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Two-dimensional Left-handed Metamaterial with a Negative Refractive Index

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Abstract. We present a two-dimensional metamaterial with left-handed properties in a certain frequency range where both effective dielectric permittivity and magnetic permeability is simultaneously negative. A wedge-shaped metamaterial structure is employed for refractive index measurements. At the left-handed frequency range the structure is shown to have negative refractive index. Phase shift between consecutive number of layers of metamaterial structures are measured and a negative refractive index value is calculated from the amount of the phase shift. The structure is shown to have negative phase velocity. The refractive index values obtained from two different and independent methods are in good agreement.

1. Introduction

Unique and novel physical properties of left-handed metamaterials (LHM) made these structures quite attractive and interesting in recent years. The response of any material to the electromagnetic (EM) wave is mainly determined by two important parameters, dielectric permittivity (ε) and magnetic permeability (μ). In general ε and μ are known to be both positive in ordinary materials. But recent studies showed that for certain artificial structures, however, both the effective permittivity (ε_{eff}) and effective permeability (μ_{eff}) can have negative values. In such media, the electric, magnetic and wavevector components form a left-handed (LH) coordinate system, hence the name left-handed material is used for description. The phase and group velocities are oriented in opposite directions such that the direction of the propagation is reversed with respect to the direction of energy flow [1]. Veselago in 1968, proposed that it is possible to have such an exotic material provided that both magnetic permeability and dielectric permittivity is less than zero. In his pioneering work he also investigated various optical properties such as Doppler shift, and Cherenkov radiation for negative refractive index structures [1].

Periodically arranged thin metallic wires are shown to exhibit plasma frequencies at the microwave frequency (GHz) regime [2]. These structures can be used as negative ε_{eff} media, since dielectric permittivity is less than zero below the plasma frequency. It has been more than 30 years to realize the possibility of negative magnetic permeability, therefore the concept of LHMs had to wait for Pendry to find out a material exhibiting $\mu_{\text{eff}} < 0$ [3]. The material is artificially constructed and named split ring resonators (SRRs). SRRs are shown to exhibit a negative μ_{eff} for frequencies close to the magnetic resonance frequency (ω_m) [3]. Experimental investigation of LHMs are done by constructing a composite metamaterial consisting of two components which have $\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$ simultaneously over a certain frequency range [4-8].

Measuring refraction through wedge shaped structures is the typical experimental method used for observation of left-handed properties in LHMs [9-11]. However, reversal of phase velocity can also be used as an indication of LH behavior [12,13]. In this paper, we first show the existence of left-handed transmission band in our two-dımensional (2D) LHM structure. Then we present direct experimental evidence that both the phase velocity and the refractive index is negative within the LH pass band of a

LHM. A comparison between the negative refractive index values obtained by using two different techniques is provided at the end. There is a quite good agreement between the measured refractive index values from wedge structures and the calculated index of refraction from phase shift experiments.

2. Left-handed transmission

LHM structures are generally composed of SRRs and thin wire grids. Periodic thin wire media is responsible for the negative effective permittivity, whereas the periodic SRR structure provides negative effective permeability. The SRR and wire patterns are fabricated on the front and back sides of FR4 circuit boards which have 30 μm thick deposited copper layer. The geometrical parameters of a single SRR unit can be found in our previous work [8]. The length and width of the wire structures are l = 19 cm, and w = 0.9 mm, respectively. The unit cell of the LHM consists of two SRRs and two wires in x-z planes, as depicted in the shaded regions of Fig. 1(a). The 2D CMM structure is made of $N_x = 5$, $N_y = 20$, and $N_z = 40$ unit cells, with lattice spacings $a_x = a_y = a_z = 9.3$ mm.

