

Rigorous Solutions of Large-Scale Dielectric Problems with the Parallel Multilevel Fast Multipole Algorithm

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Abstract

We present fast and accurate solutions of large-scale electromagnetics problems involving three-dimensional homogeneous dielectric objects. Problems are formulated rigorously with the electric and magnetic current combined-field integral equation (JMCFIE) and solved iteratively with the multilevel fast multipole algorithm (MLFMA). In order to solve large-scale problems, MLFMA is parallelized efficiently on distributed-memory architectures using the hierarchical partitioning strategy. Efficiency and accuracy of the developed implementation are demonstrated on very large scattering problems discretized with tens of millions of unknowns.

1 Introduction

Real-life electromagnetics problems often involve big objects, whose accurate discretizations lead to very large matrix equations involving millions of unknowns. Such large matrix equations can be solved iteratively with the multilevel fast multipole algorithm (MLFMA) in $\mathcal{O}(N \log N)$ time using $\mathcal{O}(N \log N)$ memory, where N is the number of unknowns [1]. Since MLFMA running on a single processor may not be sufficient to solve large-scale problems in reasonable times, it is essential to parallelize and employ MLFMA on clusters of processors [2]–[9]. Parallelization of MLFMA on distributed-memory architectures is particularly popular due to the relatively inexpensive costs of these architectures [2]–[6],[8],[9]. In fact, the last decade has seen enormous advances in this area, increasing the problem sizes from tens of millions to over one billion. On the other hand, parallel implementations have been developed mostly for analyzing perfectly conducting objects, and less attention has been paid to dielectric structures [5].

In this study, we consider the solution of electromagnetics problems involving three-dimensional dielectric objects with parallel MLFMA. For the efficient parallelization of MLFMA, we use a rigorous technique, namely, the hierarchical partitioning strategy, which was originally developed for perfectly conducting objects [6]. Using the hierarchical strategy for dielectric structures, tree structures associated with the outer and inner media are distributed optimally among the processors. The resulting implementation is used to solve large-scale problems discretized with tens of millions unknowns.

2 Solutions of Integral-Equation Formulations with Parallel MLFMA

Discretizations of surface integral equations for dielectric objects lead to $2N \times 2N$ dense matrix equations in the form of

$$\begin{bmatrix} \bar{\mathbf{Z}}^{(11)} & \bar{\mathbf{Z}}^{(12)} \\ \bar{\mathbf{Z}}^{(21)} & \bar{\mathbf{Z}}^{(22)} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{v}^{(1)} \\ \mathbf{v}^{(2)} \end{bmatrix}, \quad (1)$$

where \mathbf{x} and \mathbf{y} are vectors of N elements involving the coefficients for the equivalent electric and magnetic currents, respectively. Dielectric problems can be formulated rigorously with the electric and magnetic current combined-field integral equation (JMCFIE) [10]. For this formulation, matrix elements and the

elements of the right-hand-side vectors are derived as

$$\begin{aligned} Z_{mn}^{(11)} = Z_{mn}^{(22)} &= \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot (\mathcal{T}_o + \mathcal{T}_i)\{\mathbf{b}_n\}(\mathbf{r}) + \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \hat{\mathbf{n}} \times (\mathcal{K}_o - \mathcal{K}_i)\{\mathbf{b}_n\}(\mathbf{r}) \\ &\quad - \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \mathbf{b}_n(\mathbf{r}), \end{aligned} \quad (2)$$

$$\begin{aligned} Z_{mn}^{(12)} &= \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \hat{\mathbf{n}} \times (\eta_o^{-1}\mathcal{T}_o - \eta_i^{-1}\mathcal{T}_i)\{\mathbf{b}_n\}(\mathbf{r}) - \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot (\eta_o^{-1}\mathcal{K}_o + \eta_i^{-1}\mathcal{K}_i)\{\mathbf{b}_n\}(\mathbf{r}) \\ &\quad - \frac{1}{2}(\eta_o^{-1} - \eta_i^{-1}) \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \hat{\mathbf{n}} \times \mathbf{b}_n(\mathbf{r}), \end{aligned} \quad (3)$$

