NUMERICAL MODELING OF ELECTROMAGNETIC SCATTERING BY PERFECTLY CONDUCTING SURFACES OF REVOLUTION

S. Nechitaylo[1], I. Sukharevsky [2], A. Altintas[3], O. Sukharevsky[1]

[1] Kharkiv University of Air Forces, Research Centre, ul. Sumskaya 77/79, Kharkiv 61023, Ukraine
[2] Institute of Radiophysics and Electronics NASU, ul. Proskury 12, Kharkiv 61085, Ukraine
[3]Bilkent University, 06580, Ankara, Turkey
e-mail: i.sukharevsky@gmail.com

Abstract – **The integro-differential equation (IDE) of a three-dimensional (3-D) electromagnetic excitation problem of unclosed surfaces is numerically treated by means of the novel direct solver.**

I. INTRODUCTION

The integral equation technique is the only universal calculation method needed for simulating of reflectors of both small and large sizes. The electric field integral equation (EFIE) was usually discretized by Galerkin-type procedures, the method of moments (MoM) or similar method of subdomains [1-2]. However, for the vast majority of published investigations, small unclosed 3-D screens brought to calculation are of rather simple geometry (flat plates, corner reflectors etc.). The other discrete models having the advantage of controlled accuracy (usually, axial excitation problems of spherical screens depend on analytical regularization [3]), could be hardly extended to different surfaces and the cases of inclined incident waves.

Unlike EFIE-MoM techniques, the proposed numerical method is based upon expansion of surface current densities into dual Fourier series with the factor such that Meixner's conditions on the edge are satisfying "automatically". Such approach (being the generalization of associated 2-D problem [4]) allows crucially new numerical results to be obtained. RCS curves behavior as the number of members of dual Fourier series is increasing testifies the internal convergence of the numerical method. Obtained RCS patterns of spherical and parabolic caps are in good is agreement with those of physical optics (PO) and physical theory of diffraction (PTD) approximation.

II. METHODOLOGY

Consider the electromagnetic excitation of an arbitrary perfectly conducting boundary surface of revolution S by a plane electromagnetic wave $\{\vec{E}^0, \vec{H}^0\}$. Assume the time dependence of the field as $\exp(-i\omega t)$. Let's take into consideration tangential operators $\vec{D} = \vec{\nabla} - \vec{n}\frac{\partial}{\partial n}$, and $\vec{D}_0 = \vec{\nabla}_0 - \vec{n}_0\frac{\partial}{\partial n_0}$.

Then the next integral representation for the total field can be derived:

$$\vec{n}_0 \times \vec{E}(x_0) - \vec{n}_0 \times \vec{E}^0(x_0) = \int_S i\omega\mu \cdot g \cdot (\vec{n}_0 \times \vec{j})ds + \frac{1}{i\omega\varepsilon}(\vec{n}_0 \times \vec{D}_0)\int_S (\vec{D}\cdot\vec{j})g ds. \quad (1)$$

Initially consider observation point $\vec{x}_0$ to lie exterior to surface S from the side of the chosen normal direction, but nearby it.

Denote $\vec{I}_1 = \int_S g(\vec{n}_0 \times \vec{j})ds$, $\vec{I}_2 = (\vec{n}_0 \times \vec{D}_0)\int_S g(\vec{D}\cdot\vec{j})ds$. Then, after special transformation of $\vec{I}_2$, placing the observation point onto surface, and satisfying boundary conditions ($\vec{n}_0 \times \vec{E}(x_0)|_S = 0$), we obtain from (1) IDE:

$$ik_0\sqrt{\frac{\varepsilon_0}{\mu_0}}(\vec{n}_0 \times \vec{E}^0(x_0)) = k_0^2 \cdot \vec{I}_1 - \vec{I}_2. \quad (2)$$

We look for the IDE (2) solution in spherical coordinates as a truncated dual Fourier series

$$j_\theta(\theta,\varphi) = (\theta_1 - \theta)^{\frac{1}{2}} \sum_{l=0}^{L}\sum_{m=0}^{M}\left(A_{lm}^{(1)}\cos m\varphi + A_{lm}^{(2)}\sin m\varphi\right)\cos(\nu_l\theta),$$

$$j_\varphi(\theta,\varphi) = (\theta_1-\theta)^{-\frac{1}{2}}\sum_{l=0}^{L}\sum_{m=0}^{M}\left(B_{lm}^{(1)}\cos m\varphi + B_{lm}^{(2)}\sin m\varphi\right)\cos(\nu_l\theta),$$

where $A_{lm}^{(i)}, B_{lm}^{(i)}$ $(i=1,2)$ are unknown coefficients to be determined, $\nu_l = \dfrac{\pi l}{\theta_1}$, $0<\theta\le\theta_1$, $0<\varphi\le 2\pi$.

