Resonant Artificial Structures to Achieve Extraordinary Transmission at Microwaves

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Abstract— **In this contribution, the role of artificial resonant structures in increasing the transmission through sub-wavelength apertures is discussed. Those devices are capable to enhance the aperture equivalent electric and magnetic dipole moments and, consequently, the overall power transmission. The design details are given and the enhancement performances are then illustrated through the use of full-wave simulations. Such structures may find applications in different fields, such as high-resolution spatial filters, ultra-diffractive imaging systems, etc**

I. INTRODUCTION

The power transmission enhancement through electrically small apertures, has recently received a growing interest by the scientific community since the successful experimental demonstration by T. S. Ebbesen in 1999 [1], successively explained in terms of leaky waves by Oliner and Jackson in 2003 [2].

To the authors' best knowledge, the first theoretical setup reproducing the phenomenon of the extraordinary transmission at microwaves has been presented in [3]. However, according to [3], the surface mode that has to be excited on the screen, in order to obtain the maximum coupling of the impinging radiation with the aperture and achieve, thus, the maximum power transfer, turns out to be a highly directive leaky mode, characterized by a small valued imaginary part of the wave-number. This implies on one hand the need of materials with extreme-value constitutive parameters (i.e. a slab with electric relative permittivity close to zero, see [3]), and, on the other hand, the spatial occupancy of the cover around the sub-wavelength aperture has to be electrically large, significantly limiting the range of possible practical applications.

The structures presented in this contribution are aimed at reducing both the overall size of the devices and the complexity of the equivalent real-life structures, through to the use of different physical phenomena.

II. TRANSMISSION THROUGH SMALL APERTURES

The power transmission through an electrically small aperture (ESA) in a flat, indefinite, perfect conducting screen with negligible thickness has been deeply investigated by Bethe [4] during the 40*s*. According to Bethe's theory, if the aperture is sub-wavelength, only a small fraction of the incident power T_0 is transmitted through the screen and it results proportional to the fourth power of the linear electrical dimension of the aperture:

$$T_0 \sim \left(\frac{a}{\lambda}\right)^2 \tag{1}$$

being T_0 the transmitted power, a the linear geometrical dimension of the aperture and λ the wavelength of the impinging radiation. This result is achieved by neglecting the retardation effects due to the finite thickness of the screen and the higher-order multi-pole moments; in this case, in fact, the transmission through the ESA can be modelled as due to both an equivalent magnetic dipole moment p_m parallel to the screen and an equivalent electric dipole moment p_e normal to the screen, respectively:

$$\begin{cases} p_e = -(a^3/3\pi)E_0 \\ p_m = -(2a^3/3\pi)H_0 \end{cases} \tag{2}$$

being E_0, H_0 the amplitudes of the normal electric and the tangential magnetic fields at the aperture. In principle, it is enough to increase E_0, H_0 in close proximity to the ESA in order to enhance the power transmission.

III. FSSS BASED LAYOUT

An easy way to accomplish this task is to excite a strong resonance localized in close proximity of the ESA. The first layout we present is based on the employment of frequency selective surfaces (FSS).

A. FSS screen

Through an equivalent transmission-line (TL) model (see Fig. 1), it can be demonstrated that an artificial sheet characterized by a complex surface impedance Z_{FSS} is capable to locally enhance the amplitude of the impinging magnetic field when placed in front of a perfectly conductive (PEC) screen:

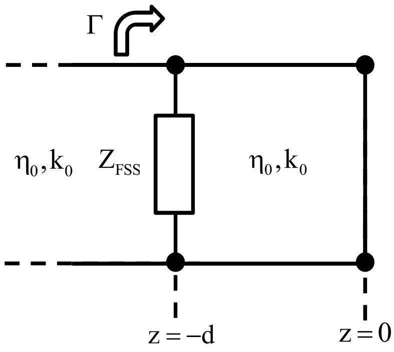


Fig. 1 TL equivalent circuit of an FSS screen placed in close proximity of a PEC surface: the FSS screen is modelled through a shunt complex impedance Z_{FSS} at a distance *d* from the PEC screen (i.e. the short-circuit).

