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A NONLINEAR CONTROL SCHEME FOR DISCRETE TIME CHAOTIC SYSTEMS

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Abstract: In this paper we consider the stabilization problem of unstable periodic orbits of discrete time chaotic systems. We consider both one dimensional and higher dimensional cases. We propose a nonlinear feedback law and present some stability results. These results show that for period 1 all hyperbolic periodic orbits can be stabilized with the proposed method. By restricting the gain matrix to a special form we obtain some novel stability results. The stability proofs also give the possible feedback gains which achieve stabilization. We also present some simulation results.

Keywords: Chaotic Systems, Chaos Control, Delayed Feedback System, Pyragas Controller, Stability.

1. INTRODUCTION

The study of dynamical systems has always been an area which attracts scientist from different disciplines including engineers, mathematicians, physicists, etc. Because of the fact that many systems exhibit chaotic behaviours, the study of such systems has received considerable attention in recent years, see e.g. (Chen and Dong, 1999), (Fradkov and Evans, 2002), and the references therein. Since chaotic systems exhibit quite interesting behaviours, various aspects of such systems have been investigated in the literature. Among these the feedback control of chaotic systems received great interests among scientists from various disciplines after the seminal work of (Ott, Grebogy and Yorke, 1990). After the latter work, various other contributions have appeared in the area of chaos control. The literature is quite rich on this subject, see e.g. (Chen and Dong, 1999), (Fradkov and Evans, 2002), and the references therein.

Chaotic systems may exhibit quite large number of interesting behaviours. One of such interesting features which is a characteristic behaviour of chaotic systems is that they usually possess attractors which are called "strange" due to various reasons. Such attractors usually contain infinitely many unstable periodic orbits, (Devaney, 1987). An interesting result first given in (Ott, Grebogy and Yorke, 1990) proved that some of these unstable orbits could be easily stabilized by applying small control inputs to such systems. Following the latter result various control schemes for the stabilization of unstable periodic orbits of chaotic systems have been proposed, see e.g. (Chen and Dong, 1999), (Fradkov and Evans, 2002), and the references therein. Among these works, the Delayed Feedback Control (DFC) scheme, first proposed by Pyragas in (Pyragas, 1992) has received considerable attention due to its various attractive features as well as its simplicity. The study of DFC revealed that this scheme has some inherent limitations, that is it cannot stabilize certain type of unstable periodic orbits, see e.g. (Morgül, 2003), (Ushio, 1996), (Nakajima, 1997), (Morgül, 2005a). We note that a recent result presented in (Fiedler *et al.*, 2007), showed clearly that under certain cases, odd number limitation property does not hold for autonomous continuous time systems. Although the subject is still open and deserves further investigation, we note that the limitation of DFC stated above holds for discrete time case, see e.g. (Ushio, 1996), (Morgül, 2003), (Morgül, 2005a).

To eliminate the limitations of DFC indicated above, different modifications and/or extensions have been proposed, see e.g. (Pyragas, 1995), (Pyragas, 2001), (Socolar *et. al.,* 1994), (Bleich, and Socolar, 1996), (Vieira, and Lichtenberg, 1996), and the references therein. Among such modifications, the periodic feedback scheme proposed in (Schuster and Stemmler, 1997) eliminates those limitations for period 1 case and it could be generalized to higher period cases in various ways. Two such generalizations are given in (Morgül, 2006), (Morgül, 2005b) and it has been shown in that any hyperbolic periodic orbit can be stabilized with these schemes. Another modification is the so-called extended DFC (EDFC), see (Socolar *et. al.,* 1994). It has also been shown that EDFC also has inherent limitations similar to the DFC. In (Vieira, and Lichtenberg, 1996), a nonlinear version of EDFC has been proposed and it was shown that an optimal version of this scheme becomes quite simple. A generalization of this scheme for arbitrary periodic orbits for one dimensional systems has been given in (Morgül, 2009a). Preliminary results of the extension of these ideas to higher dimensional case for the latter approach has been presented in (Morgül, 2009b).

