Switched PD-like Controllers for First Order Unstable Systems with Time Delay

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Abstract: A new method is proposed for the design of PD-like (first order stable) controllers for switched first order unstable systems with time delays. For this purpose, a dwell-time based stability condition of Yan and Özbay (2008) is used for the class of switched time delay systems studied here. The proposed method finds the values of PD-like controller parameters which minimize an upper bound of the dwell time, minimum time needed between the switching instants to preserve stability. The conservatism analysis of this method is done by time domain simulations. The results show that the calculated upper bound for the dwell time is close to the lower bound of the dwell time observed by simulations.

Keywords: Switched systems, time delay, PD control, stability analysis, dwell time

1. INTRODUCTION

PID controllers are the most widely used controller structures in the industry due to their simplicity, Astrom and Hagglund (2001). The user has to tune three controller parameters in this setting. According to Xu et al. (1995), PD control is most frequently used in robot position and force control because of its robustness to time delay. In this paper, we focus on PD-like (stable first order) controller design for switched first order unstable systems with time delay. A typical example of a first order unstable system with time delay is an aircraft model, Enns et al. (1992). Another example of such a system is the batch chemical reactor, see Lee et al. (2000) and Stein (2003). Different PD and PID design methods for first order unstable plants with time delays can be found in the literature, see e.g. Tan et al. (2003), Lee et al. (2000), Huang and Chen (1997) and Visioli (2000). But these designs do not consider possible switchings in the plant parameters. The impact of switchings on feedback system stability can be significant.

Over the last two decades, there has been a growing number of investigations on finding stability conditions for switched systems. Typically, a switched system consists of several candidate systems and only one them is active for a specified time period. There is a switching law to determine which candidate system would be active during the specified intervals. In general, switched systems are composed of a set of plants and a set of controllers which are designed according to corresponding plants, Sun and Ge (2005). When the plant is switched, controller also switches to ensure stability and performance at that time period. Switched systems with time delay have strong control engineering applications which are especially in network control systems (see Jiang et al. (2008) and Kim et al. (2004)) and in power systems (see Meyer et al. (2004)). Closed-loop system obtained is stable for each non-switched candidate system but may become unstable when there are infinitely many arbitrary switchings. The switched system is stable if the minimum time interval between switching instants is greater than a dwell time. For switched systems with time delay, a dwell time based stability condition is derived recently in (Yan and Özbay (2008)) and exponential stability conditions based on average dwell time technique are derived in Sun et al. (2006). For the finite dimensional case see e.g. Geromel and Colaneri (2006) and Liberzon and Morse (1999), and their references. In addition, stability analysis of switched systems with time varying delays can be found in Sun et al. (2006).

In this paper, we compute the optimal stabilizing PD-like controller parameters to minimize an upper bound of the dwell time expression derived in Yan and Özbay (2008). The rest of the paper is organized as follows: in Section 2 the switched system considered is defined with precise definition of the plant and controller classes. In section 3, the main results are given. In the last section, we make concluding remarks.

2. PROBLEM DEFINITION

Consider the switched feedback system shown in Fig. 1, where θ is an arbitrary piecewise switching signal taking values on the set $\mathcal{F} := \{1, \dots, l\}$.

In this study, we assume that $\mathcal{P} := \{P_1, \dots, P_l\}$ is known and at each switching instant, the switching signal θ selects an index $\theta \in \mathcal{F}$, so a plant is selected from $\mathcal{P}$. Each $P_\theta \in \mathcal{P}$ is a first order unstable system with time delay and can be expressed in the form

