Combined-Field Solution of Composite Geometries Involving Open and Closed Conducting Surfaces

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Abstract

Combined-field integral equation (CFIE) is modified and generalized to formulate the electromagnetic problems of composite geometries involving both open and closed conducting surfaces. These problems are customarily formulated with the electric-field integral equation (EFIE) due to the presence of the open surfaces. With the new definition and application of the CFIE, iterative solutions of these problems are now achieved with significantly improved efficiency compared to the EFIE solution, without sacrificing the accuracy.

I. Introduction

Problems arising in computational electromagnetics often involve both thin and thick conductors that need to be modelled with open and closed surfaces, respectively. Since the magnetic-field integral equation (MFIE) [1] can be formulated only on closed surfaces, the electric-field integral equation (EFIE) becomes the inevitable choice for the solution of those problems. However, the EFIE is prone to the interior-resonance problems and generates ill-conditioned matrix equations, especially when applied on closed surfaces. This leads to significant inefficiency in the solution of the composite problems with fast iterative solvers, such as the multi-level fast multipole algorithm (MLFMA) [2].

For the solution of the electromagnetic modelling problems involving only closed surfaces, CFIE is usually preferred over EFIE and MFIE mainly because it is free of the internal-resonance problem [3] and generates better-conditioned matrix equations [4]. CFIE is simply a linear combination, i.e.,

$$\mathrm{CFIE} = \alpha \mathrm{EFIE} + (1-\alpha)\mathrm{MFIE}, \tag{1}$$

so that the EFIE and MFIE can be interpreted as the two extreme cases of the CFIE, i.e., when $\alpha = 1$ and $\alpha = 0$, respectively. In this paper, we extend the definition of the CFIE to benefit from its advantages for the solution of the composite problems, such as the one shown in Fig. 1(a), which would customarily be formulated with the EFIE due to the presence of an open surface in the geometry. The proposed technique, which is based on employing a variable α in (1) so that EFIE (CFIE with $\alpha = 1$) is used for the open surfaces of the geometry while CFIE with $0 \leq \alpha < 1$ is applied for the rest of the problem, accelerates the iterative solution of the composite problems without sacrificing the accuracy.

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II. MODIFIED CFIE FORMULATION

For conducting surfaces, numerical application of the EFIE and the MFIE leads to matrix equations as

$$\sum_{n=1}^{N} Z_{mn}^{E,M} a_n = v_m^{E,M}, \qquad m = 1, ..., N. \tag{2}$$

In the above, the matrix elements, namely the interactions between the basis functions $\boldsymbol{b}_n(\boldsymbol{r})$ and the testing functions $\boldsymbol{t}_m(\boldsymbol{r})$, are derived as

$$\begin{aligned} Z_{mn}^{E} = &\int_{S_m} d\boldsymbol{r}\boldsymbol{t}_m(\boldsymbol{r}) \cdot \int_{S_n} d\boldsymbol{r'}\boldsymbol{b}_n(\boldsymbol{r'}) g(\boldsymbol{r}, \boldsymbol{r'}) \\ &- \frac{1}{k^2} \int_{S_m} d\boldsymbol{r}\boldsymbol{t}_m(\boldsymbol{r}) \cdot \int_{S_n} d\boldsymbol{r'}\boldsymbol{b}_n(\boldsymbol{r'}) \cdot \left[\nabla\nabla' g(\boldsymbol{r}, \boldsymbol{r'})\right] \end{aligned} \tag{3}$$

for the EFIE, and as

$$Z_{mn}^{M} = \int_{S_m} d\boldsymbol{r}\boldsymbol{t}_m(\boldsymbol{r}) \cdot \boldsymbol{b}_n(\boldsymbol{r}) - \int_{S_m} d\boldsymbol{r}\boldsymbol{t}_m(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \times \int_{S_n} d\boldsymbol{r'}\boldsymbol{b}_n(\boldsymbol{r'}) \times \nabla' g(\boldsymbol{r}, \boldsymbol{r'}) \tag{4}$$

for the MFIE.

To form the CFIE system, the matrix elements in Eqs. (3) and (4) are linearly combined as

$$Z_{mn}^{C} = \alpha_m Z_{mn}^{E} + (1 - \alpha_m)\frac{i}{k} Z_{mn}^{M}, \tag{5}$$

where we normalize the MFIE with i/k [2]. Prior to the CFIE implementation presented in this paper, α_m in (5) is commonly reduced to a constant α in the literature. For the solution of the composite problems, such a definition forces the constant α to be 1 since the EFIE is obligatory for open surfaces. However, we redefine the CFIE with a variable α_m, depending on the row index (m) of the impedance matrix, i.e., the index of the testing function. This provides the freedom of choosing different linear combinations for different testing functions, even using the EFIE ($\alpha_m = 1$) or the MFIE ($\alpha_m = 0$). Such a generalization allows the use of the CFIE not only for the problems of closed surfaces, but also for the composite problems involving both open and closed surfaces. This is achieved by employing the CFIE with $\alpha_m \neq 1$ for the testing functions located on the closed parts of the geometry while setting $\alpha_m = 1$ to use the EFIE on the open parts. In other words, we are proposing to model composite geometries with a hybrid CFIE-EFIE formulation, which becomes better-conditioned compared to the use of the EFIE formulation for the entire problem. As a result, we obtain faster converging iterative solutions.

