r are dominated by constraints (4.5) for shelter i_r. As we do not know, which shelters are open, we need to include these constraints for all possible shelters. Finally, constraints (4.6) compute the traffic on every arc in each scenario and constraints (4.7) and (4.8) are variable restrictions.

Deterministic constrained system optimal (DCSO) problem is a special case of the SCSO problem in which $|\Omega| = 1$ and it is NP-hard even when $\alpha = 0$ and G is bipartite. We also note that our model generalizes the classical facility location model in that the arc costs of our model are nonlinear, we assign the customers (evacuees) to facilities (shelters) and to paths to those facilities. Our model also generalizes the SO and NA traffic assignment approaches. When $\lambda = 0$ we have the NA model. When $\lambda = \infty$, we obtain a model for the SO traffic assignment and hence generalize the model given in Sherali et al. [17].

The shelter location decisions y are first-stage (design) variables and are taken in the presence of uncertainty about future realizations. In the second stage, the uncertainty is revealed and recourse decisions v are taken. However, while making the first-stage shelter location decisions, their effect on the second stage assignment decisions and total evacuation time is taken into account. The future effects of shelter location decisions is measured by taking the expectation of the recourse function on possible scenarios. Because y is a binary variable, our model has a 0-1 first stage problem. In addition, since the objective function of the second stage is nonlinear, the model has a nonlinear second stage problem.

We reformulate the SCSO as a second order conic mixed integer programming model as done in Chapter 3.

4.5 An Illustrative Case Study: Istanbul Earthquake

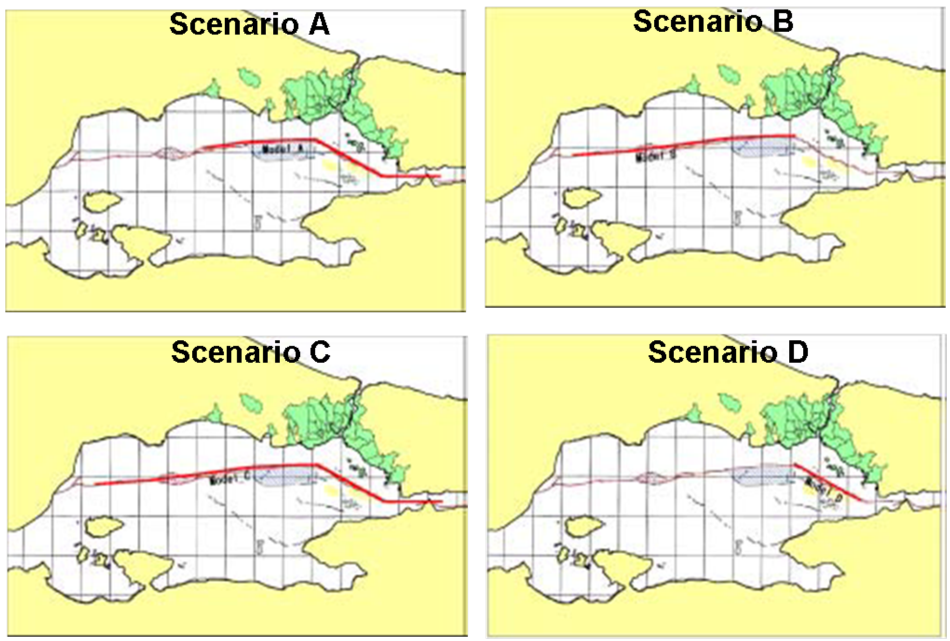
A study by Marano et al. [194] presents a quantitative and geospatial description of global losses due to earthquake-induced secondary disasters, including landslide, liquefaction, tsunami and fire, for events that took place through 1968-2008. Marano et al. [194] state in their study that 21.5% of fatal earthquakes have deaths due to secondary causes. In this study, Turkey is shown among the countries where tsunamis, landslides, and fires have been observed as secondary disasters following a major earthquake.

The 1999 Marmara (Turkey) earthquake that devastated the cities of Sakarya, Izmit and Yalova initiated the efforts for earthquake preparedness and response planning for a potential major earthquake in Istanbul. The Istanbul Metropolitan Municipality (IMM) started a disaster prevention and mitigation study that was conducted in collaboration with the Japan International Cooperation Agency (JICA) [2]. Following the potential major earthquake in Istanbul, aftershocks as observed during the 1999 Marmara earthquake and different secondary disasters such as landslides, floods, fires and possibly tsunamis can impact different geographical areas of Istanbul city and can cause further fatalities. In the technical report by IMM and JICA it is stated that there does not exist an emergency evacuation system in Istanbul, Turkey and that it is imperative that a community evacuation system be established in order to mitigate and minimize human casualties due to second or third aftershocks and secondary disasters following the earthquake. For that reason, the case study we present here assumes an emergency evacuation in the sense that we are evacuating people to protect them from the impact of aftershocks and the secondary disasters.

4.5.1 Scenarios

The IMM-JICA report determines the possible primary and secondary access and evacuation corridors and introduces a traffic management plan. In coordination with scientific institutions and researchers, IMM-JICA study determines four scenarios for the earthquake, scenarios A, B, C and D as shown in Figure 4.1.

Scenario A is referred to as the most probable scenario and scenario C as the worst case scenario. The report also includes a seismic microzonation that divides the city into smaller disaster regions with respect to a spatial risk analysis that



	Scenario A	Scenario B	Scenario C	Scenario D
Length (km)	119	108	174	37
Moment magnitude (Mw)	7.5	7.4	7.7	6.9

Figure 4.1: Possible Scenarios for a Pending Istanbul Earthquake [2]

takes into account their seismic characteristics (Peak Ground Acceleration Levels (PGA), see Figure 4.2) and damage patterns in different scenarios. Five different risk zones are determined for scenarios A, B, C, and D. Damage probabilities specific to road segments, bridges and viaducts in each risk zone are determined and potential damage to the road network, infrastructure and buildings are calculated in accordance with the PGA distribution.

The Disaster Coordination Center (DCC - AKOM in Turkish) specified 49 potential shelter sites in Istanbul to serve as safe facilities to provide the evacuees with food, accommodation and medical care [154]. The main criteria used in determining potential shelter sites are accessibility by at least two alternative routes, proximity to major highways, and availability of land [63].

We generated our instances using the data from the IMM-JICA study and DCC. As the European and Anatolian sides of Istanbul are connected by two

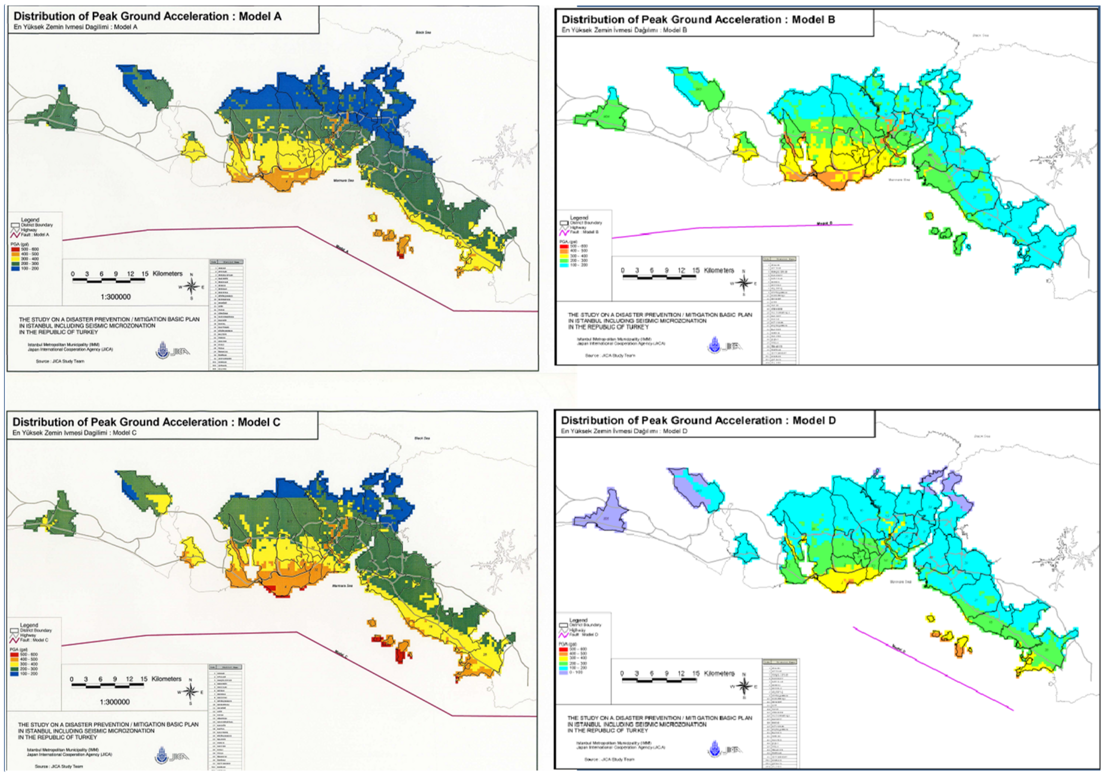


Figure 4.2: Peak Ground Acceleration Distributions for Scenarios [2]

bridges, we assume that the population living on each side will be evacuated to shelters on their own side. In the report by IMM-JICA [2], it is stated that all residents in heavily damaged buildings, half of the residents in moderately damaged buildings and 10% of residents in partially damaged buildings add up to 1.3 million citizens who require shelters in accordance with the most probable scenario. A similar number is given for each district of Istanbul by Kırıkçı [154]. We assume that only those people in need of a shelter will be evacuated and that each vehicle carries four passengers on the average. We generated the road network structure for these instances using Google Earth [195] for major highways and the secondary roads connecting these highways. We assumed that each lane has a maximum traffic flow rate (capacity) of 2000 vehicles per hour with a free flow speed specified in accordance with traffic regulations in an uninterrupted traffic flow. Figure 4.3 illustrates the road network structure, the potential shelter sites and the demand points that we use in our study (the district of Silivri with one demand point and one shelter site is out of the boundaries of Istanbul European Road Network Map).

We used the geographical distances as arc lengths in our analysis. Clearly, using geographical distances instead of real travel times as the normal length of a path in modeling evacuees' choices may result in a congested system. However, as pointed out earlier, assuming that the evacuees have full information on travel times on every possible route is not realistic. Instead, one may try to estimate the congested travel times. Our model permits the use of these estimates as normal lengths.

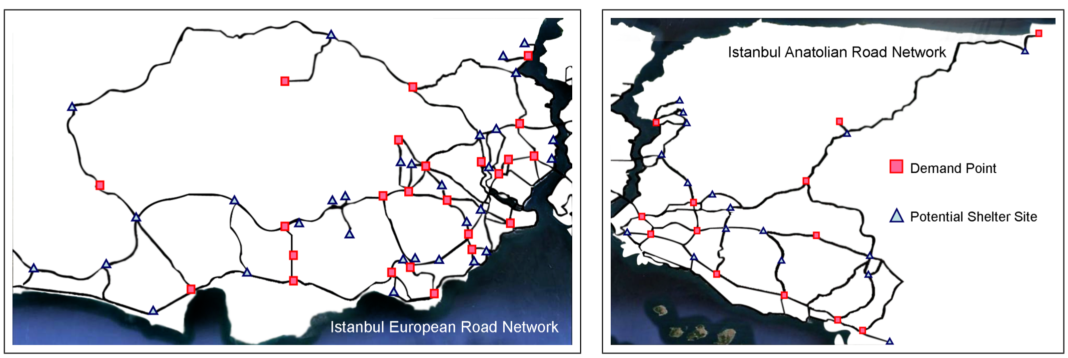


Figure 4.3: Istanbul European and Anatolian Road Networks, Potential Shelter Sites and Demand Points

We consider 12 scenarios that correspond to different severeness of the disaster. In the IMM-JICA report, the evacuation demand specific to each origin node is only given for scenario A in detail, and the information for other scenarios is not disclosed. For that reason, our base (nominal) scenario is scenario A. By taking into account the length of the fault line that is predicted to be affected and the magnitude of the earthquake in scenarios B, C and D compared to those of Scenario A, we specify the values of arc and shelter disruption probabilities for risk zones and the evacuation demand at each origin node for these scenarios. We generate three different scenarios for each of the original scenarios A, B, C and D in the report, which adds up to 12 scenarios in total. Generally, scenarios 2, 6, and 10 represent more destructive earthquake scenarios.

We assign a different disruption probability for each risk zone in every scenario. We classify the arcs and potential shelter sites of the original network into sets in accordance with which risk zone they are located in. For a particular scenario, we randomly determine if an arc (road segment) is disrupted by considering the risk zone it is located in and the probability of disruption assigned to this zone. If the arc is disrupted, we again randomly specify how much of its original capacity is degraded in multiples of a single lane capacity. For instance, if a road segment with three lanes having a capacity of 6000 vehicles per hour is disrupted, it may be degraded to a capacity of 4000, 2000 or 0 vehicles per hour. Likewise, in accordance with which risk zone a shelter site is located in and the probability assigned to that risk zone, a shelter site may get disrupted in which case it will no longer be able to serve the evacuees. We assume that the free flow travel time in a degraded (not totally disrupted) road segment is the same as the one in the original network. Each vehicle is assumed to carry four passengers on the average.

In Tables 4.1 and 4.2, we present the properties of our instances. ON is the original network when the road network and the shelters are not disrupted and the demand in ON is the demand of scenario A in IMM-JICA report. And rest of the instances in those tables are scenarios 0 through 11 with the remaining number of arcs, shelters and total road network capacity after the evacuation network is disrupted. We perform our computational tests on a personal computer with a 2.4 GHz Intel i7-3630QM CPU and 16 GB of RAM using Java ILOG CPLEX version 12.5.1.

4.5.2 Effect of the Number of Shelters and Tolerance Level

In our first experiment, we investigate the effect of the number of shelters p and the tolerance level λ on the total evacuation time and the difficulty of the problem. In Table 4.3, we report the results of the SCSO model for the Istanbul European and Anatolian instances for different values of p and λ. For each

Table 4.1: Istanbul European Instances

Scenario	$\mid N \mid$	$\mid A \mid$	Total Demand	$\mid F \mid$	Probability of Scenario	Total Road Network Capacity
ON	80	238	272,900	32		1,246,000
0	80	221	245,322	28	0.1461	1,098,000
1	80	228	219,960	30	0.0754	1,128,000
2	80	219	328,832	26	0.0436	1,074,000
3	80	213	133,161	28	0.0872	1,022,000
4	80	211	245,642	31	0.145	1,066,000
5	80	220	224,481	30	0.0439	1,072,000
6	80	223	325,988	29	0.0255	1,094,000
7	80	218	136,819	29	0.0834	1,068,000
8	80	221	245,622	30	0.1152	1,082,000
9	80	217	218,378	30	0.0897	1,080,000
10	80	223	327,002	25	0.0669	1,068,000
11	80	224	133,883	28	0.0782	1,094,000

Table 4.2: Istanbul Anatolian Instances

Scenario	$\mid N \mid$	$\mid A \mid$	Total Demand	$\mid F \mid$	Probability of Scenario	Total Road Network Capacity
ON	50	146	83,133	17		748,000
0	50	137	74,965	16	0.1461	666,000
1	50	135	63,263	17	0.0754	668,000
2	50	135	101,482	15	0.0436	676,000
3	50	133	41,920	15	0.0872	646,000
4	50	137	75,586	17	0.145	690,000
5	50	140	66,203	17	0.0439	676,000
6	50	136	103,708	17	0.0255	688,000
7	50	140	41,599	13	0.0834	700,000
8	50	136	75,299	16	0.1152	678,000
9	50	143	66,451	16	0.0897	724,000
10	50	136	97,111	13	0.0669	644,000
11	50	131	39,559	17	0.0782	642,000

instance, we report the number of shelters opened (#os), the optimal expected total evacuation time and the solution time in seconds. All instances are solved to optimality and the longest computation time is a little more than one hour. We observe that, in general, the solution times decrease with increasing p and increase with increasing λ. We observe that the expected total evacuation time decreases drastically as the tolerance parameter λ increases for both networks.

We note here that at most 20 and 13 shelters are opened in Istanbul European and Anatolian instances, respectively, even when p is larger. As also noted in Chapter 3, it is not necessarily beneficial to open more shelters as this may cause some road segments to be extremely congested.