$$\begin{aligned} Z_{mn}^{(21)} &= - \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \hat{\mathbf{n}} \times (\eta_o\mathcal{T}_o - \eta_i\mathcal{T}_i)\{\mathbf{b}_n\}(\mathbf{r}) + \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot (\eta_o\mathcal{K}_o + \eta_i\mathcal{K}_i)\{\mathbf{b}_n\}(\mathbf{r}) \\ &\quad + \frac{1}{2}(\eta_o - \eta_i) \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \hat{\mathbf{n}} \times \mathbf{b}_n(\mathbf{r}), \end{aligned} \quad (4)$$

$$v_m^{(1)} = - \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \hat{\mathbf{n}} \times \mathbf{H}^{inc}(\mathbf{r}) - \eta_o^{-1} \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \mathbf{E}^{inc}(\mathbf{r}), \quad (5)$$

$$v_m^{(2)} = \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{E}^{inc}(\mathbf{r}) - \eta_o \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \mathbf{H}^{inc}(\mathbf{r}), \quad (6)$$

where the integro-differential operators are defined for the outer ($u = o$) and inner ($u = i$) media as

$$\mathcal{T}_u\{\mathbf{b}_n\}(\mathbf{r}) = ik_u \int_{S_n} d\mathbf{r}' \mathbf{b}_n(\mathbf{r}') g_u(\mathbf{r}, \mathbf{r}') + \frac{i}{k_u} \int_{S_n} d\mathbf{r}' \nabla' \cdot \mathbf{b}_n(\mathbf{r}') \nabla g_u(\mathbf{r}, \mathbf{r}'), \quad (7)$$

$$\mathcal{K}_u\{\mathbf{b}_n\}(\mathbf{r}) = \int_{PV, S_n} d\mathbf{r}' \mathbf{b}_n(\mathbf{r}') \times \nabla' g_u(\mathbf{r}, \mathbf{r}'). \quad (8)$$

In (2)–(8), $\mathbf{t}_m(\mathbf{r})$ and $\mathbf{b}_n(\mathbf{r})$ represent the m th testing function and the n th basis function with spatial supports of S_m and S_n , respectively, $\hat{\mathbf{n}}$ is the unit normal vector at the testing point, $\eta_u = (\mu_u/\epsilon_u)^{1/2}$ is the intrinsic impedance, $k_u = \omega(\epsilon_u\mu_u)^{1/2}$ is the wavenumber, \mathbf{E}^{inc} and \mathbf{H}^{inc} are the electric and magnetic fields created by external sources in the outer medium extending to infinity, and $g_u(\mathbf{r}, \mathbf{r}') = \exp(ik_u|\mathbf{r}-\mathbf{r}'|)/|\mathbf{r}-\mathbf{r}'|$ denotes the homogeneous-space Green's function.

Matrix equation in (1) can be solved iteratively via a Krylov-subspace algorithm, where the required matrix-vector multiplications are performed efficiently with MLFMA [1]. Multilevel tree structures (one for each medium) are constructed by placing the object in a cubic box and recursively dividing the computational domain into subdomains. This way, interactions between the distant basis and testing functions are calculated in three stages, namely, aggregation, translation, and disaggregation. In the aggregation stage, radiated fields are calculated at subdomain centers from the bottom of the tree structure to the top level. In the translation stage, radiated fields are converted into incoming fields between the subdomains. Finally, in the disaggregation stage, total incoming fields are calculated from the top of the structure to the bottom level. Different tree structures are required for the outer and inner media, since radiated and incoming fields are sampled at a rate that depends on the wavenumber, hence the electrical parameters of the medium.

