Quantum Computation with Persistent-current Aharonov-Bohm Qubits and Qugates

I. O. Kulik

Department of Physics, Bilkent University, Ankara, Turkey, kulik@fen.bilkent.edu.tr

ABSTRACT

We analyse the possibility of employing mesoscopic or nanoscopic rings of a normal metal in a double degenerate persistent-current state in the presence of the Aharonov-Bohm flux equal to the half flux quantum as entangled quantum bits of information (qu-bits). The third level in a three-state qubit can be effectively used to coherently couple the qu-bit to logical gates for the reversible NOT (in a single qu-bit) and $CNOT$ (in two coupled qu-bits) operations. Further we suggest that a (hypothetic) crystal implementing conducting ring-shaped molecules, or triples of anionic vacancies (similar to F_3-centers in alkali halides) with one trapped electron, in crossed magnetic and electric fields satisfies the requirements of the proposed mechanism and may serve as a new kind of device for universal quantum computation.

Keywords: Quantum computation, qubit, persistent current, Aharonov-Bohm effect

1 QUANTUM COMPUTATION

Intriguing possibility of using massive quantum parallelism in time evolution of the collection of coherently coupled mesoscopic objects for performing transformation of information (a computation) with an unprecedented speed (exponentially faster than with a classical computer) resulted in search for solid-state realization of the elements of such "quantum computers" [1] termed quantum bits (qubits) and quantum logic gates (qugates). The basic element should have two quantum states such that transition between the states can be accomplished in a reversible coherent way with certain classical "switches". We propose that Aharonov-Bohm effect [2] can be used for that task basing on the predicted [3] and rediscovered [4] phenomenon of persistent currents in normal-metallic rings pierced by the Aharonov-Bohm flux, in case when additionally an electric field is applied perpendicular to the AB magnetic field to effect the transitions between the states of the ring via the (quenched) Rabi oscillations of state amplitudes [5].

The PC qubit organization is schematized in Fig.1. A ring of three metallic islands coupled by resonant tunneling display a Λ-shaped level configuration with

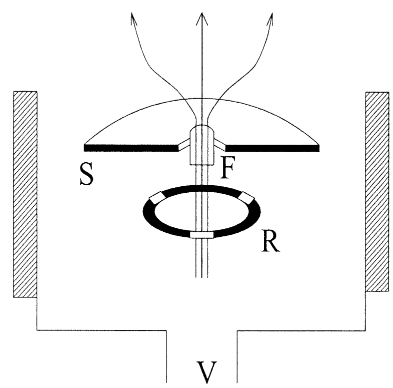

Fig.1. A sketch of persistent current qubit. **R** - ring of three resonantly coupled metallic islands (overlapping metallic films), **S** - superconducting foil focusing in its orifice a magnetic field from the trapped superconductive fluxon with help of ferromagnetic crystal **F**, and **V** - the potential electrodes for electrostatic operation of the qugate

two degenerate ground states (corresponding to clockwise and counterclockwise direction of persistent current rotation) and one elevated energy level. Virtual transitions through this level allow for one-qubit operations known as phase flip, bit flip (a NOT) and the Hadamard gate.

By shifting magnetic flux from the half-flux value we can initialize the qubit in one of its ground states and further coherently equalize populations of both bits with the Hadamard transformation. The coupling between the qubits allowing for the $CNOT$ transformation is accomplished by the nondemolition measurement of the state of one of qubits and, conditionally on its occupation, effect or not effect the transition of the second qubit by using the Quantum Hall bar in the Corbino disk geometry as shown schematically in Fig.2. An estimate shows that the requirements for the decoherence rate which would preserve the fragile quantum information stored in the qubit can be favorable, in proper organization of the three-state rings which can be either the tunneling junctions between the nanoscopic metallic

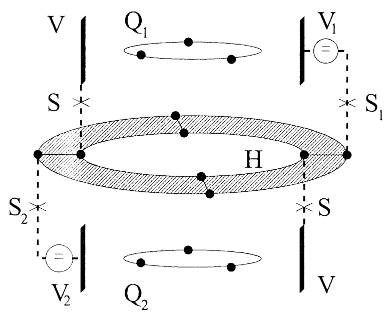

Fig.2. A sketch of the *CNOT* quantum logical gate. $\mathbf{Q_1}$, $\mathbf{Q_2}$ are qubits No.1,2, **V** are voltage electrodes and $\mathbf{V_1}$, $\mathbf{V_2}$ the voltage shift sources, $\mathbf{S_1}$, $\mathbf{S_2}$ are their respective switches. **H** is the Quantum Hall bar. Filled dots represent schematically metallic islands resonantly coupled to each other along the solid lines

islands, or a system generically similar to the triple anionic vacancy in the alkali halides known as F_3 (farben) center citeref6.