Assuming a TE(z) plane wave excitation of the following kind (an exp[jωt] time dependence is considered):

$$\begin{cases} \mathbf{E}^{TE} = \hat{x}\, E_0 \left[e^{-jk_z(z+d)} + R^{TE}\, e^{jk_z(z+d)} \right] e^{-jk_y y} \\ \mathbf{H}^{TE} = \dfrac{j}{k_0 \eta_0} \nabla \times \mathbf{E}^{TE} \end{cases} \quad (3)$$

where $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$, $k_y = k_0 \sin\theta_i$, $k_z = \sqrt{k_0^2 - k_y^2}$ and η_0 the free-space characteristic impedance. From the equivalent circuit of Fig. 1, the expression of the tangential magnetic field on the short circuit is:

$$\mathbf{H}_y^{TE}\Big|_{z=0} = \frac{\hat{y}\,\dfrac{2\,E_0\, e^{-jk_y y} k_z^2}{k_0 \eta_0}\, Z_{FSS}}{k_z Z_{FSS} \cos(k_z d) + j(k_z Z_{FSS} + k_0 \eta_0)\sin(k_z d)} \quad (4)$$

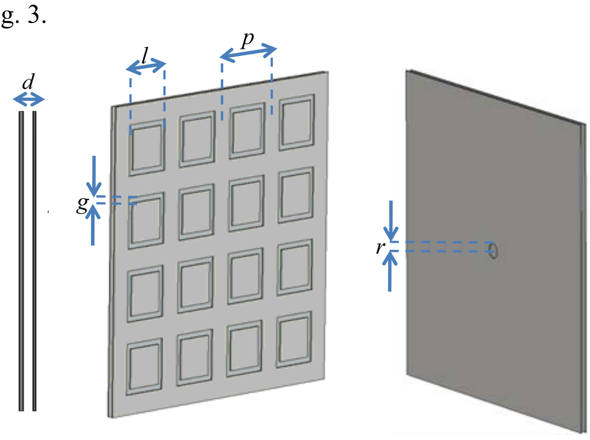
The resonance condition is, then, found by imposing the denominator of the tangential magnetic field amplitude (4) vanishes, which gives the following condition for the surface impedance Z_{FSS}^{TE} :

$$Z_{FSS}^{TE} = -\frac{k_0 \eta_0 \sin(k_z d)}{k_z} [\, j\cos(k_z d) + \sin(k_z d)] \quad (5)$$

If $k_z d \ll 1$ (i.e. assuming the gap between the FSS and the screen to be small compared to the impinging wavelength), Z_{FSS}^{TE} becomes:

$$Z_{FSS}^{TE} \simeq -d(j + k_z d) k_0 \eta_0 \simeq -jk_0 \eta_0 d \quad (6)$$

By duality, it is straightforward to derive also the design impedance in the case of a TM(z) incidence. A simple test structure, realizing the desired value of the FSS equivalent surface impedance has been designed according to the guidelines given in [5]. The resulting layout is depicted in Fig. 2 and its performance in terms of enhanced transmission in Fig. 3.



(a) (b) (c)

Fig. 2. Layout based on the use of an FSS screen. (a) Transverse section of the structure, (b) entrance side, (c) exit side. The excitation consists of a TEM(z) plane wave impinging on (b). The distance between the FSS-screen and the copper screen where the aperture is placed is *d=0.25mm*, the center to center distance between two squared elements for both the x and the y direction is *p=4mm*, the radius of the hole is *r=0.4mm*, the inner gap *g=0.4mm*, the element side *l=2.9 mm*.