$$P_\theta(s) = \frac{e^{-h_\theta s}}{s - a_\theta}. \tag{1}$$

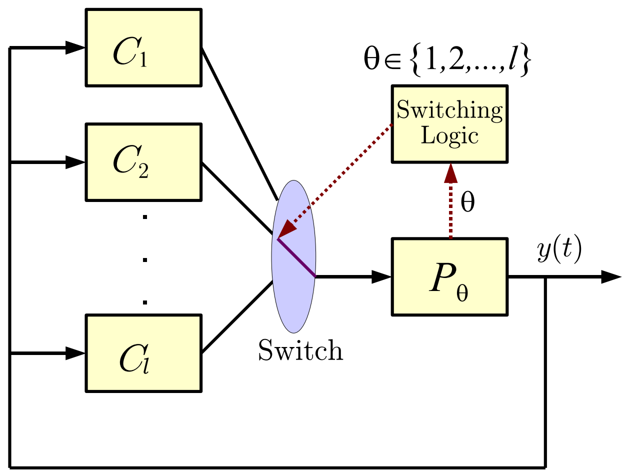

Fig. 1. Typical Switched Feedback System

where h_θ is the time delay and a_θ is the right half plane pole. As the plant switches according to the switching logic θ, controller has to switch in order to preserve stability. The controllers C_θ are proportional derivative (PD) like controllers in the form

$$C_\theta(s) = K_{p\theta} + \frac{K_{d\theta} s}{\tau_{d\theta} s + 1} \tag{2}$$

where $K_{p\theta}$ is the proportional constant, $K_{d\theta}$ is the derivative constant and $\tau_{d\theta} > 0$ is a small time constant. The derivative part of the controller is implemented as in (2) to make it a proper transfer function. Also, note that when $\tau_{d\theta}$ is an arbitrary positive number (2) represents a stable first order controller structure. Such controllers are also have practical significant importance in the framework of low order strongly stabilizing controller design for unstable time delay systems, see e.g. Gumussoy and Ozbay (2008) and Gundes and Ozbay (2007). In the light of this observation we re-write the PD-like controller as

$$C_\theta(s) = \frac{R_\theta s + K_{p\theta}}{\tau_{d\theta} s + 1} \tag{3}$$

where $R_\theta := K_{p\theta}\tau_{d\theta} + K_{d\theta}$ and for notational convenience we define $a_{c\theta} := -\tau_{d\theta}^{-1}$ as the controller pole location.

The feedback system shown in Fig. 1, runs with the initial conditions. Since each candidate plant is stabilized with a corresponding controller, the switched system will preserve its stability if the candidate plant-controller pairs run for a long enough time interval. In other words, if the switching intervals are sufficiently long, the overall switched system will be stable. The problem of computing the minimum time needed between switching instants to maintain stability (dwell time) is considered in this paper.

3. MAIN RESULTS

3.1 PD Controllers for Dwell Time Minimization

A state-space representation of the closed-loop dynamics can be written as follows:

$$\begin{bmatrix} \dot{x}_c(t) \\ \dot{x}_p(t) \end{bmatrix} = \underbrace{\begin{bmatrix} a_{c\theta} & -a_{c\theta}K_{d\theta} \\ 0 & a_\theta \end{bmatrix}}_{A_\theta} \begin{bmatrix} x_c(t) \\ x_p(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ -a_{c\theta} & a_{c\theta}K_{d\theta} - K_{p\theta} \end{bmatrix}}_{\bar{A}_\theta} \begin{bmatrix} x_c(t-h_\theta) \\ x_p(t-h_\theta) \end{bmatrix}$$

$$y(t) = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C_\theta} \begin{bmatrix} x_c(t) \\ x_p(t) \end{bmatrix} \tag{4}$$

where $x_c(t)$ and $x_p(t)$ are the states of the controller and the plant respectively. Consequently, the triplet $(A_\theta, \bar{A}_\theta, h_\theta)$ defines a candidate system of the form (4) from the set $\mathcal{A} := \{(A_i, \bar{A}_i, h_i) : i \in \mathcal{F}\}$.

An upper bound for the dwell time τ derived in Yan and Özbay (2008) can be given as follows:

$$\tau := T_d + 2h_{max}, \qquad h_{max} = \max_{i\in\mathcal{F}}\{h_i\} \tag{5}$$

where

$$T_d \leq \mu_d = \max_{i\in\mathcal{F}} \frac{1}{\sigma_{min}(S_i)} \tag{6}$$

with

$$S_i = -\{(A_i + \bar{A}_i) + (A_i + \bar{A}_i)^T + h_i\alpha_i^{-1}\bar{A}_i A_i A_i^T \bar{A}_i^T + h_i\beta_i^{-1}(\bar{A}_i)^2(\bar{A}_i^T)^2 + h_i p_i(\alpha_i + \beta_i)\} \tag{7}$$

In (7), the free parameters $p_i > 1, \alpha_i > 0$ and $\beta_i > 0$ are found by satisfying the LMIs of Lemma 2.2 of Yan and Özbay (2008). A sufficient condition on asmyptotic stability of the switched system is that for any switching rule, the switching intervals $[t_{j-1} \quad t_j), \quad j \in \mathcal{F}$ should be longer than τ.

