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TECHNICAL NOTE

Separation and Normalization in Multi-Attribute Decision Models for Investment Evaluation

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Abstract

The separation of the evaluation attributes into two distinct subsets, objective and subjective measures, is discussed. The use of normalization over the alternatives for each subjective attribute is shown to lead to erroneous choices in certain situations.

INTRODUCTION

Several publications in which multi-attribute decision analysis has been applied to investment decisions have suggested a strategy of separating the attribute set into two subsets [2],[3],[5],[7]. Following the notation in [3], one set contains the objective measures and the other the subjective measures. The suggested objective set contains only one economic attribute, either Net Present Worth or Cost (Equivalent Annual Cost or Present Worth of Costs). The subjective set contains all the other more qualitative attributes that are deemed necessary to the decision and Edward's SMART method for multi-attribute decision analysis is adapted to this subset. Except in [2], the adaption adds a normalization step.

The strategy originated as an evaluation procedure for facility site selection by Brown and Gibson [1], and was applied to investment decisions in [5] and [7]. Recently, in [3] the procedure was suggested to be a multi-attribute decision theory approach. We discuss this procedure from the decision theory viewpoint [4],[6],[8] .

Although not stated, presumably the reason for treating Net Present Worth or Cost as a single attribute in a separate objective set is that in the past managers have used these as the sole criteria for investment choices. Thus, the methodology leads managers to an understanding of how subjective factors can change their choices. This is certainly more than sufficient justification for adopting this separation approach.

If there are a total of n attributes to be used in choosing between k alternatives, let O and S denote the k-component row vectors of objective attribute values and the solution to the additive model for the subjective attributes respectively. It is surmised that the purpose of forming the two subsets is to be able to study linear combinations (i.e. $\alpha O + (1-\alpha)S$, for $\alpha \in [0,1]$) to understand the breakeven points and discover any nearly dominant alternative. See for example, Alt. 3 in Figure 1. If there is a dominant alternative i^*, then $O_{i^*} \geq O_i$ and $S_{i^*} \geq S_i$ for all $i \neq i^*$ and a strict inequality holds for at least one i. In this case, graphing is useful only as a communication aid.

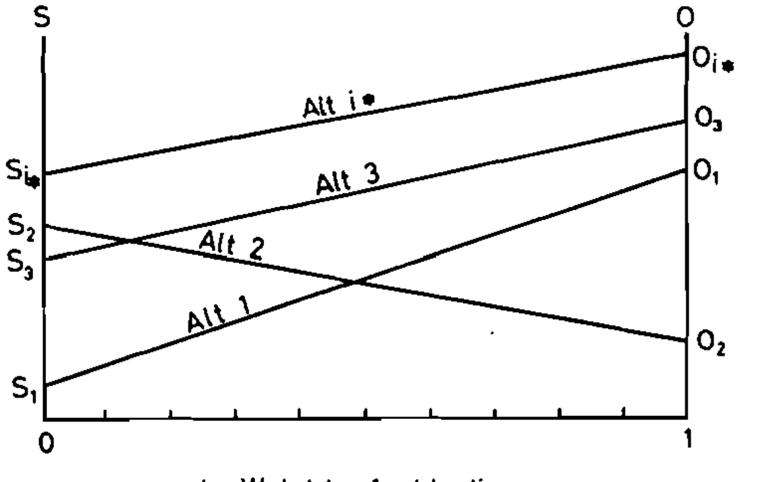

Figure 1: Graphical Analysis of the Dominance Relations Among Alternatives via Separation

The procedure delineated in [1],[3],[5],[7] normalizes the NPW or Cost attribute so that $O_k \in [0,1]$ and $\sum_k O_k = 1$. It then adds a step to normalize each of the attributes in the subjective set prior to applying the SMART method for finding S. It is the purpose of this note to show that this step can lead to an incorrect ordering of the alternatives by S and to suggest a correct method for carrying out the procedure.

The analyst should realize that he is not applying a different methodology but merely organizing the sequence of calculations of the additive decision model to perform a parameterization on the weights. To show this, let E be the $(n-1)\mathrm{x}k$ matrix of the evaluation measures for attributes in the subjective set (i.e. location measures in [4] and [8] or, the attribute measurement values assigned to alternatives) and w be an $(n-1)$-component row vector of the weights given to the attributes. Thus we have

$$wE = S. \tag{1}$$

And,

$$\alpha O + (1-\alpha)S = \alpha O + (1-\alpha)wE = w'E' = S' \tag{2}$$

where

$$E' = \begin{bmatrix} O \\ E \end{bmatrix}$$

$w' = [\alpha, (1-\alpha)w]$ and, $S' = [\alpha O + (1-\alpha)S]$.