Important difference of persistent-current AB qubits from the other suggested condensed matter structures (e.g. superconducting charge boxes) is that the full set of quantum logical gates is provided with the radiation free couplings between the qubits making the persistent-current qubit a scalable system in a network of qubits.

2 PERSISTENT-CURRENT QUBIT

In the mesoscopic ring of a normal metal of size L smaller than the phase-decoherence length of electron, the charge current is produced under the influence of Aharonov-Bohm (AB) flux [7] . Physically, the shifted energy minimum in the presence of AB flux is counterbalanced by a net charge flow producing a persistent current in the absence of resistive effects. The magnitude of persistent current in a clean metallic ring is typically given by $J_c \sim ev_F/L$ where v_F is the electron Fermi velocity. In a nanoscopic (atomically small) ring with one electron, the magnitude of maximal persistent current is $J_c \sim e|\tau|/\hbar N^2$ where N is the number of sites in a ring and τ is the electron hopping amplitude between the sites. The PC is created individually by single electrons hence the fundamental flux quantum $\Phi_0 = hc/e$ is twice larger than the Abrikosov or Josephson flux quantum $\Phi_1 = hc/2e$. This very fact may permit new effects to arise when single Josephson vortex or Abrikosov fluxon are used to manipulate the flux in a PC ring. Particularly, at $\Phi = \Phi_1$ the PC ring produces a frustrated (entangled) quantum state with current-carrying double degenerate ground states of one electron in a single spin state which we suggest below as the qu-bit.

The Hamiltonian of the free ring is

$$H = -\tau \sum_{n=1}^{N} (a_n^+ a_{n+1} e^{i\alpha} + a_{n+1}^+ a_n e^{-i\alpha}) \qquad (1)$$

where a_n^+ is a fermionic operator creating (and a_n, annihilating) electron at site $\mathbf{R}_n$, $a_{N+1} = a_1$, and α is the Aharonov-Bohm phase related to magnetic flux by $\alpha = 2\pi\Phi N/\Phi_0$. The Hamiltonian (1) is diagonalized by the angular momentum (i.e., $m = 0, 1, \ldots$) eigenstates

$$A_m^+|0> = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{2\pi imn/N} a_n^+|0> \qquad (2)$$

with energy $\varepsilon_m = -2\tau \cos\frac{2\pi}{N}(m + \frac{\Phi}{\Phi_0})$.

For half-flux quantum $\Phi = \Phi_0/2$ (which is equal to the superconducting flux quantum $\Phi_1 = hc/2e$), in a 3-site ring the energy levels become degenerate in the ground state. With the third level at higher energy, we receive the Λ-shaped energy structure. Since two ground states are degenerate at $\Phi_0/2$, they can be used as components of qu-bit while the third one is coupling the qu-bit to a qu-gate, to be discussed in the next two sections. The practical realization of qu-bit with desired architecture is sketched in Fig.1. One possibility may be a three-sectional normal-metal ring intersected by tunneling barriers. Creating strong magnetic field to operate the qu-bit at the half quantum flux is suggested with the help of superconducting fluxon trapped in a hole inside the superconducting film, with lines of magnetic field further focused by a mesoscopic ferromagnetic cylinder near the ring.

The isolated qu-bit structure can in principle be realised as a three-site defect in an insulating crystal, similar to negative-ion triple vacancy (known as F_3-center) in the alkali halide crystals (e.g., see [6]).

3 QUGATE OPERATIONS IN ONE RING

By introducing operators A_m^+, the Hamiltonian (1) is transformed to (we scale energy in units of the hopping parameter τ and reshuffle the state numbering in such a way that $m = 1, 2$ correspond to the degenerate energy level $\varepsilon = -1$, at half flux quantum, and energy state $\varepsilon = 2$ to $m = 3$)

$$H_0 = \sum_m \varepsilon_m A_m^+ A_m = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \qquad (3)$$

Assume that the ring is subject additionally to an electric field in a direction perpendicular to magnetic field as shown in Fig.1. Then the Hamiltonian $v_0 H_1$ will be added to (2) where v_0 is magnitude of the potential

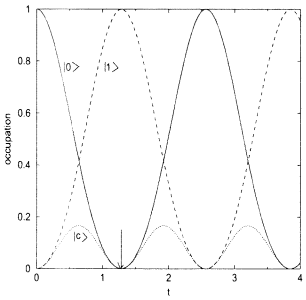

Fig.3. Qubit time evolution corresponding to NOT transformation. At point indicated by an arrow ($t = t_1$), the population of control register ($|c\rangle$) vanishes whereas the populations of the computational registers of qubit ($|0\rangle$) and ($|1\rangle$), interchange.