Our aim is to investigate the conditions on K_{pi}, K_{di} and $a_{ci} = -\tau_{di}^{-1}$ for each candidate system to ensure the stability of the switched system and obtain the corresponding values of these parameters to minimize the upper bound of the dwell time, given by (5), (6) and (7).

First, the matrix inequality in Lemma 2.2 of Yan and Özbay (2008) has to be satisfied and can be expressed in terms of plant and controller parameters as follows:

$$X = \begin{bmatrix} X_{11} & X_{21} & 0 & 0 & 0 & 0 \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} \\ 0 & X_{23} & -\alpha_i & 0 & 0 & 0 \\ 0 & X_{24} & 0 & -\alpha_i & 0 & 0 \\ 0 & X_{25} & 0 & 0 & -\beta_i & 0 \\ 0 & X_{26} & 0 & 0 & 0 & -\beta_i \end{bmatrix} < 0 \tag{8}$$

$$
\begin{aligned}
X_{11} &= 2h_i^{-1}a_{ci} + p_i(\alpha_i + \beta_i) \\
X_{21} &= -a_{ci}\ (1 + K_{di})h_i^{-1} \\
X_{22} &= 2h_i^{-1}\left(a_i - K_{pi} + a_{ci}K_{di}\right) + p_i(\alpha_i + \beta_i) \\
X_{23} &= -a_{ci}^2 \\
X_{24} &= a_{ci}^2 K_{di} - a_i\left(K_{pi} - a_{ci}K_{di}\right) \\
X_{25} &= a_{ci}\left(K_{pi} - a_{ci}K_{di}\right) \\
X_{26} &= \left(K_{pi} - a_{ci}K_{di}\right)^2
\end{aligned}
$$

In order to derive conditions on controller parameters, we recall some basic properties.

Fact 1. A $n \times n$ matrix is negative definite if and only if $\forall k \in \{1, \dots, n\}$ $(-1)^k|M_k| > 0$, where M_k's are the principal leading minors of the matrix.

Fact 2. Consider a second order polynomial with coefficients a, b and c. ($P(x) = ax^2 + bx + c$)

- $\frac{c}{a}$ is the multiplication of the roots $P(x) = 0$.
- $-\frac{b}{a}$ is the sum of the roots $P(x) = 0$.
- If the discriminant of the polynomial ($\Delta = b^2 - 4ac$) is negative and $a > 0$, then the polynomial is always positive for all x.
- If the discriminant of the polynomial ($\Delta = b^2 - 4ac$) is positive and $a > 0$, then the polynomial is intersects the x-axis and becomes negative for some x.

In order to satisfy the negative definiteness of the matrix X, Fact 1 is used.

* The determinant of the first leading minor has to be negative. (i.e $|M_1| < 0$)

$$
-\frac{2h_i^{-1}}{\tau_{di}} + p_i(\alpha_i + \beta_i) < 0 \Rightarrow 0 < p_i(\alpha_i + \beta_i) < \frac{2h_i^{-1}}{\tau_{di}}
$$
$$
\Rightarrow 0 < \tau_{di} < \frac{2h_i^{-1}}{p_i(\alpha_i + \beta_i)} \tag{9}
$$

* The determinant of the second leading minor has to be positive. (i.e $|M_2| > 0$)

$$
\begin{aligned}
&\Rightarrow p_i^2\alpha_i^2 + 2p_i\left[p_i\beta_i + h_i^{-1}\left(a_i - K_{pi} - \frac{K_{di}+1}{\tau_{di}}\right)\right]\alpha_i \\
&+ p_i^2\beta_i^2 + 2h_i^{-1}p_i\left(a_i - K_{pi} - \frac{K_{di}+1}{\tau_{di}}\right) \\
&- h_i^{-2}\left[\frac{4}{\tau_{di}}\left(a_i - K_{pi} - \frac{K_{di}}{\tau_{di}}\right) + \frac{(1+K_{di})^2}{\tau_{di}^2}\right] > 0
\end{aligned} \tag{10}
$$

