# CLOSED-FORM GREEN'S FUNCTIONS OF HED, HMD, VED, AND VMD FOR MULTILAYER MEDIA

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**Abstract**

The closed-form Green's functions of the vector and scalar potentials in the spatial domain are presented for the sources of horizontal electric, magnetic, and vertical electric, magnetic dipoles embedded in a general, multilayer, planar medium. The spectral domain Green's functions in an arbitrary layer are obtained through the Green's function of the source layer by using a recursive algorithm. Then, the spatial domain closed-form Green's functions are obtained by adding the contributions of the direct terms, asympthotic components and the complex images approximated by the Generalized Pencil of Function method.

# 1 Introduction

Various studies have been made with layered microstrip structures [1-3] due to the increased popularity of the use of multilayer transmission lines such as striplines, covered microstrip lines and suspended substrate lines.

The rigorous analysis of layered structures requires the computation of the Green's functions for multilayer media, which are usually represented by the Sommerfeld integrals and the closed-form expressions in the spatial and spectral domains, respectively. The numerical evaluation of the matrix elements of the method of moments (MoM) is very time consuming because of the numerical integration of the Sommerfeld integrals in the spatial domain, and slow convergent, highly oscillatory double integrals in the spectral domain. To circumvent this problem, closed-form Green's functions in the spatial domain were formulated for a thick subtrate by using the Sommerfeld identity and the original Prony's method [4]. This technique has further been extended to microstrip geometries with both substrate and superstrate whose thickness can be arbitrary [3].

In this paper, the closed-form Green's functions for the vector and the scalar potentials of a Horizontal Electric Dipole (HED), Horizontal Magnetic Dipole (HMD), Vertical Electric Dipole (VED) , and Vertical Magnetic Dipole (VMD) located in an arbitrary layer of a multilayered medium are presented. The spectral domain Green's functions are obtained from the Green's functions of the source layer with the use of an iterative procedure [5]. Then, the contributions of the direct terms and asymthotic terms are added analytically to the terms which are obtained from the approximation of the remaining integrand with the use of the Generalized Pencil of Function (GPOF) method [6].

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## 2 Theory

A general multilayer geometry is shown in Fig. 1, where the source (HED, HMD, VED or VMD) is embedded in region i. The z-dependence of the fields in the source region is written as the sum of the direct terms and up- and down-going waves due to the reflections from the boundaries at $z = -h$ and $z = d_i - h$, respectively. The coefficients of up- and down-going waves can be obtained in terms of the generalized reflection coefficients by applying the appropriate boundary conditions. The spectral domain Green's functions in the source layer (region i) are obtained for a horizontal electric dipole, $\vec{J}^e$, and a horizontal magnetic dipole, $\vec{J}^m$, as

$$\tilde{G}^A_{xx} = \frac{\mu_i}{2jk_{z_i}}[e^{-jk_{z_i}|z|} + A^e_i e^{jk_{z_i}z} + C^e_i e^{-jk_{z_i}z}] \qquad (1)$$

$$\tilde{G}^A_{zx} = \frac{-\mu_i}{2jk_{z_i}}[\frac{k_x k_{z_i}}{k_{\rho^2}}(A^e_i + B^e_i)e^{jk_{z_i}z} + \frac{k_x k_{z_i}}{k^2_\rho}(D^e_i - C^e_i)e^{-jk_{z_i}z}] \qquad (2)$$

$$\tilde{G}^{q_e}_x = \frac{1}{j2\epsilon_i k_{z_i}}[e^{-jk_{z_i}|z|} + \frac{k^2_{z_i}B^e_i + k^2_i A^e_i}{k^2_\rho}e^{jk_{z_i}z} + \frac{k^2_i C^e_i - k^2_{z_i}D^e_i}{k^2_\rho}e^{-jk_{z_i}z}] \qquad (3)$$

$$\tilde{G}^F_{xx} = \frac{\epsilon_i}{2jk_{z_i}}[e^{-jk_{z_i}|z|} + A^m_i e^{jk_{z_i}z} + C^m_i e^{-jk_{z_i}z}] \qquad (4)$$

$$\tilde{G}^F_{zx} = \frac{-\epsilon_i}{2jk_{z_i}}\frac{k_x k_{z_i}}{k_{\rho^2}}(A^m_i + B^m_i)e^{jk_{z_i}z} + \frac{k_x k_{z_i}}{k^2_\rho}(D^m_i - C^m_i)e^{-jk_{z_i}z}] \qquad (5)$$

$$\tilde{G}^{q_m}_x = \frac{1}{j2\mu_i k_{z_i}}[e^{-jk_{z_i}|z|} + \frac{k^2_{z_i}B^m_i + k^2_i A^m_i}{k^2_\rho}e^{jk_{z_i}z} + \frac{k^2_i C^m_i - k^2_{z_i}D^m_i}{k^2_\rho}e^{-jk_{z_i}z}] \qquad (6)$$

where the factors $A^e_i$,$B^e_i$,$C^e_i$,$D^e_i$ and $A^m_i$,$B^m_i$,$C^m_i$,$D^m_i$ are functions of the generalized reflection coefficients and the geometry and $k^2_i = k^2_\rho + k^2_{z_i}$. The superscripts $A$ and $F$ represent the magnetic and electric vector potentials, respectively, $q$ represents the scalar potentials, and $e$ and $m$ are used for the electric and magnetic sources, respectively.