In this paper we will elaborate on the nonlinear scheme proposed in (Morgül, 2009a) and (Morgül, 2009b) by considering the stabilization of arbitrary periodic orbits of multi dimensional discrete time chaotic systems. Instead of a simulation based search for the stabilizing gains used in (Morgül, 2009b), we will provide an approach which is more systematic.

This paper is organized as follows. In section 2 we will outline the basic problem considered and introduce some notation used throughout the paper. In section 3 we will introduce the nonlinear controller for the one dimensional case and present some stability results. In section 4 we will present the generalization of this scheme to higher dimensional case and provide some stability results. Then we will present some simulation results and finally we will give some concluding remarks.

2. PROBLEM STATEMENT

Let us consider the following discrete-time system

$$x(k+1) = f(x(k)) \quad , \tag{1}$$

where $k = 1,2\ldots$ is the discrete time index, $x \in \mathbf{R}^n$, $f : \mathbf{R}^n \to \mathbf{R}^n$ is an appropriate function, which is assumed to be differentiable wherever required. We assume that the system given by (1) possesses a period T orbit characterized by the set

$$\Sigma_T = \{x_1^*, x_2^*, \ldots, x_T^*\} \quad , \tag{2}$$

where $x_i^* \in \mathbf{R}^n$, $i = 1,2,\ldots,T$.

Let $x(\cdot)$ be a solution of (1). To characterize the convergence of $x(\cdot)$ to Σ_T, we need a distance measure, which is defined as follows. For x_i^*, we will use circular notation, i.e. $x_i^* = x_j^*$ for $i = j \ (mod\ (T))$. Let us define the following indices ($j = 1,\ldots,T$):

$$d_k(j) = \sqrt{\sum_{i=0}^{T-1} \|x(k+i) - x_{i+j}^*\|^2} \quad , \tag{3}$$

where $\|\cdot\|$ is the standard Euclidean norm on $\mathbf{R}^n$.

We then define the following distance measure

$$d(x(k),\Sigma_T) = \min\{d_k(1),\ldots,d_k(T)\} \quad . \tag{4}$$

Clearly, if $x(1) \in \Sigma_T$, then $d(x(k),\Sigma_T) = 0$, $\forall k$. Conversely if $d(x(k),\Sigma_T) = 0$ for some k_0, then it remains 0 and $x(k) \in \Sigma_T$, for $k \geq k_0$. We will use $d(x(k),\Sigma_T)$ as a measure of convergence to the periodic solution given by Σ_T.

Let $x(\cdot)$ be a solution of (1) starting with $x(1) = x_1$. We say that Σ_T is (locally) asymptotically stable if there exists an $\varepsilon > 0$ such that for any $x(1) \in \mathbf{R}^n$ for which $d(x(1),\Sigma_T) < \varepsilon$ holds, we have $\lim_{k\to\infty} d(x(k),\Sigma_T) = 0$. Moreover if this decay is exponential, i.e. the following holds for some $M \geq 1$ and $0 < \rho < 1$, $(k > 1)$

$$d(x(k),\Sigma_T) \leq M\rho^k d(x(1),\Sigma_T) \quad , \tag{5}$$

then we say that Σ_T is (locally) exponentially stable.

To stabilize the periodic orbits of (1), let us apply the following control law :

$$x(k+1) = f(x(k)) + u(k) \tag{6}$$

where $u(\cdot) \in \mathbf{R}^n$ is the control input. In classical DFC, the following feedback law is used $(k > T)$:

$$u(k) = K(x(k) - x(k-T)) \quad , \tag{7}$$

where $K \in \mathbf{R}^{n\times n}$ is a constant gain to be determined. It is known that the scheme given above has certain inherent limitations, see e.g. (Ushio, 1996). For simplicity, let us assume one dimensional case, i.e. $n = 1$. For Σ_T, let us set $a_i = f'(x_i^*)$. It can be shown that Σ_T cannot be stabilized with this scheme if $a = \prod_{i=1}^T a_i > 1$, see e.g. (Morgül, 2003), (Ushio, 1996), and a similar condition can be generalized to the case $n > 1$, (Nakajima, 1997), (Morgül, 2005a). A set of necessary and sufficient conditions to guarantee exponential stabilization can be found in (Morgül, 2003) for $n = 1$ and in (Morgül, 2005a) for $n > 1$. By using these results one can find a suitable gain K when the stabilization is possible.