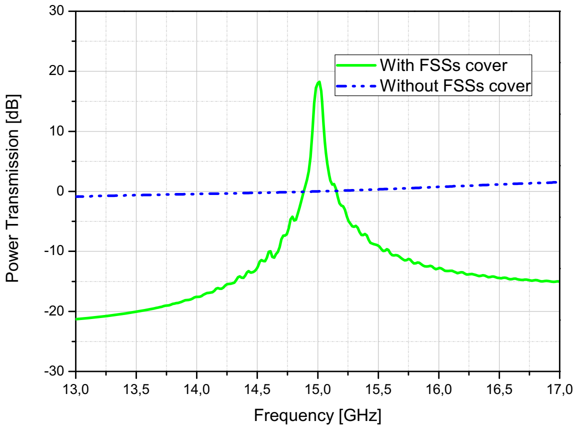


Fig. 3. Power transmission enhancement for the structure depicted in Fig. 2. The enhancement is about 18 dB with respect to the case of absence of the FSS cover.

The main drawback of the proposed setup is that, even if the structure is suitable for a simple practical implementation and the thickness is extremely reduced compared to the case of the metamaterial slab presented in [3], the transverse dimensions are still comparable to the wavelength of the impinging field.

IV. SRRS BASED LAYOUT

In order to reduce also the device transverse size, the FSS screen can be replaced by an alignment of split ring resonators (see [6]). It is known, in fact, from [7] that, pairing an epsilon negative (ENG) slab with a mu negative (MNG) slab of equal thickness satisfying the system:

$$\begin{cases} d_1 = d_2 = d \\ \varepsilon_1 = -\varepsilon \\ \varepsilon_2 = \varepsilon \\ \mu_1 = \mu \\ \mu_2 = -\mu \end{cases} \tag{7}$$

where ε₁, μ₁, ε₂, μ₂ are the relative permittivity and permeability of the first and second cover, respectively, while d_1 and d_2 are the cover thicknesses, a strong resonance arises at the interface between the two complex media. This resonance may be explained in terms of the sign flip of both the electric and magnetic tangential field component first derivatives, as reported in Table I.

TABLE I

BOUNDARY CONDITIONS AT THE INTERFACE BETWEEN TWO SINGLE NEGATIVE MATERIALS ON THE HOLE PLANE.

TEM	TE_z	TM_z
$\left.\frac{1}{\mu_1}\frac{dE_x}{dz}\right\|_{hole} = \left.\frac{1}{\mu_2}\frac{dE_x}{dz}\right\|_{hole}$	$\left.\frac{1}{\mu_1}\frac{dE_x}{dz}\right\|_{hole} = \left.\frac{1}{\mu_2}\frac{dE_x}{dz}\right\|_{hole}$	$\left.\frac{\varepsilon_1}{k_{z1}^2}\frac{dE_y}{dz}\right\|_{hole} = \left.\frac{\varepsilon_2}{k_{z2}^2}\frac{dE_y}{dz}\right\|_{hole}$
$\left.\frac{1}{\varepsilon_1}\frac{dH_y}{dz}\right\|_{hole} = \left.\frac{1}{\varepsilon_2}\frac{dH_y}{dz}\right\|_{hole}$	$\left.\frac{\mu_1}{k_{z1}^2}\frac{dH_y}{dz}\right\|_{hole} = \left.\frac{\mu_2}{k_{z2}^2}\frac{dH_y}{dz}\right\|_{hole}$	$\left.\frac{1}{\varepsilon_1}\frac{dH_x}{dz}\right\|_{hole} = \left.\frac{1}{\varepsilon_2}\frac{dH_x}{dz}\right\|_{hole}$

If a metal screen with a sub-wavelength aperture is placed between the ENG-MNG slabs, a resonance arises at the ESA plane, thus enhancing the transmitted power.

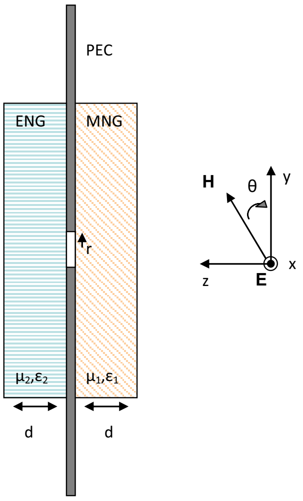


Fig. 4. Layout based on an MNG-ENG double cover: a sub-wavelength hole in a PEC thin screen covered by a pair of homogeneous single negative slabs.