Table 4.3: Results for the Istanbul European and Anatolian Instances

	Istanbul European				Istanbul Anatolian			
λ	p	#os	Exp.Tot.Evac.Time	Sol.Time	p	#os	Exp.Tot.Evac.Time	Sol.Time
0	10	10	25,448,247	300.60	5	5	78,850	72.46
	20	20	25,160,535	163.35	10	10	56,621	36.61
	25	20	25,135,322	199.10	13	12	56,592	29.11
	30	20	25,135,322	166.04	15	12	56,574	26.53
	32	20	25,135,322	215.57	17	12	56,574	27.86
0.05	10	10	22,291,176	780.28	5	5	43,547	77.75
	20	20	22,033,781	389.07	10	10	32,512	30.14
	25	20	22,008,431	337.63	13	13	32,410	28.21
	30	20	22,008,431	444.20	15	13	32,397	29.60
	32	20	22,008,431	385.59	17	13	32,397	28.34
0.1	10	10	10,293,567	2,471.33	5	5	42,366	97.67
	20	20	10,167,674	880.59	10	10	32,306	47.48
	25	20	10,160,242	934.62	13	13	32,212	41.93
	30	20	10,160,242	1,304.19	15	13	32,202	40.69
	32	20	10,160,242	755.29	17	13	32,202	39.01
0.15	10	10	6,432,920	3,048.37	5	5	40,913	119.37
	20	20	6,390,771	2,344.98	10	10	32,318	54.62
	25	20	6,385,797	2,682.86	13	13	32,218	47.10
	30	20	6,385,797	3,634.32	15	13	32,193	46.41
	32	20	6,385,797	3,522.80	17	13	32,193	42.54

4.5.3 Effect of the Number Scenarios

Next, we investigate the effect of number of scenarios on solution times. In addition to the instances of Table 4.3, we generated instances with 20, 30, 40, and 50 scenarios for both networks. We present the results in Tables 4.4 and 4.5 for Istanbul European and Istanbul Anatolian instances, respectively. Istanbul European instances cannot be solved for $\lambda \geq 0.2$ when we have 12, for $\lambda \geq 0.15$

when we have 20, 30 and 40 scenarios and for $\lambda \geq 0.1$ when we have 50 scenarios due to memory issues. We are able to solve Istanbul Anatolian network for all instances. The solution times increase for both of the networks as we increase the number of scenarios.

Table 4.4: Performance of the Model with respect to Number of Scenarios Used, Istanbul European

p	λ	20 Scenarios	30 Scenarios	40 Scenarios	50 Scenarios
10	0	1,356.57	3,251.88	4,678.79	10,105.45
20	0	1,045.88	1,694.39	1,660.19	2,956.82
25	0	968.22	2,416.30	2,337.38	2,655.69
30	0	871.98	1,800.57	2,263.78	3,258.51
32	0	842.68	2,061.41	2,229.47	2,503.97
10	0.05	2,904.32	4,219.94	8,200.14	10,024.99
20	0.05	1,381.12	2,365.43	4,246.90	3,690.56
25	0.05	1,422.64	2,010.96	2,934.63	2,695.69
30	0.05	1,538.21	2,236.61	3,079.87	4,994.38
32	0.05	1,484.87	2,951.45	3,545.91	4,795.99
10	0.1	3,426.88	4,411.65	3,689.71	
20	0.1	2,395.78	3,249.89	5,764.08	
25	0.1	2,427.97	4,644.29	7,288.15	Out of Memory
30	0.1	3,263.28	4,465.89	5,943.04	
32	0.1	5,143.12	4,599.89	7,245.78	

4.5.4 Effect of β

The β parameter determines the steepness of the BPR function. It is a measure of how fast the travel time on a road segment increases depending on the congestion. We reformulate our problem by taking $\beta = 3$, $\beta = 2$, $\beta = 1$ (the quadratic case) and $\beta = 0$ (the linear case), and represent the nonlinearity in the objective function as we have done in Section 4.4.

In Tables 4.6 and 4.7, we report the total evacuation times and the solution times for different β values. As the level of nonlinearity decreases as we decrease β, the solution times decrease, as well. We notice that the shelters opened differ by 0-20 % between subsequent β values. The percentage of different shelters opened gets larger as we move farther away from one β to the other in sequence. As β decreases, the penalty for exceeding the road capacity gets significantly smaller and total evacuation times decrease.

Table 4.5: Performance of the Model with respect to Number of Scenarios Used, Istanbul Anatolian

p	λ	20 Scenarios	30 Scenarios	40 Scenarios	50 Scenarios
5	0	135.16	342.35	319.23	1,275.82
10	0	72.73	131.39	176.62	501.84
13	0	66.56	115.65	52.93	469.10
15	0	61.63	116.28	60.03	485.93
17	0	55.05	129.83	63.91	521.02
5	0.05	183.55	421.66	459.73	1,925.80
10	0.05	102.63	190.36	203.59	861.83
13	0.05	88.04	187.26	97.89	745.92
15	0.05	105.66	143.71	81.44	667.46
17	0.05	101.85	145.11	76.61	762.15
5	0.1	292.16	499.98	738.75	1,878.17
10	0.1	157.16	210.16	152.46	1,049.99
13	0.1	119.44	192.46	130.91	1,141.32
15	0.1	153.53	173.75	109.25	1,027.57
17	0.1	138.80	195.26	106.63	855.33
5	0.15	234.87	665.10	880.47	2,229.39
10	0.15	169.25	229.27	253.03	1,254.10
13	0.15	147.80	229.05	190.14	1,206.51
15	0.15	126.20	174.11	155.11	1,187.39
17	0.15	180.42	197.92	148.48	1,208.96
5	0.2	251.30	1,027.82	915.87	3,000.22
10	0.2	175.29	243.71	311.96	1,300.65
13	0.2	109.98	198.64	144.90	1,304.73
15	0.2	119.95	190.11	146.56	1,533.68
17	0.2	114.39	191.83	140.27	1,387.06

Table 4.6: Performance of the Model with respect to β, Istanbul European

p	λ	Evac.Time	Sol.Time	Evac.Time	Sol.Time	Evac.Time	Sol.Time	Evac.Time	Sol.Time	Evac.Time	Sol.Time
		$\beta = 4$		$\beta = 3$		$\beta = 2$		$\beta = 1$		$\beta = 0$	
10	0	25,448,247	300.60	1,448,226	221.89	76,117	202.30	14,000	26.26	9,540	0.58
20	0	25,160,535	163.35	1,399,871	104.98	69,525	58.21	11,423	13.63	7,695	0.40
25	0	25,135,322	199.10	1,399,446	79.12	69,204	43.26	11,132	5.23	7,472	0.36
32	0	25,135,322	215.57	1,399,446	87.49	69,170	46.64	11,082	4.56	7,472	0.37
10	0.05	22,291,176	780.28	1,278,486	620.60	74,309	356.24	13,870	48.80	9,540	0.93
20	0.05	22,033,781	389.07	1,232,201	223.80	69,372	137.49	11,328	24.22	7,695	0.75
25	0.05	22,008,431	337.63	1,231,458	230.80	69,106	85.99	11,081	16.77	7,471	0.72
32	0.05	22,008,431	385.59	1,231,458	246.13	69,061	90.07	11,062	10.77	7,471	0.78
10	0.1	10,293,567	2,471.33	740,677	688.97	63,827	642.24	13,873	123.67	9,540	1.86
20	0.1	10,167,674	880.59	712,732	460.33	58,284	242.36	11,135	49.39	7,689	1.76
25	0.1	10,160,242	934.62	710,956	523.64	58,220	238.17	10,896	42.34	7,465	1.61
32	0.1	10,160,242	755.29	710,956	262.27	58,168	268.08	10,875	29.90	7,465	1.58
10	0.15	6,432,920	3,048.37	518,188	2,228.97	62,207	2,676.37	13,853	1,228.35	9,540	6.05
20	0.15	6,390,771	2,344.98	510,273	1,293.28	56,719	856.67	11,041	190.60	7,676	5.72
25	0.15	6,385,797	2,682.86	507,893	1,245.46	56,654	581.46	10,804	175.48	7,452	5.66
32	0.15	6,385,797	3,522.80	507,893	1,010.60	56,654	627.48	10,772	145.57	7,452	5.67

Table 4.7: Performance of the Model with respect to β, Istanbul Anatolian

p	λ	Evac.Time	Sol.Time	Evac.Time	Sol.Time	Evac.Time	Sol.Time	Evac.Time	Sol.Time	Evac.Time	Sol.Time
		$\beta = 4$		$\beta = 3$		$\beta = 2$		$\beta = 1$		$\beta = 0$	
5	0	78,850	72.46	20,729	23.52	7,710	27.97	4,919	5.33	4,269	0.18
10	0	56,621	36.61	14,575	32.16	5,735	9.14	3,741	0.84	3,241	0.08
13	0	56,592	29.11	14,265	7.24	5,709	9.20	3,630	0.57	3,129	0.07
17	0	56,574	27.86	14,265	8.35	5,702	7.66	3,615	0.39	3,129	0.09
5	0.05	43,547	77.75	15,271	25.85	7,407	31.78	4,916	7.43	4,252	0.14
10	0.05	32,512	30.14	11,187	30.73	5,214	6.21	3,723	1.34	3,241	0.11
13	0.05	32,410	28.21	10,624	7.83	5,147	6.92	3,617	0.79	3,125	0.10
17	0.05	32,397	28.34	10,624	9.95	5,138	7.23	3,599	0.55	3,125	0.10
5	0.1	42,366	97.67	15,010	32.73	7,335	32.33	4,884	10.60	4,246	0.29
10	0.1	32,306	47.48	11,071	39.29	5,192	16.75	3,720	1.66	3,237	0.15
13	0.1	32,212	41.93	10,652	43.08	5,127	8.89	3,612	1.02	3,125	0.15
17	0.1	32,202	39.01	10,652	46.76	5,101	6.17	3,579	0.70	3,112	0.16
5	0.15	40,913	119.37	15,343	46.03	7,270	40.42	4,890	11.31	4,239	0.26
10	0.15	32,318	54.62	11,074	56.33	5,185	11.76	3,713	2.04	3,237	0.20
13	0.15	32,218	47.10	10,610	37.74	5,129	11.09	3,612	1.37	3,125	0.20
17	0.15	32,193	42.54	10,610	42.48	5,101	6.75	3,579	0.89	3,112	0.20
5	0.2	34,024	180.11	13,317	49.46	6,977	40.14	4,883	17.33	4,239	0.34
10	0.2	28,462	63.80	10,219	130.98	4,957	16.16	3,665	4.55	3,237	0.30
13	0.2	28,312	59.98	9,646	29.33	4,880	12.87	3,565	1.78	3,125	0.29
17	0.2	28,299	46.90	9,646	45.37	4,873	9.29	3,552	1.30	3,112	0.59

4.5.5 Quality of Stochastic Programming Solutions

If it were possible to wait until all the uncertainties are revealed and solve the evacuation problem with perfect information in hand for each scenario, this would be called a wait and see solution (WSS) [196]. On the other hand, one could solve the stochastic problem by replacing all the random parameters with their means, this solution is called the mean value solution (MVS). Expected value of perfect information (EVPI) is the difference between the expected total evacuation times of WSS and the stochastic programming solution and value of stochastic solution (VSS) is the difference between the expected total evacuation times of the stochastic programming and mean value solutions. EVPI measures the value of knowing the future with certainty, while VSS assesses the value of knowing and using distributions of future outcomes [193].

To measure the EVPI and VSS, we solve our evacuation problem first by considering a specific scenario, i.e., employ the WSS. We also solve the evacuation problem by taking the mean of the stochastic parameters in each scenario, which results in the MVS. Given the shelter locations from the WSS for a specific scenario and the MVS, we solve our problem for each scenario and obtain the total evacuation times in these scenarios separately. We adopt an approach that includes only the shelter location decisions from WSS and MVS to solve the model for each scenario in order to compute EVPI and VSS, and show the quality of our stochastic programming solution.

For the mean value solution, the demand for each origin is taken as the mean of the demand values at these origins in all possible scenarios. For the potential shelter sites, $|F|$, we take a shelter as disrupted if it is disrupted in two or more scenarios. As for the road segments, if a road segment is disrupted in three or more scenarios it is removed from the graph of mean value scenario, and if this is not the case we take the average of the arc capacities over all possible scenarios to find the capacity of the road segment as a multiple of lanes. This results in having 9 disrupted shelters and 23 disrupted road segments for Istanbul European network and 4 disrupted shelters and 7 disrupted road segments for Istanbul Anatolian network.

For the case when shelters are uncapacitated, we compare the quality of 15 different solutions, named as 0, 1,...,11, ON, M, S. Among these, 0, 1,...,11 are the optimal solutions corresponding to the original scenarios, ON represents the solution obtained from using the original undisrupted network with the original demand from scenario A in IMM-JICA report, and M and S represents the mean value solution and stochastic programming solution, respectively. The regret of a solution in a given scenario is the difference between the total evacuation time for that solution in that scenario and the minimum total evacuation time for that scenario. The maximum regret for a solution is the maximum of its regrets over all scenarios.

In Table 4.8, we present the total evacuation times for all 15 solutions and 12 scenarios, as well as the expectation of the total evacuation times. We take $\lambda = 0.1$ and $p = 32$ for the European network and $\lambda = 0.2$ and $p = 17$ for the Anatolian network. Table 4.9 gives the WSS, EVPI and VSS for both networks. Finally, in Table 4.10, we present the regrets of the solutions in all 12 scenarios and the maximum regrets.

Table 4.8: Total Evacuation Times

	Realized Scenario												
Sol.	0	1	2	3	4	5	6	7	8	9	10	11	Expectation
Eur													
0	401,261	13,105,568	3,542,209	100,045	618,028	186,761	3,043,580	53,916	872,288	309,581	2,175,269,495	237,142	147,062,200
1	5,182,927	599,421	9,582,876	271,566	5,747,337	245,165	26,199,557	925,467	6,489,759	6,290,393	2,178,326,729	86,659	149,882,018
2	1,584,639	644,151	1,221,336	836,989	620,289	1,110,192	8,592,209	321,992	7,071,255	2,403,956	5,036,314,169	116,299	338,759,711
3	1,236,185	977,131	7,069,495	77,090	1,774,549	3,215,344	7,919,391	319,085	21,916,560	2,016,525	5,036,376,277	234,567	340,853,840
4	501,085	12,970,605	3,557,423	357,217	192,609	703,203	3,038,228	62,706	1,080,314	495,758	998,767,044	82,879	68,371,867
5	1,241,925	743,241	9,584,396	233,115	1,826,228	166,686	25,151,462	924,985	912,536	1,411,165	2,175,245,104	230,655	147,439,960
6	761,586	13,097,345	3,450,679	102,303	866,572	3,481,574	1,012,426	54,291	1,032,087	303,566	2,175,257,308	511,325	147,277,842
7	402,008	13,106,206	3,545,843	347,278	657,467	3,289,101	3,046,602	51,658	1,404,953	302,015	2,175,245,104	508,196	147,306,123
8	5,182,927	732,691	9,582,876	271,566	19,922,789	245,165	25,134,765	925,467	516,151	4,414,396	2,178,326,729	508,477	151,096,905
9	429,509	13,095,126	3,245,833	100,839	617,673	312,604	3,029,981	52,094	832,955	288,826	2,175,198,104	231,126	147,046,022
10	156,825,960	8,217,426	171,431,615	431,443,643	9,570,567	1,977,081,898	608,062,355	4,314,125	141,098,313	48,409,756	113,229,230	27,449,743	202,993,673
11	4,178,835	18,565,606	24,761,353	490,517	1,471,541	3,397,331	35,097,250	668,328	21,992,283	635,031	998,729,233	82,676	73,857,901
ON	1,589,878	641,838	1,224,002	357,957	617,264	1,350,865	7,514,343	318,667	1,305,142	507,915	999,528,680	215,897	67,813,519
M	189,897,690	31,080,870	59,034,477	3,554,501	17,429,254	32,368,706	47,062,832	2,385,485	1,185,932,246	94,420,333	180,651,595	12,282,184	196,453,643
S	630,561	1,028,932	3,956,950	4,499,696	760,151	12,948,939	6,006,103	324,008	1,339,799	846,470	124,211,377	343,626	10,160,347
Ana													
0	7,513	77,406	103,189	3,726	14,438	6,986	56,313	2,926	95,121	8,777	500,602	2,527	61,271
1	8,319	4,540	44,636	3,435	8,780	5,434	19,382	2,705	112,868	5,217	503,132	2,729	53,378
2	12,479	53,731	36,576	3,112	8,984	8,171	44,880	2,789	10,389	7,457	1,490,450	3,240	112,609
3	12,091	53,746	59,917	2,909	9,413	7,775	36,367	3,128	10,420	6,631	1,489,162	5,117	113,400
4	8,410	51,492	36,576	3,164	8,541	6,184	22,232	2,508	15,388	5,217	500,265	2,527	45,174
5	8,319	4,540	44,636	3,435	8,780	5,434	19,382	2,705	112,868	5,217	503,831	2,729	53,425
6	8,319	4,540	44,636	3,435	8,780	5,434	19,382	2,705	112,868	5,217	503,041	2,729	53,372
7	10,442	52,791	59,703	9,548	11,367	8,359	43,511	2,071	20,007	8,281	291,919	2,571	35,017
8	12,062	53,720	59,702	2,909	9,380	7,462	35,792	3,020	10,385	6,540	1,489,162	5,104	113,329
9	8,319	4,540	44,636	3,435	8,780	5,434	19,382	2,705	112,868	5,217	501,065	2,729	53,240
10	18,646	85,480	3,206,787	6,772	30,174	16,963	115,207	4,310	28,776	20,501	193,259	3,207	176,327
11	11,688	78,675	91,212	6,908	17,511	9,261	76,688	2,625	99,511	11,821	499,907	2,195	63,478
ON	8,322	4,542	44,661	3,435	8,780	5,434	19,382	2,707	113,000	5,217	504,208	2,728	53,467
M	12,091	53,746	59,917	2,909	9,413	7,775	36,367	3,128	10,420	6,631	1,489,162	5,117	113,400
S	10,498	52,785	59,687	6,237	11,359	8,359	50,594	2,105	19,999	8,210	193,354	2,401	28,303

It is clearly seen that, the stochastic programming solution provides a much better evacuation planning compared to ON solution. With the stochastic programming solutions, we obtain about 7 and 2 times better results with respect to expectation, compared to the ON solutions for Istanbul European and Istanbul Anatolian networks, respectively. With respect to maximum regret, the results are more striking. Stochastic Programming Solution has almost 70 and 7 times better maximum regret values for Istanbul European and Istanbul Anatolian networks, respectively.