By equalities $\vec{e}_{\varphi_0}(\vec{n}_0\times\vec{E}^0(\vec{x}_0)) = \vec{e}_{\theta_0}\cdot\vec{E}^0(\vec{x}_0)$, $\vec{e}_{\theta_0}(\vec{n}_0\times\vec{E}^0(\vec{x}_0)) = -\vec{e}_{\varphi_0}\cdot\vec{E}^0(\vec{x}_0)$, we derive from (2) system of two equations:

$$\begin{cases}\dfrac{ik_0}{120\pi}(\vec{e}_{\theta_0}\cdot\vec{E}^0(\vec{x}_0)) = k_0^2(\vec{e}_{\varphi_0}\cdot\vec{I}_1)-(\vec{e}_{\varphi_0}\cdot\vec{I}_2),\\ -\dfrac{ik_0}{120\pi}(\vec{e}_{\varphi_0}\cdot\vec{E}^0(\vec{x}_0)) = k_0^2(\vec{e}_{\theta_0}\cdot\vec{I}_1)-(\vec{e}_{\theta_0}\cdot\vec{I}_2).\end{cases}$$

Having regard to linear representations of $\vec{I}_1$ and $\vec{I}_2$ through coefficients $A_{lm}^{(i)}, B_{lm}^{(i)}$ $(i=1,2)$ and taking the number of the observation angles $(\theta_{0i},\varphi_{0j})$ greater than the number of unknown coefficients, we obtain, as a result, overdetermined system of linear algebraic equations relatively $A_{lm}^{(i)}, B_{lm}^{(i)}$.

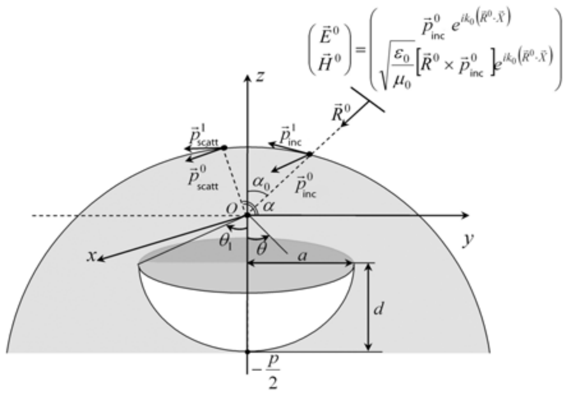

Fig. 1. Problem geometry.

Calculation of integrals appeared in $\vec{I}_1$ and $\vec{I}_2$ of (3) reduces to integrals:

$$\int_S g(r)f(\theta)\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix}d\theta d\varphi = \int_0^{2\pi}\int_0^{\theta_1} g(r)f(\theta)\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix} = \begin{Bmatrix}\cos m\varphi_0\\ \sin m\varphi_0\end{Bmatrix}U_c(\theta_0), \tag{3}$$

and their partial derivatives with respect to θ_0 and φ_0. Here $U_c(\theta_0) = \int_0^{2\pi}\cos m\psi\, d\psi\int_0^{\theta_1} g(r)f(\theta)d\theta$, $g(r) = \dfrac{e^{ik_0r}}{4\pi r}$,

$(\theta_0,\varphi_0)\notin S$, $f(\theta)$ - integrable function, possibly contained a singularity of a type $(\theta_1-\theta)^{-\frac{1}{2}}$, and r - a distance between observation and integration points. Consider the derivative $\partial U_c(\theta_0)/\partial\theta_0$ of (3), which has the greatest difficulty to be calculated. Further assessments for integrals and derivatives will be concretised for the case of a paraboloid of revolution S.

Since $(\theta_0,\varphi_0)\notin S$ by the assumption, the internal integral takes the form

$$\Phi(\psi;\theta_0) = \int_0^{\theta_1}\frac{\partial g(\theta,\psi;\theta_0)}{\partial\theta_0}f(\theta)d\theta = \delta(\theta_0)\int_{-\frac{\theta_0}{2}}^{\frac{\theta_1-\theta_0}{2}}\left(-t+\tilde{\beta}(\theta_0)\sin^2\frac{\psi}{2}\right)\cdot\left(t^2+\tilde{\mu}(\theta_0)\sin^2\frac{\psi}{2}\right)^{-3/2}R(t,\psi;\theta_0)dt, \tag{4}$$

where $R(t,\psi;\theta_0)$ has singularity with respect to t not higher than square singularity on the upper limit of integration; $\delta(\theta_0)$, $\tilde{\beta}(\theta_0)$, and $\tilde{\mu}(\theta_0)$ are non-singular well-behaved functions.

After limiting transition $(\theta_0,\varphi_0)\to S,$ (4) is to be considered in points $\left\{\psi:\sin\dfrac{\psi}{2}=0\right\}$ as a value principle. Subtracting the main singularity in (4), we obtain an integral considered as a value principle, which is to be calculated by a quadrature formula with a singular weight $\left(-t^2+\tilde{\beta}(\theta_0)\sin^2\dfrac{\psi}{2}t\right)\cdot\left(t^2+\tilde{\mu}(\theta_0)\sin^2\dfrac{\psi}{2}\right)^{-3/2}$ and by step-constant approximation of non-singular function. External integration in (5) can be calculated using 5-dot Gauss formula.