However, in order to provide a reference design suitable for a practical implementation, as in the case of the FSS one, such a layout has to be modified. It is well known from literature, in fact, that the phenomena involving interface effects between complex materials are hardly verified in experimental setups, due to the limitations imposed by the homogenization procedure [7]. However, some extra conditions have to be added to the ones shown in Table I, due to the presence of the conductive screen (see Fig 3):

$$\begin{cases} \left.\dfrac{\partial H_{\text{tan gential}}}{\partial z}\right|_{\text{hole plane}} \simeq 0 \\ \left.\dfrac{\partial E_{\text{tan gential}}}{\partial z}\right|_{\text{hole plane}} \neq 0 \end{cases} \tag{8}$$

By adding (8) to the conditions in Table I, it is clear that only the sign flip of the magnetic permeability plays a role in achieving a resonance at the ESA plane. The two single negative covers can be, then, replaced by a single cover made of split-ring resonators, exhibiting an effective negative permeability at the design frequency. In addition, according to the electromagnetic behaviour of the MNG-ENG pair, the resonance condition is expected to be achieved even if the transverse dimensions of the cover are electrically small. A typical design is reported in Fig. 5 and the related performances in Fig. 6.

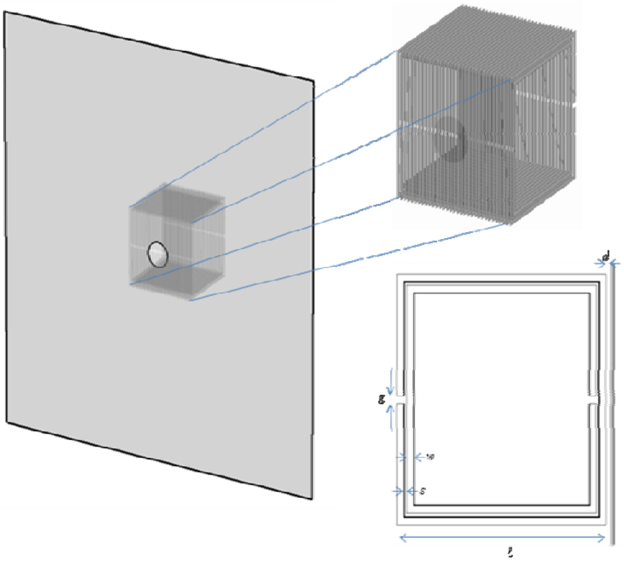


Fig. 5. Layout based on the use of SRRs. The alignment is made of 19 SRR with the following dimensions: *ℓ = 6.475 mm*, *w = 0.185 mm*, *g = 0.37 mm*, *s = 0.185 mm*, *d = 0.46 mm*. The spacer between two consecutive SRR is *0.37 mm* and the hole radius is *r = 1 mm*.

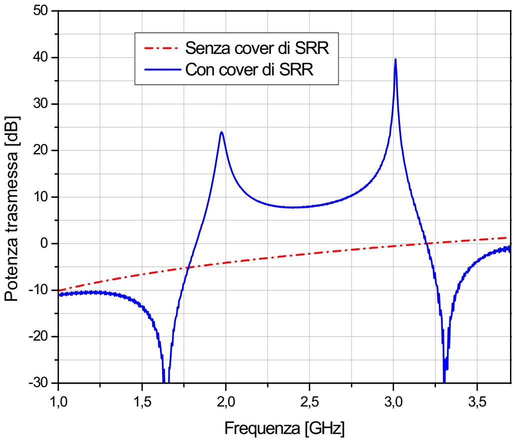


Fig. 6. Power transmission enhancement for the structure depicted in Fig. 5. The enhancement is about 40 dB respect to the case of absence of the SRRs.

Differently from the FSS based layout, in this case the device size is comparable to the ESA dimensions and, in addition, it exhibits two different working frequencies: the one at about 2 GHz is due to the SRR proper resonance, and the one at 3 GHz is due to the equivalent “negative permeability behavior” of the SRR alignment ($\mu_r \simeq -1$ at the working frequency, see [6]).