For both Istanbul European and Anatolian networks, the stochastic programming solution performs best in terms of the expected total evacuation time and in terms of the maximum regret. When we use the stochastic programming solution, on the average, a vehicle spends about 10 hours more and 7 minutes more to reach its shelter, compared to what it would have spent if we had perfect information about the disaster for Istanbul European and Anatolian networks, respectively. If we look at a specific scenario, say, if we assume we have perfect information for scenario 10 in Istanbul Anatolian network and if we locate the shelters and make evacuation plans accordingly, and if indeed this scenario realizes, an evacuee will reach a safe shelter about at the same time he/she reaches safety in the stochastic programming solution. The VSS is very high, especially for the Istanbul European network. Indeed, the ratio of the expected total evacuation time for the mean value solution to the one of the stochastic programming solution is 19.3 and 4.0 for Istanbul European and Anatolian networks, respectively. We also observe that planning according to a single scenario can lead to very poor performance. For Istanbul European network, if we plan according to scenario 10, the expected total evacuation time is about 20 times higher than the expected total evacuation time for the stochastic programming solution.

Table 4.9: WSS, EVPI, VSS

	Istanbul European	Istanbul Anatolian
WSS	7,896,033	20,197
EVPI	2,264,314	8,106
VSS	186,293,296	85,097

Table 4.10: Regrets

Sol	0	1	2	3	4	5	6	7	8	9	10	11	Max Regret
Eur													
0	0	12,506,147	2,320,873	22,955	425,419	20,075	2,031,154	2,258	356,137	20,755	2,062,040,265	154,466	2,062,040,265
1	4,781,665	0	8,361,540	194,476	5,554,728	78,479	25,187,131	873,808	5,973,607	6,001,567	2,065,097,499	3,982	2,065,097,499
2	1,183,378	44,730	0	759,899	427,680	943,506	7,579,783	270,334	6,555,104	2,115,131	4,923,084,938	33,623	4,923,084,938
3	834,924	377,710	5,848,159	0	1,581,940	3,048,658	6,906,964	267,426	21,400,408	1,727,699	4,923,147,046	151,891	4,923,147,046
4	99,824	12,371,184	2,336,087	280,127	0	536,517	2,025,802	11,048	564,163	206,932	885,537,813	203	885,537,813
5	840,664	143,820	8,363,060	156,025	1,633,619	0	24,139,036	873,327	396,385	1,122,339	2,062,015,873	147,979	2,062,015,873
6	360,325	12,497,924	2,229,343	25,212	673,963	3,314,888	0	2,633	515,936	14,740	2,062,028,077	428,648	2,062,028,077
7	747	12,506,785	2,324,508	270,188	464,858	3,122,415	2,034,176	0	888,802	13,189	2,062,015,873	425,519	2,062,015,873
8	4,781,665	133,270	8,361,540	194,476	19,730,180	78,479	24,122,339	873,808	0	4,125,570	2,065,097,499	425,800	2,065,097,499
9	28,248	12,495,705	2,024,497	23,749	425,064	145,918	2,017,555	436	316,803	0	2,061,968,874	148,450	2,061,968,874
10	156,424,699	7,618,005	170,210,279	431,366,553	9,377,958	1,976,915,212	607,049,929	4,262,467	140,582,162	48,120,930	0	27,367,067	1,976,915,212
11	3,777,574	17,966,185	23,540,017	413,427	1,278,932	3,230,645	34,084,823	616,670	21,476,132	346,205	885,500,002	0	885,500,002
ON	1,188,616	42,417	2,666	280,866	424,655	1,184,179	6,501,917	267,008	788,990	219,089	886,299,449	133,221	886,299,449
M	189,496,429	30,481,449	57,813,141	3,477,411	17,236,645	32,202,020	46,050,406	2,333,826	1,185,416,095	94,131,507	67,422,364	12,199,508	1,185,416,095
S	229,299	429,511	2,735,614	4,422,606	567,542	12,782,253	4,993,677	272,350	823,648	557,644	10,982,147	260,950	12,782,253
Ana													
0	0	72,866	66,613	817	5,897	1,552	36,931	855	84,736	3,560	307,343	332	307,343
1	806	0	8,060	526	239	0	0	634	102,483	0	309,873	534	309,873
2	4,966	49,191	0	203	443	2,737	25,498	718	4	2,240	1,297,191	1,045	1,297,191
3	4,577	49,206	23,341	0	872	2,342	16,985	1,058	35	1,414	1,295,903	2,922	1,295,903
4	896	46,952	0	255	0	750	2,851	438	5,003	0	307,006	332	307,006
5	806	0	8,060	526	239	0	0	634	102,483	0	310,572	534	310,572
6	806	0	8,060	526	239	0	0	634	102,483	0	309,782	534	309,782
7	2,929	48,250	23,127	6,639	2,826	2,925	24,129	0	9,621	3,064	98,660	376	98,660
8	4,549	49,180	23,126	0	839	2,028	16,410	949	0	1,324	1,295,903	2,909	1,295,903
9	806	0	8,060	526	239	0	0	634	102,483	0	307,806	534	307,806
10	11,133	80,940	3,170,211	3,863	21,633	11,529	95,825	2,240	18,391	15,284	0	1,012	3,170,211
11	4,174	74,134	54,636	3,999	8,970	3,828	57,306	554	89,126	6,604	306,648	0	306,648
ON	808	2	8,085	526	239	0	0	637	102,615	0	310,950	533	310,950
M	4,577	49,206	23,341	0	872	2,342	16,985	1,058	35	1,414	1,295,903	2,922	1,295,903
S	2,984	48,245	23,110	3,328	2,817	2,925	31,212	34	9,614	2,993	95	206	48,245

We also investigate the performance of the solutions for different values of tolerance levels. We report in Table 4.11 the total evacuation time versus λ for both networks when $p = 32$ and $p = 17$, respectively for scenario 10. We observe that the total evacuation time for the stochastic programming solution is almost as good as solution 10 for scenario 10. We notice that ON solution tends to give better solutions for small λ values compared to mean value solution. Neither ON nor M solution is able provide a better total evacuation time compared to stochastic programming solution. The solutions ON and M provide are about 3.1 and 33.2 times and 13.2 and 2.4 times worse than that of stochastic programming solution when $\lambda = 0$ and $\lambda = 0.15$, respectively. For Istanbul Anatolian network these numbers are about 8, 2.5 when $\lambda = 0.1$ and 2.6, 7.7 when $\lambda = 0.2$.

Table 4.11: Total Evacuation Time for Various Levels of Tolerance for Scenario 10

λ	Istanbul European							
	0	1	2	3	4	5	6	7
0	2,175,259,598	998,806,214	10,728,808,279	10,728,822,320	998,822,735	2,175,259,598	2,175,322,600	2,175,259,598
0.05	2,175,256,062	2,175,320,421	10,728,761,795	10,728,702,790	753,768,923	2,175,256,062	2,175,315,663	2,175,256,062
0.1	2,175,269,495	2,178,326,729	5,036,314,169	5,036,376,277	998,767,044	2,175,245,104	2,175,257,308	2,175,245,104
0.15	2,175,269,190	2,175,198,536	5,036,312,908	2,175,249,906	753,716,960	2,175,269,190	2,175,255,531	2,175,245,104
	8	9	10	11	ON	M	S	
0	2,175,265,854	753,696,271	323,291,760	998,743,086	1,002,780,099	10,728,776,770	323,365,972	
0.05	2,175,260,370	753,694,397	276,769,362	753,689,274	1,002,668,638	3,332,666,041	276,828,692	
0.1	2,178,326,729	2,175,198,104	113,203,246	998,729,233	999,528,680	180,651,595	124,211,377	
0.15	2,175,198,890	753,632,213	74,770,390	753,679,132	999,527,218	181,278,134	75,875,383	
	Istanbul Anatolian							
	0	1	2	3	4	5	6	7
0	1,585,374	2,679,390	660,747	1,241,800	811,153	1,585,374	660,747	1,645,442
0.05	1,577,796	687,758	1,577,370	1,503,328	1,503,964	1,763,994	1,585,505	1,418,292
0.1	501,214	589,880	1,577,623	501,214	500,721	1,583,700	1,588,090	292,361
0.15	501,155	1,499,699	1,490,518	1,489,320	500,761	501,155	1,499,699	1,551,573
0.2	500,602	503,132	1,490,450	1,489,162	500,265	503,831	503,041	291,919
	8	9	10	11	ON	M	S	
0	1,585,370	2,420,442	547,125	817,167	661,202	1,577,324	547,212	
0.05	1,503,800	1,582,057	197,445	1,509,893	1,590,550	1,503,800	197,449	
0.1	501,214	589,880	197,047	501,931	1,588,809	500,814	197,260	
0.15	1,489,320	1,499,699	197,041	501,807	1,500,406	1,489,241	197,129	
0.2	1,489,162	501,065	193,259	499,907	504,208	1,489,162	193,354	

Finally, we use the performance measures "maximum latency (or maximum experienced travel time)", and "percentage of evacuees reaching safety up to a specified time T" to evaluate the performance of our solutions. Table 4.12 illustrates the maximum latency incurred by the evacuees for scenario 10 of Istanbul Anatolian network, for different tolerance levels, when $p = 17$. After solution 10, which is the optimal for the realized scenario, the stochastic programming solution has the least maximum latency, many times better than that of any other solution. For some λ values the maximum latency incurred in stochastic programming solution is the same as that of solution 10. In terms of maximum latency, stochastic programming solution provides 4.4 times and 14.6 times better solutions than those of the ON and mean value solutions, respectively. Figure 4.4 also depicts the percentage of evacuees reaching safety across time for different strategies, for scenario 10 of Istanbul Anatolian instance, when $p = 17$ and $\lambda = 0.2$. Since realized scenario is 10, solution 10 evacuates 100% of the population in less than six hours. While the stochastic programming solution evacuates almost everyone to safety within the same time as solution 10, the original network solution ON and the mean value solution M, can only evacuate 85% and 69% of the population within the same time. Solution strategy 7 starts really poorly in the beginning but then evacuates everyone within almost the same times as solution strategy 10 and stochastic programming solution.

Table 4.12: Maximum Latency for Various Levels of Tolerance for Scenario 10, Istanbul Anatolian

λ	0	1	2	3	4	5	6	7	8	9	10	11	ON	M	S
0	76.01	99.46	21.95	73.44	21.95	76.01	21.95	76.01	76.01	76.01	14.12	21.95	21.96	76.01	14.12
0.05	76.01	24.77	76.01	73.44	73.44	76.01	76.01	76.01	73.44	76.01	5.18	73.44	76.03	73.44	5.18
0.1	21.95	21.95	76.01	21.95	21.95	76.01	76.01	5.16	21.95	21.95	5.13	21.95	76.03	21.95	5.16
0.15	21.95	73.53	73.53	73.53	21.95	21.95	73.53	73.53	73.53	73.53	5.13	21.95	73.54	73.53	5.17
0.2	21.95	21.95	73.53	73.53	21.95	21.95	21.95	5.13	73.53	21.95	5.03	21.95	21.96	73.53	5.03

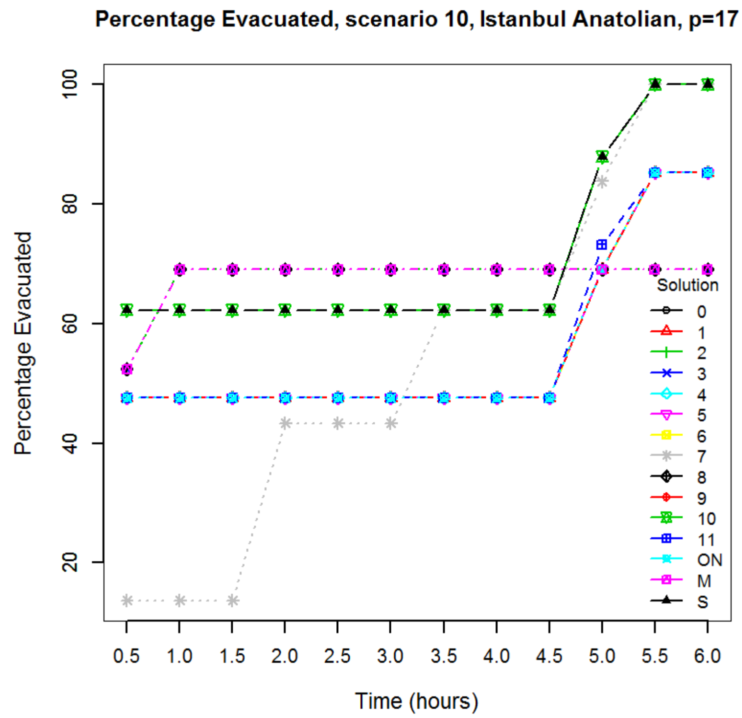


Figure 4.4: The Percentage of Evacuees Reaching Safety across Time for Different Strategies

4.5.6 Capacitated Shelters

To add capacity constraints to the SCSO model, we omit the constraint $\sum_{s\in F} y_s \leq p$ and add the capacity constraints $\sum_{r\in O}\sum_{\pi\in P^{\lambda}_{rs}(\omega)} w_r(\omega)v_\pi(\omega) \leq K_s y_s, \forall\omega\in\Omega, s\in F(\omega)$ where K_s is the capacity of shelter s. We refer to the resulting model as "stochastic constrained system optimal model with capacitated shelters" and abbreviate it with SCSO-CS. We take shelter capacities as used in Kırıkçı [154]. In Table 4.13, we present the results for the Istanbul European and Anatolian Instances when shelters are capacitated. With original capacities, the problem is infeasible for both Istanbul European and Istanbul Anatolian networks for all λ values up to 0.2. We increase the capacities of some of the shelters so that the problem becomes feasible for some λ values. With the updated capacities, the problem is feasible for Istanbul European network for $\lambda \geq 0.05$ and for Istanbul Anatolian network for $\lambda \geq 0.1$.

Table 4.13: Results for the Istanbul European and Anatolian Instances when Shelters are Capacitated

	p	λ	#os	Exp. Tot. Evac. Time	Solution Time
Istanbul European					
	32	0.05	26	22,008,492	118.04
	32	0.1	24	11,609,623	231.91
	32	0.15	20	8,946,248	2,741.05
Istanbul Anatolian					
	17	0.1	10	114,819	36.44
	17	0.15	10	102,865	34.54
	17	0.2	13	49,498	35.05

For the case when shelters are capacitated, we compare the quality of 14 different solutions, named as 0, 1,...11, M, S, with 0, 1,...11 being the optimal solutions corresponding to the original scenarios, M and S representing the mean value solution and stochastic programming solution, respectively.

In Table 4.14, we present the total evacuation times for all 14 solutions and 12 scenarios, as well as the expectation of the total evacuation times, when shelters are capacitated. We take $\lambda = 0.1$ and $p = 32$ for the Istanbul European network and $\lambda = 0.2$ and $p = 17$ for the Istanbul Anatolian network. Table 4.15 gives the WSS, EVPI and VSS for both networks. Finally, in Table 4.16, we present the regrets of the solutions in all 12 scenarios and the maximum regrets over all scenarios.

