COMPARATIVE EVALUATION OF ABSORBING BOUNDARY CONDITIONS USING GREEN'S FUNCTIONS FOR LAYERED MEDIA

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Abstract

Absorbing boundary conditions are comparatively studied using the Green's functions of the vector and scalar potentials for multilayer geometries and general sources. The absorbing boundaries are introduced as additional layers with predefined reflection coefficients into the calculation of the Green's functions. The Green's functions are calculated using different reflection coefficients corresponding to different absorbing boundaries and compared to those obtained with no absorbing boundary. This approach provides an absolute measure of the effectiveness of different absorbing boundaries.

I. Introduction

Application of the numerical techniques based on the differential equations in unbounded regions, such as the finite difference time domain (FDTD) and the finite element methods (FEM), requires the truncation of the solution domain with artificial boundaries. Ideally, these boundaries are supposed to absorb all the incident waves, that is, there should be no reflected waves, so they are called absorbing or radiation boundaries [1]-[3]. However, there is always some reflected waves due to imperfect cancelation of the impinging waves on these artificial boundaries, because these boundary conditions are mathematical approximations to the partial differential equation for the one-way wave equation. The level of the reflection depends upon the absorbing boundaries used and the order of the approximation.

Since different absorbing boundaries give rise to different levels of reflected waves, one needs to examine these boundary conditions comparatively to decide on the type and the order of the absorbing boundary condition (ABC) to be used, to improve the accuracy of the results. For the purpose of comparison, numerical experiments can be performed on the geometry of interest, but here we propose the use of the Green's functions to assess the level of imperfections of the ABCs for planar geometries. The Green's functions of the vector and scalar potentials are obtained for multilayer media by including the reflections from each boundary [5]. Therefore, it facilitates the use of the absorbing boundary as an additional layer for which the reflection coefficients can be derived explicitly, and makes it possible to compare the effect of the ABCs on the Green's functions, providing an absolute measure of the merit of the ABCs. This is because the approach proposed here

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eliminates the extraneous problems caused by the discretizations required by the techniques used in conjunction with the ABCs, and therefore, gives true comparison of different types and orders of ABCs.

In this paper, the Green's function based comparison of the ABCs is given for the sources of a horizontal electric dipole (HED) and a horizontal magnetic dipole (HMD) in multilayer media. The comparison is based on the absorbing boundaries formulated by [2] for normal incidence, by [3] for arbitrary angle of incidence and "the perfectly matched layer (PML)" [6] proposed by Berenger.

II. Formulation

Since the Green's functions of the planar multilayer geometries are used in the comparison of the ABCs, the results are restricted to planar geometries, and examples are given for Fig. 1. It should be noted that layer-2 and layer-3 have the same electrical properties but they are separated by an artificial boundary which is supposed to absorb the up-going waves.

The analytical ABCs can be derived by operator factoring and the corresponding reflection coefficients can be obtained as described in [4]. The reflection coefficient expression is given here for the second- order ABC as

$$R = \frac{-\frac{k_{z_i}}{k_i} + p_o + p_2\frac{k_\rho^2}{k_i^2}}{\frac{k_{z_i}}{k_i} + p_o + p_2\frac{k_\rho^2}{k_i^2}} \tag{1}$$

where the subscript i stands for the layer number, k_ρ is the wave number in the transverse direction, and p's and q's depend upon the type of the approximation used in the derivation of the ABC and are given in [4]. A more general absorbing boundary, which is designed to absorb the plane waves incident at arbitrary angles, can be represented by the following reflection coefficient [3];

$$R = -\prod_{j=1}^{order} \frac{\cos\alpha_j - \frac{k_{z_i}}{k_i}}{\cos\alpha_j + \frac{k_{z_i}}{k_i}} \tag{2}$$

where the perfect absorption occurs at the angles of α_j's. "The perfectly matched layer (PML)" of Berenger involves creation of a non-physical absorber adjacent to the outer grid boundary that has a wave impedance independent of the angle of incidence and frequency of outgoing scattered waves. Berenger proposes a free-space computational zone surrounded by a PML layer backed by a perfectly conducting (PEC) wall yielding a PML reflection factor of

$$R(\theta) = e^{-2\sigma_{max}\delta\cos\theta/(n+1)\epsilon_o c} \tag{3}$$

where $\sigma(\rho) = \sigma_{max}(\frac{\rho}{\delta})^n$, ρ is the depth, δ is the thickness and σ is the electric conductivity of the PML layer.