Using Fact 2, since the discriminant and the coefficient of the second order term of the polynomial in (10) are positive, it has two real roots. By definition α is positive and consequently multiplication of the roots of the polynomial in (10) is positive which means the constant term of the polynomial is positive.

$$
\begin{aligned}
&p_i^2\,\beta_i^2 + 2h_i^{-1}p_i\left(a_i - K_{pi} - \frac{K_{di}+1}{\tau_{di}}\right) \\
&- h_i^{-2}\left[\frac{4}{\tau_{di}}\left(a_i - K_{pi} - \frac{K_{di}}{\tau_{di}}\right) + \frac{(1+K_{di})^2}{\tau_{di}^2}\right] > 0
\end{aligned} \tag{11}
$$

Similarly, by definition β is positive; the discriminant and the coefficient of the second order term of the polynomial in (11) are positive, then it has two positive real roots. Therefore, multiplication of the roots of the polynomial in (11) is positive which means the constant term of the polynomial is positive. Since $h_i > 0$ and $\tau_{di} > 0$, this term can be expressed as follows:

$$
4(a_i - K_{pi})\tau_{di} + (1 - K_{di})^2 < 0 \tag{12}
$$

In order to satisfy the inequality (12), $K_{pi} > a_i$ must hold. Similarly, a bound for K_{di} could be found from inequality (12) which is as follows:

$$
1 - 2\sqrt{(K_{pi} - a_i)\tau_{di}} < K_{di} < 1 + 2\sqrt{(K_{pi} - a_i)\tau_{di}} \tag{13}
$$

It can be shown that a P controller stabilizes a first order unstable process with time delay if and only if $a_ih_i < 1$, (Huang and Chen (1997)). Thus, the sufficient conditions upon the plant and the controller parameters are defined and the remaining problem is to find the values of these parameters in the defined intervals which minimizes the pre-defined dwell time expression. Since the expressions given are too complex to solve analytically, we tried to find the set of values of the corresponding parameters which minimizes the dwell time by a numerical search in the parameter space restricted by the inequalities derived above.

Our first assumption was that the candidate systems inside the set $\mathcal{A}$ are known, which means the plant parameters a_i and h_i are known. By dividing the intervals for controller parameters in (9), (12) and (13) into certain number of points, a set of parameters is obtained consisting of values of $(K_{pi}, K_{di}, \tau_{di})$. We tried to reach positive T_d values defined in (6) and store them by searching upon the variables α_i, β_i and p_i. After the search is completed among the whole parameter space, global minimum point for T_d and the corresponding parameters are obtained.

Let us illustrate the results on an example with the plant

$$
P(s) = \frac{e^{-h_is}}{s-1}
$$

which means the right half plane pole of the plant is set to 1 and only the delay parameter of the plant switches. Note that the plant (1) with an arbitrary a_θ, for any $\theta = i \in \mathcal{F}$ can be written as:

$$
P_i(\hat{s}) = \frac{e^{-h_ia_i\hat{s}}}{\hat{s}-1} \tag{14}
$$

where $\hat{s} = \frac{s}{a_i}$ is the normalized Laplace transform variable. Therefore, without the loss of generality, we can consider $a_i = 1$ and discuss controllers for switched parameter $\hat{h}_i = h_ia_i$.

Our numerical calculations for minimizing the upper bound of the dwell time show that the controller can be written in the following form which is valid for $h_i \in (0.0032, 0.155)$:

$$
C_i(s) = \frac{R_is + K_{pi}}{\tau_{di}s + 1} \tag{15}
$$

where $R_i = (\tau_{di} + 1.65 + 3h_i)$. Note that the controller is determined by two parameters K_{pi} and τ_{di} whose values are shown in Table 1.

For small delay values, the time constant of the system is small and hence the system response is fast. Therefore, dwell time obtained is obviously small. As delay is increasing, the time constant of the system is higher which results in a slower system and hence dwell time gets larger. The parameters of the controller which are K_{pi} and τ_{di} are shown in Fig. 2 and 3 and it can be seen from the figures that K_{pi} is rapidly decreasing while τ_{di} is increasing with the increasing delay.