III. RESULTS

To demonstrate the improvements obtained with the use of the novel definition of the CFIE, we present the results of a scattering problem involving a perfectly conducting sphere of radius 30 cm and a square patch with edges of 60 cm as depicted in Fig. 1(a). The patch is located at a distance of 50 cm from the center of the sphere. The structure is illuminated by a plane wave propagating in the $-x$ direction with the electric field polarized in the y direction. The problem is solved with an MLFMA solver employing the Rao-Wilton-Glisson [5] functions as the basis and testing functions defined on the triangular domains.

At 600 MHz, triangulation size of about $\lambda/10$ leads to 1302 unknowns on the sphere and 465 unknowns on the patch. Fig. 1(b) demonstrates the iteration counts for the solution of the problem with the conjugate gradient squared (CGS) algorithm with respect to the value of α_m in (5) applied on the closed parts of the geometry (surface of the sphere), while keeping $\alpha_m = 1$ on the open parts of the geometry (surface of the patch). In Fig. 1(b), the dashed curve represents the number of iterations required to reach 10^{-3} residual error while employing a block-diagonal preconditioner (BDP) with 31,787 nonzero elements to accelerate the iterative solution. The solid curve in Fig. 1(b) represents the iteration counts required to reach 10^{-6} residual error while employing a stronger filtered near-field preconditioner (NFP) with 123,605 nonzero elements. Both curves are observed to be minimized when α_m is about 0.2–0.3, with significant improvement in the convergence compared to the pure EFIE solution of the problem ($\alpha_m = 1 \quad \forall m$), which is not shown in the figure due to the extremely high iteration counts.

In Fig. 2(a), the residual error is observed with respect to the number of iterations. The curves in this figure are obtained for two types of accelerations: (i) BDP, and (ii) NFP with 358,595 nonzero elements obtained by keeping all of the near-field interactions in the impedance matrix. This figure also demonstrates that the convergence is significantly improved with the use of the modified CFIE with $\alpha_m = 0.2$ on the sphere and $\alpha_m = 1$ on the patch. Fig. 2(b) shows the normalized radar cross section (RCS/λ^2 in dB) values on the z-x plane with respect to θ, where we observe that the modified CFIE remains highly accurate while reducing the number of iterations.

Fig. 3(a) demonstrates the acceleration of the iterative solutions over a frequency band of 500–1200 MHz. This figure presents the iteration counts required by the EFIE and the modified CFIE with $\alpha_m = 0.2$ to reach a residual value of 10^{-6} with the CGS algorithm employing the NFP. In addition to the overall inefficiency, the EFIE suffers from the internal resonance problems as manifested by the peaks in the number of iterations. Finally, the same problem is also solved at 6 GHz, when the size of the geometry becomes relatively large, i.e, $\lambda/10$ triangulation leads to 132,336 unknowns on the sphere and 49,200 unknowns on the patch. Fig. 3(b) shows that the EFIE solution does not converge even using the strong NFP with 35,158,068 nonzero elements, whereas the modified CFIE solution reaches the 10^{-6} error in only 35 iterations.

IV. Conclusion

A novel modification of the CFIE formulation is presented for the solution of the composite problems involving both open and closed conducting surfaces. The novel formulation obliterates the necessity to use EFIE for the whole geometry even though only a small part of the geometry is an open surface. As a consequence of the improved conditioning of the matrix equation, solutions of the composite problems are accelerated without sacrificing the accuracy.

References

[1] A. J. Poggio and E. K. Miller, “Integral equation solutions of three-dimensional scattering problems,” in *Computer Techniques for Electromagnetics*, R. Mittra, Ed. Oxford: Permagon Press, 1973, Chap. 4.

[2] W. C. Chew, J.-M. Jin, E. Michielssen, and J. Song, *Fast and Efficient Algorithms in Computational Electromagnetics*. Boston: Artech House, 2001.

[3] J. R. Mautz and R. F. Harrington, “H-field, E-field, and combined field solutions for conducting bodies of revolution,” *AEÜ*, vol. 32, no. 4, pp. 157–164, Apr. 1978.

[4] L. Gürel and Ö. Ergül, “Comparison of FMM implementations employing different formulations and iterative solvers,” *2003 IEEE AP-S Int. Symposium*, Columbus, Ohio, vol. 1, pp. 19–22, June 2003.

[5] S. M. Rao, D. R. Wilton, and A. W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape,” *IEEE Trans. Antennas Propagat.*, vol. AP-30, no. 3, pp. 409–418, May 1982.