Table 4.14: Total Evacuation Times

	Realized Scenario												
Sol.	0	1	2	3	4	5	6	7	8	9	10	11	Expectation
Eur													
0	619,554	∞	∞	4,496,552	∞	∞	∞	269,078	∞	∞	∞	∞	∞
1	∞	606,132	∞	∞	5,806,220	10,803,413	∞	2,687,014	6,615,177	19,334,866	∞	1,029,248	∞
2	∞	∞	1,623,788	∞	∞	∞	∞	∞	∞	2,285,037	∞	167,911	∞
3	∞	∞	∞	99,261	∞	∞	∞	∞	∞	∞	∞	∞	∞
4	∞	∞	∞	4,496,319	261,194	∞	∞	322,572	28,860,304	2,329,049	∞	1,029,384	∞
5	∞	∞	∞	∞	∞	6,693,301	∞	2,637,143	∞	∞	∞	487,205	∞
6	∞	∞	∞	4,496,716	∞	∞	3,024,328	331,004	28,868,756	2,379,558	∞	1,036,876	∞
7	∞	∞	∞	873,149	956,690	∞	∞	266,813	∞	∞	∞	153,834	∞
8	∞	∞	∞	∞	∞	∞	∞	2,728,662	1,012,485	7,677,118	∞	∞	∞
9	887,680	∞	∞	∞	∞	∞	∞	360,880	∞	918,019	∞	∞	∞
10	7,656,602	12,081,369	∞	9,572,308	∞	720,625,162	∞	4,079,307	∞	∞	135,952,177	392,754	∞
11	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	82,540	∞
M	7,611,940	8,865,691	∞	2,018,039	∞	715,498,719	∞	3,481,212	∞	∞	∞	284,377	∞
S	637,163	611,072	3,921,490	4,500,243	264,425	12,952,928	4,763,189	324,303	7,214,394	2,357,504	135,952,177	126,355	11,605,694
Ana													
0	7,676	77,569	∞	4,007	∞	7,105	∞	2,928	∞	∞	∞	2,588	∞
1	57,968	30,213	∞	8,615	57,942	32,438	∞	12,237	91,637	25,506	∞	7,664	∞
2	19,239	57,556	83,104	6,511	19,725	10,616	59,691	3,109	15,095	12,092	∞	3,788	∞
3	∞	55,165	∞	2,909	∞	∞	∞	∞	∞	∞	∞	9,114	∞
4	10,405	52,885	∞	6,224	11,341	8,329	∞	2,073	19,815	∞	∞	2,194	∞
5	7,676	77,569	∞	4,007	∞	7,105	∞	2,928	∞	∞	∞	2,588	∞
6	9,265	79,816	∞	4,615	26,605	7,277	39,603	2,928	23,030	14,824	345,159	3,196	∞
7	∞	52,891	∞	∞	∞	8,356	∞	2,073	∞	∞	∞	2,570	∞
8	∞	55,139	∞	2,909	∞	∞	∞	∞	13,676	∞	∞	5,188	∞
9	158,450	36,332	∞	7,894	19,866	9,625	48,515	3,355	23,416	9,541	∞	3,031	∞
10	17,800	∞	∞	6,774	∞	∞	∞	4,279	∞	∞	342,800	3,212	∞
11	11,682	78,716	∞	∞	∞	9,259	∞	2,627	∞	∞	∞	2,194	∞
M	∞	55,067	∞	2,936	∞	∞	∞	∞	13,657	∞	∞	5,210	∞
S	22,509	59,101	83,104	6,766	25,832	12,861	239,070	3,223	20,818	13,438	344,465	2,770	49,498

For both Istanbul European and Anatolian networks, there is no single solution other than the stochastic programming solution that is feasible over all possible scenarios, when shelters are capacitated. In Tables 4.14 and 4.16, "∞" basically means that the proposed solution is not feasible for that specific scenario. For that reason the stochastic programming solution is superior compared to all other solutions.

When we use the stochastic programming solution, on the average, a vehicle spends about 7 hours more and a little bit more than 11 minutes more to reach its shelter, compared to what it would have spent if we had perfect information about the disaster for Istanbul European and Anatolian networks, respectively. If we look at a specific scenario, say, if we assume we have perfect information about scenario 10, a possible worst case scenario, and if we locate the shelters and make evacuation plans accordingly, and if indeed this scenario realizes, an evacuee will reach a safe shelter at the same time he/she reaches safety in accordance with the stochastic programming solution, in Istanbul European network.

For both Istanbul European and Anatolian networks the VSS is infinite. We also observe that planning according to a single scenario leads to much worse performance compared to the case when shelters are uncapacitated. Whichever scenario is chosen for evacuation planning purposes, the solution becomes infeasible for some scenarios in both Istanbul European and Anatolian networks. For that reason the maximum regret is infinite for all other solutions except that of the stochastic programming solution for both Istanbul European and Anatolian networks.

Table 4.15: WSS, EVPI, VSS for the Capacitated Shelters Case

	Istanbul European	Istanbul Anatolian
WSS	9,947,393	35,952
EVPI	1,658,300	13,546
VSS	∞	∞

Table 4.16: Regrets when Shelters are Capacitated

Sol	0	1	2	3	4	5	6	7	8	9	10	11	Max Regret
Eur													
0	0	∞	∞	4,397,291	∞	∞	∞	2,265	∞	∞	∞	∞	∞
1	∞	0	∞	∞	5,545,026	4,110,112	∞	2,420,201	5,602,692	18,416,847	∞	946,708	∞
2	∞	∞	0	∞	∞	∞	∞	∞	∞	1,367,018	∞	85,371	∞
3	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	∞
4	∞	∞	∞	4,397,058	0	∞	∞	55,760	27,847,819	1,411,030	∞	946,845	∞
5	∞	∞	∞	∞	∞	0	∞	2,370,330	∞	∞	∞	404,665	∞
6	∞	∞	∞	4,397,455	∞	∞	0	64,191	27,856,271	1,461,539	∞	954,336	∞
7	∞	∞	∞	773,888	695,496	∞	∞	0	∞	∞	∞	71,294	∞
8	∞	∞	∞	∞	∞	∞	∞	2,461,849	0	6,759,099	∞	∞	∞
9	268,126	∞	∞	∞	∞	∞	∞	94,068	∞	0	∞	∞	∞
10	7,037,048	11,475,238	∞	9,473,047	∞	713,931,861	∞	3,812,494	∞	∞	0	310,215	∞
11	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	∞
M	6,992,386	8,259,559	∞	1,918,778	∞	708,805,418	∞	3,214,399	∞	∞	∞	201,837	∞
S	17,609	4,940	2,297,702	4,400,982	3,231	6,259,627	1,738,861	57,491	6,201,908	1,439,485	0	43,815	6,259,627
Ana													
0	0	47,355	∞	1,098	∞	0	∞	856	∞	∞	∞	394	∞
1	50,292	0	∞	5,706	46,601	25,333	∞	10,165	77,961	15,965	∞	5,470	∞
2	11,563	27,342	0	3,602	8,384	3,512	20,088	1,036	1,419	2,551	∞	1,594	∞
3	∞	24,951	∞	0	∞	∞	∞	∞	∞	∞	∞	6,920	∞
4	2,729	22,672	∞	3,315	0	1,225	∞	0	6,139	∞	∞	0	∞
5	0	47,355	∞	1,098	∞	0	∞	856	∞	∞	∞	394	∞
6	1,589	49,603	∞	1,706	15,264	172	0	856	9,354	5,283	2,359	1,002	∞
7	∞	22,677	∞	∞	∞	1,252	∞	0	∞	∞	∞	376	∞
8	∞	24,925	∞	0	∞	∞	∞	∞	0	∞	∞	2,994	∞
9	150,774	6,119	∞	4,986	8,524	2,520	8,912	1,282	9,739	0	∞	837	∞
10	10,125	∞	∞	3,866	∞	∞	∞	2,206	∞	∞	0	1,018	∞
11	4,007	48,503	∞	∞	∞	2,155	∞	554	∞	∞	∞	0	∞
M	∞	24,854	∞	27	∞	∞	∞	∞	-19	∞	∞	3,016	∞
S	14,834	28,887	0	3,857	14,491	5,757	199,466	1,150	7,141	3,897	1,665	576	199,466

4.6 Conclusion

Evacuation planning is done in the presence of uncertainty without exact or complete information. Due to the fact that the place, the time and the scale of a disaster can not be easily predicted, it is very difficult to estimate the evacuation demand accurately. Similarly, road capacities and shelter sites may be affected (degraded or disrupted), but the questions of "which ones" and "how much" are difficult to answer.

In this chapter, we proposed a novel model that captures this stochasticity in evacuation planning by taking account of the uncertainty in demand, road capacities and availability of shelters. In our case study, we observed that planning in accordance with a single scenario such as the worst case scenario or most probable scenario, or solving the problem with mean value approach do not produce effective solutions and that using stochastic programming can lead to significant improvements. We further observed that when shelters are capacitated, the superiority of stochastic programming solution is emphasized, as planning in accordance with a single scenario or mean value approach will possibly generate infeasible solutions for some scenarios, unlike in stochastic programming solution.

Chapter 5

Shelter Location and Evacuation Route Assignment under Uncertainty: A Benders Decomposition Approach

To be able to model a stochastic evacuation planning problem more realistically one needs to consider a large number of scenarios. As the number of scenarios grows the extended formulation (EF) developed in Chapter 4 may not be solved within reasonable CPU times or can not be solved at all.

In this chapter, our aim is to propose an exact algorithm based on Benders decomposition [197] to solve the formulation proposed in Chapter 4 with large number of scenarios. The second stage of the model is a second order cone programming problem since the nonlinear objective function is represented with second order cone programming. To the best of our knowledge, the algorithm we propose is the first in the literature that uses duality results for second order cone programming in a Benders decomposition setting. We solve practical size problems with up to 1000 scenarios in moderate CPU times. We investigate methods such as adopting a multi-cut strategy, deriving pareto-optimal cuts, using a reduced primal subproblem and preemptive priority multiobjective program to enhance the proposed algorithm. Computational results confirm the efficiency of our algorithm as it is considerably faster and can solve instances with larger number of scenarios compared to solving the EF with an off-the-shelf solver.

The rest of the chapter is organized as follows. In Section 5.1 we present solution methodologies used for stochastic evacuation problems. In Section 5.2 we state our contributions. In Section 5.3 we propose a Benders decomposition approach and explore ways to improve it in Section 5.4. We present the computational results of our study in Section 5.5 and conclude in Section 5.6.

5.1 Solution Methodologies

There is a vast amount of literature that proposes new ideas, models or solution methodologies to support evacuation management decisions. However, despite the fact that evacuation planning is typically characterized by great uncertainties, the studies in the literature mostly rely on deterministic models that adopt a single hazard scenario such as worst-case or most probable scenario.

The evacuation studies in the literature that takes into consideration uncertainty mostly focus on demand uncertainty (Rui et al. [189], Yao et al. [8], Huibregtse et al. [190], Ng and Waller [137], Yazıcı and Özbay [120], Kulshrestha et al. [21]) and/or capacity uncertainty (Shen et al. [100], Ng and Waller [137], Yazıcı and Özbay [120]).

Shen et al. [100] develop two different scenario-based, stochastic, bilevel models that minimize the maximum UE travel time among all node shelter pairs by locating shelters at the upper level and assigning evacuees to shelters and routes in a UE manner at the lower level. To solve this first model they present a genetic algorithm based approach. The second model is proposed to model real time decision making during evacuations and a simulation based approach that uses successive shortest path algorithm is developed to solve it. Yao et al. [8] solve their model using an off-the-shelf solver. Rui et al. [189] present a model that generates an optimum transit schedule and passenger pick up routes during an emergency evacuation. They employ hybrid genetic algorithms, artificial neural network, and hill climbing heuristic algorithms to solve their model. Huibregtse et al. [190] optimize evacuation measures to increase the efficiency of an evacuation plan by considering uncertainty in demand, the behavior of people and the hazard in a scenario-based setting. They use an off-the-shelf solver to solve their problem. Ng and Waller [137] consider demand and capacity uncertainty together in a scenario-based evacuation planning model. They provide a framework that determines the amount of demand inflation and supply deflation necessary to ensure a user-specified reliability level. They solve their problem using an off-the-shelf solver for a range of nine scenarios. Yazıcı and Özbay [120] as well, take into consideration the uncertainty in demand and capacity simultaneously. They propose a CTM-based SO DTA formulation with probabilistic constraints. They use a P-Level Efficient Points method by Prékopa [198] to write the deterministic equivalent formulation of the problem and solve it with an off-the-shelf solver for three scenarios. Kulshrestha et al. [21] focuses on demand uncertainty by developing a robust bi-level model that minimizes the total cost to establish and operate shelters at the upper level while assigning evacuees to shelters and routes in a UE manner at the lower level. They confine the uncertain demand to an uncertainty set and determine the optimal locations and capacities of shelters from the worst case scenario. Their model is formulated as a mathematical program with complementarity constraints and is solved by a cutting plane algorithm, for a total of three nominal, low and high demand scenarios. Li et al. [101] propose a scenario based model that identifies a set of shelter locations among potential ones, which are robust for a range of hurricane events, by considering disruption in shelter sites. They develop heuristic algorithms based on Lagrangian relaxation and present a case study for the state of North Carolina for 33 hurricane scenarios.

In the previous chapter we introduced a novel scenario-based model that decides simultaneously on the locations of shelters and the allocations of evacuees to shelters and routes under uncertainty. The model we proposed incorporates evacuees' preferences and fairness considerations by routing the evacuees on paths that are not much longer than the shortest paths to the nearest shelters. We showed that our model can solve practical size problems exactly by using second order cone programming approach for a range of up to 50 scenarios by using an off-the-shelf solver.

5.2 Our Contribution

We base the work in this chapter on the stochastic CSO (SCSO) model we presented in the Chapter 4. The evacuation models in the literature that take into account congestion by employing a nonlinear objective function are generally solved by heuristic methodologies especially for large instances, i.e., large evacuation road networks. Against this background, we propose an exact algorithm based on Benders decomposition. We test various ways of enhancing the algorithm and report the results of our computational experiments. The results show that the algorithm can solve problems with up to 1000 scenarios and is faster compared to solving the EF with an off-the-shelf solver.

5.3 Benders Decomposition Approach

In our model the shelter location decisions y are taken in the presence of uncertainty. For that reason, they are called as first-stage (design) variables. In the second stage, the uncertainty is revealed and recourse decisions, i.e., shelter and route assignment decisions v are taken. However, while making decisions about where to locate the shelters in the first stage, their effect on the second stage assignment decisions and total evacuation time is taken into account. The future effects of shelter location decisions is measured by taking the expectation of the recourse function on possible scenarios. Our model has a 0-1 first stage problem and a nonlinear second stage problem.

To be able to model a stochastic evacuation problem more realistically, one needs to consider a large number of scenarios. As the number of scenarios increases, the EF developed in Chapter 4 may not be solved within reasonable CPU times or can not be solved at all. For that reason, we develop a Benders decomposition (BD) [197] based approach to solve our problem considering a large number of scenarios and explore methodologies to accelerate our BD algorithm. This algorithm can also be considered as an L-Shaped Algorithm developed by Van Slyke and Wets [199] to solve stochastic programs with recourse. The first stage variables in our problem, i.e., shelter location decisions, are the complicating variables and their number is less than that of the second-stage non-complicating variables. In BD approach the main idea is to project out these non-complicating variables. The resulting problem is called the master problem (MP) and it contains fewer variables but a large number of constraints. These constraints are known as Benders cuts (BC) and most of them are not active at an optimal solution. Because of this fact, the most natural strategy to solve the MP is through relaxation [200]. An iterative solution methodology is pursued, by solving the relaxed MP at every iteration and passing the optimal solution to sub-problems that are basically the dual of the original problem with shelter location decisions temporarily fixed to the values obtained from the MP and adding the violated BCs to MP until all of them are satisfied at a relaxed MP solution. Since the MP is a relaxation, its optimal value provides a lower bound on the optimal value and an upper bound is given by the expectation of the optimal values of the sub-problems.

We project out the non-complicating second-stage variables v and x through parametrization of the complicating first-stage variables y. This results in the following primal sub-problems, one for each scenario $\omega \in \Omega$.