For the layers different from the source layer, the amplitudes of the up- and down-going waves are related to those in the adjacent layers by utilizing an iterative algorithm as

$$A^-_{i-m} = A^-_{i-m+1}\frac{T_{i-m+1,i-m}e^{j(k_{z_{i-m+1}}-k_{z_{i-m}})(-h-z_{-m+1})}}{1 - R_{i-m,i-m+1}\tilde{R}_{i-m,i-m-1}e^{-jk_{z_{i-m}}2(z_{-m}-z_{-m+1})}} \qquad (7)$$

where $R_{j,k}$ and $\tilde{R}_{j,k}$ represent the Fresnel reflection coefficients and the generalized reflection coefficients ,respectively, and $T_{j,k}$ is the transmission coefficient. Therefore, starting from the source layer, the field expressions in an arbitrary layer can be calculated iteratively . The closed-form expressions of the spatial domain Green's functions are then obtained by adding the contributions of direct terms and asympthotic components by using the Sommerfeld identity and approximating the remaining integrand by the GPOF method which is more efficient than the Prony's method [7] in terms of its noise sensitivity and the requirement for the additional analytical manipulations. While approximating the integrand by the GPOF method, the z variaton of $\tilde{G}_{zx}$ and $\tilde{G}_\rho$ are kept in explicit form for the proper use of the Green's functions in MOM applications.

## 3 Applications

Various layered microstrip geometries have been studied to demonstrate the validity of the technique. As a typical example, the closed-form Green's functions for the HED and HMD of a covered microstrip line with an air-gap between the two dielectric substrates are given in Figs. 2(a) and 2(b), respectively. The amplitudes and phases of the Green's functions of the vector potentials $\tilde{G}^{A}_{xx}$ and $\tilde{G}^{F}_{xx}$ calculated using both the closed forms (approximate) and the numerical integration (exact) are shown in Figs. 2(a) and 2(b) for $\epsilon_{r_1}=\epsilon_{r_3}=10.2$, $d_1=d_3=0.13$cm, $\epsilon_{r_2}=1$ and $d_2=0.05$cm. In all the cases studied, the approximate results are found to be in excellent agreement with the exact ones for both vertical and horizontal sources, and for all the components of the Green's functions.

## 4 Conclusions

The closed-form Green's funcitions in the spatial domain are presented for a general, planar, multilayer medium for the HED, HMD, VED and VMD. The computational efficiency in the calculation of the spatial domain Green's functions is significantly increased with the use of the GPOF method. A very good agreement is observed between the approximate and exact Green's functions.

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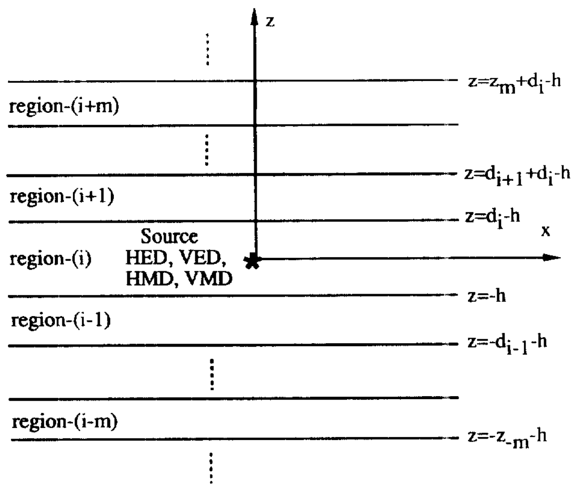


Fig. 1. A source embedded in a multilayered medium

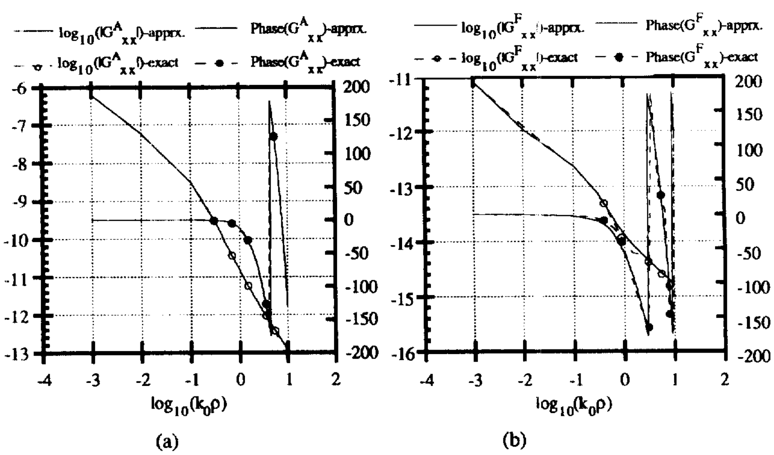


Fig. 2. The Green's functions for the horizontal (a) electric and (b) magnetic potentials in the source layer (i=2); magnitude and phase. f=3 GHz, $\varepsilon_{r1}=\varepsilon_{r3}=10.2$, $\varepsilon_{r2}=\varepsilon_{r4}=1.0$, $d_1=d_3=0.13$cm, $d_2=0.05$cm, layer-0 is PEC.