The Green's functions used in this study are obtained for general multilayer media for the sources of an HED and HMD [5], where the reflection coefficients at each boundary are used according to the definition of the generalized reflection coefficient [5]. Therefore, the reflection coefficients given in (1)-(3) can easily be incorporated into the formulation of the Green's functions. The Green's functions of the vector and scalar potentials are employed in the study of the ABCs for the purpose of distinguishing the effects of the ABCs on the far and near fields.

III. Results and Discussion

The approach discussed in Section 2 can be applied to any multilayer geometry with arbitrary layer parameters, such as thicknesses, permittivities and permeabilities. For the sake of illustration, the following parameters have been chosen for the geometry shown in Fig.1: the dielectric constant of the substrate $\epsilon_{r_1}=4.0$; the thickness of the substrate $d_1=0.02032$ cm (8.0 mils); the frequency of operation $f=1.0$ GHz; the distance of the ABC from the air-dielectric interface $d_2=10.0$ cm; layer-0 PEC.

Ideally, absorbing boundaries are supposed to absorb all the waves impinging upon them but since the ABCs are approximations to the ideal case, inevitably there is always some reflection. Therefore, the ideal absorbing boundary corresponds to, in the case presented here, no absorbing boundary at all. To study the effect of the absorbing boundaries, the scalar potential at the air-dielectric interface in the presence of the absorbing boundaries is calculated and compared to those obtained with no absorbing boundary.

Figures 2 and 3 show the magnitude and the phase of the Green's function of the scalar potential, $G_x^{q_e}$, with and without the absorbing boundaries. It is observed that, depending on the type and the order of the ABC approximation used, the scalar potential showed some deviations from the ideal case. The third order Padè approximation [4] showed significant improvement as compared to the second order Padè approximation (Eq.1) both of which absorb only the normal-incident plane waves. It might be claimed that if the absorbing boundary annihilates the incoming waves at different angles of incidence, it would improve the overall performance of the ABC without increasing the order of approximation. As a matter of fact, an improvement is observed in the magnitude of the scalar potential, Fig. 2 where the angles of exact absorption are set to 0° and 60° in (2) for the second-order approximation, but the improvement in the phase is not significant. On the other hand, the Green's functions obtained using the PML ABC showed perfect agreement with those obtained with the ideal ABC. The distance between the substrate and the PML was reduced down to $\lambda/60$, even the agreement between the Green's functions with PML ABC and without PML was perfect. Here it can be concluded that the PML ABC is far superior compared to the others included in this paper. It is observed that the same arguments are valid for the magnetic source.

IV. Conclusions

The use of the Green's functions for the study of the ABCs have been demonstrated for planar media. Three different ABCs have been compared with each other and with the ideal case, but this approach can also be applied to other ABCs provided that the analytical expressions of the associated reflection coefficients are available. The strength of this approach over the numerical comparison of the ABCs is the analytical nature of the comparison, which does not depend on the numerical technique used, and is to provide an absolute comparison between different ABCs.

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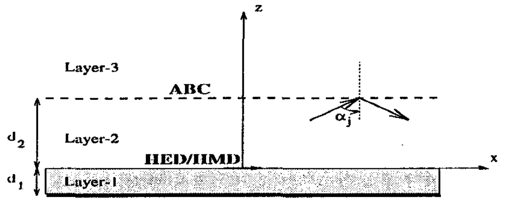

Figure 1: A typical planar geometry

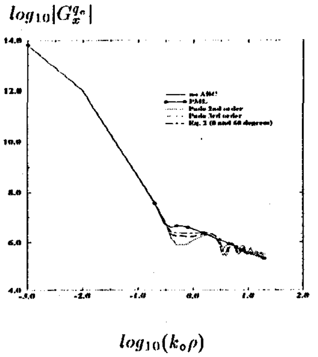

Figure 2: Magnitude of G_x^{qe}

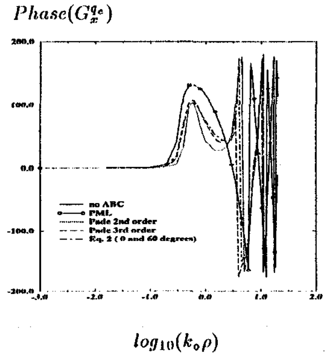

Figure 3: Phase of G_x^{qe}