3. A NONLINEAR GENERALIZATION OF DFC

To simplify our analysis we first consider one dimensional case, i.e. $n = 1$ throughout this section. Consider the system given by (1). First consider a period 1 orbit Σ_1 of (1) (i.e. fixed point of $f : \mathbf{R} \to \mathbf{R}$) given by $\Sigma_1 = \{x_1^*\}$. Instead of control law given by (7), let us consider the following control law :

$$u(k) = \frac{K}{K+1}(x(k) - f(x(k)) \quad , \qquad (8)$$

where $K \in \mathbf{R}$ is a constant gain to be determined. Obviously we require $K \neq -1$. By using (8) in (6), we obtain :

$$x(k+1) = \frac{1}{K+1} f(x(k)) + \frac{K}{K+1} x(k) \ . \qquad (9)$$

Obviously on Σ_1, we have $u(k) = 0$, see (8). Furthermore if $x(k) \to \Sigma_1$ (i.e. when Σ_1 is asymptotically stable) we have $u(k) \to 0$ as well. Therefore, the scheme proposed in (8) enjoys the similar properties of DFC.

To analyze the stability of Σ_1, let us define $a = a_1 = f'(x_1^*)$. By using linearization, (9) and the classical Lyapunov stability analysis, we can easily show that Σ_1 is locally exponentially stable for (9) if and only if

$$\mid \frac{K+a}{K+1} \mid < 1 \quad . \qquad (10)$$

It can easily be shown that if $a \neq 1$, then any Σ_1 can be stabilized by choosing K appropriately to satisfy (10), see e.g. (Morgül, 2009a) and (Morgül, 2009b). This shows that any hyperbolic periodic orbit can be stabilized with the proposed scheme for $T = 1$ case.

The scheme proposed above for $T = 1$ case could be generalized to period m case as follows :

$$u(k) = \frac{K}{K+1}(x(k-m+1) - f(x(k)) \quad , \qquad (11)$$

where $K \in \mathbf{R}$ is a constant gain to be determined. If we use (11) in (6), we obtain :

$$x(k+1) = \frac{1}{K+1}(f(x(k)) + Kx(k-m+1)) . (12)$$

Now let us assume that period m orbit Σ_m of (1) be given as in (2). Let us define $a_i = f'(x_i^*)$, $i = 1, 2, \ldots, n$, and $a = \prod_{i=1}^{i=n} a_i$. Let us define the following characteristic polynomial $p_m(\cdot)$ associated with the system given by (6) and (11) as follows :

$$p_m(\lambda) = (\lambda - \frac{K}{K+1})^m - \frac{a}{(K+1)^m} \lambda^{m-1} \quad . (13)$$

A polynomial is called as Schur stable if all of its roots are strictly inside the unit disc of the complex plane, i.e. the roots have magnitude strictly less than 1. As is well known, by using Lyapunov stability theory local stability can be analyzed by using the Schur stability of an appropriately defined characteristic polynomial, see e.g. (Khalil, 2002). The next theorem is a result of such an analysis.

Theorem 1 : Let Σ_m given by (2) be a period $T = m$ orbit of (1) and set $a_i = f'(x_i)$, $i = 1, 2, \ldots, m$, $a = \prod_{i-1}^{m} a_i$. Consider the control scheme given by (6) and (11). Then :

i : Σ_m is locally exponentially stable if and only if $p_m(\lambda)$ given by (13) is Schur stable. This condition is only sufficient for asymptotic stability.

ii : If $p_m(\lambda)$ has at least one unstable root, i.e. outside the unit disc, then Σ_m is unstable as well.

iii : If $p_m(\lambda)$ is marginally stable, i.e. has at least one root on the unit disc while the rest of the roots are inside the unit disc, then the proposed method to test the stability of Σ_m is inconclusive.