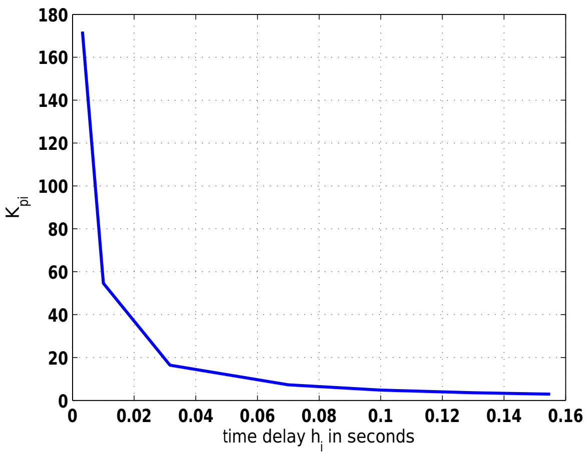

Fig. 2. The parameters of the controller versus delay

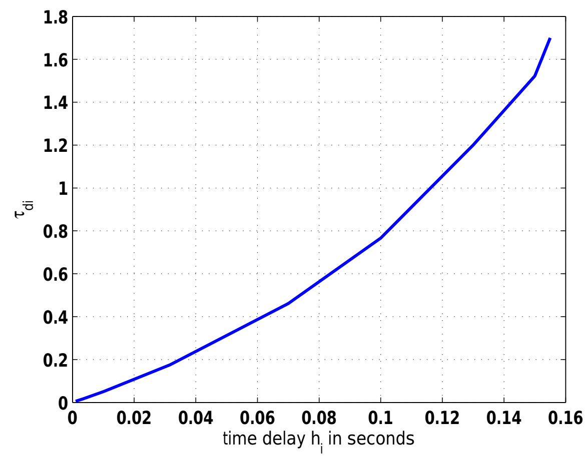

Fig. 3. The parameters of the controller versus delay

The minimum dwell time calculated versus time delay graph is as shown in Fig. 4. From this figure, we can conclude that as the delay is increasing, the dwell time is increasing exponentially and for $h_i > 0.155$, a finite dwell time can not be found with this approach.

Table 1. The minimum dwell time τ versus delay

h_i	τ	K_{pi}	τ_{di}
0.0032	0.0188	172	0.0155
0.01	0.0591	54.6	0.05
0.0316	0.2040	16.4	0.175
0.07	0.575	7.22	0.461
0.1	1.068	4.77	0.766
0.13	2.469	3.55	1.2
0.15	8.696	3.04	1.522
0.155	22.003	2.89	1.7

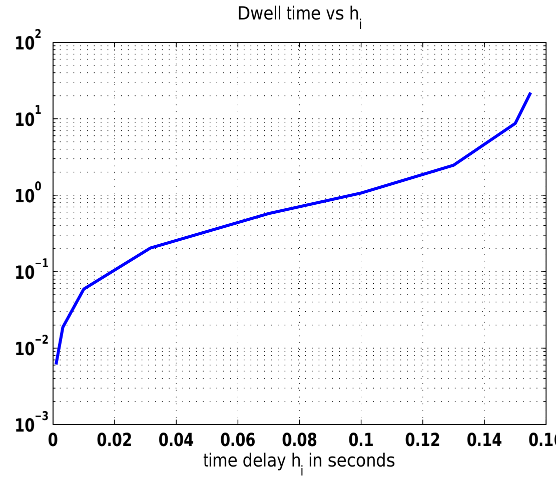

Fig. 4. The minimum dwell time versus delay

3.2 Conservatism Analysis and Simulations

In this section, the conservativeness of the LMI-based stability test suggested in Yan and Özbay (2008) for the switched time delay system is analyzed. That means a bound lower than the calculated one is searched for the minimum dwell time to point out how conservative the results are. Time domain simulations and analysis are carried out in order to accomplish this goal. The switched time delay system is simulated by doing arbitrary switching to find the highest value of the minimal switching time instant that causes instability is observed and by this way, the conservativeness of the calculated value is realized.