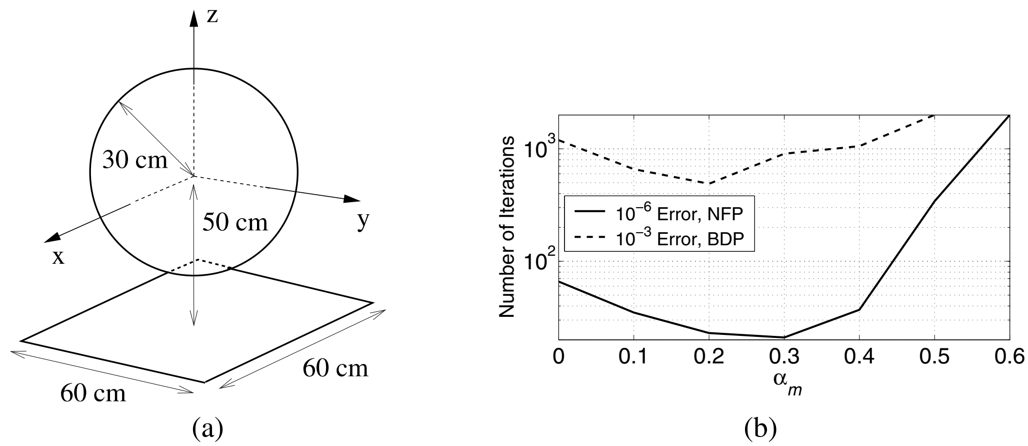

Fig. 1. (a) Composite geometry involving a perfectly conducting sphere of radius 30 cm (closed surface) and a square patch with edges of 60 cm (open surface). (b) Iteration counts for the solution of the problem with the CGS algorithm with respect to the value of α_m.

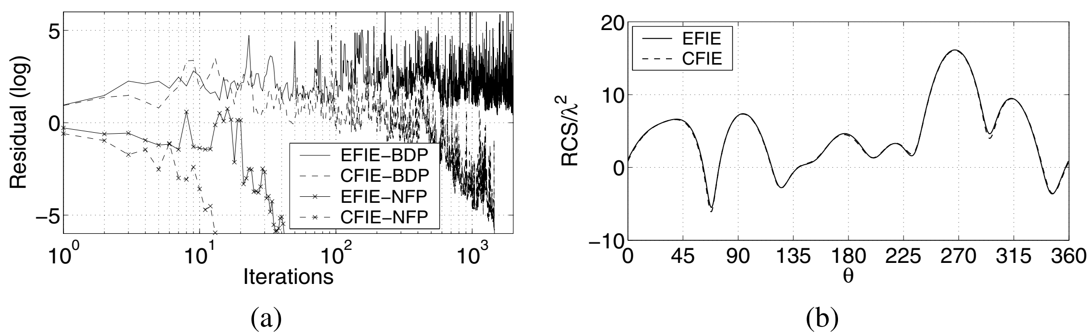

Fig. 2. (a) Comparison of convergence characteristics of the EFIE and the CFIE formulations for the scattering problem in Fig. 1(a) at 600 MHz. (b) Normalized radar cross section (RCS/λ^2 in dB) obtained at 600 MHz on the z-x plane with respect to θ.

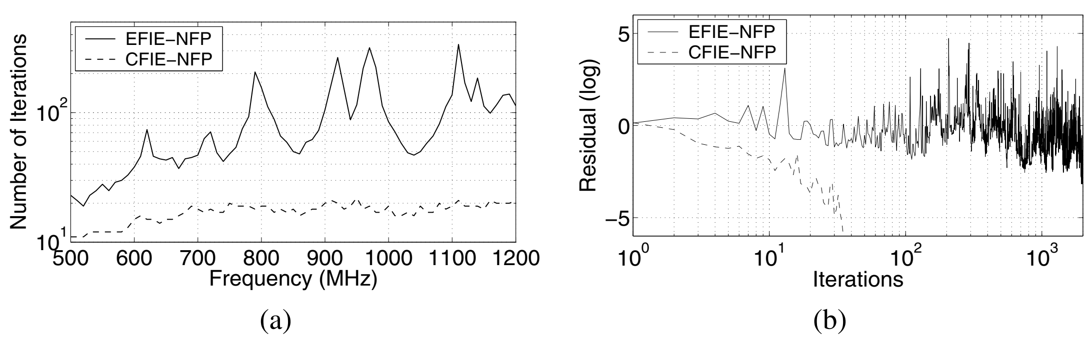

Fig. 3. (a) Iteration counts required to reach 10^{-6} residual error in the solution for the scattering problem in Fig. 1(a) in the frequency range from 500 MHz to 1200 MHz. (b) Comparison of convergence characteristics of the EFIE and the CFIE formulations for the scattering problem in Fig. 1(a) at 6 GHz.