Proof : The proof of this Theorem easily follows from standard Lyapunov stability arguments, see e.g. (Khalil, 2002), and (Morgül, 2003), (Morgül, 2005a), (Morgül, 2009a) and (Morgül, 2009b) for similar arguments. □

Associated with (13), let us define the following constants

$$K_{cr} = -0.5 + 0.5(\mid a \mid)^{1/m} \quad , \qquad (14)$$

$$a_{mcr} = (\frac{m}{m-2})^m \quad . \qquad (15)$$

Given a and m, by studying the relation between the roots of $p_m(\cdot)$ given by (13) and the gain K, we obtain the following results.

Theorem 2 : Let a and m be given and consider the polynomial $p_m(\cdot)$ given by (13).

i : If K is a stabilizing gain, then $K + 1 > 0$.

ii : If $a > 1$, then stabilization is not possible, (i.e. $p_m(\cdot)$ is not Schur stable for any K).

iii : If $\mid a \mid < 1$, $p_m(\cdot)$ is Schur stable for any $K \geq 0$.

iv : For $K \leq K_{cr}$, stabilization is not possible.

v : If $-a_{mcr} < a < 1$, then there exits a $K_m > K_{cr}$ such that $p_m(\cdot)$ is Schur stable for $K_{cr} < K < K_m$.

Proof : For stability of $p_m(\cdot)$, a necessary condition is to have $\mid p_m(0) \mid < 1$, see e.g. (Elaydi, 1996). This implies $K + 1 > 0$ should hold, which proves **i**. Another necessary condition for stability is that $p_m(1) > 0$ should hold, see e.g. (Elaydi, 1996). Together with $K + 1 > 0$, this implies that $1 - a > 0$ should hold, which proves **ii** . When $\mid a \mid < 1$, Σ_m is already stable for $K = 0$. This could also be seen from (13), since $m-1$ roots of $p_m(\cdot)$ are at 0 and the last one is at a when $K = 0$. By analyzing the roots of $p_m(\cdot)$ (e.g. by using Rouchè's theorem), it can be shown that all of the roots of (13) are inside the unit disc for $K \geq 0$, which proves **iii**. Another necessary condition for Schur stability is to have $(-1)^m p_m(-1) > 0$, see e.g. (Elaydi, 1996). By using the latter, together with $K + 1 > 0$, we obtain $K > K_{cr}$, which proves **iv**. To prove **v**, one can show that when $K = K_{cr}$, $m - 1$ roots of $p_m(\cdot)$ are strictly inside the unit disc and the last one is at -1. Then by using some continuity arguments. we can show that for $K = K_{cr} + \varepsilon$ where $\varepsilon > 0$ is sufficiently small, $p_m(\cdot)$ is Schur stable, which proves **v**. □

4. EXTENSION TO HIGHER DIMENSIONAL CASE

The stabilization scheme given in the previous section can be generalized to higher dimensional case by changing K from being a scalar to a gain matrix. For motivation, as in the previous section let us consider a period 1 orbit Σ_1 of (1) given by $\Sigma_1 = \{x_1^*\}$, where $x_1^* \in \mathbf{R}^n$ is a fixed point of $f : \mathbf{R}^n \to \mathbf{R}^n$. One possible generalization of the control law given by (11) is the following :

$$u(k) = (K+I)^{-1}K(x(k) - f(x(k))) \quad , \qquad (16)$$

where $K \in \mathbf{R}^{n\times n}$ is a constant gain matrix to be determined, and I is $n \times n$ identity matrix. Obviously $K+I$ should be nonsingular, i.e. K should not have an eigenvalue -1. By using (16) in (6) we obtain :

$$x(k+1) = (K+I)^{-1}(f(x(k)) + Kx(k)) \quad . \quad (17)$$

For stability analysis, let us define :

$$J = \frac{\partial f}{\partial x}\,|_{x=x_1^*} \quad . \qquad (18)$$

Let us define $e = x - x_1^*$. By using linearization, from (17) we obtain :

$$e(k+1) = Ae(k) \quad , \qquad (19)$$

where $A = (I+K)^{-1}(J+K)$. Clearly the error dynamics given by (19) is locally exponentially stable if A given above have all of its eigenvalues in the unit disc. The characteristic polynomial associated with A can be given as :