MLFMA can be parallelized efficiently using a hierarchical partitioning strategy [6], which is based on the simultaneous partitioning of both subdomains and their fields at each level of the tree structure. Specifically, for a parallelization among p processes, subdomains and field samples are divided into p_s and $p_f = p/p_s$ partitions, respectively, where p_s and p_f are determined by load-balancing algorithms. Changing the partitioning scheme between levels bears an additional cost, but in general, the hierarchical strategy improves the load-balancing and reduces the communication time, compared to other available strategies. In this study, the hierarchical strategy is applied for the first time to solve dielectric problems.

3 Numerical Examples

All solutions presented in this paper are performed on a cluster of Intel Xeon Nehalem quad-core processors with 2.80 GHz clock rate. MLFMA is parallelized into 64 processes using the hierarchical strategy.

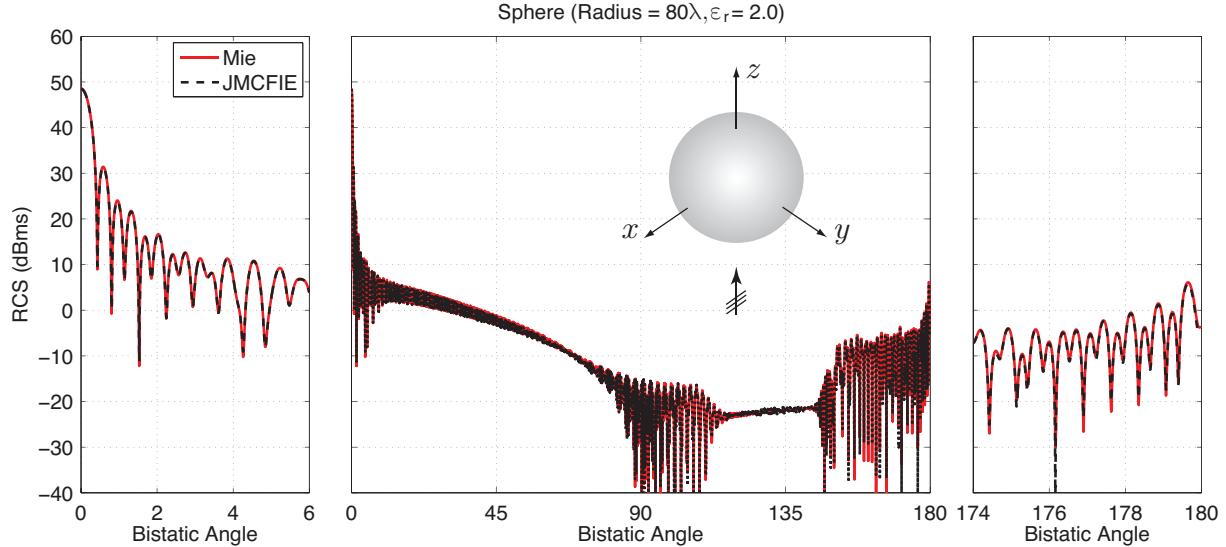


Figure 1: Solution of a scattering problem involving a dielectric sphere of radius 0.3 m at 80 GHz. The relative permittivity of the sphere is 2.0. The total RCS is plotted as a function of bistatic angle from 0° to 180° , where 0° corresponds to the forward-scattering direction.

Discretizations are performed with the Rao-Wilton-Glisson functions on $\lambda/10$ triangles, where $\lambda = 2\pi/k_o$ is the wavelength in the outer medium. Both near-field and far-field interactions are computed with maximum 1% error. Iterative solutions are performed with the biconjugate-gradient-stabilized (BiCGStab) algorithm and iterations are carried out until the residual error is reduced to below 0.005. Convergence of iterations is accelerated using block-diagonal preconditioners.

Fig. 1 presents the solution of a scattering problem involving a dielectric sphere of radius 0.3 m at 80 GHz discretized with 46,811,328 unknowns. The relative permittivity of the sphere is 2.0 and it is illuminated by a plane wave propagating in the z direction with the electric field polarized in the x direction in free space. The solution requires 107 iterations and the total time required for both setup and solution is 29 hours. Fig. 1 depicts the bistatic radar cross section (RCS) values (in dB meter square) on the z - x plane as a function of the bistatic angle θ from 0° to 180° . RCS values around the forward-scattering (0°) and back-scattering (180°) directions are focused in separate plots. We observe that computational values obtained with the parallel MLFMA perfectly agree with the analytical Mie-series results.