III. Numerical Results

To demonstrate method abilities, current density distributions and radiation patterns of spherical and parabolic screens has been calculated for two polarizations of an incident plane wave with a scanning angle α_0 and wavelength $\lambda = 0{,}03m$. Unit vector of polarization $\vec{p}^{\,0}_{inc}$ is perpendicular to the plane of the propagation vector $\vec{R}^0 = (0, -\sin\alpha_0, -\cos\alpha_0)$ and the axis of a screen; $\vec{p}^{\,1}_{inc}$ lies in this plane (Fig. 1). RCS was calculated by a formula $\sigma = \lim_{r\to\infty} 4\pi r^2 \left| \frac{(\vec{p}^{\,i}_{scatt} \cdot \vec{E}^{scatt})}{(\vec{p}^{\,i}_{inc} \cdot \vec{E}^{0})} \right|^2$. Fig. 2 demonstrates the internal convergence of the numerical method. RCS' values agree well with PTD approximation [5] as the number of azimuthal harmonics increases.

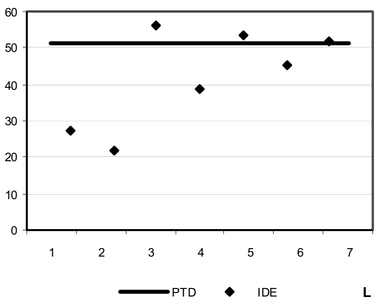

Fig. 2

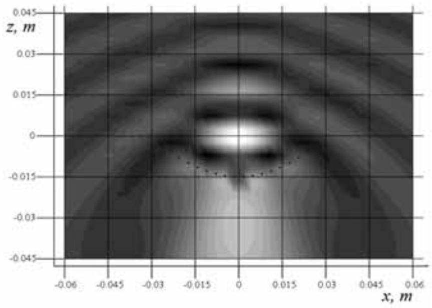

Fig. 3a

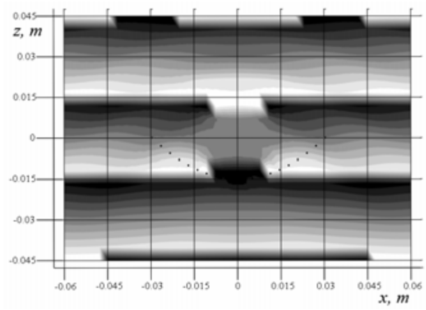

Fig. 3b

Fig. 2. Backscattering from an axially excited spherical screen (RCS as a function of a number of azimuthal harmonics; $k_0 a = 9, \theta_0 = 165^0, M = 1$).

Fig. 3. Near-field radiation (E_φ amplitude – a, phase – b) pattern for the axially excited parabolic screen ($a = \lambda, d = 0{,}5\lambda$) in a plane orthogonal to the polarization.

Near-field radiation patterns for the parabolic reflector excited by a plane wave are represented in Fig. 3. Near-field radiation was obtained with formulae:

$$\vec{p}\vec{E}(x_0) = \vec{p}\vec{E}^0(x_0) + \frac{1}{i\omega}\int_S \vec{j}(x)\vec{E}^e_0(x/x_0, \vec{p})ds \cdot$$

Here x_0 is a radius-vector of an observation point, x is an integration point, $\vec{p}$ is a unit vector to project the field, $\vec{E}^e_0$ is a field of the electric dipole with a vector-moment $\vec{p}$ in the point x_0, and ω is the radiation radial frequency. One can see the energy splash in a reflector focus, and the lightened domain (Poisson's spot) decreasing on its amplitude as distance from the top increases behind the screen.

IV. Conclusion

The developed methodology of integro-differential equation is described for the numerical analyses of electromagnetic excitation of unclosed PEC surfaces. To validate the accuracy and efficiency of the algorithm in this paper, RCS of spherical and parabolic screens are being calculated. The numerical solutions agree well with PO and PTD solutions. The offered method allows calculating the fields scattered by screens on various incident wave sources and can be easily extended to a problem of electromagnetic scattering by arbitrary surfaces.

References

[1.] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 409–418, May 1982.

[2.] A. G. Davydov, E. V. Zakharov, and Yu. V. Pimenov, "Method of numerical solution of the problems of electromagnetic wave diffraction by non-closed surfaces of arbitrary shape," *Dokl. Acad. Nauk USSR*, vol. 276:1, pp. 96-100, 1984 (in Russian).

[3.] S.S. Vinogradov, P. D. Smith, and E.D. Vinogradova, *Canonical Problems in Scattering and Potential Theory Part II: Acoustic and Electromagnetic Diffraction by Canonical Structures,* monographs and surveys in applied and pure mathematics. London: Chapman & Hall/CRC, 2002.

[4.] S.V. Kukobko, A.Z. Sazonov, and I.O. Sukharevsky, "Iterative calculation method for two-dimensional model of reflector-type antenna with sharp nose radome", *Proc. Int. Conf. on Mathematical Methods in Electromagnetic Theory,* Dniepropetrovsk, pp. 409-411, 2004.

[5.] P. Ya. Ufimtsev, Theory of edge diffraction in electromagnetics. Encino, California: Tech Science Press, 2003.