$$\begin{aligned} p_1(\lambda) &= \det(\lambda I - A) \qquad (20) \\ &= \det[(\lambda I - (I+K)^{-1}K) \\ &\qquad -(I+K)^{-1}J] \end{aligned}$$

which is similar to (13) for $n = 1$. However, establishing a similar relation for the case $m > 1$ is not straightforward. First note that if J does not have an eigenvalue 1, then by choosing K appropriately, $p_1(\lambda)$ can be made stable. Indeed, if Δ is any Schur stable matrix, then $K = (I-\Delta)^{-1}(\Delta - J)$ is such a stabilizing gain matrix, (Morgul, 2009b). Hence the limitations of DFC are greatly eliminated with the proposed scheme; in fact any hyperbolic fixed point can be stabilized with the proposed approach.

Moreover, if K is constrained to the form $K = \varepsilon I$, then (20) reduces to :

$$p_1(\lambda) = \det[(\lambda - \frac{\varepsilon}{\varepsilon+1})I - \frac{1}{\varepsilon+1}J] \quad , \quad (21)$$

for which the similarity with (13) is more apparent. In this case, the roots of $p_1(\lambda)$ given by (21) are the same as the eigenvalues of $J_\varepsilon = \frac{\varepsilon I + J}{\varepsilon+1}$. Let the eigenvalues of J be given as $\lambda_1, \ldots, \lambda_n$. Then the eigenvalues of J_ε are $\frac{\varepsilon+\lambda_i}{\varepsilon+1}$, $i = 1, 2, \ldots, n$. Clearly for stabilization we require : $\mid \frac{\varepsilon+\lambda_i}{\varepsilon+1} \mid < 1$, $i = 1, 2, \ldots, n$. If all λ_i are real, and $\lambda_i < 1$, then by choosing $\varepsilon > 0$ sufficiently high, we can always find a stabilizing gain of the form $K = \varepsilon I$. On the other hand, if some eigenvalues of J are complex, then a similar analysis would be more complicated and due to space limitations this part of the analysis is omitted here.

The control law given by (16) could be generalized to higher order periods $T = m > 1$ as follows :

$$u(k) = (K+I)^{-1}K(x(k-m+1) - f(x(k)) \quad .(22)$$

Let $\Sigma_m = \{x_1^*, \ldots, x_m^*\}$ be such a period m orbit of (1). Let us define the following Jacobians :

$$J_i = \frac{\partial f}{\partial x}\,|_{x=x_i^*} \quad , \ i = 1,..,m \ , \ J = J_1.J_2..J_m \ . (23)$$

Stability of Σ_m for the system given by (6) and (22) becomes rather complex. We follow the methodology given in (Morgul, 2003), (Morgul, 2005a). Let us define the variables x_i as follows :

$$x_i(k) = x(k-m+i) \ , \ i = 1, \ldots, m \ . \qquad (24)$$

By using (24), let us define $z = (x_1^T..x_m^T)^T \in \mathbf{R}^{nm}$. Let us define the following variables (for $i = 1, \ldots, m$) :

$$Y_0 = x_m \quad , \qquad (25)$$

$$Y_i = (I+K)^{-1}f(Y_{i-1}) + (I+K)^{-1}Kx_i \quad . \quad (26)$$

Now let us define a map $F : \mathbf{R}^{nm} \to \mathbf{R}^{nm}$ as follows :

$$F(z) = (x_2^T x_3^T ... x_m^T Y_1^T)^T \quad . \qquad (27)$$

It can easily be shown that $F^m(z) = (Y_1^T ... Y_m^T)^T$. Also it can be shown that Σ_m now corresponds to a fixed point of F^m. More precisely, corresponding to Σ_m, let us define a vector $z^* = (x_1^{*T}..x_m^{*T})^T$. It is easy to show that z^* is a fixed point of F^m. It then can easily be shown that the stability of Σ_m for (6) and (30) can be analyzed by considering the stability of the fixed point z^* of F^m. For the latter, let us define the following Jacobian matrix :

$$J_F = \frac{\partial F^m}{\partial z}\,|_{z=z^*} \quad , \qquad (28)$$

and define the characteristic polynomial $p_m(\cdot)$ as follows :

$$p_m(\lambda) = \det(\lambda I - J_F) \quad . \qquad (29)$$

Theorem 3 : Let Σ_m be a period m orbit of (1). Consider the system given by (6) and (22). Then, Σ_m is locally exponentially stable if and only if the characteristic polynomial given by (29) is Schur stable.