The closed loop system in (4) is simulated in time domain with nonzero initial conditions and this simulation could not be done precisely with internal time delay. Therefore, for simplicity as the first step, the time delay of the plant is approximated by 2^{nd} order Pade approximation, as follows:

$$e^{-h\theta s}X(s) \approx \left(\frac{1 - \frac{h_i}{2}s + \frac{h_i^2}{12}s^2}{1 + \frac{h_i}{2}s + \frac{h_i^2}{12}s^2} I \right) X(s)$$
$$= (C_{di} + (sI - A_{di})^{-1}B_{di} + D_{di})X(s) \quad (16)$$

where

$$A_{di} = \begin{bmatrix} 0 & I \\ -\frac{12}{h_i^2}I & -\frac{6}{h_i}I \end{bmatrix} \quad B_{di} = \begin{bmatrix} 0 \\ I \end{bmatrix}$$
$$C_{di} = \begin{bmatrix} 0 & -\frac{12}{h_i^2}I \end{bmatrix} \quad D_{di} = I$$

Then, the time delay part is converted to state space with internal state $z(t)$ by the following equations;

$$\dot{z}(t) = A_{di}z(t) + B_{di}x(t)$$
$$x(t-h_i) = C_{di}z(t) + D_{di}x(t) \quad (17)$$

and the overall switched system can be expressed as follows:

$$\begin{bmatrix} \dot{z}(t) \\ \dot{x}(t) \end{bmatrix} = \begin{bmatrix} A_{di} & B_{di} \\ \bar{A}_iC_{di} & (A_i + \bar{A}_iD_{di}) \end{bmatrix} \begin{bmatrix} z(t) \\ x(t) \end{bmatrix} \quad (18)$$

The instability of the system can be realized from the norm of the state vector. If the norm of the states goes to infinity as time goes to infinity, then the system is unstable and if the norm of the states goes to zero as time goes to infinity, then the system is stable.

Two systems are selected from the set $\mathcal{A}$ and simulations are started with arbitrary initial conditions for $x(t)$ and zero initial condition for $z(t)$. At the beginning, the first system runs t_1 seconds with the specified initial conditions. When $t = t_1$, the plant and the controller are switched to the second system in the set, which then runs t_2 seconds with the states at $t = t_1$ as initial condition. This is an infinite loop, meaning that switching from one system to the other continues as time goes to infinity. Actually, the switching intervals should be arbitrary. But in this case, we applied this constant interval switching rule to find a lower bound of the dwell time.

The minimum of t_1 and t_2 values for which the system goes from instability to stability yields the dwell time. This can be illustrated on an example of the previous section. Assume the plant is $P(s) = \frac{e^{-h_i s}}{s-1}$, the delay parameters that construct the set of candidate plants are $h_1 = 0.01$ and $h_2 = 0.07$.

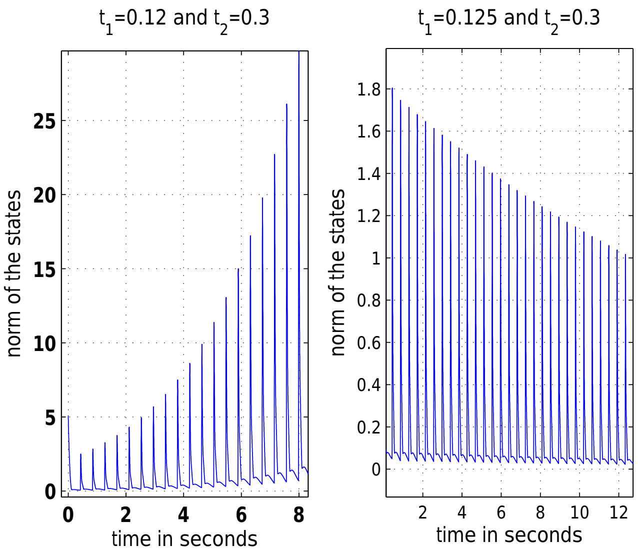

Fig. 5. Dwell time from simulations

From Fig. 5, it is obvious that the graph on the left belongs to an unstable system and the graph on the right belongs to a stable system and a lower bound of the dwell time is between 0.12 and 0.125 seconds for this example. Whereas the computed dwell time from Yan and Özbay (2008) is 0.575.