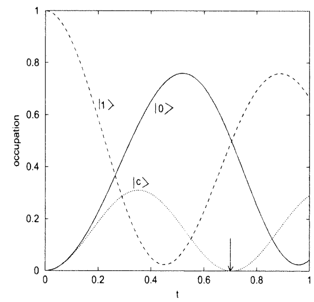

Fig.4. Qubit time evolution corresponding to Hadamard transformation. At point indicated by an arrow ($t = t_3$), the population of control register ($|c\rangle$) vanishes whereas the computational basis of the qubit, originally in a state ($|1\rangle$, equally populates to states $|0\rangle$ and $|1\rangle$.

$v_n = v_0 \cos(2\pi n/3)$, and

$$H_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}. \tag{4}$$

Under this Hamiltonian and for $v_0 = -\tau$, the qu-bit originally in a state $|\Psi_{\text{in}}>= (1,0,0) = |\uparrow>$ will evolve, after a properly chosen time t_1, to a state $|\Psi_{\text{out}}>= -(0,1,0) = e^{\pi i}|\downarrow>$. The evolution is due to a unitary operator $U(t) = \exp(-i(H_0 + v_0 H_1)t)$ with the time of evolution (at $\tau = 1$)

$$t_1 = \pi/\sqrt{6} = 1.2825. \tag{5}$$

Indeed, direct evaluation of $U(t)$ gives

$$U(t) = \frac{1}{2}h_0 + \frac{1}{2}h_1 \cos(t\sqrt{6}) + \frac{i}{\sqrt{6}}h_2 \sin(t\sqrt{6}) \tag{6}$$

where h_0, h_1 and h_2 are matrices

$$\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}. \tag{7}$$

The evolution corresponding to the Rabi oscillations is presented in Fig.3. The transformation between the degenerate states $|\uparrow>, |\downarrow>$ at $t = t_1$ is achieved through the virtual transition to third level (a control state), $|c>= (0,0,1)$. At the end of the transition, the latter remains depopulated, and stays so after any transformation which we consider later. In that respect, the three-state qu-bits are not in any way distinguishable from the conventional qubits. To compensate for the phase shift π relative to the shift $2\pi t_1$ which the other qu-bits accumulated, while idling during time t_1, it suffices to apply an extra voltage v_1 to a given qu-bit for a time t_1' such that $v_1 t_1' = \pi(1 - 2t_1)$. Therefore, the combination of both operations is equivalent to the bit flip transition between $|\uparrow>$ and $|\downarrow>$ states, or the reversible NOT operation.

Another one-qubit operation is the Hadamard gate [1] which equally populates two ground states provided it was originally in one of states $|\uparrow>$ or $|\downarrow>$. As shown in [5], such transformation in fact accomplishes at electrical potential $v_0 = -2.4867\tau$ with the evolution time $t = 0.7043/\tau$. The corresponding evolution diagram is presented in Fig.4. The transitions corresponding to the NOT ant the Hadamard gates occur at commensurate situation at which the difference between two ground states (no longer degenerate, if $v_0 \neq 0$) is equal, or multiple (with a factor K) of the distance from the higher of ground states to the excited level. AS shown in [5], the NOT corresponds to $K = 1$ and the Hadamard gate to $K = 3$.

4 QUGATE OPERATIONS WITH TWO COUPLED RINGS

The objective is to implement quantum mechanically the "control-NOT" operation which flips the state of one of the two qu-bits (the control) provided the second (target) qu-bit is in one of its particular states. This means, for example, that the bit No.1 should be nondemolition-measured and, if up, the second bit is flipped. The two states $|\uparrow>$ and $|\downarrow>$ differ in the direction of their currents. We use this to design an interaction between the qu-bits $\hat{\jmath}^{(1)} \otimes \hat{H}_1^{(2)}$ where $\hat{\jmath}$ is a current operator (in proper units) $\hat{\jmath} = diag(1,-1,0)$ in the representation of operators A_m^+, and upper indices (1,2) correspond to the qu-bits No.1,2.

The realization of the controlled operations with double qubits is an essential requirement of any mechanism of quantum computation. It is possible to obtain a CNOT gate in the quantum system we propose. Both three level systems are initially prepared to be in their qubit subspaces and they are connected by a quantum nondemolitional measurement device which reads the first qubit and depending on its state, induces a static potential $V_0^{(1)}$ in the second qubit to perform the bit flip. The experimental scheme is schematized in Fig.2 which employs two mesoscopic rings, a Hall bar in the form of a Corbino disk [8] in the full quantum regime and the superconducting loop. The persistent current J_1 in the loop of qubit $\mathbf{Q_1}$ creates a current in the superconducting loop $J_1' = \eta J_1$ where η is the efficiency of current transformation and converts it to voltage

$$V = R_{xy} J_1' = n \frac{h}{e_2} J_1' \tag{8}$$

on the center of n-th Hall plateau. [The system is assumed to be initiated such that current in a loop is zero at zero persistent current in a qubit loop; the other possibility could be to include the $-\Phi_0/2$ compensating coil between $\mathbf{Q_1}$ and $\mathbf{H}$ to exclude the large static flux $\Phi_0/2$ in the qubit.] Estimate shows that due to a large value of R_{xy} ($27k\Omega$ on the main Hall plateau), the voltage V is large enough to drive the qubit at the efficiency $\eta \sim 0.1$.