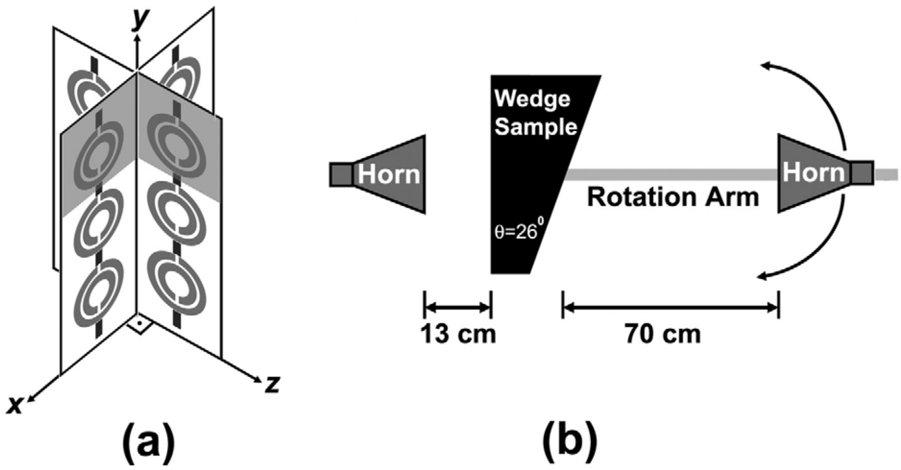

Figure 1. (a) Schematic illustration of a two-dimensional left-handed metamaterial with split ring resonators on one side and wire structures on the other side of dielectric board. Shaded region displays the unit cell of LHM, **(b)** Schematic drawing of top-view of experimental setup used for measuring refractive index values of LHM.

Transmission measurements are performed in free space. The experimental measurement setup consists of a HP 8510C network analyzer, and microwave horn antennas. The incident electromagnetic (EM) wave propagates along the x direction, while **E** is along the y direction, and **H** is along the z direction (see Fig. 1(a) for directions). Transmitter and receiver horn antennas are connected to the HP-8510C network analyzer to measure the transmission coefficients. First, we measured the transmission spectrum in free space (i.e. without a LHM structure). This data was used as the calibration data for the network analyzer. Then, we inserted our LHM sample between the two horn antennas, and performed the transmission measurements by maintaining distance between the fixed transmitter and receiver antennas.

Figure 2 shows the measured transmission spectra of periodic SRRs (green line), wires (blue line) and 2D LHM (red line) between 3-6 GHz. The bandgap of periodic SRR media between 3.55-4.05 GHz is due to magnetic resonance of SRRs, hence $\mu(\omega) < 0$ for this frequency range [8]. The LHM pass band coincides with the stop band of SRR. A left-handed transmission band is observed between 3.73 and 4.05 GHz. This frequency range covers the region where both ε and μ are simultaneously negative. The transmission peak measures -9.88 dB at 3.86 GHz, which is significantly higher than the previously reported 2D LHM structures [4,7].

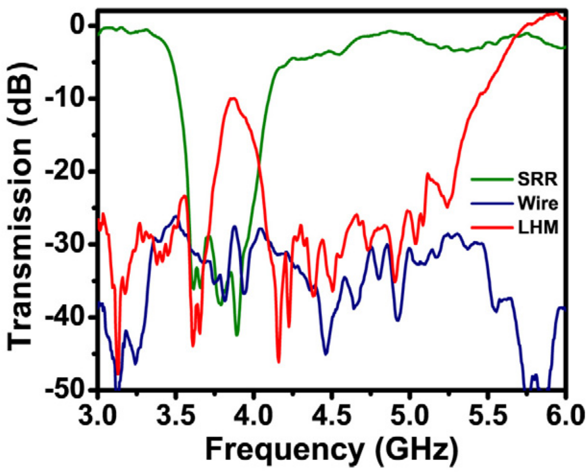

Figure 2. Measured transmission spectra of periodic arrangements of only SRR (green line), only wire (blue line) and LHM (red line) media.

By using same SRR design we have reported high transmission (-1.2 dB) for one-dimensonal (1D) LHM structures [8] so it is not surprising to obtain high transmission for the 2D case. This relatively high transmission can be explained by better impedance matching between air and CMM for this particular CMM design [14]. The transmission band starting from 5.3 GHz is due to downward plasma frequency shift. This is due to the electric response of split-ring resonator structures [8]. In ref. 8 we have successfully shown the true left-handed behaviour for LHMs. The achievements are mainly: (i) we have shown the magnetic resonance of SRR structures by using a ring resonator structure where the splits of SRR are closed, (ii) we have verified the effect of SRRs electric response to the plasma frequency of the composite metamaterial, and (iii) we have observed a left-handed transmission band within the frequency region where both dielectric permittivity and magnetic permeability is negative.

3. Negative refractive index

Wedge (prism) shaped samples can be employed for measuring refractive index values of a sample. For this purpose we have constructed a prism shaped 2D LHM structure. The minimum and maximum number of unit cells at the propagation direction is 3 and 19, which results in a wedge angle of θ = 26°. Figure 1(b) depicts the schematic drawing of the top view of the experimental setup. The source is 13 cm (~2λ) away from the first interface of the wedge sample. Note that the full width at half maximum of the incident beam (9.5 cm) at the first interface is smaller than the size of the incident surface (34 cm). Receiver antenna is mounted on a rotating arm to obtain the angular distribution of the transmitted electromagnetic wave. Receiver antenna is located at a distance of 70 cm (~10λ) away from the second interface of the wedge. We measure the intensity of the electric field.