Primal Sub Problem ($PSP(\overline{y}, \omega)$)

$$\min \sum_{a \in A(\omega)} t_a^0 \left(1 + \alpha \left(\frac{x_a(\omega)}{c_a(\omega)} \right)^{\beta} \right) x_a(\omega) \tag{5.1}$$

$$\text{s.t.} \sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^{\lambda}(\omega)} v_\pi(\omega) = 1 \qquad \forall r \in O, \tag{5.2}$$

$$\sum_{\pi \in P_{rs}^{\lambda}(\omega)} v_\pi(\omega) \leq \overline{y}_s \qquad \forall r \in O, s \in F(\omega), \tag{5.3}$$

$$\sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^{\lambda}(\omega): d^\pi > (1+\lambda) d_{ri}^*(\omega)} v_\pi(\omega) \leq 1 - \overline{y}_i \qquad \forall r \in O, i \in F(\omega), \tag{5.4}$$

$$x_a(\omega) = \sum_{r \in O} \sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^{\lambda}(\omega): a \in \pi} w_r(\omega) v_\pi(\omega) \qquad \forall a \in A(\omega), \tag{5.5}$$

$$v_\pi(\omega) \geq 0 \qquad \forall \pi \in \cup_{r \in O, s \in F(\omega)} P_{rs}^{\lambda}(\omega), \tag{5.6}$$

where $\overline{y} \in V \subseteq Y$, is a fixed vector for the complicating variables, $V \subseteq Y$ is the set of vectors y that renders $PSP(\overline{y}, \omega)$ feasible and Y is the set of all possible shelter location decisions y. Given the locations of the shelters, each sub-problem is a nonlinear CSO shelter and traffic assignment problem. We reformulate the $PSP(\overline{y}, \omega)$ as a second order conic programming model as in Chapter 4. By employing SOCP, the nonlinearity is transferred to the constraint set in the form of second order quadratic constraints. Hence, the subproblem is still a nonlinear programming problem but it can be solved efficiently once represented as SOCP. The resulting conic PSP with SOCP constraints (CPSP) is given below.

Conic Primal Subproblem ($CPSP(\overline{y}, \omega)$)

$$\min \sum_{a \in A(\omega)} \left(t_a^0 x_a(\omega) + \frac{t_a^0 \alpha}{c_a(\omega)^\beta} \mu_a(\omega) \right) \tag{5.7}$$

$$\text{s.t.} \quad \sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^\lambda(\omega)} v_\pi(\omega) = 1, \qquad \forall r \in O, \tag{5.8}$$

$$\sum_{\pi \in P_{rs}^\lambda(\omega)} v_\pi(\omega) \leq \overline{y}_s, \qquad \forall r \in O, s \in F(\omega), \tag{5.9}$$

$$\sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^\lambda(\omega): d^\pi(\omega) > (1+\lambda) d_{ri}^*(\omega)} v_\pi(\omega) \leq 1 - \overline{y}_i, \qquad \forall r \in O, i \in F(\omega), \tag{5.10}$$

$$x_a(\omega) = \sum_{r \in O} \sum_{s \in F(\omega)} \sum_{\pi \in P_{rs}^\lambda(\omega): a \in \pi} w_r(\omega) v_\pi(\omega), \qquad \forall a \in A(\omega), \tag{5.11}$$

$$x_a^{'}(\omega)^2 + \rho_a(\omega)^2 \leq \delta_a(\omega)^2, \qquad \forall a \in A(\omega), \tag{5.12}$$

$$\theta_a^{'}(\omega)^2 + \sigma_a(\omega)^2 \leq \phi_a(\omega)^2, \qquad \forall a \in A(\omega), \tag{5.13}$$

$$u_a^{'}(\omega)^2 + \gamma_a(\omega)^2 \leq \eta_a(\omega)^2, \qquad \forall a \in A(\omega), \tag{5.14}$$

$$-x_a^{'}(\omega) + 2x_a(\omega) = 0, \qquad \forall a \in A(\omega), \tag{5.15}$$

$$-\rho_a(\omega) + \theta_a(\omega) = 1, \qquad \forall a \in A(\omega), \tag{5.16}$$

$$-\delta_a(\omega) + \theta_a(\omega) = -1, \qquad \forall a \in A(\omega), \tag{5.17}$$

$$-\theta_a^{'}(\omega) + 2\theta_a(\omega) = 0, \qquad \forall a \in A(\omega), \tag{5.18}$$

$$-\sigma_a(\omega) + u_a(\omega) - x_a(\omega) = 0, \qquad \forall a \in A(\omega), \tag{5.19}$$

$$-\phi_a(\omega) + u_a(\omega) + x_a(\omega) = 0, \qquad \forall a \in A(\omega), \tag{5.20}$$

$$-u_a^{'}(\omega) + 2u_a(\omega) = 0, \qquad \forall a \in A(\omega), \tag{5.21}$$

$$-\gamma_a(\omega) - x_a(\omega) + \mu_a(\omega) = 0, \qquad \forall a \in A(\omega), \tag{5.22}$$

$$-\eta_a(\omega) + \mu_a(\omega) + x_a(\omega) = 0, \qquad \forall a \in A(\omega), \tag{5.23}$$

$$v_\pi(\omega) \geq 0, \qquad \forall \pi \in \bigcup_{r \in O, s \in F(\omega)} P_{rs}^\lambda(\omega) \tag{5.24}$$

$$x_a(\omega), x_a^{'}(\omega), \theta_a(\omega), \theta_a^{'}(\omega), u_a(\omega), u_a^{'}(\omega), \delta_a(\omega),$$
$$\eta_a(\omega), \phi_a(\omega), \mu_a(\omega) \geq 0, \qquad \forall a \in A(\omega), \tag{5.25}$$

Objective function (5.7) is modified from the original objective function as defined above and constraints (5.8)-(5.11) are the original constraints from $PSP(\overline{y}, \omega)$. Constraints (5.12)-(5.14) define the three second order quadratic cones. And constraints (5.15)-(5.23) are generated by replacing each term (the two terms on the left hand side inside the norms and the term on the right hand side) of SOCP constraints (3.18)-(3.20) by a single auxiliary variable to help derive the dual of the $CPSP(\overline{y}, \omega)$. Constraints (5.24) and (5.25) are variable restrictions.

We associate the dual variables $z_r(\omega)$, $\mathit{\Gamma}_{rs}(\omega)$, $\Lambda_{ri}(\omega)$, $\psi_a(\omega)$, $c_{1a}(\omega)$, $c_{2a}(\omega)$, $c_{3a}(\omega)$, $c_{4a}(\omega)$, $c_{5a}(\omega)$, $c_{6a}(\omega)$, $c_{7a}(\omega)$, $c_{8a}(\omega)$, $c_{9a}(\omega)$ for constraints (5.8)-(5.11), (5.15)-(5.23) respectively. And the resulting dual subproblem (DSP) is formulated as follows:

Dual Sub Problem ($DSP(\overline{y}, \omega)$)

$$\max \sum_{r \in O} z_r(\omega) + \sum_{r \in O} \sum_{s \in F(\omega)} \mathit{\Gamma}_{rs}(\omega)\overline{y}_s + \sum_{r \in O} \sum_{i \in F(\omega)} \Lambda_{ri}(\omega)(1 - \overline{y}_i)$$

$$+ \sum_{a \in A(\omega)} c_{2a}(\omega) - \sum_{a \in A(\omega)} c_{3a}(\omega) \tag{5.26}$$

$$\text{s.t. } z_r(\omega) + \mathit{\Gamma}_{rs}(\omega) + \sum_{i \in F(\omega): d^{\pi}(\omega) > (1+\lambda) d^{*}_{ri}(\omega)} \Lambda_{ri}(\omega) - \sum_{a \in A(\omega): a \in \pi} w_r(\omega)\psi_a(\omega) \leq 0, \quad \forall r \in O, s \in F(\omega), \pi \in P^{\lambda}_{rs}(\omega), \tag{5.27}$$

$$\psi_a(\omega) + 2c_{1a}(\omega) - c_{5a}(\omega) + c_{6a}(\omega) - c_{8a}(\omega) + c_{9a}(\omega) \leq t^0_a, \quad \forall a \in A(\omega), \tag{5.28}$$

$$c_{8a}(\omega) + c_{9a}(\omega) \leq \frac{t^0_a \alpha}{c_a(\omega)^{\beta}}, \quad \forall a \in A(\omega), \tag{5.29}$$

$$c_{2a}(\omega) + c_{3a}(\omega) + 2c_{4a}(\omega) \leq 0, \quad \forall a \in A(\omega), \tag{5.30}$$

$$c_{5a}(\omega) + c_{6a}(\omega) + 2c_{7a}(\omega) \leq 0, \quad \forall a \in A(\omega), \tag{5.31}$$

$$c_{1a}(\omega)^2 + c_{2a}(\omega)^2 \leq c_{3a}(\omega)^2, \quad \forall a \in A(\omega), \tag{5.32}$$

$$c_{4a}(\omega)^2 + c_{5a}(\omega)^2 \leq c_{6a}(\omega)^2, \quad \forall a \in A(\omega), \tag{5.33}$$

$$c_{7a}(\omega)^2 + c_{8a}(\omega)^2 \leq c_{9a}(\omega)^2, \quad \forall a \in A(\omega), \tag{5.34}$$

$$\mathit{\Gamma}_{rs}(\omega) \leq 0, \quad \forall r \in O, s \in F(\omega), \tag{5.35}$$

$$\Lambda_{ri}(\omega) \leq 0, \quad \forall r \in O, i \in F(\omega), \tag{5.36}$$

$$c_{3a}(\omega), c_{6a}(\omega), c_{9a}(\omega) \geq 0, \quad \forall a \in A(\omega) \tag{5.37}$$

The DSP is also a SOCP problem, i.e., it is the maximization of a linear function (5.26) over the intersection of an affine set (5.27)-(5.31) and the product of second order quadratic cones (5.32)-(5.34). Note that when the $CPSP(\overline{y}, \omega)$ is feasible, we can also find a point for which it is strictly feasible. Since it is also bounded, by strong duality theorem for SOCP problems [138], the $DSP(\overline{y}, \omega)$ is feasible and bounded and strong duality holds, i.e., $CPSP(\overline{y}, \omega)$ and $DSP(\overline{y}, \omega)$ attain the same optimal values. Since Y is a finite discrete set, the generalized BD procedure generates finitely many cuts and terminates in a finite number of steps [200].

We ensure that the MP generates shelter location decisions that render every subproblem feasible. Hence, we only add optimality cuts as deemed necessary. We write the Benders cuts (optimality cuts) as:

$$\theta \geq \sum_{\omega \in \Omega} p(\omega) \left(\sum_{r \in O} z_r^g(\omega) + \sum_{r \in O} \sum_{s \in F(\omega)} \Gamma_{rs}^g(\omega) y_s + \sum_{r \in O} \sum_{i \in F(\omega)} \Lambda_{ri}^g(\omega)(1 - y_i) \right)$$

$$+ \sum_{\omega \in \Omega} p(\omega) \left(\sum_{a \in A(\omega)} c_{2a}^g(\omega) - \sum_{a \in A(\omega)} c_{3a}^g(\omega) \right) \forall g \in G$$

where $G = \cup_{\overline{y} \in V} G(\overline{y})$ is the set of optimal multiplier vectors, $G(\overline{y})$ is the set of optimal multiplier vectors for a given $\overline{y} \in V \subseteq Y$ and θ is the surrogate variable that represents the subproblems in the MP objective function and is a lower bound on the expected total evacuation time (Van Slyke and Wets [199], Birge and Louveaux [193]).

MP is as follows:

Master Problem (MP)

$$\min \theta \tag{5.38}$$

$$\text{s.t.} \quad \sum_{s \in F} y_s = p, \tag{5.39}$$

$$\sum_{s \in \overline{F}_r(\omega)} y_s \geq 1, \qquad \forall r \in O, \omega \in \Omega \tag{5.40}$$

$$\theta \geq \sum_{\omega \in \Omega} p(\omega) \left(\sum_{r \in O} z_r^g(\omega) + \sum_{r \in O} \sum_{s \in F(\omega)} \Gamma_{rs}^g(\omega) y_s \right)$$
$$+ \sum_{\omega \in \Omega} p(\omega) \left(\sum_{r \in O} \sum_{i \in F(\omega)} \Lambda_{ri}^g(\omega)(1 - y_i) \right)$$
$$+ \sum_{\omega \in \Omega} p(\omega) \left(\sum_{a \in A(\omega)} c_{2a}^g(\omega) - \sum_{a \in A(\omega)} c_{3a}^g(\omega) \right) \qquad \forall g \in G \tag{5.41}$$

$$\theta \geq l \tag{5.42}$$

$$y_s \in \{0, 1\} \qquad \forall s \in F \tag{5.43}$$

The objective function (5.38) of MP minimizes the value of the surrogate variable that represents the expected total evacuation time. Constraint (5.39) limits the number of shelter sites open to a pre-specified number p. By adding induced constraints (5.40) in the MP we ensure that SPs are always feasible, i.e., there exists at least an open and reachable shelter for each $r \in O$, in every scenario $\omega \in \Omega$, where $\overline{F}_r(\omega)$ is the set of undisrupted and reachable shelters for demand point $r \in O$ in scenario $\omega \in \Omega$. Constraint set (5.41) are the optimality cuts. Constraints (5.42) set a lower bound l on the auxiliary variable θ. We compute such a lower bound with a very simple heuristic method. In a given scenario, we find shortest path to the closest shelter for each origin r and compute the total travel time of the vehicles on this path using the free flow travel time. Sum of the total travel times on these paths gives us a lower bound for that specific scenario. We take their expected value to compute the lower bound l. Constraints (5.43) define the types of variables.

5.4 Improving the Performance of the BD Algorithm

Since Benders [197] introduced the Benders decomposition method in 1962, many researchers have investigated methods in order to improve its performance. Geoffrion and Graves [201] propose a branch and bound framework in which they solve the MP in an ϵ-optimal fashion instead of solving it to optimality at every iteration. McDaniel and Devine [202] present a methodology that solves the LP relaxation of the integer subproblem for some initial number of iterations to generate Benders cuts to reduce the computational burden. Magnanti and Wong [203] introduce the first study on accelerating the BD algorithm by generating strong (pareto-optimal) optimality cuts from the alternate optima of the Benders subproblem. Papadakos [204], Fischetti et al. [205], Saharidis and Ierapetritou [206] and Sherali and Lunday [207] propose alternative methodologies on deriving strong nondominated Benders cuts. Van Roy [208] proposes a cross decomposition method that unifies Benders decomposition and Lagrangian relaxation into a single framework. Saharidis et al. [209] demonstrate the effectiveness of a new strategy, which they refer to as covering cut bundle generation. The method they propose is based on the idea of generating multiple cuts that involve as many complicating variables as possible or directly generating a high density pareto cut by lifting the pareto-optimal cuts [210]. Saharidis et al. [211] and Tang et al. [210] work on deriving valid inequalities to improve the lower bounds obtained by the MP. Fischetti and Lodi [212] and Rei et al. [213] show how local branching can be used to accelerate the classical BD algorithm.

5.4.1 Multicut Strategy

In our initial experiments we observed that generating a single cut aggregated from the optimal multiplier vectors of each subproblem results in slow convergence of the BD algorithm. By adding disaggregate cuts, more detailed information is given to the first stage, which often results in fewer iterations compared to the single cut method [193]. Hence, we employ a multi-cut strategy, i.e., we add an optimality cut for every subproblem related to a scenario in case a violation is identified. Therefore, for any optimal solution of $DSP(\overline{y}, \omega)$, the Benders optimality cuts are redefined as follows:

$$\theta(\omega) \geq p(\omega) \left(\sum_{r \in O} z_r^g(\omega) + \sum_{r \in O} \sum_{s \in F(\omega)} \mathit{\Gamma}_{rs}^g(\omega) y_s + \sum_{r \in O} \sum_{i \in F(\omega)} \Lambda_{ri}^g(\omega)(1 - y_i) \right)$$

$$+p(\omega) \left(\sum_{a \in A(\omega)} c_{2a}^g(\omega) - \sum_{a \in A(\omega)} c_{3a}^g(\omega) \right) \forall \omega \in \Omega, g \in G.$$

The objective function of the MP is modified as $\sum_{\omega \in \Omega} \theta(\omega)$, where $\theta(\omega)$ is a surrogate variable that represents a subproblem related to a specific scenario $\omega \in \Omega$ and we set a lower bound for each subproblem, i.e., we modify constraint (5.42) as $\theta(\omega) \geq l(\omega)$ for all $\omega \in \Omega$.

5.4.2 Implementing a Callback Routine

In classic implementation of BD, the current relaxed MP is solved to optimality at every iteration of the algorithm and for that reason a search tree is generated from scratch every time the relaxed MP is solved [214]. Consequently, valuable time may be expended towards proving the optimality of a solution that is going to be cut off and re-evaluating the same nodes that have already been visited in previous iterations.

An outline of classical Benders [197] decomposition method for SCSO model is depicted in Figure 5.1 and Algorithm 1, in which $v(MP)$ and $v(SP)$ denote the current optimal value of the MP and SP, respectively; UB and LB are the current upper and lower bounds respectively and ϵ is a given gap tolerance. Furthermore $\overline{y}^k$ and $(z^k, \mathit{\Gamma}^k, \Lambda^k, \psi^k, (c_1^k) - (c_9^k))$ are the solutions from the MP and DSP at iteration k, respectively and G^k is the restricted set of optimal multiplier vectors of G generated up to iteration k.