Proof : The proof of this Theorem easily follows from standard Lyapunov stability arguments, see e.g. (Khalil, 2002), and (Morgül, 2003), (Morgül, 2005a), (Morgül, 2009a) for similar arguments. □

Obtaining a better expression for $p_m(\cdot)$ given by (29) for an arbitrary gain matrix K is not straightforward. For the special case of $K = \varepsilon I$, after some straightforward calculations we obtain the following :

$$p_m(\lambda) = \det[(\lambda - \frac{\varepsilon}{\varepsilon+1})^m I - \frac{\lambda^{m-1}}{(\varepsilon+1)^m} J] \quad ,(30)$$

where J is given by (23). Clearly, if $n = 1$, then (30) reduces to (13).

For further development, let us assume that J has only real eigenvalues. Let $\lambda_1, \ldots, \lambda_n$ be these eigenvalues. Also let us define the following polynomials (for $i = 1, \ldots, n$) :

$$p_{mi}(\lambda) = (\lambda - \frac{\varepsilon}{\varepsilon+1})^m - \frac{\lambda^{m-1}\lambda_i}{(\varepsilon+1)^m} \quad . \qquad (31)$$

It can easily be shown that the following holds :

$$p_m(\lambda) = \prod_{i=1}^{n} p_{mi} \quad . \qquad (32)$$

If we can find an ε such that all $p_{mi}(\cdot)$ are Schur stable, then $K = \varepsilon I$ will be a stabilizing gain. Based on this observation, we can state the following result.

Theorem 4 : Let Σ_m be a period m solution of (1), and consider the system given by (6), (22). Consider the Jacobian J associated with Σ_m as defined in (23) and assume that J has only real eigenvalues given as $\lambda_1, \ldots, \lambda_n$. If $-a_{mcr} < \lambda_i < 1$, $i = 1, \ldots, n$, where a_{mcr} is given by (15), then there exist two constants $\varepsilon_{max} > \varepsilon_{min}$ such that $K = \varepsilon I$ is a stabilizing gain for Σ_m; here $\varepsilon_{min} < \varepsilon < \varepsilon_{max}$.

Proof : The proof follows from the results given in section 3 and the developments given above. See also Theorem 2. □

5. SIMULATION RESULTS

As a simulation example, we consider the coupled map lattices, which exhibit various interesting dynamical behaviours. We will use the following coupled lattice system :

$$x(k+1) = f(x(k)) + \alpha(f(y(k)) - f(x(k))), \quad (33)$$

$$y(k+1) = f(y(k)) + \alpha(f(x(k)) - f(y(k))), \quad (34)$$

where $f(\cdot)$ is the tent map given as $f(z) = mz$ for $z \leq 0.5$, and $f(z) = m - mz$ for $0.5 < z \leq 1$. For $m = 1.9$ and $\alpha = 0.1$, this map has period 3 solution characterized by the set $\Sigma_3 = \{w_1^*, w_2^*, w_3^*\}$ where $w_i^* = (\,x_i^* \;\; y_i^*\,)^T$, $i = 1,2,3$ and $x_1^* = 0.8966, y_1^* = 0.642793$, $x_2^* = 0.24467, y_2^* = 0.6304685$, $x_3^* = 0.48859, y_3^* = 0.678386$. The associated Jacobian matrix J given by (23) could easily be found. The eigenvalues of J can be found as $\lambda_1 = -4.3898$, $\lambda_2 = -5.4872$. Note that a_{3cr} can be found from (15) as 27, hence the conditions in the Theorem 4 are satisfied. By using (14), the critical gains K_{1cr} and K_{2cr} corresponding to λ_1 and λ_2 could be found as $K_{1cr} = 0.3187$ and $K_{2cr} = 0.3819$. By using the polynomials defined in (31) on can show that stabilization is possible for gains of the form $K = \varepsilon I$ where $0.3819 < \varepsilon < 90$. A typical simulation results were given in Figures 1-4. In these simulations, we chose $\varepsilon = 0.4$, $x(0) = 0.8, y(0) = 0.6$. In Figure 1, we show $d(w(k), \Sigma_3)$ versus k, and as can be seen the decay is exponential. Figure 2 shows $x(k)$ versus $y(k)$ plot for $k \geq 800$. As can be seen, solutions converge to Σ_3. Finally Figure 3 and 4 show $u_1(k)$ and $u_2(k)$ vs. k.

