

# Parallel Dedicated Machine Scheduling With A Single Server: Full Precedence Case

Scott R. Schultz  
Department of Mechanical and Industrial Engineering  
Mercer University  
Macon, GA 31207

Mehmet R. Taner  
Department of Industrial Engineering  
Bilkent University  
Bilkent TR-06800 Ankara, Turkey

## Abstract

The motivation for this study was the observation of a practical scenario that involves scheduling of two parallel machines attended by a single setup crew so as to minimize the makespan. This problem is known in scheduling literature as the parallel machine scheduling problem with a single server,  $P2S1 // C_{\max}$ . In order to gain insight on this problem, we analyzed a constrained version of it. In this constrained case, jobs are dedicated to each machine, and the processing sequence on each machine is given and fixed. The problem is thus referred to as the parallel, dedicated machine scheduling problem with a single server and full precedence,  $PD2S1/full\ prec./ C_{\max}$ . We explore the combinatoric structure of the problem, and develop a branch and bound procedure and five heuristic algorithms.

## Keywords

parallel machines: single server: scheduling

**1. Introduction:** This research was motivated by a scheduling problem observed at an aircraft manufacturing plant. The plant's primary process is the fabrication of skin panels. Once fabricated, these skin panels are shipped to another plant for assembly. The bottleneck operation in the skin panel fabrication process is the drill and rivet operation. This operation consists of two machines in parallel serviced by a single setup crew. Forty unique panels must be processed by either of these two machines. After pursuing setup time reduction initiatives and assessing the cost of dedicated setup crews, the plant is now interested in developing a dynamic scheduling system for this operation that will help minimize makespan.

A more general version of this problem is known as the parallel (identical, non-dedicated) machine scheduling problem with a single server,  $P2S1 // C_{\max}$ . Both Glass *et al.* [3] and Hall *et al.* [4] studied the complexity of parallel machine scheduling with a single common server under a variety of assumptions about setup and processing times. Among other results, Hall *et al.* show that  $P2S1/s_j = 1/C_{\max}$  is binary *NP-hard* and that  $P2S1/s_j = s/C_{\max}$  is unary *NP-hard*. Glass *et al.* address a closely related problem where the set of all jobs is partitioned in advance into two subsets. This problem is referred to as the parallel dedicated machine problem with a single server,  $PD2S1 // C_{\max}$ . Glass *et al.* show that the  $PD2S1/s_j = s/C_{\max}$  and  $PD2S1/p_j = p/C_{\max}$  versions of this problem are *NP-hard* in the strong sense. In addition, Glass *et al.* present a heuristic for  $PD2S1 // C_{\max}$  that guarantees a tight worst-case ratio of 3/2. Brucker *et al.* [2] continue studying the complexity aspects of the parallel machine problem with a single server. They derive complexity results for additional special cases.

Abdekhodae and Wirth [1] provide a computational study for  $P2S1 // C_{\max}$ . They present several heuristics and compare their results to lower bounds over a range of problems. They also present an integer program that can solve problems with up to 12 jobs.

In order to gain insight to the general problem observed in the aircraft manufacturing plant, we investigate a constrained version of the  $PD2S1 // C_{\max}$  problem analyzed by Glass *et al.* The problem investigated here assumes the ordering of jobs on each machine is also specified. In other words, a full precedence relationship is given for all jobs. This problem is denoted as  $PD2S1/full\ prec./C_{\max}$ . We were not able to find any studies in the literature that specifically address the  $PD2S1/full\ prec./C_{\max}$  problem investigated in this paper.