The angular refraction spectrum is scanned by $\Delta\theta$= 2.5° steps between 3.73 GHz to 4.05 GHz, where the transmission band is shown to have left-handed characteristics. Figure 3 gives the refraction spectrum as a function of frequency and refraction angle. The point "0" corresponds to the normal of the wedge sample. Negative angles mean that the refraction takes place at the other side of the normal, when compared to that of positive refraction. It is evident from figure 3 that the transmitted EM wave is refracted with negative refraction angles. The refraction index is found to be negative for the entire LH transmission band. At lower frequencies the EM waves are refracted at higher negative refraction angles, which results in a higher negative refractive index. The refraction index is lowered if we go to higher frequencies.

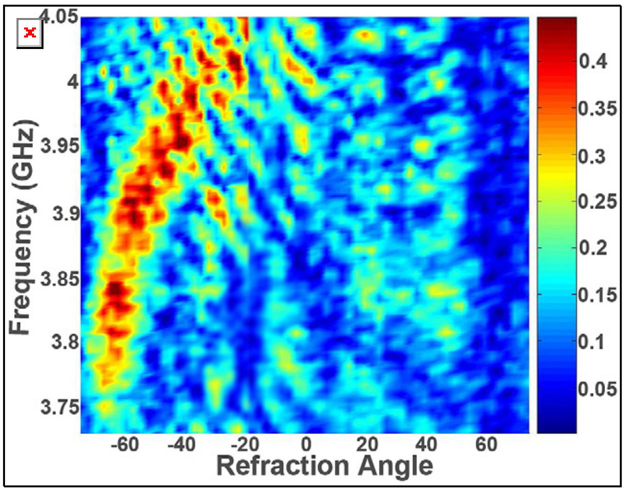

Figure 3. Measured refraction spectrum as a function of the frequency. The frequency region covers the left-handed frequencies, and different refraction angles are observed for different frequencies.

By employing Snell's law ($n_{CMM} \sin\theta_i = n_{air} \sin\theta_r$) an effective refractive index can be defined for the LHM. For $\theta_i = 26°$, EM wave is refracted at an angle of $\theta_r = 60°$ for 3.86 GHz, where the transmission is measured to be maximum. Then from Snell's law we obtain n_{eff} = -1.98 at 3.86 GHz. Snell's law is an effective way to calculate the refractive indices of the LHM structure.

4. Negative phase velocity

One of the characteristics of LHMs is that they do not only have negative refractive index but also possess negative phase velocities. The transmitted phase of LHMs is measured to investigate the phase velocity within the left-handed (3.73-4.05 GHz) transmission band. Phase measurements are performed on rectangular slabs of LHMs, with various numbers of layers.

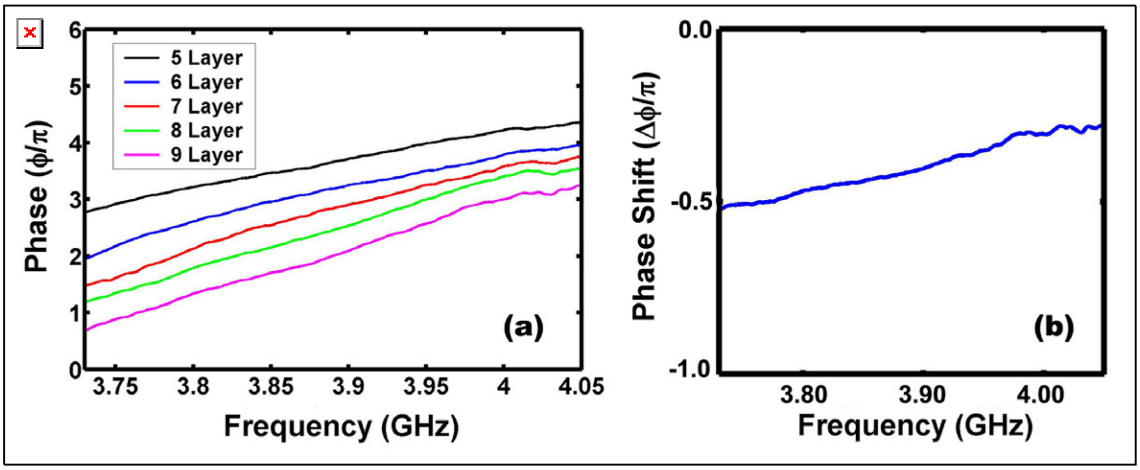

Figure 4. (a) Measured phase of transmitted wave through different number of layers of LHM structures. Phase decreases with increasing number of layers which is a sign for negative refraction. **(b)** Average phase shift between consecutive layers of LHMs.

