Extension of Forward Backward Method with DFT Based Acceleration Algorithm for The Efficient Analysis of Large Periodic Arrays with Arbitrary Boundaries

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Introduction

In this paper, an extension of Forward-Backward (FB) with DFT (Discrete Fourier Transform) based acceleration approach is presented to provide a relatively efficient analysis of EM radiation/scattering from an electrically large, planar, periodic, finite arrays with arbitrary boundaries, such as arrays with circular or elliptical boundaries. Computational complexity of this new approach is $O(N_{tot})$, where N_{tot} is the number of unknowns in the array.

Recently, fast but accurate analysis of large array problems has been of great interest to researchers due to their wide variety of applications. Numerical methods such as MoM and FEM, when used alone in a conventional fashion, become rapidly inefficient for solving large array problems as a result of the extremely large number of unknowns that need to be solved. Several efficient approaches have been proposed to accelerate MoM solution and to reduce the memory storage requirements. Some of the MoM based works are infinite array solution, element by element with infinite array assumption and window function that includes the effects of truncation of the array [1], the hybrid UTD-MoM approach [2], and MoM solution based on DFT representation of currents that reduce the number of unknowns [3], and forward-backward approach which reduces iterations [4]. The FB is an iterative algorithm which accelerates the MoM computation by decomposing the current vector and the impedance matrix into forward and backward field contributions. FB algorithm has an $O(N_{tot}^2)$ computational complexity whereas the computational complexity of the direct solution of MoM matrix equation is $O(N_{tot}^3)$. Some acceleration techniques have been employed to reduce the computational complexity of FB to $O(N_{tot})$, such as FB with a spectral acceleration algorithm (NSA) [4]. Furthermore, FB approach with an acceleration algorithm based on DFT has been developed for and implemented to two dimensional freestanding dipole arrays [5] and dipole arrays on grounded substrate [6]. In DFT based approach, contributions to every receiving element are divided into two interaction groups, namely strong and weak contributions. The contributions from the strong group are calculated via an element-by-element fashion whereas the weak group interaction is obtained from the DFT representation of the entire current distribution. It has been shown that only very few significant DFT terms are sufficient to provide accurate results.

Formulation

In this section, the implementation of DFT-FB approach in the analysis of large periodic arrays with arbitrary boundaries is briefly described. To demonstrate the concept, FBM is implemented by considering a periodic array of identical short and thin perfectly conducting wire dipole elements oriented in the $\hat{y}$ direction at z=0 plane in air as illustrated in Fig. 1 (black dipoles in elliptical boundary). For a simple thin-wire half-wavelength dipole array, sinusoidal basis functions are sufficient to express the current

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distributions on the nm^{th} dipole. Applying MoM to the electric field integral equation of this problem, the following matrix equation is obtained

$$\sum_{n=-N}^{N}\sum_{m=-M}^{M} A_{nm} Z_{mn,pq} = V_{pq} e^{-j\beta_x pd_x} e^{-j\beta_y qd_y}, \quad -N < p < N, -M < q < M \qquad (1)$$

where A_{nm} is the unknown coefficient to be determined,

$$\beta_x = k_0 \sin\theta_i \cos\phi_i, \quad \beta_y = k_0 \sin\theta_i \sin\phi_i \qquad (2)$$

k_0 is wave number, and (θ_i, ϕ_i) gives the direction of the scanning beam for the radiation problem or the direction of the incident field for the scattering problem.

In order to implement the FBM and employ the DFT based acceleration algorithm in a very efficient fashion, the array shown in Fig. 1 (black dipoles in elliptical boundary) is mathematically extended into a rectangular array with virtual elements shown by blue dipoles located external to the array boundary. From a mathematical point of view, it is apparent that the virtual elements have zero current distributions, zero excitations and zero mutual impedance with any of the other elements, i.e.,

$$Z_{nm,pq} = 0 \quad (3) \quad \text{and} \quad V_{pq} = 0 \quad (4)$$

if $nm \neq pq$, ($Z_{nm,nm} \neq 0$) and either nm or pq indicates that the virtual element is located external to the array boundary. The assumptions given by (3) and (4) will assure $A_{nm} = 0$ for virtual elements.

FB method can be employed to solve eqn. (1) by first decomposing $\bar{I}$ and $\bar{\bar{Z}}$ into forward and backward components. Then, the original matrix equation (1) is transformed into two matrix equations

$$\bar{\bar{Z}}^s \bar{I}^f = \bar{V} - \bar{\bar{Z}}^f \left(\bar{I}^f + \bar{I}^b \right) \quad \text{and} \quad \bar{\bar{Z}}^s \bar{I}^b = -\bar{\bar{Z}}^b \left(\bar{I}^f + \bar{I}^b \right) \qquad (5)$$

where $\bar{I}^f$ is the contribution due to wave propagation in forward direction and $\bar{I}^b$ is its backward correction. $\bar{\bar{Z}}^s$ is formed by diagonal elements of $\bar{\bar{Z}}$, and $\bar{\bar{Z}}^f$ and $\bar{\bar{Z}}^b$ are lower and upper triangular parts of $\bar{\bar{Z}}$ with $\bar{\bar{Z}}^s$ subtracted. Equations in (5) are solved iteratively for $\bar{I}^f$ and $\bar{I}^b$, starting with zero $\bar{I}^{b,(0)}$. The solution for $\bar{I}^f$ and $\bar{I}^b$ is obtained iteratively until convergence is achieved. FBM requires $O(N_{tot}^2)$ computational complexity and memory storage. To reduce the computational complexity, an acceleration algorithm based on the DFT representation of the induced currents on the array has been proposed, [5,6]. This algorithm divides the contributing elements in front of the receiving element into strong and weak interaction regions. Contributions from the strong region are evaluated using an element-by-element fashion whereas, the contributions from the weak region are computed by using few significant DFT terms from the DFT representation of the entire array currents. In general, to represent a strong group, it is sufficient to select elements that are within a few wavelength distance from the receiving element. The size of the strong group remains the same for most of the receiving elements during the forward procedure, which potentially results in $O(N_{tot})$