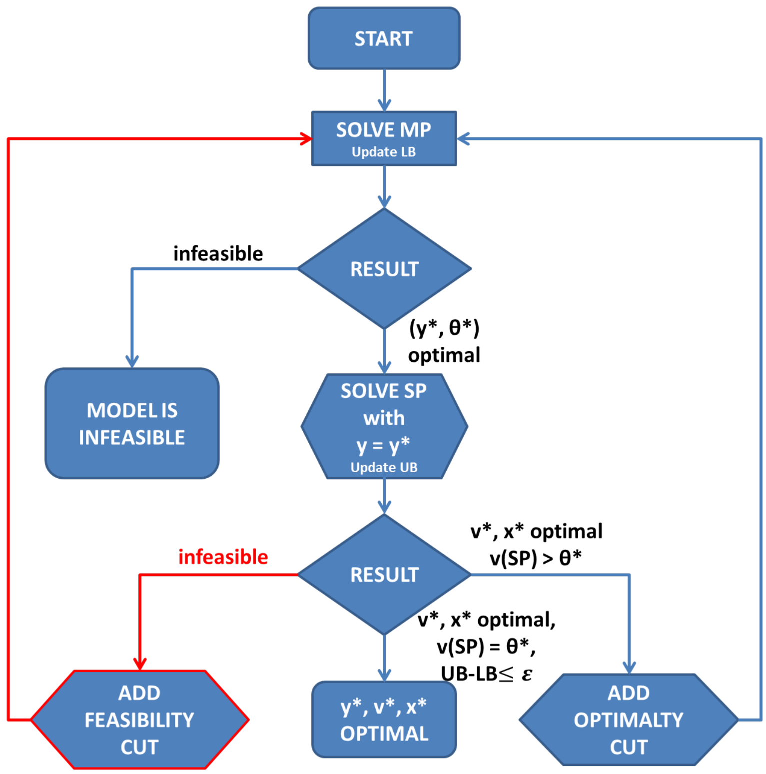


Figure 5.1: Classical Benders Decomposition Algorithm

The idea of solving the MP only once in the form of BD has been first presented by Thorsteinsson [215] as a Branch and Check framework and has been realized by the lazy constraint technology offered by the state-of-the-art solvers allowing us to execute the entire algorithm on a single search tree [216]. The lazy constraint callback routine is invoked for every candidate integer incumbent solution. The candidate incumbent solution is either certified as valid by the callback procedure or a Benders cut violated by this candidate incumbent solution is identified and added as a lazy constraint into the model. As a result, as the algorithm proceeds, no integer solution is evaluated more than once. Further, by informing the solver that the added cuts are lazy, the capabilities of the solver are fully exploited and the generated Benders cuts are added by the solver whenever necessary [3]. The BD algorithm with callback routine is illustrated in Figure 5.2 and Algorithm

Algorithm 1: Classical Benders Decomposition Algorithm

Let $UB = +\infty$, $LB = -\infty$, $k = 1$
while $UB - LB > \epsilon$ **do**
begin
  Solve the MP (5.38)-(5.43)
  $LB \leftarrow v(MP)$
  $\overline{y}^k \leftarrow y^*$ and $\overline{\theta}(\omega)^k \leftarrow \theta(\omega)^*$
  for $\omega \in \Omega$ **do**
  begin
    Solve the $SP(\overline{y}^k, \omega)$ (5.7)-(5.25)
    if $\overline{\theta}(\omega)^k < p(\omega)v(SP(\overline{y}^k, \omega)$ **then**
    begin
      $G^{k+1} \leftarrow G^k \cup (z^k, \mathit{\Gamma}^k, \Lambda^k, \psi^k, (c_1^k) - (c_9^k))$ to generate an optimality cut
    end
  end
  $UB \leftarrow \min \left\{UB, \sum_{\omega\in\Omega} p(\omega)v(SP(\overline{y}^k, \omega))\right\}$
  $k \leftarrow k + 1$
end

2. We refer to the version of BD algorithm in which we solve the basic DSP employing the multicut strategy and the callback routine as BD_DSP.

5.4.3 Defining Strong (Pareto-Optimal) Cuts

When the PSP of BD algorithm is a network optimization problem such as facility location on networks, shortest route and transshipment, it is commonplace to get degenerate solutions to the Benders PSP, which leads to multiple optimal solutions for the DSP [203]. Due to this fact, cuts of different strength can be generated. Although any of these are valid optimality cuts, defining strong (pareto-optimal) ones at every iteration of the algorithm may significantly decrease the number of iterations and hence improve the overall solution time.

As the $PSP(\overline{y}, \omega)$ is a network optimization problem, i.e., a shelter and route assignment problem, the selection of good cuts at every iteration is an issue for our problem, as well. We therefore need to define a relation comparing

Algorithm 2: Benders Decomposition Algorithm with Callback Routine

Let $UB = +\infty$, $LB = -\infty$, $k = 1$
while $UB - LB > \epsilon$ **do**
begin
  Solve the MP (5.38)-(5.43)
  $LB \leftarrow v(MP)$
  $\overline{y}^k \leftarrow \overline{y}$ and $\overline{\theta}(\omega)^k \leftarrow \overline{\theta}(\omega)$ incumbent soln
  ctrl = true
  for $\omega \in \Omega$ **do**
  begin
    Solve the $SP(\overline{y}^k, \omega)$ (5.7)-(5.25)
    if $\overline{\theta}(\omega)^k < p(\omega)v(SP(\overline{y}^k, \omega))$ **then**
    begin
      ctrl = false
      $G^{k+1} \leftarrow G^k \cup (z^k, \mathit{\Gamma}^k, \Lambda^k, \psi^k, (c_1^k) - (c_9^k))$ to generate an optimality cut
    end
  end
  if *ctrl* **then**
  begin
    $UB \leftarrow \min\left\{UB, \sum_{\omega\in\Omega} p(\omega)v(SP(\overline{y}^k, \omega))\right\}$
  end
  $k \leftarrow k + 1$
end

the strength of cuts corresponding to different multiplier vectors (dual variables) $(z, \mathit{\Gamma}, \Lambda, \psi, c_2, c_3)$. Given two multiplier vectors $(z^1, \mathit{\Gamma}^1, \Lambda^1, \psi^1, c_2^1, c_3^1)$ and $(z^2, \mathit{\Gamma}^2, \Lambda^2, \psi^2, c_2^2, c_3^2)$ it is said that the first dominates the second if and only if:

$$\sum_{r\in O} z_r^1(\omega) + \sum_{r\in O}\sum_{s\in F(\omega)} \mathit{\Gamma}_{rs}^1(\omega) y_s + \sum_{r\in O}\sum_{i\in F(\omega)} \Lambda_{ri}^1(\omega)(1-y_i) + \sum_{a\in A(\omega)} c_{2a}^1(\omega) - \sum_{a\in A(\omega)} c_{3a}^1(\omega) \geq$$

$$\sum_{r\in O} z_r^2(\omega) + \sum_{r\in O}\sum_{s\in F(\omega)} \mathit{\Gamma}_{rs}^2(\omega) y_s + \sum_{r\in O}\sum_{i\in F(\omega)} \Lambda_{ri}^2(\omega)(1-y_i) + \sum_{a\in A(\omega)} c_{2a}^2(\omega) - \sum_{a\in A(\omega)} c_{3a}^2(\omega)$$

$\forall y \in V$ and strict for at least one point $y \in V$. A cut is said to be pareto-optimal, if it is not dominated by any other cut. Let y^0 be a point in the relative interior

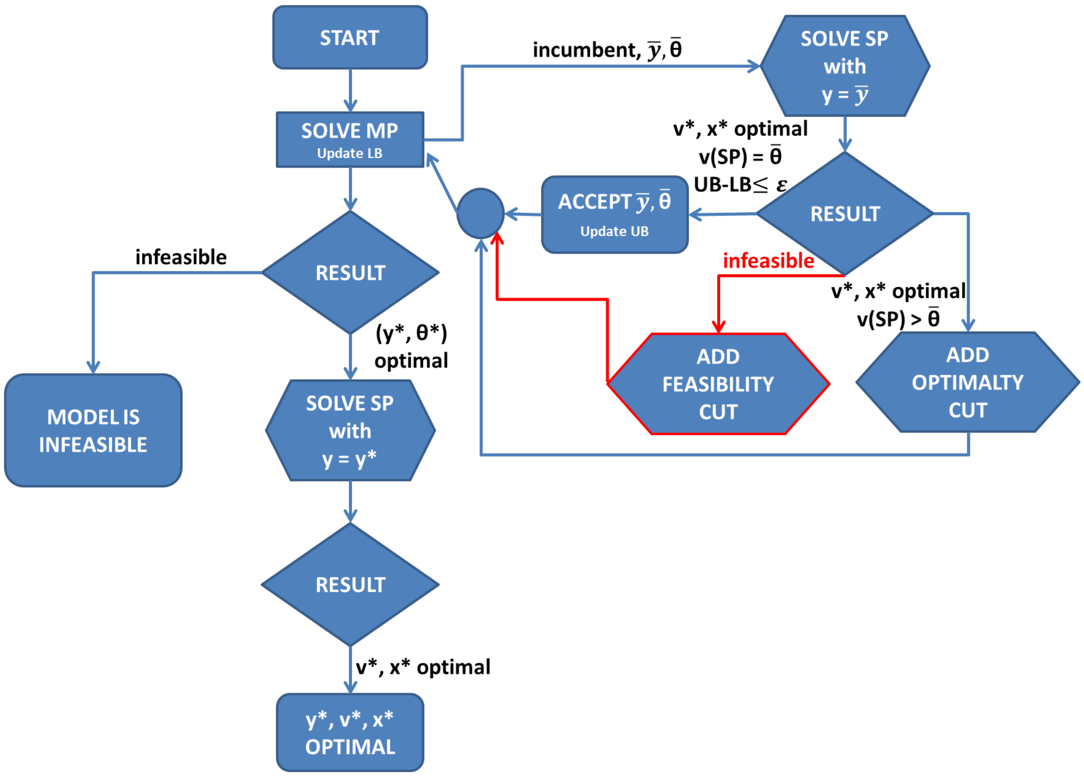


Figure 5.2: Benders Decomposition Algorithm with Callback Routine, [3]

of the convex hull of feasible location vectors and $v(DSP(\overline{y}, \omega))$ be the optimal value of $DSP(\overline{y}, \omega)$. To generate a pareto-optimal cut, we solve the following auxiliary problem:

Magnanti Wong (MW) Problem, Magnanti and Wong [203]

$$\begin{aligned} \max \sum_{r \in O} z_r(\omega) + \sum_{r \in O} \sum_{s \in F(\omega)} \mathit{\Gamma}_{rs}(\omega) y_s^0 + \sum_{r \in O} \sum_{i \in F(\omega)} \Lambda_{ri}(\omega)(1 - y_i^0) \\ + \sum_{a \in A(\omega)} c_{2a}(\omega) - \sum_{a \in A(\omega)} c_{3a}(\omega) \end{aligned} \tag{5.44}$$

$$\text{s.t.} \quad (5.27) - (5.37)$$

$$\begin{aligned} \sum_{r \in O} z_r(\omega) + \sum_{r \in O} \sum_{s \in F(\omega)} \mathit{\Gamma}_{rs}(\omega) \overline{y}_s + \sum_{r \in O} \sum_{i \in F(\omega)} \Lambda_{ri}(\omega)(1 - \overline{y}_i) \\ + \sum_{a \in A(\omega)} c_{2a}(\omega) - \sum_{a \in A(\omega)} c_{3a}(\omega) = v(DSP(\overline{y}, \omega)) \end{aligned} \tag{5.45}$$

Constraint (5.45) in the dual auxiliary problem MW ensures that one chooses an optimal multiplier vector from among alternative ones. And the objective function of the MW problem chooses among these multiplier vectors the one that generates the strongest cut to be added to MP. Algorithm 3 depicts how our BD algorithms proceeds when we employ the MW auxiliary problem. We denote this algorithm by BD_DSP_MW.

Algorithm 3: BD Algorithm with MW Problem

Let $UB = +\infty$, $LB = -\infty$, $k = 1$
while $UB - LB > \epsilon$ **do**
begin
    Solve the MP (5.38)-(5.43)
    $LB \leftarrow v(MP)$
    $\overline{y}^k \leftarrow \overline{y}$ and $\overline{\theta}(\omega)^k \leftarrow \overline{\theta}(\omega)$ incumbent soln
    ctrl = true
    for $\omega \in \Omega$ **do**
    begin
        Solve the $SP(\overline{y}^k, \omega)$ (5.7)-(5.25)
        if $\overline{\theta}(\omega)^k < p(\omega)v(SP(\overline{y}^k, \omega))$ **then**
        begin
            ctrl = false
            Find a MW core point y^0
            Use y^0 and $v(SP(\overline{y}^k, \omega))$ to solve the MW problem
            $G^{k+1} \leftarrow G^k \cup (z^k, \Gamma^k, \Lambda^k, \psi^k, (c_1^k) - (c_9^k))$ to generate an optimality cut
        end
    end
    if *ctrl* **then**
    begin
        $UB \leftarrow \min\left\{UB, \sum_{\omega\in\Omega} p(\omega)v(SP(\overline{y}^k, \omega))\right\}$
    end
    $k \leftarrow k + 1$
end

Papadakos [204] points out that the dependency of MW method on the Benders subproblem and on a MP core point may sometimes decrease the performance of the algorithm and that it may not always be easy to find a readily available core point. Another issue with the MW problem they discuss is the numerical unboundedness caused by constraint (5.45), which they show can be eliminated from the MW problem in order to generate a pareto-optimal cut. That way an enhanced MW (EMW) method independent of the subproblem, which enables adding a useful cut before solving the MP is proposed.

Papadakos [204] prove that y^0 does not have to be a core point or even a point of V. Further, since y^0 only modifies the objective function and does not alter the feasible region of the MW problem, choosing a y^0 which is not in $ri(CH(V))$ still generates a valid optimality cut, although it may not be pareto-optimal. For our case, we can choose y^0 in such a way that, $y^0 \in cone(p - \{0,1\}^{|F|}) = [0,\infty)^{|F|} \supseteq Y$, as proposed in the study by Papadakos [204]. We refer to the BD algorithm in which we employ the EMW method as BD_DSP_EMW. We start this algorithm with $y^0 = 1$ and update this point at every iteration using the equation, $y_i^0 = \frac{1}{2}y_i^0 + \frac{1}{2}\overline{y}_i$. In Algorithm 4 we show how our BD_DSP_EMW algorithm works.

Both BD_DSP_MW and BD_DSP_EMW algorithms solve an auxiliary dual subproblem in order to generate the pareto-optimal cuts. The main drawback of these two algorithms is that one has to solve the dual subproblem and the MW auxiliary problem at every iteration for every scenario, which may result in long CPU times. However, since the EMW problem is independent of the subproblem, we can take advantage of adding initial cuts to the MP before we begin solving it and then continue as in the Algorithm 2 or apply the procedure in Algorithm 4 without lines 17 and 18. We denote this algorithm by BD_DSP_IC.

Sherali and Lunday [207] propose a procedure that generates maximal nondominated Benders cuts. Instead of solving an auxiliary MW problem evey time you solve a DSP, which brings a computational burden and increases the CPU times, they consider solving a preemptive priority multiple objective program, where they wish to solve the original DSP optimally with the first priority, and among alternative optimal solutions to this problem, they wish to maximize (5.44). They also point out that, there exists a $\zeta > 0$ small enough such that this preemptive priority multiple objective program can be equivalently represented as a weighted-sum problem [217], which is also equivalent to solving the PSP with right hand sides perturbed. Within this context, the DSP we solve can be modified as follows:

Modified Sherali and Lunday DSP

$$\max\ (5.26) + \zeta\left(\sum_{r\in O} z_r(\omega) + \sum_{r\in O}\sum_{s\in F(\omega)} \Gamma_{rs}(\omega) y_s^0 + \sum_{r\in O}\sum_{i\in F(\omega)} \Lambda_{ri}(\omega)(1-y_i^0)\right)$$
$$+\,\zeta\left(\sum_{a\in A(\omega)} c_{2a}(\omega) - \sum_{a\in A(\omega)} c_{3a}(\omega)\right) \tag{5.46}$$
$$\text{s.t. } (5.27) - (5.37) \tag{5.47}$$

where y_s^0, is a positive weight vector, i.e., a positive core point solution as we defined previously. We begin with a core point and update it as we described in EMW method. We take $\zeta = 10^{-11}$. Our algorithm proceeds as we described in Algorithm 2 and is denoted by BD_DSP_SL.