Figure 4(a) shows the transmitted phase of LHMs (with varying number of layers) between frequencies 3.73-4.05 GHz, where LHM acts as a left-handed medium. As shown in figure 4(a), the phase of the transmitted EM wave decreases, when the number of layers along the propagation direction increases. However one expects the phase to increase with increasing number of layers in an ordinary positively refracting medium. This behaviour is not observed for a positive index media, i.e right-handed materials. The observed characteristic is due to the left-handed properties of LHM. Figure 4(b) gives the average phase shift for the LH frequency range. The phase shift is negative between LHMs with consecutive numbers of layers, indicating that the phase velocity is negative.

Snell's law is not only way to calculate refractive index values. We can also use phase shift to find refractive indices. Phase velocity is defined as $v_{ph} = c/n$, and also given by $v_{ph} = \omega/k$. Then, refraction index can be defined as $n = k.c/\omega$, where $k = \Delta\Phi/\Delta L$. We then obtain the refraction index formula in terms of phase difference, length difference and as:

$$n = \frac{\Delta\phi}{\Delta L}.\frac{c}{\omega} \quad . \qquad \text{(Eq. 1)}$$

At f = 3.86 GHz, the average phase shift between CMM layers is $\Delta\Phi = -0.44\ \pi$. By employing Eq.1, n_{eff} is obtained to be -1.94, which is in good agreement with the value of -1.98 obtained from the refraction experiment through wedge sample.

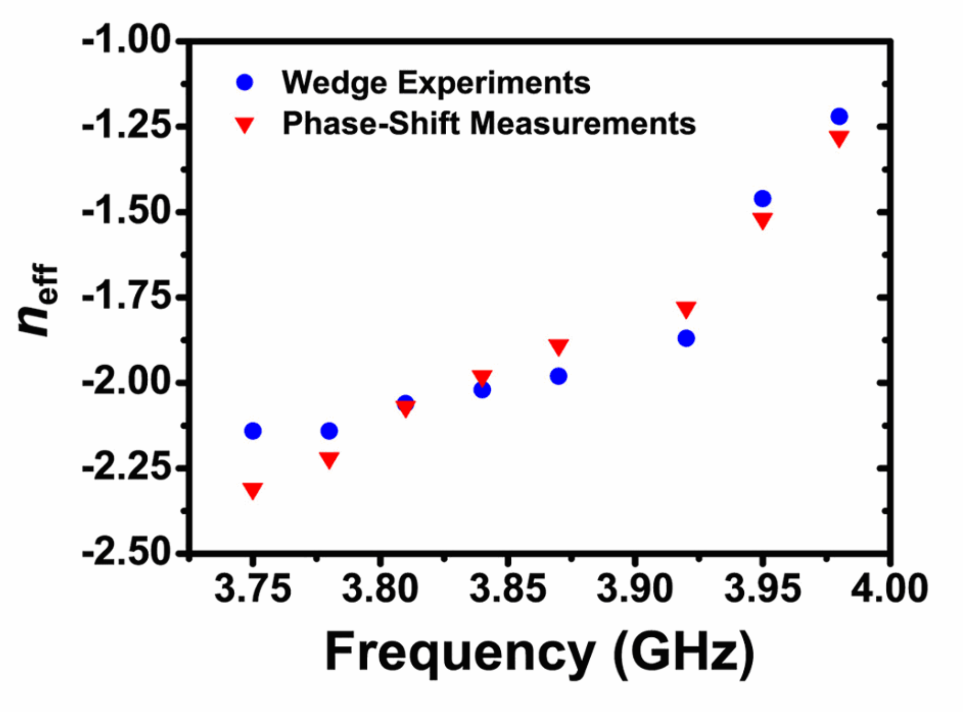

Figure 5. Comparison of refractive index values obtained by using two different techniques namely: Refraction through wedge shaped LHMs (blue) and phase-shift experiments (red).

We have calculated refractive indices for eight different frequencies by using Snell's law for refraction through wedge shaped samples and Eq. 1 for phase-shift experiments. The calculated index of refraction values are given in figure 5. As clearly seen from the figure the results are in good agreement for two different techniques.

We can also find the value of phase velocity for within the LHM. The measured phase velocity at 3.86 GHz is negative and equal to -0.51 c. Therefore we have verified both negative refractive index and negative phase velocity by using the phase differences between consecutive number of layers for a LHM structure.

5. Conclusion

In conclusion, we have demonstrated a left-handed transmission band for 2D LHM structure in free space with a high transmission peak. We experimentally confirmed that 2D LHM has negative refractive index at the entire left-handed frequency range (3.73-4.05 GHz). Phase shift and therefore phase velocity is shown to be negative, and the values of negative refractive indices obtained from the refraction experiments and the phase measurements are in good agreement.

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