The difference between the dwell time from calculation and simulation could be due to the Pade approximation or the conservativeness of the LMI-based analysis. Therefore, we have investigated the role of the Pade approximation by increasing the Pade order and applying the same process.

From Fig. 6, a lower bound of the dwell time is between 0.39 sec. and 0.41 sec. and as we can see from Figure 7, as the Pade order increases, the dwell time value from simulations get closer to the calculated dwell time. In conclusion, for this example, the exact minimum dwell time is between 0.39 (lower bound found from simulations) and 0.57 (upper bound found from the formula given

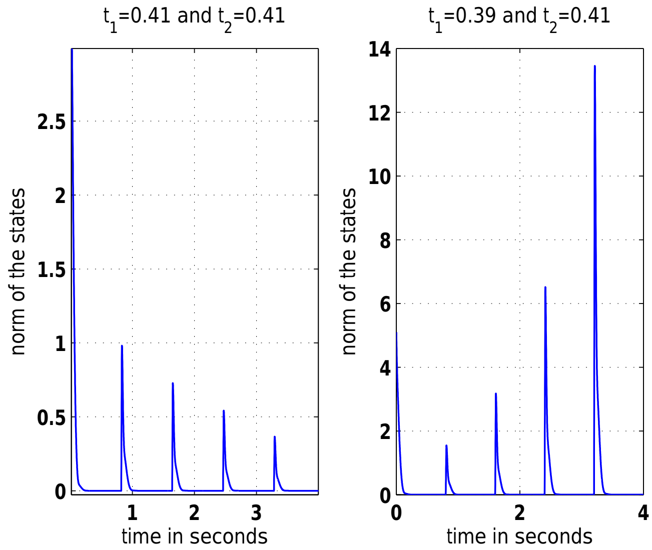

Fig. 6. Dwell time from simulations when Pade order=8

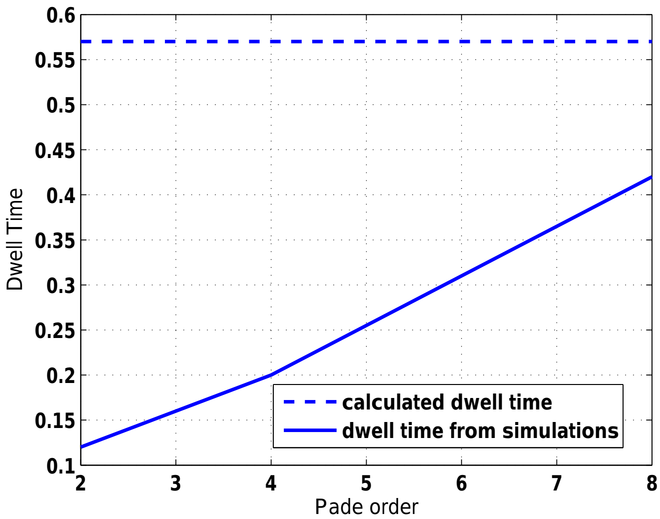

Fig. 7. Dwell time versus Pade order

in Yan and Özbay (2008)). This illustrates the level of conservativeness in the dwell time computation for this type of plants and controllers.

4. CONCLUSIONS

In this paper a PD-like controller design is considered for first order unstable switched systems with time delay. The controller parameters are optimized to minimize an upper bound of the dwell time expression derived in Yan and Özbay (2008). This optimization is done by a numerical search in the feasible parameter space. The PD-like controller proposed for the normalized plant

$$P_i(s) = \frac{e^{-h_i s}}{s-1}$$

is in the form

$$C_i(s) = \frac{R_i s + K_{pi}}{\tau_{di} s + 1}$$

where $R_i = (\tau_{di} + 1.65 + 3h_i)$. The parameters K_{pi} and τ_{di}, for various values of the time delay h_i are given in Table 1, which shows that τ_{di} is small (i.e. the controller is close to a PD controller) when h_i is small; but when $h_i \geq 0.03$sec., the free parameter τ_{di} is not negligible, hence the controller cannot be considered as a PD controller. In all cases, the proposed controller, shown above, is a stable first order system. It guarantees stability of the system under arbitrary switching in the plant parameters provided that the controller is switched synchronously and the smallest time interval between consecutive switching instants is greater than the computed dwell time (see Table 1).

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