The Hall voltage generated in the bar is designed so that either $V_0^{(1)}$ or zero voltage is produced corresponding to the fixed value of the current flowing in one or the other direction. The Hall bar is connected to the V electrodes of qubit $\mathbf{Q_2}$. If the voltage is $V_0^{(1)}$, the bit flip of the second qubit is realized after time t_1 or if the voltage is zero no change is made. The procedure may in principle be executed in a totally reversible way if the Hall bar operates in the manifestly quantum regime. According to measurements [8], longitudinal currents in the contactless realization of the quantum Hall effect (the Corbino geometry) persist for hours, i.e. the longitudinal resistance R_{xx} is extremely small, practically a vanishing quantity.

In conclusion, we showed the possibility of using the persistent-current loops of normal metal as quantum bits (qu-bits) of information coherently coupled to the qu-gates in a quantum computer. The qu-bits suggested differ from the qubits currently investigated by a quantum-computing community (trapped ions [9], superconducting Cooper-pair islands or π-Josephson junctions, [10], [11], [12], [13], [14], [**?**], etc.) in that respect that the auxiliary register of the qu-bit provides a possibility to coherently couple its operational $\uparrow$ and $\downarrow$ registers to (or the mere reversible execution of) the logical reversible *NOT* and *CNOT* gates. The realization may include mesoscopic normal metal islands connected in a ring by tunneling, or nanoscopic three-site defects, generically similar to F_3-centers in alkali halides, with one trapped electron in one spin state. The main disadvantage of the first is in the unavoidable parasitic states due to impurities, imperfections, etc. The only disadvantage of the second is in the extremely high magnetic and electric fields required to operate qu-bits. We may hope however that the implementation of high-μ, high-ϵ materials in the vicinity of the rings may reduce the necessary values of the external fields to a reasonable level.

The persistent-current scheme is flexible for modifications including the addition of extra registers for quantum error-correction, and is expected to better fit the requirements of keeping quantum coherence within the operational cycle of computer by addressing in the same quantum unit the memory and the processor registers.

REFERENCES

[1] M. A. Nielsen, I. L. Chuang, "Quantum Computation and Quantum Information", Cambridge Univ. Press, 2000.

[2] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485, 1959.

[3] I. O. Kulik, JETP Lett. **11**, 275, 1970.

[4] M. Buttiker, Y. Imry and R. Landauer, Phys. Lett. **A96**, 365, 1983.

[5] I. O. Kulik, T. Hakioglu, and A. Barone, European Physical Journ., **B30**, 219, 2002.

[6] C. Kittel, Introduction to Solid State Physics. J. Wiley, New York, 1996.

[7] I. O. Kulik, "Non-decaying currents in normal metals", in: "Quantum computation and spin electronics, in: Quantum Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics, p.259. I. O. Kulik and R. Ellialtioglu, eds. Kluwer, Dordrecht, 2000.

[8] V. T. Dolgopolov, A. A. Shashkin, N. B. Zhitenev, S. I. Dorozhkin, and K. von Klitzing, Phys. Rev. **46**, 12560, 1992.

[9] J. J. Cirac, P. Zoller, Phys. Rev. Lett. **74**, 4091, 1985.

[10] V. Bouchiat, D. Vion, P. Joyez, D. Esteve, and M. H. Devoret, Phys. Scripta **T76**, 165 (1998).

[11] T. P. Orlando, J. E. Mooij, Lin Tian, C. H. van der Wal, L. S. Levitov, S.Lloyd, and J. J. Mazo, Phys. Rev.**B60**, 15398 (1999); Science, **285**, 1036 (1999).

[12] A. Shnirman, G. Schön, and Z.Hermon, Phys. Rev. Lett, **79**, 2371 (1997).

[13] L. V. Ioffe, V. B. Geshkenbein, M. V. Feigelman, A. L. Fauchere, and G. Blatter, Nature **398**, 679 (1999).

[14] A. M. Zagoskin, "A scalable, tunable qubit, based on a clean DND or grain boundary D-D junction", preprint cond-mat/9903170.