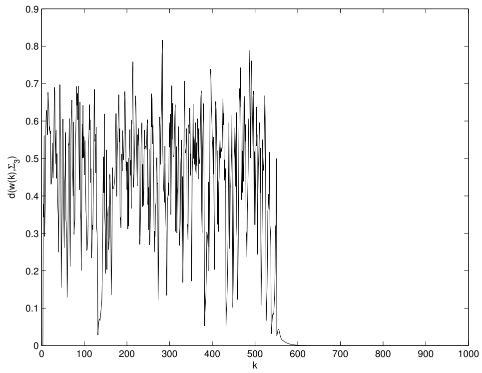

Fig. 1. $d(w(k), \Sigma_3)$ vs. k

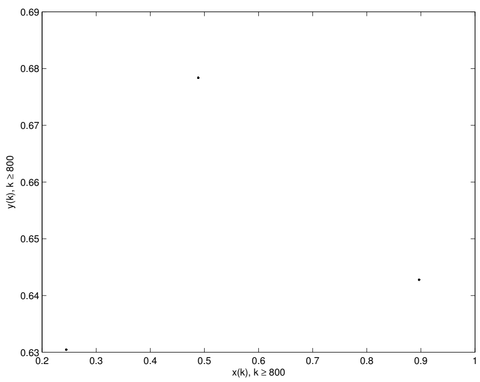

Fig. 2. $x(k)$ vs. $y(k)$ for $k \geq 800$

6. CONCLUSIONS

In this paper, we considered a generalization of the DFC scheme proposed in (Morgul, 2009a) to multidimensional case. Such an attempt was first made in (Morgul, 2009b), but the results presented in the latter were rather preliminary in nature. In the present paper, we considered first one dimensional case for the proposed method and presented some conditions

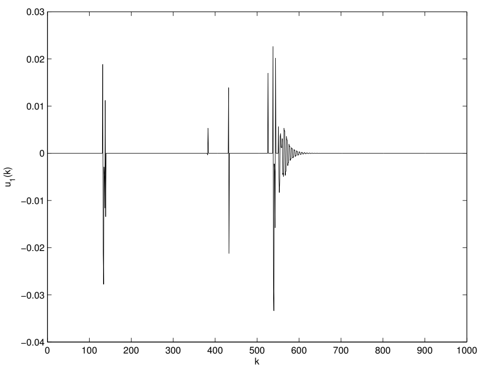

Fig. 3. $u_1(k)$ vs. k

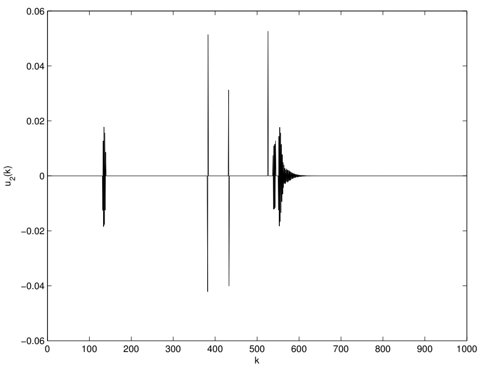

Fig. 4. $u_2(k)$ vs. k

which guarantee the existence of a stabilizing gain. Based on these results, obtaining bounds for the stabilizing gains are rather straightforward. Then by using these results in multidimensional case we obtained some conditions which guarantee the existence of a stabilizing gain in the form $K = \varepsilon I$. We also presented some simulation results.

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