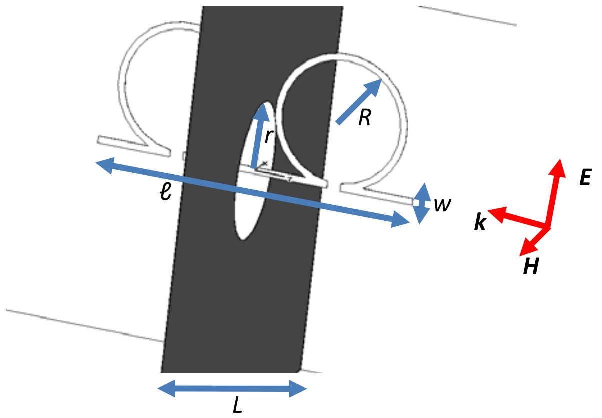
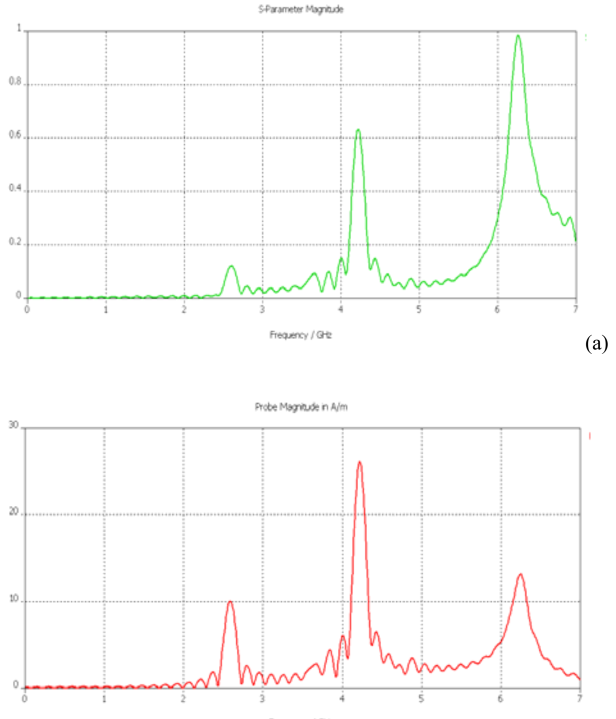


Fig. 7. Layout based on the use of a double omega inclusion with the following dimensions: *ℓ = 12 mm*, *w = 0.5 mm*, *R = 5 mm*, r = *4 mm*, reference screen size *L* x *L =40 mm* x *40 mm*



(a)

(b)

Fig. 8. (a) Power transmission enhancement for the structure depicted in Fig. 7. (b) Magnetic probe amplitude along the axis of the omega loop; the transmission peaks correspond to the multiple resonances of the double omega inclusion.

V. SINGLE INCLUSIONS BASED LAYOUT

The results shown in Fig. 6 suggest the use of a single resonating inclusion as the most suitable device for transmission enhancement through a sub-wavelength aperture. In particular, results for a single SRR placed in front of the hole can be found in [8], showing how it is possible to enhance the power transmission by the means of a single resonant magnetic inclusion placed in close proximity of the aperture.

Moreover, we introduce here the use of a double omega inclusion, with the loop plane normal to the magnetic field vector of the impinging wave, in order to increase at given frequencies the equivalent tangential magnetic dipole moment and the normal electric dipole moment of the hole (Fig. 7). The corresponding results are reported in Fig. 8. Further details on the behaviour of this design will be given at the conference.

VI. CONCLUSIONS

In this contribution, different resonant approaches for power transmission enhancement trough sub-wavelength apertures at microwave frequencies have been presented.

The aim was to avoid the use of leaky-wave-related phenomena in order to design compact devices suitable for cost-effective practical applications.

From the FSS cover to the double omega inclusion, a set of suitable metallic structures have been presented, characterized by a decreasing overall space occupancy and complexity and increasing performances. Those structures may find applications in different fields, such as high-resolution spatial filters, ultra-diffractive imaging systems, through-the-wall imaging, etc.

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