Thus, these suggested methods are really just differently organized calculation procedures for the additive decision model. This should alert the analyst to be careful in selecting the attribute measurement scales against which the alternatives are to be judged.

There are two interpretations of the weights. They can be considered "exchange rates between location measures" [4] where location refers to the value on the attribute measurement scale assigned to an alternative. The most often used concept to operationize this interpretation is *importance*. Importance is the basic criteria used to rank the attributes which is the common first step in all methods for determining weights. This is usually done independent of the attribute measurement scale selection thus implicitly assuming all attributes are measured on the same scale and neglecting the possible interaction between the weights and the measurement scales. The interpretation of weights as exchange rates does not preclude using different attribute measurement scales, but the weights have to be adjusted to account for the scale differences. The swing weights estimating procedure was developed to account for potential interactions [8].

In the second interpretation, Keeney & Raiffa [6] suggest weights are "rescaling parameters necessary to match the units of one single attribute value function to the units of another" [8]. This interpretation explicitly admits the possibility of differing attribute measurement scales. If there are no scale differences, the scaling constants represent relative importance of the attributes as in [4].

Consider two alternatives, A and B, for which two of the attributes, b and d, are considered equally important. Suppose b is measured on [0, 80] and d on [0,100], A locates on the top of the b scale, B at the top of the d scale, and each is identically located on the other attribute. Thus, B receives extra value from these two attributes toward being the favored alternative if the weights have equal numerical value. This contradicts the assumption that the attributes are equally important.

These problems of ensuring the correct relationships between weights and scales do not occur when the more sophisticated model development approaches suggested by Keeney & Raiffa [6],[8] are used. However, when the linear model is assumed a priori, the analyst must be aware of the potential interactions.

If each attribute is first measured on its own natural scale, the alternatives are located on each scale, and the minimum (maximum) location values are extrapolated to a common value scale, then importance weights can be determined independently. This procedure does cause some errors which are usually small and therefore neglected [4]. When value functions are directly estimated in this manner, the resultant model often exhibits the threshold effect, i.e. the model will show small numerical differences between two alternatives for which the decision maker is actually indifferent. If accuracy is important then the analyst must use the more complex procedures for value function estimation. See [6] and [8].

In [5] and [7] the authors start with attribute measurement scales which differ between some of the attributes, however, in [3] a common initial scale is chosen. In all of these models the location values are normalized over the alternatives for each attribute before applying the additive decision model calculations. This in effect changes each attribute measurement scale so that, when the additive model is applied, the attributes are scaled differently. This can change the ordering of alternatives as shown in the appendix by analyzing a 2x2 E matrix and with a numerical example. Presumably the reason for this normalization is to obtain an S which is measured on [0,1] to complement the fact that their O is measured on [0,1], ensuring that the two ordinates in Figure 1 are scaled identically. If a common scale of $[0,M]$ is used, this same effect can be obtained by multiplying S by $1/M$.

APPENDIX

Let E be the $(n-1)$xk matrix of evaluation measures and any element $e_{i,j}$ of E represent the value of attribute i assigned to alternative j. Let w be an $(n-1)$-component row vector of the weights given to the subjective attributes. Then, we have $wE = S$ as in (1) where, the score vector S, is a component row vector which provides the order of the alternatives.

The purpose of this appendix is to show that the normalization (over the values assigned to alternatives) for each attribute can change the order in the score vector S and thus, can lead to erroneous results in certain conditions. First, a 2x2 evaluation matrix is analyzed to show the effect of normalization and then, an example problem is investigated by numerical means to provide some practical insights.

It is well known and can easily be proven that normalization of w has no effect on the order in the score vector. Thus for simplicity, the vector of weights w is assumed to be [1,1].