Fischetti et al. [205] propose a new selection criterion for Benders cuts, in particular when both violations of feasibility and optimality cuts exist. They represent the PSP as a pure feasibility problem. Given a MP solution $(\overline{y}, \overline{\theta}(\omega))$ a violated cut can be identified if and only if the following modified subproblem is infeasible:

PSP as a Pure Feasibility Problem

$$\min\ 0 \tag{5.48}$$
$$\text{s.t. } \sum_{a\in A(\omega)}\left(t_a^0 x_a(\omega) + \frac{t_a^0 \alpha}{c_a(\omega)^\beta}\mu_a(\omega)\right) \le \overline{\theta}(\omega), \tag{5.49}$$
$$(5.8) - (5.25) \tag{5.50}$$

We associate the dual variable $(-\xi)$ to newly added constraint (5.49). Equivalently a violated cut can be generated if and only if the following modified DSP is unbounded:

Modified Fischetti DSP

$$\max\ (5.26) - \xi\overline{\theta}(\omega) \tag{5.51}$$

$$\text{s.t.}\ \ (5.27)$$

$$\psi_a(\omega) + 2c_{1a}(\omega) - c_{5a}(\omega) + c_{6a}(\omega) - c_{8a}(\omega) + c_{9a}(\omega) \leq \xi t_a^0, \quad \forall a \in A(\omega), \tag{5.52}$$

$$c_{8a}(\omega) + c_{9a}(\omega) \leq \xi \frac{t_a^0 \alpha}{c_a(\omega)^{\beta}}, \qquad \forall a \in A(\omega), \tag{5.53}$$

$$(5.30) - (5.37)$$

$$\sum_{r \in O} z_r(\omega) + \sum_{r \in O} \sum_{s \in F(\omega)} (\mathit{\Gamma}_{rs}(\omega) + \Lambda_{rs}(\omega)) + \sum_{a \in A(\omega)} (\psi_a(\omega) + c_{1a}(\omega) + ... + c_{9a}(\omega)) = 1 \tag{5.54}$$

$$\xi \geq 0, \tag{5.55}$$

In this formulation constraint (5.54) normalizes the unbounded objective function, i.e., the cut violation to be maximized, to a positive value. If the optimal value of this modified DSP is positive, we generate and add the cut (5.26) $\leq \xi\overline{\theta}(\omega)$. We denote the algorithm in which we use the above methodology by BD_FP.

Solving the DSP instead of PSP yields much better results in terms of the CPU time. Due to this reason, in our algorithms we solve the dual of the subproblems to generate optimality cuts. However, once we have an incumbent solution $\overline{y}$ from the MP, constraints (5.9) and (5.10) in PSP become redundant when we modify constraints (5.8) and (5.11). The resulting reduced primal subproblem (RPSP) is formulated as follows:

Reduced Primal Subproblem

$$\min \sum_{a \in A(\omega)} t_a^0 \left(1 + \alpha \left(\frac{x_a(\omega)}{c_a(\omega)}\right)^{\beta}\right) x_a(\omega) \tag{5.56}$$

$$\text{s.t.} \sum_{\pi \in P_r'(\omega, \overline{y})} v_\pi(\omega) = 1 \qquad \forall r \in O, \tag{5.57}$$

$$x_a(\omega) = \sum_{r \in O(\omega)} \sum_{\pi \in P_r'(\omega,\overline{y}): a \in \pi} w_r(\omega) v_\pi(\omega) \qquad \forall a \in A(\omega), \tag{5.58}$$

$$v_\pi(\omega) \geq 0 \qquad \forall \pi \in \cup_{r \in O} P_r^{'}(\omega, \overline{y}), \tag{5.59}$$

where $P_r'(\omega, \overline{y}) = \{\pi \in \cup_{s \in F'(\omega,\overline{y})} P_{rs}^{\lambda}(\omega) : d^\pi(\omega) \leq (1+\lambda) \min_{i \in F'(\omega,\overline{y})} d_{ri}^*(\omega)\}$ is the set of alternative paths between origin r and open and functioning shelters in scenario ω and $F'(\omega, \overline{y}) = \{s \in F(\omega) : y_s = 1\}$ is the open and functioning shelters in scenario $\omega \in \Omega$. This RPSP has a considerably smaller number of variables and constraints, and employing it in the BD algorithm may result in smaller CPU times, instead of solving the original PSP. We combine this with the MW or EMW procedure to generate optimality cuts and denote the resulting algorithm with *BD_RPSP_MW*.

5.5 Computational Study

In our early experiments we observed that the solution times of the classical Benders decomposition where the master problem is solved as an integer problem at each iteration is much worse than our implementation in which we employ a lazy constraint callback. Likewise the versions of our BD algorithm that employ the aggregate cut rather than a multi-cut strategy and/or solving the PSP instead of the DSP to generate the optimality cuts are not promising in terms of CPU times, either. For both *BD_DSP_MW* and *BD_DSP_EMW* algorithms, we notice that the number of iterations and the total number of optimality cuts generated generally decreases compared to the *BD_DSP*. However, the CPU times worsen as a result of solving the dual subproblem and the MW auxiliary problem, at every iteration and for every subproblem when there is a violation. Another algorithm with similar results is BD_FP. Unfortunately this methodology does not yield solution in good CPU times. Because of the same reasons we explained above for the MW and EMW versions of our algorithms, the solution times of BD_RPSP_MW algorithm in which we solve a reduced primal subproblem is not promising either. Hence, we do not report our computational studies for these algorithms.

Among all the versions we experimented on, there are three algorithms that solve our problem in considerably good solution time. Below we summarize these algorithms.

Algorithm BD_DSP**:** In this version of our BD algorithm we solve the DSP to generate the optimality cuts implementing the lazy constraint callback feature of ILOG CPLEX and we adopt a multi-cut strategy. We follow the procedures as depicted in Algorithm 2.

Algorithm BD_DSP_IC**:** This is the version of our algorithm in which we solve the DSP to identify the violated optimality cuts, with a lazy constraint callback and multi-cut strategy. In this algorithm we take the advantage of the EMW problem that can be solved independently from the DSP and solve the EMW problem only once before we begin solving the MP to generate an initial set of valid cuts. We set the core point $y^0 = 1$ to generate the initial set of cuts. We apply the procedure in Algorithm 4 without lines 17 and 18.

Algorithm BD_DSP_SL**:** We employ the DSP, lazy constraint callback and multi-cut strategy for this algorithm, as well. We solve the original DSP optimally with the first priority, and among alternative optimal solutions to this problem, we maximize (5.44). To achieve this, we use a weighted-sum of (5.26) and (5.44) with a weight vector of $(1, \zeta)$, as our modified objective function of DSP. We begin with a core point $y^0 = 1$ and update it at every iteration using the equation $y_i^0 = \frac{1}{2}y_i^0 + \frac{1}{2}\overline{y}_i$. We take $\zeta = 10^{-11}$. Our algorithm proceeds as we described in Algorithm 2.

We generated our instances, i.e., Istanbul Anatolian and Istanbul European instances, using the real data from a disaster prevention and mitigation study conducted by the Istanbul Metropolitan Municipality (IMM) and Japan International Cooperation Agency (JICA) [2] for earthquake preparedness and response planning for an impending major earthquake in Istanbul, Turkey, as in Chapter 4.

The specifics of the instances used in the computational study are shown in Table 5.1. Here $|O-F|$ is the number of origin destination pairs that are connected with a directed path in the original undisrupted graph.

We perform our computational tests on a workstation with 2 Xeon E5-2609 4C 2.4GHz CPU and 96GB RAM by using Java ILOG CPLEX version 12.5.1. We used the geographical distances as arc lengths in our analysis. Instead, one may try to estimate the congested travel times. Our model permits the use of these estimates as normal lengths.

In Table 5.2 and 5.3 we compare the computational efficiencies of the three Benders decomposition algorithms and the EF for different values of p and λ, where $\#Scen$ is the number of scenarios used in that specific instance. For each instance, we report the gap between the UB and LB at a solution, the number of iterations the BD algorithm performed, the number of optimality cuts added and the solution times. We set a time limit of 5 hours for our experiments. We are able to solve the Istanbul Anatolian instances with up to 1000 scenarios without any memory problems using any of the BD algorithms. For all of the Istanbul Anatolian instances, BD algorithms solve the problem to optimality. On the other hand, using the EF with CPLEX does not generate a solution within 5 hours of time limit even at the root node, for the instances with 1000 scenarios. All of the BD algorithms perform much better compared to the EF in terms of the CPU times. The BD_DSP algorithm performs at least 1.38 times better than the EF, this rate increases up to 18.78 and marks 5.42 on the average, not including the actual solution times of the EF for the instances with 1000 scenarios since these instances hit the time limit. The BD_DSP_SL algorithm performs even better, with 1.52, 19.85 and 5.77 values as the minimum, maximum and average rates

Algorithm 4: BD Algorithm with EMW Method

Let $UB = +\infty$, $LB = -\infty$, $k = 1$
while $UB - LB > \epsilon$ **do**
begin
  Find a MW core point y^0
  Use y^0 to solve the EMW problem
  $G^{k+1} \leftarrow G^k \cup (z^k, \Gamma^k, \Lambda^k, \psi^k, (c_1^k) - (c_9^k))$ to generate a valid cut
  Solve the MP (5.38)-(5.43)
  $LB \leftarrow v(MP)$
  $\overline{y}^k \leftarrow \overline{y}$ and $\overline{\theta}(\omega)^k \leftarrow \overline{\theta}(\omega)$ incumbent soln
  ctrl = true
  for $\omega \in \Omega$ **do**
  begin
    Solve the $SP(\overline{y}^k, \omega)$ (5.7)-(5.25)
    if $\overline{\theta}(\omega)^k < p(\omega)v(SP(\overline{y}^k, \omega))$ **then**
    begin
      ctrl = false
      Find a MW core point y^0
      Use y^0 to solve the EMW problem
      $G^{k+1} \leftarrow G^k \cup (z^k, \Gamma^k, \Lambda^k, \psi^k, (c_1^k) - (c_9^k))$ to generate an optimality cut
    end
  end
  if *ctrl* **then**
  begin
    $UB \leftarrow \min \left\{UB, \sum_{\omega \in \Omega} p(\omega)v(SP(\overline{y}^k, \omega))\right\}$
  end
  $k \leftarrow k + 1$
end

Table 5.1: Specifics of the Instances Used in the Computational Study

Instance	|N|	|A|	|O|	|F|	Total Demand	|O-F|
Istanbul Anatolian	50	146	13	17	83,133	221
Istanbul European	80	238	25	32	272,900	800

respectively. These numbers for *BD_DSP_IC* algorithm are 1.85, 20.53, 5.73 respectively. The *BD_DSP_SL* generally performs better than the *BD_DSP* algorithm for the Istanbul Anatolian case. In 38 of the 45 total instances for Istanbul Anatolian network *BD_DSP_SL* has smaller CPU times and generally ends in smaller number of iterations adding smaller number of optimality cuts. This rate is 28 to 45 for *BD_DSP_IC* algorithm. *BD_DSP_IC* performs better than *BD_DSP_SL* in 20 of 45 instances in terms of CPU times. For all of the instances of the Istanbul Anatolian network *BD_DSP* and *BD_DSP_SL* achieve the same optimal results. In Figure 5.3 we illustrate how the UB and LB are updated across iterations for *BD_DSP*, *BD_DSP_SL* and *BD_DSP_IC* algorithms when the number of scenarios is 1000, $\lambda = 0.2$, and $p = 10$.

Table 5.2: Comparison of Different Algorithms w.r.t. Computational Effectiveness, Istanbul Anatolian Instances

			BD_DSP				*BD_DSP_SL*				*BD_DSP_IC*				*EF(SCSO)*	
# Scen	p	λ	Gap	#Iter	# Opt. Cuts	Sol. Time	Gap	#Iter	# Opt. Cuts	Sol. Time	Gap	#Iter	# Opt. Cuts	Sol. Time	Gap	Sol. Time
50	5	0	0	94	4,012	242	0	93	3,964	224	0	88	3,851	226	0	1,120
50	10	0	0	92	3,844	229	0	91	3,840	214	0	96	3,674	236	0	544
50	15	0	0	21	745	54	0	21	745	49	0	20	703	47	0	170
50	5	0.05	0	79	3,282	214	0	86	3,289	232	0	91	4,025	255	0	1,281
50	10	0.05	0	116	4,457	302	0	116	4,457	290	0	94	3,799	247	0	457
50	15	0.05	0	20	693	55	0	20	693	48	0	15	565	38	0	94
50	5	0.1	0	92	4,171	271	0	92	4,171	269	0	94	4,321	270	0	1,266
50	10	0.1	0	97	4,190	261	0	97	4,193	256	0	86	3,890	245	0	567
50	15	0.1	0	21	713	58	0	21	713	53	0	15	559	41	0	90
50	5	0.15	0	93	4,030	309	0	64	2,795	206	0	71	3,144	220	0	1,061
50	10	0.15	0	99	3,824	264	0	99	3,824	259	0	93	3,663	255	0	621
50	15	0.15	0	24	777	67	0	24	777	61	0	14	533	40	0	93
50	5	0.2	0	104	4,514	368	0	104	4,514	364	0	97	4,481	356	0	2,062
50	10	0.2	0	95	4,075	270	0	96	4,125	266	0	90	3,891	258	0	776
50	15	0.2	0	16	642	50	0	16	642	43	0	16	636	45	0	113
100	5	0	0	62	4,985	325	0	62	4,985	307	0	66	5,172	335	0	6,100
100	10	0	0	87	6,095	413	0	87	6,095	398	0	91	6,864	459	0	1,693
100	15	0	0	24	1,775	124	0	24	1,775	111	0	22	1,757	105	0	600
100	5	0.05	0	79	6,575	421	0	79	6,575	414	0	66	5,383	370	0	5,525
100	10	0.05	0	87	7,112	466	0	87	7,112	434	0	87	6,864	465	0	2,322
100	15	0.05	0	30	2,269	159	0	30	2,269	147	0	31	2,275	169	0	540
100	5	0.1	0	98	8,258	575	0	98	8,258	565	0	69	6,476	427	0	8,767
100	10	0.1	0	131	10,589	688	0	130	10,484	664	0	120	9,371	662	0	2,176
100	15	0.1	0	23	1,856	128	0	23	1,856	116	0	22	1,908	125	0	487
100	5	0.15	0	81	7,585	521	0	80	7,381	501	0	96	8,473	607	0	6,380
100	10	0.15	0	93	7,660	518	0	95	7,496	534	0	124	9,895	688	0	2,231
100	15	0.15	0	25	1,730	147	0	25	1,730	130	0	19	1,533	105	0	581
100	5	0.2	0	66	5,883	487	0	66	5,883	472	0	68	5,685	519	0	5,777
100	10	0.2	0	90	7,994	545	0	90	7,994	503	0	96	8,001	603	0	2,397
100	15	0.2	0	9	799	62	0	9	799	50	0	11	791	63	0	537
1000	5	0	0	69	58,667	3,719	0	57	48,741	2,919	0	54	45,825	3,026	NS	18,000*
1000	10	0	0	96	71,622	4,944	0	96	71,693	4,801	0	89	65,506	4,909	NS	18,000*
1000	15	0	0	36	20,521	1,864	0	36	20,521	1,750	0	36	21,752	1,889	NS	18,000*
1000	5	0.05	0	67	54,698	4,045	0	61	51,723	3,438	0	78	63,416	4,886	NS	18,000*
1000	10	0.05	0	80	66,505	4,542	0	105	77,038	5,876	0	74	58,588	4,244	NS	18,000*
1000	15	0.05	0	25	19,557	1,394	0	25	19,557	1,355	0	18	13,498	986	NS	18,000*
1000	5	0.1	0	66	57,680	4,358	0	72	62,446	4,594	0	56	47,557	3,630	NS	18,000*
1000	10	0.1	0	74	66,560	4,457	0	82	71,534	4,894	0	132	106,110	8,413	NS	18,000*
1000	15	0.1	0	32	23,556	1,849	0	48	23,573	2,625	0	45	30,612	2,685	NS	18,000*
1000	5	0.15	0	75	64,640	5,224	0	71	61,675	4,936	0	71	59,705	5,196	NS	18,000*
1000	10	0.15	0	102	83,721	6,602	0	135	85,520	8,129	0	119	95,995	7,745	NS	18,000*
1000	15	0.15	0	30	19,234	1,819	0	30	19,235	1,772	0	31	19,860	1,928	NS	18,000*
1000	5	0.2	0	76	62,679	5,986	0	75	61,703	5,620	0	67	55,737	5,313	NS	18,000*
1000	10	0.2	0	101	91,607	6,726	0	84	74,192	5,504	0	117	94,250	7,583	NS	18,000*
1000	15	0.2	0	18	13,669	1,148	0	18	13,669	1,014	0	16	13,628	974	NS	18,000*