in the computational complexity. By using the DFT representation of A_{nm}, the weak region contributions to the pqth element can be expressed as

$$E_{weak} = \sum_k \sum_l B_{kl} \sum_{n,m \in weak} Z_{nm,pq} e^{-j\beta_x nd_x} e^{-j\beta_y md_y} e^{-j\frac{2\pi kn}{2N+1}} e^{-j\frac{2\pi lm}{2M+1}} \quad (6)$$

where B_{kl}'s are DFT coefficients.

It has been observed that DFT representation of practical large array currents is very compact, [3], such that only few of these DFT coefficients are nonzero. Since the contribution of weak region provides slight corrections, it is sufficient to use few significant DFT terms in calculation of (6). Sufficient DFT terms are selected based on the criteria given in [3].

Eqn. (6) can be rewritten in the following form

$$E_{weak} = \sum_{kl \in Q} B_{kl} C_{kl,pq} \quad (7)$$

where Q is the set of significant DFT terms,

$$C_{kl,pq} = \sum_{nm \in weak} Z_{nm,pq} e^{-j\beta_x ndx} e^{-j\beta_y mdy} e^{-j2\pi\frac{kn}{2N+1}} e^{-j2\pi\frac{lm}{2M+1}} \quad (8)$$

$C_{kl,pq}$ can be calculated very efficiently in an iterative fashion apart from the usual FB iterations, [6]. As shown in [6], calculation of $C_{kl,pq}$ requires only $O(N_{tot})$ operation. Hence, the overall computational complexity is $O(N_{tot})$. It is noted that in the computation of (8), $Z_{nm,pq}$'s between virtual elements and real elements are identical to those between real elements. Since A_{nm}'s of virtual elements are taken as zero in computation of B_{kl}'s, utilization of nonzero $Z_{nm,pq}$'s will not cause any problem if all DFT terms are employed. In reality, insignificant errors might occur. However, insignificant error in the computation of contribution from weak region will not affect the overall accuracy.

Numerical Results

To validate the accuracy and efficiency of the method, some numerical results for free standing dipole array and printed dipole array with circular and elliptical boundaries obtained using the FB approach with DFT based acceleration algorithm are presented and compared with the direct solution of MoM. Due to space limitations, only the results of printed dipole arrays with elliptical boundaries are given. As an example uniformly fed 749 element printed dipole array with elliptical boundary is considered. When it is extended to rectangular array, it has 41x25 elements. Dipoles are oriented in x-direction. Fig.2 shows the current distributions of 3rd and 9th rows. The strong region consists of 5x5 elements and 13 DFT terms are used. There is a good agreement between the two curves. There are some minor errors which can be expected due to nonzero currents on virtual elements. 3 iterations are sufficient for convergence, and error is less than 1.5%. In Fig.3 CPU times of DFT-FBM approach and MoM with LU decomposition are compared for the analysis of circular printed dipole arrays. The required CPU time for DFT-FBM approach is very small compared to that required in

MoM especially when N_{tot} is large. More numerical results will be shown in the presentation.

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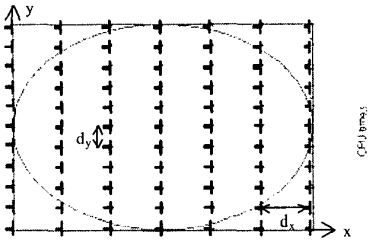


Fig.1 Geometry of the array and extended array

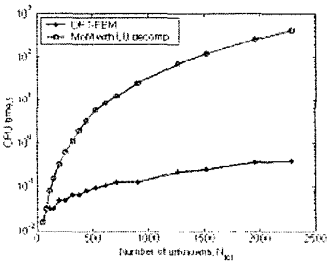
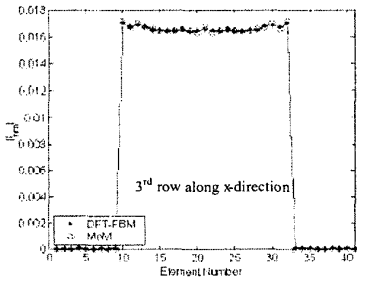


Fig.3 Comparison of CPU times



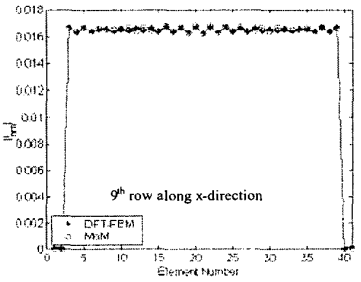


Fig.2 Comparison of current distributions obtained by MoM and DFT-FBM on 749 element elliptical printed dipole array ; $d_x = d_y = 0.5\lambda_0$, length of dipole$=0.3\lambda_0$, width of dipole$=0.01\lambda_0$, $\varepsilon_r = 2.55$, substrate thickness$=0.06\lambda_0$