Fig. 2 presents the solution of a scattering problem involving a dielectric hemisphere lens with 25 mm radius. The lens has a relative permittivity of 4.8 and it is investigated at 120 GHz and 960 GHz, corresponding to radii of 10λ and 80λ , respectively. Discretizations at these frequencies lead to matrix equations involving 615,456 and 39,389,184 unknowns, respectively. At both frequencies, the lens is illuminated by a plane wave propagating in the $-z$ direction. Fig. 2(a) depicts the total electric field in the vicinity of the lens at 120 GHz, where focusing is clearly observed in the transmission region. Fig. 2(b) depicts the total electric field from $z = -40$ mm to 40 mm at 960 GHz. Focusing is again observed at around -9 mm. For the larger problem, the number of iterations is 68, whereas the total time is 22 hours.

4 Conclusion

This study presents rigorous solutions of large-scale electromagnetics problems involving homogeneous dielectric objects. Problems are formulated with JMCFIE and solved iteratively via MLFMA. For large-scale computations, MLFMA is parallelized using the hierarchical partitioning strategy. We show that dielectric problems discretized with tens of millions of unknowns can be solved accurately and efficiently with the developed implementation.

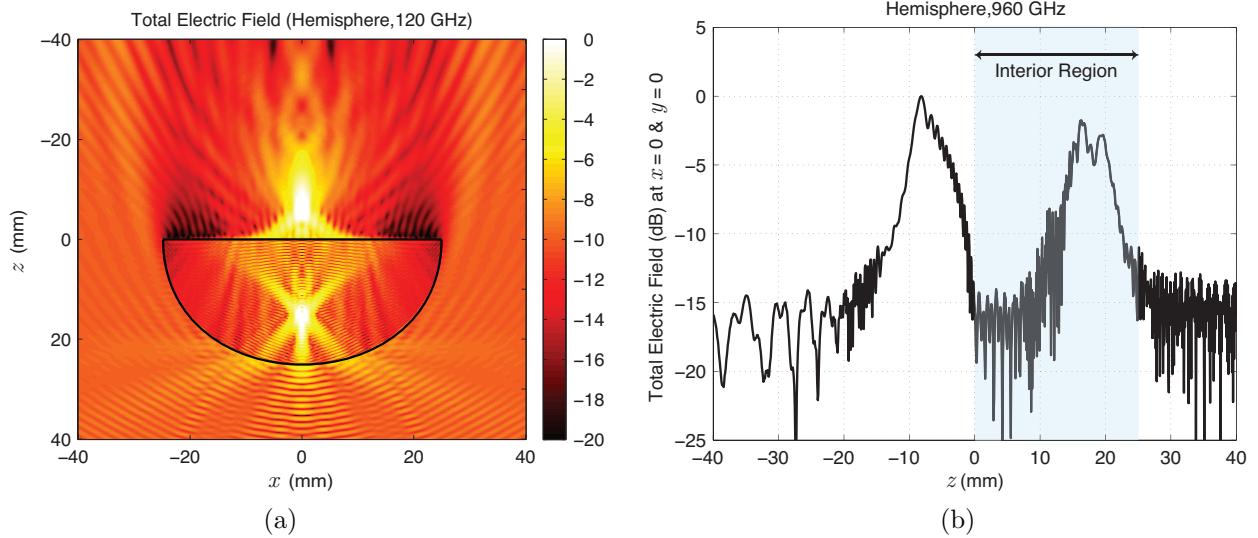


Figure 2: Solutions of scattering problems involving a dielectric hemisphere lens with 25 mm radius. The lens has a relative permittivity of 4.8. (a) The total electric field in the vicinity of the lens at 120 GHz. (b) The total electric field from $z = -40$ mm to 40 mm at 960 GHz.

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