Fast Solution of Electromagnetic Scattering Problems with Multiple Excitations Using the Recompressed Adaptive Cross Approximation

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Abstract—We present an algebraic compression technique to accelerate the computation of multiple monostatic radar cross sections of arbitrary 3-D geometries. The method uses adaptive cross approximation, followed by a recompression technique to reduce the CPU time and the memory consumption. Each scattering problem due to a single excitation is solved with the multilevel fast multipole algorithm. The numerical results demonstrate the efficiency and accuracy of the proposed method.

II. IMPLEMENTATION

The numerical solution of an electromagnetic scattering problem using the method of moments (MoM) requires a discretization that yields a dense \( N \times N \) matrix equation. To accelerate the solution, various methods are proposed that exploit the redundancies in the MoM matrix. One of the most efficient methods is MLFMA, which has a complexity of \( O(N \log N) \) [8]. Alternatively, there are purely algebraic methods. A subset of these methods is composed of solvers based on hierarchical matrices [9]. We use MLFMA as a fast solver for this work.

Although MLFMA may solve a problem with one RHS rapidly, solving multiple MRCS problems is time consuming. The matrix equation with an excitation matrix in the RHS can be represented as

\[
Z_{(N \times N)} \cdot X_{(N \times M)} = V_{(N \times M)}, \tag{1}
\]

where \( Z \in \mathbb{C}^{N \times N} \) is the matrix of interactions between \( N \) testing and basis functions, \( X \in \mathbb{C}^{N \times M} \) contains the unknown coefficient vectors corresponding to the excitation vectors in the RHS matrix \( V \in \mathbb{C}^{N \times M} \), and \( M \) is the number of excitations. The matrix \( V_{(N \times M)} \) in (1) may contain linearly dependent columns. For a given threshold \( \epsilon \), we use ACA to compress and factorize the RHS matrix. Applying ACA on \( V \) leads to

\[
V_{(N \times M)} \approx \tilde{A}_{(N \times k)} \cdot B_{(k \times M)}^H, \tag{2}
\]

where \( \tilde{A}_{(N \times k)} \in \mathbb{C}^{N \times k} \), \( B_{(M \times k)} \in \mathbb{C}^{M \times k} \), and \( k \) denotes the effective rank of \( V \). To obtain a decomposition of \( V \) with an effective rank \( k' \) \((k' < k)\), we employ a recompression technique that incorporates the QR decomposition and computes SVD in an efficient manner [6]. We apply a QR factorization for \( \tilde{A} \) to get \( \tilde{A}_{(N \times k)} = Q_A(N \times k) \cdot R_A(k \times k) \) and for \( B \) to obtain \( B_{(M \times k)} = Q_B(M \times k) \cdot R_B(k \times k) \), where matrices \( Q_A \) and \( Q_B \) are unitary matrices. Then, we perform a truncated SVD with a given threshold \( \epsilon \) on the product of \( R_A \) and \( R_B^H \). After this recompression, the final low-rank approximation will be in the form of

\[
V_{(N \times M)} \approx \tilde{A}_{(N \times k')} \cdot (B_{(k' \times M)}^R)^H, \tag{3}
\]

where \( k' \) is the effective rank of the RHS matrix and is smaller than the rank \( k \) achieved by ACA. An approximate solution
to (1) can be achieved by substituting (3) in (1) and rewriting it as
\[ \mathbf{X} \approx (\mathbf{Z}^{-1} \cdot \mathbf{A}) \cdot \mathbf{B}^H, \tag{4} \]
where \( \mathbf{Z}^{-1} \) denotes a standard solution, i.e., MLFMA in this work, but other fast methods can also be used to accelerate the solution.

III. NUMERICAL RESULTS

In this section, we investigate the 2-D MRCS values of the Flamme geometry at a frequency of 4 GHz. As shown in Fig. 1, the Flamme has a maximum length of 0.6 m. The surface of the geometry is discretized with planar triangles with an average mesh size of \( \lambda/10 \), which leads to 13,386 unknowns. The Flamme geometry is illuminated with \( \theta \)-polarized plane waves incident from an angular sector defined by \([\theta_{\min}, \theta_{\max}] = [30^\circ, 50^\circ]\) and \([\phi_{\min}, \phi_{\max}] = [45^\circ, 65^\circ]\). The angular resolution is 0.5°. The brute-force (BF) simulations require 1681 runs. Figure 2 illustrates multiple 2-D MRCS results. The results obtained from RACA are illustrated in Fig. 2(a). The relative error between the proposed method (RACA) and BF runs is calculated via
\[ \frac{\sigma_R - \sigma_{\text{BF}}}{\sigma_{\text{BF}}} \]
with \( \sigma_{\text{BF}} \) being the ground truth. The relative error of the RACA. (d) The relative error of the CS interpolation. The results show that RACA is more accurate and more efficient than interpolation techniques. The method incorporates the QR decomposition and computes SVD in an efficient manner. Due to the purely algebraic nature of the proposed method, it can be combined with any fast solver.

IV. CONCLUSION

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