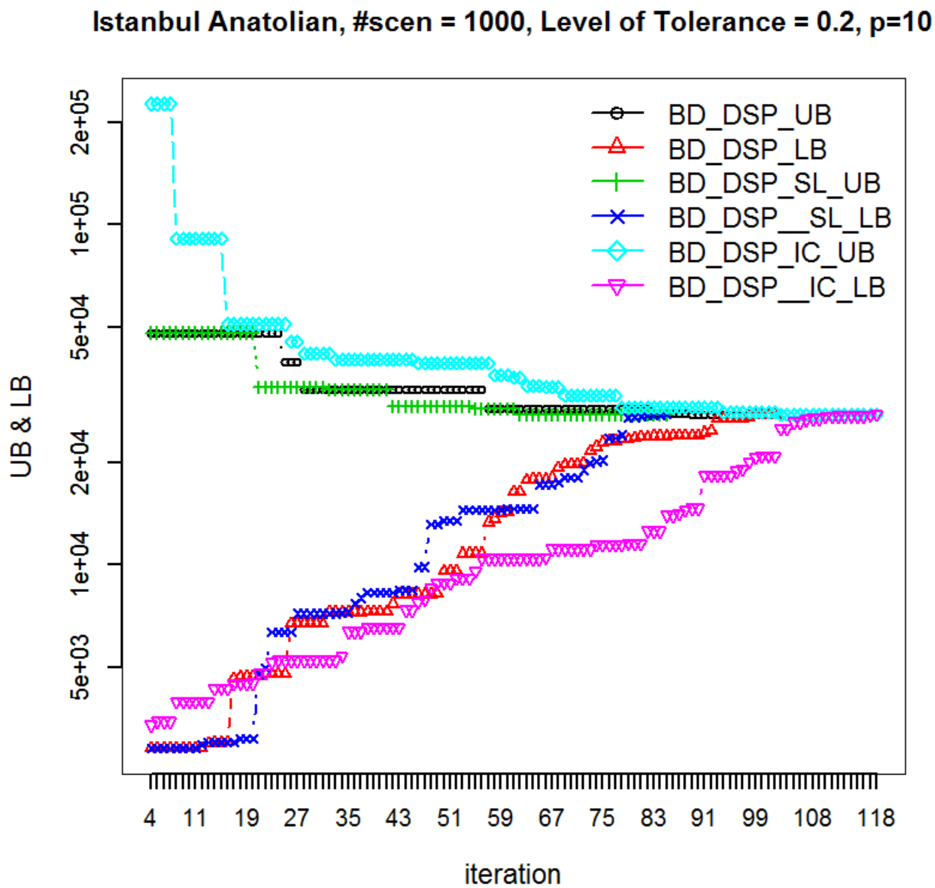


Figure 5.3: Iteration vs. UB & LB for Istanbul Anatolian Network, $\#Scen = 1000$, $\lambda = 0.2$, $p = 10$

For the Istanbul European network, EF runs into memory problems for the instances ($\#Scen = 50, \lambda = 0.2$), ($\#Scen = 100, \lambda \geq 0.15$) and ($\#Scen = 1000, \lambda \geq 0.1$). When the number of scenarios is 100, $\lambda = 0.05$ and $p = 5$ and $p = 10$, the EF can obtain a solution with a gap of 97 % and 98 % respectively, within the 5 hour time limit. When $\lambda = 0.1$ and $p = 10$ EF can not obtain a solution at the root node within the time limit. For all of the instances that the EF does not encounter the memory issue when the number of scenarios is 1000, there is no solution obtained at the root node within the time limit. BD algorithms run into memory problems for the instances ($\#Scen = 100, \lambda = 0.2$) and ($\#Scen = 1000, \lambda \geq 0.15$). Excluding the instances with memory problems, BD_DSP and

BD_DSP_IC algorithms solve all of the the instances to optimality except for two instances with ($\#Scen = 1000, \lambda = 0.1, p = 10, p = 25$) where they run into the time limit. BD_DSP_SL algorithm solves all of these instances to optimality. All of the BD algorithms outperforms the EF in terms of the memory problems and the CPU times. Except for the single instance with $\#Scen = 50$, $\lambda = 0.15$ and $p = 30$ where EF performs better and excluding the instances with memory problems and the ones that hit the time limit, BD_DSP algorithm performs at least 1.04 times better than the EF, this rate increases up to 11.04 and marks 4.21 on the average. For the BD_DSP_SL algorithm these numbers are 1.09, 11.38 and 4.25 as the minimum, maximum and average rates respectively. These numbers are 1.36, 10.70 and 4.0 for BD_DSP_IC, respectively and BD_DSP_IC is the single algorithm that outperforms EF in every instance. In 22 of the 36 instances the BD_DSP_SL algorithm performs relatively better compared to the BD_DSP in terms of CPU times. This rate is 15 to 36 for BD_DSP_IC algorithm. And BD_DSP_IC performs better than BD_DSP_SL in 13 of 36 instances in terms of CPU times. The algorithms outperform each other for different instances in terms of the number of iterations. BD_DSP adds generally less number of optimality cuts with small number of scenarios, but starting with bigger λ values when the number of scenarios is 100, and for bigger number of scenarios, BD_DSP_SL adds generally less number of optimality cuts. For all of the instances of the Istanbul European network BD_DSP and BD_DSP_SL achieve the same optimal results, as well.

Table 5.3: Comparison of Different Algorithms w.r.t. Computational Effectiveness, Istanbul European Instances

			BD_DSP				BD_DSP_SL				BD_DSP_IC				EF(SCSO)	
# Scen	p	λ	Gap	#Iter	# Opt. Cuts	Sol. Time	Gap	#Iter	# Opt. Cuts	Sol. Time	Gap	#Iter	# Opt. Cuts	Sol. Time	Gap	Sol. Time
50	10	0	0	114	4,930	603	0	114	4,930	601	0	136	5,584	743	0	2,532
50	25	0	0	210	8,823	1,076	0	224	9,006	1,119	0	189	7,478	954	0	18000*
50	30	0	0	50	1,843	249	0	50	1,843	241	0	46	1,793	226	0	415
50	10	0.05	0	108	4,916	612	0	119	5,324	655	0	121	5,297	700	0	3,919
50	25	0.05	0	158	5,895	808	0	153	5,874	784	0	158	6,447	834	0	8,923
50	30	0.05	0	20	716	106	0	20	716	99	0	21	668	106	0	163
50	10	0.1	0	145	6,661	1,107	0	151	6,904	1,168	0	156	7,052	1,233	0	6,263
50	25	0.1	0	144	6,064	1,039	0	144	6,064	1,025	0	126	5,639	931	0	8,071
50	30	0.1	0	24	781	179	0	24	781	171	0	16	648	117	0	187
50	10	0.15	0	139	6,367	2,569	0	141	6,406	2,625	0	129	5,926	2,447	0	9,031
50	25	0.15	0	131	4,921	2,337	0	111	3,628	1,955	0	120	4,353	2,172	0	3,538
50	30	0.15	0	14	546	279	0	14	546	252	0	9	342	160	0	217
50	10	0.2	0	144	6,781	10,906	0	157	7,464	12,025	0	193	9,002	14,719		
50	25	0.2	0	87	3,520	6,375	0	106	4,074	7,782	0	134	5,126	10,676	OM	
50	30	0.2	0	6	199	443	0	6	199	451	0	7	195	506		
100	10	0	0	91	7,805	1,006	0	111	9,412	1,175	0	160	13,607	1,841	0.97	18000*
100	25	0	0	134	10,893	1,379	0	135	10,999	1,359	0	160	11,789	1,650	0	11,175
100	30	0	0	20	1,578	218	0	20	1,578	200	0	41	2,538	419	0	916
100	10	0.05	0	136	12,387	1,655	0	143	12,705	1,703	0	152	13,750	1,870	0.98	18000*
100	25	0.05	0	263	19,549	2,843	0	262	19,346	2,759	0	181	13,368	1,976	0	18000*
100	30	0.05	0	32	2,569	343	0	36	2,679	365	0	24	1,958	262	0	1,189
100	10	0.1	0	167	15,733	2,684	0	173	14,097	2,782	0	214	18,999	3,480	NS	18000*
100	25	0.1	0	159	13,144	2,299	0	173	13,167	2,480	0	176	14,332	2,551	0	18000*
100	30	0.1	0	27	2,117	402	0	27	2,117	391	0	25	1,912	358	0	934
100	10	0.15	0	235	21,003	8,586	0	211	19,249	7,802	0	229	21,258	8,666		
100	25	0.15	0	200	16,515	6,804	0	204	16,444	6,873	0	233	18,121	8,263	OM	
100	30	0.15	0	29	2,211	1,028	0	29	2,211	980	0	30	2,422	1,063		
1000	10	0	0	80	67,617	9,614	0	80	68,714	9,434	0	68	54,556	8,040	NS	18000*
1000	25	0	0	163	115,143	18000*	0	148	117,292	16,594	0	124	87,597	13,565	NS	18000*
1000	30	0	0	26	19,508	2,931	0	26	19,508	2,755	0	49	30,516	5,268	NS	18000*
1000	10	0.05	0	88	74,661	11,590	0	75	62,676	9,624	0	139	109,263	17,982	NS	18000*
1000	25	0.05	0	159	125,186	18000*	0	161	89,657	18000*	0	150	112,432	17,624	NS	18000*
1000	30	0.05	0	40	27,770	4,472	0	72	27,818	7,777	0	39	26,908	4,370	NS	18000*
1000	10	0.1	0.54	81	70,752	18000*	0	67	60,844	15,151	0.45	90	71,645	18000*		
1000	25	0.1	0.03	85	74,706	18000*	0	89	69,677	18000*	0.04	86	72,163	18000*	OM	
1000	30	0.1	0	42	27,961	8,287	0	59	27,978	10,752	0	69	38,991	13,465		

5.6 Conclusion

For a more realistic evacuation planning, one has to take into account the uncertainties regarding the evacuation demand, road network structure and the possible disruption in shelters and consider as many scenarios as needed. In evacuation planning context, addressing the shelter location and traffic management decisions separately may lead to suboptimal results and hence they should be handled simultaneously.

In this study, we proposed an exact algorithm based on a Benders decomposition of a formulation that generates a fair and efficient evacuation plan. We employed duality results for second order cone programming in a Benders decomposition setting. We developed different BD algorithms that can solve practical size problems with up to 1000 scenarios in moderate CPU times. We investigated methods such as adopting a multi-cut strategy, using lazy constraint callback feature, deriving pareto-optimal cuts, using a reduced primal subproblem and preemptive priority multiobjective program to enhance the proposed algorithm. Computational results confirm the efficiency of our algorithm as it is considerably faster and can solve instances with larger number of scenarios compared to solving the extended formulation (EF) with an off-the-shelf solver.

Chapter 6

Conclusion and Future Research

The decision of where to locate the shelters is a strategic decision taken before the disaster takes place and has an impact on routing of evacuees. Addressing the problems of locating shelters and planning the evacuation traffic separately may lead to suboptimal results as the shelter location decisions affect the evacuation traffic management and have a critical effect on the success of an evacuation plan. For an efficient evacuation planning, evacuation management authority minimizes the total evacuation time, i.e., applies the SO approach. To impose the results of such an approach on the evacuees may not be applicable as it may send some evacuees to very distant shelters assigning them to very long routes. UE approach may not be suitable either, as disasters are rare events and it is not possible for evacuees to know the traffic conditions on the road network during a disaster. In the turmoil of a disaster, the NA approach will rather be a reasonable approach as the evacuees will tend to reach the nearest shelter by taking a shortest route (or shortest free flow time route). But this approach will probably cause congestion in the network resulting in an inefficient evacuation plan. However, if we incorporate the fairness concept into the model, evacuees can be convinced and guided by the evacuation management authority. If evacuees know that they are being treated fairly among others and also that their relatively small sacrifice by taking a route within a tolerance level instead of the shortest route will contribute to them and to the overall benefit of the evacuation process in a great deal, they can consent to the CSO solution.

We proposed a novel model that captures this human behavior by combining shelter location decisions with evacuation traffic assignment and generating fair and efficient solutions by compromising the needs of the evacuees and the evacuation management authority. Our results show that the decision of how many shelter sites to open and where to locate them is critical to the evacuation planners. For our instances, we observed that as we open more shelter sites and convince the evacuees for a higher level of tolerance, the total evacuation time and the maximum latency decrease and the percentage of people evacuated up to a specified time increases. This guarantees having a more efficient evacuation plan, compared to the case where evacuees act selfishly and take the shortest route to the nearest shelter site. However, one needs to be careful about opening more shelter sites when geographical distances or free flow travel times are used as the normal length of a path, i.e., if the potential shelter sites are not chosen properly it may not be advantageous to open more shelters. As the level of tolerance increases, so does unfairness, both in terms of paths and shelters. By considering the trade off between unfairness and price of fairness, a carefully chosen level of tolerance can be a balance between these two conflicting objectives.

The difficulty in evacuation planning is that it has to be done in the presence of uncertainty without exact or complete information. If we ignore how vulnerable the road network structure and/or the shelters are, the damage may result in an inefficient evacuation plan possibly resulting in chaos and panic among the evacuees and further losses. For that reason, we extended our work to incorporate uncertainty using two stage stochastic programming where the shelter sites are decided in the first stage and routing decisions are taken in the second stage. We proposed a second model that captures this stochasticity in evacuation planning by taking account of the uncertainty in demand, road capacities and availability of shelters. We used our model on a case study for an impending earthquake in Istanbul, Turkey. In our case study, we observed that the solution of our stochastic model leads to a significant decrease in the total evacuation time compared to the deterministic and mean value solutions. We further observed that when shelters are capacitated, the superiority of stochastic programming solution is emphasized, as planning in accordance with a single scenario or mean value approach will possibly generate infeasible solutions for some scenarios, unlike in stochastic programming solution.

To have a more realistic evacuation planning that considers uncertainty, we need to consider a large number of scenarios. The two-stage stochastic model can not solve practical size problems with a large number of scenarios by using an off-the-shelf solver in reasonable CPU times or can not solve them at all. To overcome this, we developed exact algorithms based on Benders Decomposition. We employed duality results for second order cone programming, as the second stage of our problem is a second order cone programming problem. We tested our algorithms using practical size problems with up to 1000 scenarios. We investigated methods such as adopting a multi-cut strategy, using lazy constraint callback feature, deriving pareto-optimal cuts, using a reduced primal subproblem and preemptive priority multiobjective program to enhance the proposed algorithm. Computational results confirmed the efficiency of our algorithm as it is considerably faster and can solve instances with larger number of scenarios compared to solving the EF with an off-the-shelf solver.

Specifying which routes are going to play a major role during an evacuation is a question emergency evacuation planners are trying to answer. It is important that these routes are large enough to serve a demand surge during an evacuation and that they survive the destructive impact of a disaster. For that reason expanding and/or strengthening these routes subject to a budget would contribute to the effectiveness of an evacuation plan. As a continuation of this study capacity expansion and/or retrofit decisions can be incorporated in the first stage and the trade off between the expansion/retrofit decisions and the total evacuation time can be analyzed.

In our model the uncertainty is caused by nature. Another interesting extension would be a game theoretic approach, where there is an attacker (a terrorist organization using a chemical/biological bomb) whose aim is to give the defender maximum possible harm by delaying the evacuation as much as possible by attacking critical road segments or bridges and shelter sites subject to a limited budget. Defender on the other hand, tries to minimize this effect by selecting the shelter locations and assigning the evacuees to these shelters and to routes that reach them in such a way that the total evacuation time is minimized.

We observed in our experiments that when we solve the dual subproblem instead of the primal subproblem to generate optimality cuts, we have better solution times. The reduced primal subproblem has considerably less amount of variables and constraints compared to the original primal subproblem. Instead of solving the dual subproblem, one can solve the simpler reduced primal subproblem and employ the complementary slackness conditions for second order cone programming in order to generate the optimality cut, which we believe could improve the solution times.

In our model we pregenerate all possible feasible paths by using an algorithm by Byers and Waterman [149]. As evacuation network size and tolerance level gets larger, this means generating millions of paths in advance, which eventually causes memory problems. Incorporating uncertainty with a large number of scenarios certainly makes the problem more difficult. In that case, it may be advantageous not to work with large relaxations that involve variables for all possible paths and to generate these variables when required within a branch and price framework for the deterministic problem. Similarly, a column generation or a cutting plane framework can be used for SP of the BD algorithm, depending on whether we solve the (R)PSP or DSP to generate optimality cuts, respectively.

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