Let a typical evaluation matrix be,

$$E = \begin{bmatrix} a_1 - b & a_1 + b \\ a_2 + c & a_2 - c \end{bmatrix}$$

In the *unnormalized case* the score vector will be,

$$S = \begin{bmatrix} a_1 + a_2 - b + c \\ a_1 + a_2 + b - c \end{bmatrix}$$

Thus, the order in S depends only on the values of b and c which are actually the discrepencies from the row means a_1 and a_2 respectively. On the other hand, in the *normalized case*,

$$E^N = \begin{bmatrix} \frac{a_1-b}{2a_1} & \frac{a_1+b}{2a_1} \\ \frac{a_2+c}{2a_2} & \frac{a_2-c}{2a_2} \end{bmatrix}$$

and the score vector will be,

$$S^N = \begin{bmatrix} 1 + \frac{-ba_2+ca_1}{2a_1a_2} \\ 1 - \frac{-ba_2+ca_1}{2a_1a_2} \end{bmatrix}$$

Note that here the order in S^N depends on a_1 and a_2 as well as b and c. It is also possible to represent the *normalized* score vector as,

$$S^N = \begin{bmatrix} 1 - \frac{b}{2a_1} + \frac{c}{2a_2} \\ \\ 1 + \frac{b}{2a_1} - \frac{c}{2a_2} \end{bmatrix}$$

Hence, it is possible to come up with the following conclusions:

- In the *unnormalized case* only b and c are important in terms of generating the order in S. Here, b and c represent the *absolute* discrepencies from the row means.
- However, in the *normalized case* b/a_1 and c/a_2 happen to be important in the generation of the order in S. Note that, these terms represent the *relative* discrepencies from the row means.

Consider a hypothetical example where the set of subjective attributes consisting of A, B, C, D, E are considered important for selecting one of the two investment alternatives. The weights assigned to the attributes and the evaluation measures assigned to alternatives for each attribute on a common [0,10] scale are depicted in Table 1.

Note that Table 1 also provides the *unnormalized* score vector [7.30, 7.42]. The *normalized* case is shown in Table 2. Here, the evaluation measures in Table 1 are normalized over the alternatives for each attribute. Note that, the order in the *normalized* score vector [0.4947, 0.5053] is the same as in Table 1. That is, the second alternative is preferred in both cases.

Table 3 provides a summary of six solutions to this example for a parametric analysis changing only the evaluation value of attribute A assigned to the second alternative.

TABLE 1

UNNORMALIZED CASE

		Evaluation Measure		Weighted Evaluation	
Attribute	Weight	Alt. 1	Alt. 2	Alt. 1	Alt. 2
B	0.40	6.0	7.0	2.40	2.80
A	0.28	7.5	9.0	2.10	2.52
C	0.12	10.0	7.5	1.20	0.90
D	0.08	8.0	6.0	0.64	0.48
E	0.12	8.0	6.0	0.96	0.72
SUM	1.00			7.30	7.42

TABLE 2

NORMALIZED CASE

		Evaluation Measure		Weighted Evaluation	
Attribute	Weight	Alt. 1	Alt. 2	Alt. 1	Alt. 2
B	0.40	0.4615	0.5385	0.1846	0.2154
A	0.28	0.4545	0.5455	0.1273	0.1527
C	0.12	0.5714	0.4286	0.0686	0.0514
D	0.08	0.5714	0.4286	0.0457	0.0343
E	0.12	0.5714	0.4286	0.0686	0.0514
SUM	1.00			0.4947	0.5053

TABLE 3

SUMMARY OF ANALYSIS

Solution No	$e_{2,2}$	Unnormalized Solution	Normalized Solution
I	9.00	Alt. 2	Alt. 2
II	8.57	Indifferent	Alt. 2
III	8.50	Alt. 1	Alt. 2
IV	8.40	Alt. 1	Alt. 2
V	8.35	Alt. 1	Indifferent
VI	8.30	Alt. 1	Alt. 1

The following conclusions are drawn by the aid of Table 3:

- It is possible to obtain contradictory results for a certain range of the evaluation measure (i.e. $e_{2,2} \in [8.35, 8.57]$). That is, the selected alternative in the *unnormalized case* is not the same as the one selected in the *normalized case.*

- The switch of the choice from the second alternative to the first happens slower in the *normalized case.* This is because normalizing decreases the effects of marginal changes in the evaluation measures.

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BIOGRAPHICAL SKETCHES

Charles H. Falkner is a Visiting Professor of Industrial Engineering at Bilkent University. He has previously taught at the University of Wisconsin-Madison and at Marquette University. His current area of research and teaching is the applications of Operations Research to Engineering Economic Analysis with particular emphasis on the justification of advanced manufacturing technologies using multi-attribute decision theory. He has industrial experience with Westinghouse Electric, Aerojet-General, Lockheed Missiles and Space, and General Motors Companys. He is a member of ORSA, TIMS, SME, and IIE, is listed in Who's Who in Technology Today, is a registered Professional Engineer, and has over 25 publications. His degrees (Industrial Engineering) are from Stanford University (B.S., Ph.D.) and University of California-Berkeley (M.S.).

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